different ranges, the pseudopotential would be real, energy-dependent, and nonlocal.

[10]This differs from the coupling of two real channels, in which case elastic unitarity holds only up to the inelastic threshold. Cf. H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958).

[11]See, for example, R. G. Newton, Ref. 5, Chap. 17.

[12]Implications of the breakdown of global analyticity on high-energy phenomenology are currently under investigation (private communication from M. G. Gundzik).

[13]E. C. G. Sudarshan, Fields and Quanta (to be published).

[14]Some progress has been made in solvable Lee-type models with shadow states. Cf. C. A. Nelson and E. C. G. Sudarshan, University of Texas at Austin Report No. CPT-94, 1971 (unpublished).

PHYSICAL REVIEW D VOLUME 4, NUMBER 12 15 DECEMBER 1971

Broken Coupling Constants, Smoothness, and Scalar Mesons in the Gell-Mann–Oakes–Renner Model

Ranabir Dutt* and Shalom Eliezer
Department of Physics, Technion–Israel Institute of Technology, Haifa, Israel

and

Priyabrata Nanda
Department of Physics and Astrophysics, University of Delhi, Delhi-7, India

(Received 21 June 1971)

Considering SU(3)-octet broken scalar-pseudoscalar meson (*PPS*) couplings as well as "smoothness" in the chiral model of Gell-Mann, Oakes, and Renner (GMOR), we obtain the spectrum of scalar mesons in terms of the known masses of the speudoscalar mesons only. The agreement of the predicted spectrum with experiment is reasonably good. The broken pieces of the coupling constants are determined as well and are found to be comparable to their corresponding symmetric pieces. The effects of the broken couplings in the numerical results, such as decay widths, have also been examined. We have demonstrated that the usual assumption of the Goldstone bosons required by exact chiral symmetry emerges automatically from the smoothness relations. A few other interesting aspects of our work are finally discussed.

I. INTRODUCTION

During recent years various studies based on theories of chiral symmetry[1-11] and broken scale invariance[12-14] have indicated convincingly that scalar mesons have indeed a significant role in determining the coupling constants and masses of the hadrons. Working in different formalisms, many authors have predicted the masses and decay rates of scalar particles; however, none of these predictions can be tested decisively because of the lack of precise experimental information. In a broken chiral $SU(3)\otimes SU(3)$ model, Glashow[3] first obtained the scalar-meson spectrum in terms of the known masses of pseudoscalar mesons assuming that the mass term of the Lagrangian has the simple $(3, \overline{3})\oplus(\overline{3}, 3)$ transformation under the group $SU(3)\otimes SU(3)$ and is dominated by scalar-meson tadpoles. His predicted spectrum is not realistic in the sense that it includes a scalar particle with complex mass and the scalar kaon (so-called κ meson) with a very low mass ($\approx$770 MeV) which is not admitted by present experimental evidence.[15]

In subsequent works, several authors[5-7] have shown that a large mass of the κ meson is favored in the GMOR model of chiral symmetry.[1] In this model, one obtains a simple algebraic relation between the κ-meson mass (m_κ) and the coupling constants f_K and f_π, which are experimentally determined from the rates of $K\to l\nu$ and $\pi\to l\nu$ decays, respectively. For a reasonable value of the ratio $f_K/f_\pi \simeq 1.2$, m_κ is found to be approximately 1200 MeV,[6,7] in close agreement with experiment.

On the other hand, the complete spectrum of the scalar mesons has been obtained by Pande[8] who worked in the same model (GMOR), utilizing the constraints of various subsymmetries contained in chiral $SU(3)\otimes SU(3)$ symmetry.[9] He also considered the effect of SU(3) breaking through conventional mixing phenomena and predicted a small mixing angle ($\theta_S \approx 11°$) for scalar particles. This work is, however, open to criticism as it incorporates a very drastic assumption that masses of scalar and pseudoscalar singlets are identical in the chiral-symmetry limit. This assumption is questionable, and no theory has so far revealed such peculiar degeneracy in nature.

