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Nonlinear Hadron Couplings from Divergence Conditions. III. SU(3) Multiplets

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The Lagrangian density for a system of the pseudoscalar-meson nonet, the vector-meson nonet, and the spin-$\frac{1}{2}$-baryon octet is obtained by the SU(3) extension of the nonlinear Lagrangian density for pions, ρ mesons, and nucleons. The nonpolynomial Lagrangian density for the pseudoscalar mesons is derived by a general procedure, which is applicable to all models and includes a rigorous treatment of the mass term. The effect of the SU(3)-symmetry breaking is taken into account by introducing the mass matrix, and the broken divergence conditions for the vector and axial-vector currents are given in an explicit form.

I. INTRODUCTION

The nonlinear couplings of pions, ρ mesons, and nucleons were derived in two earlier papers[1,2] by imposing suitable divergence conditions on the source functions in the π- and ρ-field equations. We shall now conclude our investigation by extending the results to the pseudoscalar-meson nonet P, the vector-meson nonet V_μ, and the spin-$\frac{1}{2}$-baryon octet B.

The extensive literature on the SU(3) extension of nonlinear Lagrangian densities has been reviewed, for instance, by Weinberg[3] and by Gasiorowicz and Geffen,[4] where the main complication arises from the nonpolynomial nature of the pion Lagrangian density. For, in the SU(3) extension the role of the 2×2 matrix

$$\tau_i\pi_i\,,$$

with $i=1, 2, 3$, must be replaced by the 3×3 matrix

$$\lambda_i P_i\,,$$

with $i=0, 1, 2, \cdots, 8$, where the λ_i are Gell-Mann's SU(3) matrices.[5] This transition is not entirely straightforward owing to the fact that while $(\tau_i\pi_i)^2$ is a multiple of the unit matrix, $(\lambda_i P_i)^2$ does not possess this simple property. Because of the mathematical difficulties, the SU(3) extension of the nonlinear pion Lagrangian density has so far been given only for specific models, and the treatment of the pseudoscalar-meson mass term is especially inadequate in the existing literature. We shall, however, describe a general scheme that will enable us to carry out the SU(3) extensions of all pion models.

As we shall see, the divergence conditions that apply to the ρ-π-N system can be maintained for the V-P-B system as long as the SU(3) symmetry is preserved, but they are no longer valid when this symmetry is broken. We shall also investigate the effect of the SU(3)-symmetry breaking on the divergence conditions, and for this purpose we shall follow the symmetry-breaking mechanism of an earlier paper,[6] which is not only remarkably simple but also gives the symmetry-breaking terms explicitly rather than merely specifying their transformation properties.

We shall generally follow the same notation as in Refs. 1 and 2 with appropriate extensions for the SU(3) multiplets. The pseudoscalar-meson nonet, the vector-meson nonet, and the baryon octet will be denoted either by the usual 3×3 matrices P, V_μ, and B or by the nine-component vectors P_i, $V_{\mu,i}$, and B_i, the relationship between the matrix and component forms being given by

$$P=\lambda_i P_i/\sqrt{2}\,,\quad V_\mu=\lambda_i V_{\mu,i}/\sqrt{2}\,,\quad B=\lambda_i B_i/\sqrt{2}\,, \tag{1.1}$$

where $B_0 = 0$ to ensure that B represents an octet.

Because of the complexity of the mathematical formalism, we have found it necessary to define special commutators and anticommutators with a semicolon in the following manner: If

$$O = \sum_i \alpha_i A_i , \quad O' = \sum_j \alpha'_j A'_j , \tag{1.2}$$

where A_i and A'_j are SU(3) matrices while α_i and α'_j are Dirac matrices or spinors, then

$$[O; O'] = \sum_{i,j} \alpha_i \alpha'_j [A_i, A'_j], \quad \{O; O'\} = \sum_{i,j} \alpha_i \alpha'_j \{A_i, A'_j\}. \tag{1.3}$$

The above relations indicate that $[O; O']$ or $\{O; O'\}$ represents a commutator or anticommutator with respect to the SU(3) matrices, while it represents just a product as far as the Dirac matrices or spinors are concerned.

Also note that when Tr appears before a product of the Dirac and SU(3) matrices, the trace extends only over the SU(3) matrices.

II. PSEUDOSCALAR-MESON NONET

We shall first consider the SU(3) extension of the pion Lagrangian density to that of the pseudoscalar-meson nonet with SU(3) symmetry.