In a recent paper, Carruthers[16] made a systematic analysis of the existing data of scalar-meson decays, using the SU(3)-symmetric structure of the scalar-pseudoscalar-meson vertex (*PPS*) and the known masses of scalar mesons. He has shown that experimental evidence is consistent with a choice of a large mixing angle ($\theta_S \approx 23^\circ$) due to SU(3) breaking between the octet and the singlet components of the scalar nonet. We must mention that this conclusion cannot be taken seriously since it is apparent that the mixing angle may be altered, as desired, by varying several input quantities used in this calculation, such as the masses and the decay rates of a few scalar particles.

In this paper we shall demonstrate that without using ambiguous and unconfirmed experimental information, the complete spectrum of scalar mesons can be reproduced in terms of the well-known masses of pseudoscalar mesons just from the general assumptions of *smoothness and octet breaking of the PPS vertex* in the GMOR model. The idea of considering broken coupling constants, which have so far been ignored in previous calculations based on chiral symmetry,[1,17] is mainly motivated by the recent success of the work of Brown *et al.*[18] These authors have shown large octet splitting of *VVP* couplings from the analysis of vector-meson decays. In our calculation we neglect altogether the effects of particle mixing both for scalar and pseudoscalar mesons and assume *a priori* that lack of mixing effects will be compensated by the SU(3)-octet splitting of the coupling constants.

In Sec. II we invoke the smoothness assumption for certain three-point functions in the GMOR model and give the outline of the derivation of the sum rules among the broken coupling constants and the masses of scalar and pseudoscalar mesons. The general solutions of these relations are presented in Sec. III. We predict the masses of scalar mesons in terms of the known masses of pseudoscalar particles; the complete spectrum is in reasonable agreement with the experiment.[15] We further calculate the relative strengths of the octet broken parts of the couplings to their respective invariant pieces, and we find that SU(3) breaking of couplings is quite considerable. A number of decay widths have been computed to demonstrate the influence of octet-broken couplings in the numerical results. Our predicted decay widths are comparable to those obtained by Carruthers,[16] using large mixing angle ($\approx 23^\circ$) for scalar mesons and symmetric *PPS* vertices. In Sec. IV we discuss two important features of our approximations:

(a) In the GMOR model, use of SU(3)-invariant coupling constants together with smoothness *demands* the SU(3) symmetry of the chirally broken Hamiltonian and thus forbids the use of physical masses of scalar and pseudoscalar mesons.

(b) In the limit of exact chiral symmetry, the smoothness relations give *explicitly* either of the two possibilities: (i) eight zero-mass pseudoscalar mesons (Goldstone bosons); (ii) seventeen degenerate scalar and pseudoscalar mesons, whereas the mass of the pseudoscalar singlet remains always finite and distinct from the degenerate masses. The first possibility is usually taken as the starting assumption in the conventional chiral-symmetry models.[1,11,19]

We mention a few general remarks about our work in Sec. V.

II. SMOOTHNESS AND BROKEN-COUPLING-CONSTANT SUM RULES

We start with the broken chiral model of GMOR, in which one defines a set of scalar and pseudoscalar nonets $u_i(x)$ and $v_i(x)$ ($i = 0, 1, 2, \ldots, 8$) transforming as the $(3, \overline{3}) \oplus (\overline{3}, 3)$ representation of the group $SU(3) \otimes SU(3)$, and consider the symmetry-breaking Hamiltonian density

$$H_{\rm SB} = \epsilon_0 u_0 + \epsilon_8 u_8 \,. \tag{1}$$

The bare scalar and pseudoscalar densities u_i and v_i's may be associated with the physical scalar- and pseudoscalar-meson states through wave-function renormalization constants, Z's, defined as

$$\begin{aligned} \langle 0 | u_i | S_j \rangle &= \delta_{ij} Z_{S_i}{}^{1/2} , \\ \langle 0 | v_i | P_j \rangle &= \delta_{ij} Z_{P_i}{}^{1/2} , \quad i, j = 0, 1, 2, \ldots, 8 \,. \end{aligned} \tag{2}$$

We shall show that simple relations between Z_S and Z_P exist if we assume the maximal smoothness condition for certain three-point vertex functions. This approximation means that the vertex functions are as smooth functions of momenta as possible after the removal of the pole singularities. We consider the following three-point vertex functions:

$$\begin{aligned} G_{P_i P_j S_k}(p_1{}^2, q^2, p_2{}^2) \equiv {} & (p_1{}^2 - m_{P_i}{}^2)(q^2 - m_{P_j}{}^2)(p_2{}^2 - m_{S_k}{}^2)(Z_{P_i} Z_{P_j} Z_{S_k})^{-1/2} \\ & \times \int d^4x\, d^4y\, e^{i p_1 \cdot x - i p_2 \cdot y} \langle 0 | T\{ v_i(x) v_j(0) u_k(y)\} | 0 \rangle \,, \end{aligned} \tag{3}$$

$$i(p_1+p_2)_\mu K^+_{ijk}(p_1^2, q^2, p_2^2)+iq_\mu K^-_{ijk}(p_1^2, q^2, p_2^2) \equiv -(p_1^2 - m_{P_i}^2)(p_2^2 - m_{S_k}^2)(Z_{P_i}Z_{S_k})^{-1/2} \times \int d^4x\, d^4y\, e^{-iq\cdot x - ip_2\cdot y}\langle 0|T\{v_i(0)u_k(y)A^j_\mu(x)\}|0\rangle, \quad (4)$$

where $q_\mu = (p_1 - p_2)_\mu$ and i, j are the pseudoscalar indices, and k is the scalar index.

When the particles are on the mass shells, $G_{P_iP_jS_k}$ is the physical coupling constant of a scalar and two pseudoscalar fields, and $K^{\pm}_{ijk}$'s are the physical form factors of the axial-vector current between a scalar and a pseudoscalar state.

Equations (3) and (4) may be related by taking the divergence of Eq. (4), and using the PCAC (partially conserved axial-vector current) relation given by

$$\partial_\mu A^\mu_i(x) = (\epsilon_0 d_{0ik} + \epsilon_8 d_{8ik})v_k(x), \quad i=1,2,\ldots,8; \quad k=0,1,2,\ldots,8. \quad (5)$$

We consider that the couplings and the form factors are independent of momenta in the limit $q^2 \to 0$ (smoothness) and equate the coefficients of p_1^2 and p_2^2 to obtain[20]

$$Z_S = Z_P = Z, \quad (6)$$

$$Z^{1/2}\left[(\tfrac{2}{3})^{1/2}\epsilon_0 G_{P_iP_jS_k}\left(\frac{1}{m_{P_i}^2}+\frac{1}{m_{P_j}^2}\right)+\sum_{l=0}^{8}\frac{\epsilon_8}{m_{P_l}^2}(d_{8jl}G_{P_iP_lS_k}+d_{8il}G_{P_jP_lS_k})\right] = d_{ijk}(m_{P_i}^2+m_{P_j}^2-2m_{S_k}^2), \quad (7)$$

where in Eq. (7), we have symmetrized between the pseudoscalar indices i and j.

The sum rule in Eq. (7) is valid for all possible choices of the indices i, j, and k from 0, 1, 2, ..., 8, except for those cases where $i=0$, $j=0$ may occur simultaneously.

We shall now consider the general forms of the SU(3)-octet splitting of the PPS coupling constants given as

$$G_{P_iP_jS_k} = g_a d_{ijk} + h_{a1}\sum_{l=0}^{8} d_{ijl}d_{l8k} + h_{a2}\sum_{l=0}^{8}(d_{ikl}d_{l8j}+d_{jkl}d_{l8i}) + h_{a3}\delta_{ij}\delta_{k8}, \quad (8a)$$

$$G_{P_0P_jS_k} = g_b\delta_{jk} + h_b d_{jk8}, \quad (8b)$$

$$G_{P_iP_jS_0} = g_c\delta_{ij} + h_c d_{ij8}, \quad (8c)$$

$$G_{P_0P_0S_0} = g_d, \quad i,j,k = 1,2,\ldots,8 \quad (8d)$$

where the g's are the SU(3)-invariant parts and the h's are the octet-broken parts of the coupling constants.