By postulating that the source function in the pion-field equation be expressible as a complete divergence, it was shown in Ref. 1 that the nonlinear pion Lagrangian density is given by

$$L_\pi = -(1/16f^2)\,\mathrm{Tr}(\partial_\mu U \partial_\mu U^{-1}) + L(m_\pi) \tag{2.1}$$

with

$$L(m_\pi) = \frac{m_\pi^2}{4f} \int_0^{\pi_i^2} \frac{s'}{t} d(\pi_i^2), \tag{2.2}$$

where Tr denotes the trace over the isospin matrices, U is a unitary function of $i\gamma_5 \vec{\tau} \cdot \vec{\pi}$, and s and t are Hermitian functions of $\vec{\pi}^2$ such that

$$U(i\gamma_5 \vec{\tau} \cdot \vec{\pi}) = 1 + s(\vec{\pi}^2) + 2i\gamma_5 \vec{\tau} \cdot \vec{\pi} t(\vec{\pi}^2). \tag{2.3}$$

In view of the unitary nature of U, the above relation also implies that

$$U^{-1} = 1 + s - 2i\gamma_5 \vec{\tau} \cdot \vec{\pi} t \tag{2.4}$$

and

$$\begin{aligned} &(1+s)^2 + 4t^2 \vec{\pi}^2 = 1, \\ &(1+s)s' + 2(t^2 + 2tt'\vec{\pi}^2) = 0, \end{aligned} \tag{2.5}$$

s' and t' being the derivatives of s and t with respect to $\vec{\pi}^2$. In the SU(3) extension $U(i\gamma_5 \vec{\tau} \cdot \vec{\pi})$ can be replaced by $U(i\gamma_5 \lambda_i P_i)$, but $U(i\gamma_5 \lambda_i P_i)$ cannot be expressed in a form analogous to (2.3) because of the more complicated commutation properties of the λ_i matrices. It is, therefore, necessary to express (2.2) also in terms of U instead of s and t. For this purpose we note that, according to (2.3),

$$\frac{\partial U}{\partial \pi_i} = 2s'\pi_i + 4i\gamma_5 (\vec{\tau} \cdot \vec{\pi}) t' \pi_i + 2i\gamma_5 \tau_i t,$$

which, together with (2.4), gives

$$\mathrm{Tr}\left(i\gamma_5 \vec{\tau} \cdot \vec{\pi} U^{-1} \frac{\partial U}{\partial \pi_i}\right) = 4[2s't\vec{\pi}^2 - 2(1+s)t'\vec{\pi}^2 - (1+s)t]\pi_i$$

or, after simplification with the help of (2.5),

$$\mathrm{Tr}\left(i\gamma_5 \vec{\tau} \cdot \vec{\pi} U^{-1} \frac{\partial U}{\partial \pi_i}\right) = \left(\frac{2s'}{t}\right) \pi_i . \tag{2.6}$$

Hence, it is possible to put (2.2) in the form

$$L(m_\pi) = \frac{m_\pi^2}{4f} \int_0^{\pi_i} \mathrm{Tr}\left(i\gamma_5 \vec{\tau} \cdot \vec{\pi} U^{-1} \frac{\partial U}{\partial \pi_i}\right) d(\pi_i). \tag{2.7}$$

The pion-field equation resulting from (2.1) is expressible as[1]

$$(\Box^2 - m_\pi^2)\vec{\pi} = \partial_\mu \vec{J}_{\mu 5}(\vec{\pi}), \tag{2.8}$$

where $\vec{J}_{\mu 5}(\vec{\pi})$ is given, in terms of s and t, by

$$\begin{aligned} \vec{J}_{\mu 5}(\vec{\pi}) = {} & \partial_\mu \vec{\pi} - (t/f)(1+s)\partial_\mu \vec{\pi} \\ & + (2/f)[ts' - t'(1+s)](\vec{\pi} \cdot \partial_\mu \vec{\pi})\vec{\pi}. \end{aligned} \tag{2.9}$$

Using (2.3) and (2.4), and observing that

$$\partial_\mu s = 2s'(\vec{\pi} \cdot \partial_\mu \vec{\pi}), \quad \partial_\mu t = 2t'(\vec{\pi} \cdot \partial_\mu \vec{\pi}),$$

it can be verified that (2.9) can also be expressed in terms of U as

$$\vec{J}_{\mu 5}(\vec{\pi}) = \partial_\mu \vec{\pi} + (1/8f)\,\mathrm{Tr}[i\gamma_5 \vec{\tau}(U^{-1}\partial_\mu U - U \partial_\mu U^{-1})]. \tag{2.10}$$

The SU(3) generalization of the above results can be carried out in the following way: In order to obtain the Lagrangian density we replace π_i and τ_i by P_i and λ_i in (2.1) and (2.7), which gives

$$L_P = -(1/16f^2)\,\mathrm{Tr}(\partial_\mu U \partial_\mu U^{-1}) + L(m_P) \tag{2.11}$$

with

$$L(m_P) = \frac{m_P^2}{4f} \int_0^{P_i} \mathrm{Tr}\left(i\gamma_5 \lambda_j P_j U^{-1} \frac{\partial U}{\partial P_i}\right) dP_i , \tag{2.12}$$

where Tr denotes the trace over the SU(3) matrices, and U is a unitary function of $i\gamma_5 \lambda_i P_i$. Moreover, the field equation can be obtained by a similar replacement in (2.8) and (2.10), so that we have

$$(\Box^2 - m_P^2)P_i = \partial_\mu J_{\mu 5, i}(P) \tag{2.13}$$

with

$$J_{\mu 5, i}(P) = \partial_\mu P_i + (1/8f)\,\mathrm{Tr}[i\gamma_5 \lambda_i (U^{-1}\partial_\mu U - U \partial_\mu U^{-1})]. \tag{2.14}$$

It is still necessary to verify that the Lagrangian density (2.11) indeed yields the field equation (2.13). Such a verification requires some interesting mathematical manipulations, which are given in Appendix A.