Combining Eqs. (7) and (8) and considering various combinations of definite i, j, and k, we get a set of relations among the g's, h's, and the masses of the scalar and pseudoscalar mesons. Out of these relations only 11 are independent since a couple of them are related to the others by Gell-Mann–Okubo (GMO) mass relations both for scalar and pseudoscalar mesons[21]

$$4m_\kappa^2 - 3m_{S^*}^2 - m_{\pi_N}^2 = 0, \quad (9a)$$

$$4m_K^2 - 3m_\eta^2 - m_\pi^2 = 0. \quad (9b)$$

The suitable forms of these eleven equations are the following:

$$Ex(\sqrt{3}g_c + h_c) = \sqrt{6}(m_\pi^2 - m_\epsilon^2), \quad (10)$$

$$Ex(2\sqrt{3}g_c - h_c) = 2\sqrt{6}(m_K^2 - m_\epsilon^2), \quad (11)$$

$$E\alpha[\sqrt{3}g_d m_\eta^2 + m_X^2(\sqrt{3}g_c - h_c)] = 0, \quad (12)$$

$$E\left[\left(\frac{\sqrt{2}}{m_X^2}+x\right)(\sqrt{3}g_b+h_b)+\left(\frac{2}{3}\right)^{1/2}\frac{\alpha}{m_\eta^2}(\sqrt{3}g_a+h_{a1}+2h_{a2})\right] = \sqrt{6}(m_X^2+m_\pi^2-2m_{\pi_N}^2), \quad (13)$$

$$E\left[2\left(\frac{\sqrt{2}}{m_X^2}+x\right)(2\sqrt{3}g_b-h_b)-\left(\frac{2}{3}\right)^{1/2}\frac{\alpha}{m_\eta^2}(2\sqrt{3}g_a-h_{a1}-11h_{a2})\right] = 4\sqrt{6}(m_X^2+m_K^2-2m_\kappa^2), \quad (14)$$

$$E\left[\left(\frac{\sqrt{2}}{m_X^2}+x\right)(\sqrt{3}g_b-h_b)+\left(\frac{2}{3}\right)^{1/2}\frac{\alpha}{m_\eta^2}(-\sqrt{3}g_a+3h_{a1}+6h_{a2}+3h_{a3})\right] = \sqrt{6}(m_X^2+m_\eta^2-2m_{S^*}^2), \quad (15)$$

$$Ex(2\sqrt{3}g_a - h_{a1} + h_{a2}) = 3(m_\pi^2 + m_K^2 - 2m_\kappa^2), \quad (16)$$

$$Ex(\sqrt{3}g_a + h_{a1} - h_{a2}) = 3(m_K^2 - m_{\pi_N}^2), \quad (17)$$

$$Ex(\sqrt{3}g_a + h_{a1} + 2h_{a2} + 3h_{a3}) = 3(m_\pi^2 - m_{S^*}^2), \quad (18)$$

$$E\left[2x(\sqrt{3}g_a + h_{a1} + 2h_{a2}) + \sqrt{6}\frac{\alpha}{m_X^2}(\sqrt{3}g_b + h_b)\right] = 3(m_\pi^2 + m_\eta^2 - 2m_{\pi_N}^2), \quad (19)$$

$$E\left[x(2\sqrt{3}g_a - h_{a1} - 11h_{a2}) - \sqrt{6}\frac{\alpha}{m_X^2}(2\sqrt{3}g_b - h_b)\right] = 3(m_K^2 + m_\eta^2 - 2m_\kappa^2), \quad (20)$$

where

$$E = Z^{1/2}\epsilon_0,$$

$$\alpha = \epsilon_8/\epsilon_0 = -2\sqrt{2}(m_K^2 - m_\pi^2)/(2m_K^2 + m_\pi^2), \quad (21)$$

$$x = (\sqrt{2} + \alpha)/m_\pi^2 = (2\sqrt{2} - \alpha)/2m_K^2 = (\sqrt{2} - \alpha)/m_\eta^2 = 3\sqrt{2}/(2m_K^2 + m_\pi^2).$$

III. SOLUTIONS FOR BROKEN COUPLINGS AND SCALAR-MESON SPECTRUM

From Eqs. (10)–(20) we can eliminate E and determine, in principle, ten unknown quantities in terms of the known masses of pseudoscalar mesons. Equations (13)–(15) together with Eq. (9) give a simple relation among the broken coupling constants,

$$h_{a1} + h_{a2} + h_{a3} = 0. \quad (22)$$

From Eqs. (16), (17), (19), and (20) we have

$$\frac{1}{m_X^2}\frac{h_b}{g_b}(m_K^2 - m_\pi^2)(2m_\kappa^2 + m_{\pi_N}^2 - 2m_K^2 - m_\pi^2) = 0. \quad (23)$$

Since m_X is not infinite, $m_K \neq m_\pi$ and the second bracket cannot be taken to be zero as it causes infinite couplings [see Eqs. (33)], we have

$$h_b = 0. \quad (24)$$

Using Eq. (24) in Eqs. (13)–(19) and eliminating g's and h's, we obtain two equations in the variables m_κ and m_{π_N}:

$$2m_\kappa^2(7m_K^2 - m_\pi^2) - 4m_{\pi_N}^2(5m_K^2 - 2m_\pi^2) = 3(m_K^2 - m_\pi^2)^2, \quad (25)$$

$$2m_\kappa^2\left(1 - \frac{a}{3xm_X^2}\right) - 2m_{\pi_N}^2\left(1 + \frac{b}{3xm_X^2}\right) = \frac{c}{3xm_X^2} + m_\pi^2 - m_K^2, \quad (26)$$

where

$$a = 4x(m_K^2 - m_\pi^2) - 3\sqrt{2} - (\alpha/m_\eta^2)(m_K^2 - m_\pi^2), \quad (27a)$$

$$b = 2x(m_K^2 - m_\pi^2) + 3\sqrt{2} + (\alpha/m_\eta^2)(m_K^2 - m_\pi^2), \quad (27b)$$

$$c = (m_K^2 - m_\pi^2)[3\sqrt{2} + (\alpha/m_\eta^2)(m_K^2 - m_\pi^2) - 2x(2m_K^2 + m_\pi^2)]. \quad (27c)$$

From Eqs. (26) and (27) we predict

$$m_{\pi_N} \approx 1000 \text{ MeV}, \quad m_\kappa \approx 1200 \text{ MeV}, \quad (28)$$

using known masses of pseudoscalar mesons. Our predictions are in excellent agreement with the experimental masses of $\pi_N(980)$ or $\pi_N(1016)$ and $\kappa(1080–1260)$, respectively.[15,16] It is important to mention that we do not get real solutions for these two masses if we use $E(1422)$ instead of $X(958)$ in Eq. (26).

The mass of the scalar η (S^*) can be predicted from Eqs. (9a) and (28). We get

$$m_{S^*} \approx 1260 \text{ MeV}. \quad (29)$$

As far as the mass of the scalar singlet (ϵ meson) is concerned, we can obtain its value only in the *chiral-symmetry limit*. Equations (10)–(12) give

$$g_d/g_c = -\left(\frac{m_X}{m_\eta}\right)^2 \frac{3m_\epsilon^2 - 4m_K^2 + m_\pi^2}{3m_\epsilon^2 - 2m_K^2 - m_\pi^2}. \quad (30)$$

If we compare Eq. (30) with the *SU(3) ⊗ SU(3)-invariant ratio*[16]

$$g_d/g_c = -2, \quad (31)$$

we get

$$m_\epsilon \approx 840 \text{ MeV}. \quad (32)$$

This value is approximately 20% higher than the experimental value 700 MeV. Since in reality chiral symmetry is broken, this discrepancy may be reasonably attributed to the remnant effect of SU(3)⊗SU(3) symmetry. For practical purposes, we shall, henceforth, use $\epsilon(700)$ instead of $\epsilon(840)$, which is obtained in an ideal world.

Having determined the spectrum of the scalar mesons, we utilize our smoothness relations to express $g_{b,c,d}$ and h's in terms of the known masses of scalar and pseudoscalar particles and the symmetric coupling g_a. In our theory, g_a remains the only undetermined quantity. We give the explicit forms:

$$h_{a1}/g_a = \sqrt{3}(m_{\pi_N}^2 - m_{S^*}^2 + m_\pi^2 - m_K^2)/D, \quad (33a)$$

$$h_{a2}/g_a = \sqrt{3}(2m_\kappa^2 - m_{\pi_N}^2 - m_{S^*}^2)/D, \quad (33b)$$

$$h_{a3}/g_a = -(h_{a1} + h_{a2})/g_a\,, \quad (33c)$$

$$h_c/g_a = 2\sqrt{2}(m_K^2 - m_\pi^2)/D\,, \quad (33d)$$

$$g_b/g_a = \frac{(\frac{3}{2})^{1/2} m_X^2 (2m_\kappa^2 - 2m_{\pi_N}^2 - m_K^2 + m_\pi^2)}{(m_K^2 - m_\pi^2)D}\,, \quad (33e)$$

$$g_c/g_a = (\tfrac{2}{3})^{1/2}(3m_\epsilon^2 - 2m_K^2 - m_\pi^2)/D\,, \quad (33f)$$

$$g_d/g_a = -\left(\frac{2}{3}\right)^{1/2}\left(\frac{m_X}{m_\eta}\right)^2 (3m_\epsilon^2 - 4m_K^2 + m_\pi^2)/D\,, \quad (33g)$$

where

$$D = 2m_\kappa^2 + m_{\pi_N}^2 - 2m_K^2 - m_\pi^2\,. \quad (34)$$

From Eqs. (22), (24), (28), (29), (33), and (34) we obtain numerically

$$\begin{aligned} &g_a : h_{a1} : h_{a2} : h_{a3} = 1 : -0.60 : 0.22 : 0.38\,,\\ &h_b = 0\,,\\ &g_c : h_c = 1 : 0.82\,,\\ &g_a : g_b : g_c : g_d = 1 : 1.36 : 0.33 : -0.49\,. \end{aligned} \quad (35)$$

It is clear from Eq. (35) that octet splitting of the *PPS* couplings are quite considerable in comparison with their symmetric values. The effect of the large splittings of the couplings can be studied directly in decays of scalar and pseudoscalar mesons. We shall consider the following decay modes:

(a) $\pi_N \to \pi\eta$, (b) $\epsilon \to \pi\pi$, (c) $\kappa \to K\pi$,

(d) $\kappa \to K\eta$, (e) $\pi_N \to K\bar{K}$.

The vertices which we need to describe these processes are given as

$$\begin{aligned} &G_{\pi_N\pi\eta} = (g_a/3)(\sqrt{3} + h_{a1}/g_a + 2h_{a2}/g_a)\,,\\ &G_{\epsilon\pi\pi} = (ga/\sqrt{3})(\sqrt{3}g_c/g_a + h_c/g_a)\,,\\ &G_{\kappa K\pi} = (ga/4\sqrt{3})(2\sqrt{3} - h_{a1}/g_a + h_{a2}/g_a)\,,\\ &G_{\kappa K\eta} = (ga/12)(-2\sqrt{3} + h_{a1}/g_a + 11h_{a2}/g_a)\,,\\ &G_{\pi_N K\bar{K}} = (ga/2\sqrt{3})(\sqrt{3} + h_{a1}/g_a - h_{a2}/g_a)\,. \end{aligned} \quad (36)$$

To calculate the absolute decay rates for these processes, we must know the unknown parameter g_a from a known input. We assume the width $\Gamma(\pi_N \to \pi\eta)$ to be 50 MeV, the same as considered by Carruthers[16] in his analysis. Using Eqs. (35) and (36), we compute the decay rates and show the results in Table I along with the predictions of Carruthers. Our predictions[22] with *broken couplings* and *unmixed* mesons are found to be more or less of the same order as given by a possible *large mixing* of scalar mesons and *symmetric coupling constants*. This fact justifies (at least qualitatively) our *a priori* conjecture that the effect of particle mixing may be compensated for by the broken coupling constants.

IV. CONSEQUENCES OF SMOOTHNESS APPROXIMATION IN GMOR MODEL

So far we have concentrated mainly on the numerical aspects of the smoothness approximation, together with the SU(3) breaking of coupling constants. Here we shall point out a few characteristic features of our work, which may be of theoretical interest:

(1) In general, when SU(3) symmetry is broken, many authors[1, 16, 17] use SU(3)-symmetric coupling constants together with the physical masses of the

TABLE I. Predictions of scalar-meson decay widths using broken *PPS* couplings.

Decay modes	Predicted decay widths (Γ in MeV): Present calculation with broken couplings and no mixing	Analysis of Carruthers[a] using large mixing and symmetric coupling	Experimental widths[b,c] (MeV)
$\pi_N \to \pi\eta$	50 (input)	50 (input)	60^{+16}_{-10}
$\kappa \to K\pi$	180	140	90–400
$\epsilon \to \pi\pi$	510	320	500
$\kappa \to K\eta$	2	0	<10
$\pi_N \to K\bar{K}$	3	0	<5

[a]Reference 16.

[b]Reference 15.

[c]R. Ammar, W. Kropoc, H. Yarger, R. Davis, J. Mott, B. Werner, M. Derrick, T. Fields, F. Schweingruber, D. Hodge, and D. D. Reeder, Phys. Rev. Letters 21, 1832 (1968); Phys. Rev. D 2, 430 (1970); H. Genz and Frank Steiner, Lett. Nuovo Cimento 1, 355 (1971). See other references given therein for the width of the ϵ meson.

particles. We find that this possibility is not permissible by the smoothness assumption. In other words, if we assume that the *PPS* vertex is SU(3)-invariant, our smoothness relations give rise to the degeneracies of the octets of scalar and pseudoscalar mesons separately. To show it explicitly, we make all h's (symmetry-broken parts) zero; from Eqs. (9)–(21) we then get

$$\begin{aligned} &\epsilon_8 = 0 , \\ &m_\pi = m_K = m_\eta \equiv m_P , \qquad (37) \\ &m_{\pi_N} = m_\kappa = m_{S^*} \equiv m_S , \end{aligned}$$

and m_X and m_ϵ are finite but not equal to the degenerate masses of their corresponding octets. In this case, we have only two independent equations:

$$m_P^2(m_P^2 - m_S^2) = (\tfrac{2}{3})^{1/2} Z^{1/2} \epsilon_0 g_a , \qquad (38a)$$

$$m_P^2(m_P^2 - m_\epsilon^2) = Z^{1/2} \epsilon_0 g_c . \qquad (36b)$$

(2) If we further impose the condition of exact chiral symmetry of the Hamiltonian, i.e., $\epsilon_0 = \epsilon_8 = 0$, ($H_{SB} = 0$), Eqs. (38) give two alternatives:

A. $m_P = 0$,

B. $m_P \neq 0, \quad m_P = m_S = m_\epsilon$.

Case A corresponds to the usual Goldstone picture (zero-mass octet of pseudoscalar mesons), while case B gives another possibility of realizing the chiral-symmetry limit through the appearance of 16 degenerate scalar and pseudoscalar mesons. In all chiral-symmetry calculations[1, 11, 19] the validity of the first one is *assumed,* whereas we have shown that it follows also from the smoothness approximation. What we have found cannot be considered as consistency checks because we have not made use of either of these assumptions at any stage of our calculation.

V. DISCUSSION

We have outlined in the previous sections a possible way of determining the scalar-meson spectrum and the effects of SU(3)-octet breaking on the *PPS* coupling constants. Working in the GMOR model, we have been able to reproduce the scalar-meson masses and the relative strengths of the broken couplings to their symmetric counterparts, in terms of the known masses of the pseudoscalar mesons. Apart from the "smoothness" approximation, we have not used any arbitrary input information which can make our predictions uncertain and ambiguous. The important results and conclusions of our work may be summarized in the following way:

(1) We predict the masses of π_N and κ mesons to be 1000 MeV and 1200 MeV, respectively. These are in excellent agreement with recent experiment. The same mass of κ meson has been obtained in a different way by other authors for a reasonable value for $f_K/f_\pi \simeq 1.2$.

(2) In our work, the mass of the scalar singlet (ϵ meson) can be predicted only in the limit of exact chiral symmetry, and this is found to be 840 MeV. If we consider that the experimental mass of the ϵ meson is 700 MeV, then Eq. (30) gives $g_d/g_c = -1.5$. Using the same $\epsilon(700)$ and fitting experimental data, Carruthers obtains the ratio to be -6.1. This is quite different from the SU(3) $\otimes$ SU(3)-invariant ratio $g_d/g_c = -2$, which is true when the nonets of scalar and pseudoscalar mesons belong to the $(3, \bar{3}) \oplus (\bar{3}, 3)$ representation. Closeness of our result with the symmetric ratio is reasonable because we work in a model where the mesons are specifically attributed to this particular representation of the SU(3)$\otimes$SU(3) group.

(3) It has been demonstrated that SU(3)-octet breaking of the *PPS* couplings may be indeed large, and its effect in the numerical results cannot be disregarded. Further, we have shown that the symmetric coupling g_c is finite in contrast to GMOR, who find it to be nearly zero. A finite value of this coupling is also required by Carruthers.

(4) In some special cases, our sum rules [Eqs. (9)–(21)] lead to a few important observations:

(a) Use of SU(3)-symmetric couplings together with "smoothness" requires the GMOR Hamiltonian to be SU(3)-invariant, i.e., $\epsilon_8 = 0$, and consequently the masses of scalar and pseudoscalar octets are separately degenerate.

(b) If exact chiral symmetry of the Hamiltonian is demanded [i.e., $\epsilon_0 = \epsilon_8 = 0$ in Eq. (1)], we obtain explicitly either eight zero-mass pseudoscalar mesons or 17 degenerate scalar and pseudoscalar mesons (complete scalar nonet and pseudoscalar octet), the mass of the pseudoscalar singlet (X meson) always remaining finite and distinct.

In the second case, the mass of the ϵ meson is required to be approximately 370 MeV (degenerate mass of the pseudoscalar octet) while, as we have shown from a different standpoint, its value should be 840 MeV in the chiral-symmetry limit. These two observations are mutually contradictory, and hence we must disregard the second possibility. Therefore, the first one seems to be realistic as it is commonly believed that Goldstone bosons are required for the realization of chiral symmetry in nature. Our results thus indicate a consistency between "smoothness" and the basic requirement (spontaneous breakdown) of chiral symmetry. One may guess that the way we have achieved this correspondence is perhaps analogous to the fashion in which soft-meson current-algebra predictions are consistent with the lowest-order results of phenomenological chiral-symmetry models.

ACKNOWLEDGMENTS

It is a great pleasure for us to thank Professor Paul Singer for his interest and encouragement in this work. One of us (P.N.) wishes to thank Professor S. N. Biswas for interesting comments and suggestions.

---

*Postdoctoral Research Fellow from the Department of Physics and Astrophysics, University of Delhi, Delhi-7, India.

[1]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[2]S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[3]S. L. Glashow, in *Hadrons and Their Interactions*, edited by A. Zichichi (Academic, New York, 1968), pp. 83–140.

[4]R. Gatto, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, Berlin, 1970), Vol. 53.

[5]P. R. Auvil and N. G. Deshpande, Phys. Rev. 183, 1463 (1969).

[6]L. Schülke, Nucl. Phys. B14, 619 (1969).

[7]J. Ellis, Nucl. Phys. B13, 153 (1969).

[8]L. K. Pande, Lett. Nuovo Cimento 4, 401 (1970); Phys. Rev. Letters 25, 777 (1970).

[9]S. Okubo and V. S. Mathur, Phys. Rev. D 1, 2046 (1970).

[10]V. S. Mathur, S. Okubo, and J. Subba Rao, Phys. Rev. D 1, 2058 (1970); R. Dutt and S. Eliezer, *ibid.* 4, 180 (1971).

[11]R. Dashen, Phys. Rev. 183, 1245 (1969).

[12]M. Gell-Mann, in *Particle Physics*, edited by W. A. Simmons and S. F. Tuan (Western Periodicals, Los Angeles, Calif., 1970).

[13]G. Mack, Nucl. Phys. 85, 499 (1968); P. Carruthers, Phys. Rev. D 2, 2265 (1970).

[14]J. Ellis, Nucl. Phys. B22, 478 (1970).

[15]Particle Data Group, Phys. Letters 33B, 1 (1970).

[16]P. Carruthers, Phys. Rev. D 3, 959 (1971).

[17]R. W. Griffith, Phys. Rev. 176, 1705 (1968).

[18]L. M. Brown, H. Munczek, and P. Singer, Phys. Rev. Letters 21, 707 (1968); Phys. Rev. 180, 1474 (1969). See also P. Singer, Phys. Rev. D 1, 86 (1970).

[19]L. Bessler, T. Muta, H. Umezawa, and D. Welling, Phys. Rev. D 2, 349 (1970).

[20]The equality of Z's given in Eq. (6) has also been obtained by other authors, e.g., see Refs. 5 and 10. The validity of this condition has been disproved from a different standpoint – by Brandt *et al.* [R. A. Brandt and G. Preparata, Phys. Rev. Letters 26, 1605 (1971)].

[21]We denote the scalar η meson by S^*.

[22]We obtain very broad widths for the decays $S^* \to \pi\pi$ and $S^* \to K\bar{K}$.