From the above general scheme, the results for various models described in Ref. 1 can be obtained by an appropriate choice of U. In particular, $U(i\gamma_5\lambda_i P_i)=U(i\gamma_5\sqrt{2}P)$ is to be chosen as

$$U=e^{2\sqrt{2}if\gamma_5 P} \tag{2.15}$$

for model A,

$$U=(1-8f^2P^2)^{1/2}+2\sqrt{2}\,if\gamma_5 P \tag{2.16}$$

for model B, and

$$U=\frac{1+\sqrt{2}\,if\gamma_5 P}{1-\sqrt{2}\,if\gamma_5 P} \tag{2.17}$$

for model C. Substitutions of these values of U into (2.11) give the Lagrangian densities for models A, B, and C, and it can be shown with the help of (2.12) that

$$L(m_P)=-\tfrac{1}{2}m_P{}^2\,\mathrm{Tr}(P^2) \tag{2.18}$$

for model A,

$$L(m_P)=-(m_P{}^2/8f^2)\,\mathrm{Tr}[1-(1-8f^2P^2)^{1/2}] \tag{2.19}$$

for model B, and

$$L(m_P)=-(m_P{}^2/4f^2)\,\mathrm{Tr}[\ln(1+2f^2P^2)] \tag{2.20}$$

for model C.

III. P-B SYSTEM

The general form of the nonlinear Lagrangian density for the π-N system can be expressed as[1]

$$L_{\pi+N}=L_\pi+L_N+L_{\pi N}, \tag{3.1}$$

where L_π is given by (2.1) and (2.7), and

$$L_N=-\bar{N}(\gamma_\mu\partial_\mu+m_N)N, \tag{3.2}$$

$$\begin{aligned}L_{\pi N}=&-\tfrac{1}{2}\,\bar{N}\gamma_\mu(U^{1/2}\partial_\mu U^{-1/2}+U^{-1/2}\partial_\mu U^{1/2})N\\&-\tfrac{1}{2}(g_N/f)\bar{N}\gamma_\mu(U^{1/2}\partial_\mu U^{-1/2}-U^{-1/2}\partial_\mu U^{1/2})N.\end{aligned} \tag{3.3}$$

Moreover, the pion-field equation resulting from (3.1) takes the form

$$(\Box^2-m_\pi{}^2)\vec{\pi}=\partial_\mu\vec{J}_{\mu5}(\vec{\pi},N) \tag{3.4}$$

with

$$\vec{J}_{\mu5}(\vec{\pi},N)=\vec{J}_{\mu5}(\vec{\pi})+\vec{J}_{\mu5}(N), \tag{3.5}$$

where $\vec{J}_{\mu5}(\vec{\pi})$ is given by (2.9) or (2.10), while $\vec{J}_{\mu5}(N)$ is given in terms of s and t by

$$\begin{aligned}\vec{J}_{\mu5}(N)=&\,g_N(1+s)(\bar{N}i\gamma_\mu\gamma_5\vec{\tau}N)-2ft(\bar{N}i\gamma_\mu\vec{\tau}\times\vec{\pi}N)\\&-g_N(s/\vec{\pi}^2)(\bar{N}i\gamma_\mu\gamma_5\vec{\tau}\cdot\vec{\pi}N)\vec{\pi},\end{aligned} \tag{3.6}$$

which can also be shown to be expressible in terms of U as

$$\begin{aligned}\vec{J}_{\mu5}(N)=&\,\tfrac{1}{2}f\,\bar{N}i\gamma_\mu\gamma_5(U^{1/2}\vec{\tau}U^{-1/2}-U^{-1/2}\vec{\tau}U^{1/2})N\\&+\tfrac{1}{2}g_N\bar{N}i\gamma_\mu\gamma_5(U^{1/2}\vec{\tau}U^{-1/2}+U^{-1/2}\vec{\tau}U^{1/2})N.\end{aligned} \tag{3.7}$$

It should be noted that the π-Ξ system can be treated in the same way as the π-N system, and thus the π-Ξ coupling is given by

$$\begin{aligned}L_{\pi\Xi}=&-\tfrac{1}{2}\,\bar{\Xi}\gamma_\mu(U^{1/2}\partial_\mu U^{-1/2}+U^{-1/2}\partial_\mu U^{1/2})\Xi\\&-\tfrac{1}{2}(g_\Xi/f)\bar{\Xi}\gamma_\mu(U^{1/2}\partial_\mu U^{-1/2}-U^{-1/2}\partial_\mu U^{1/2})\Xi.\end{aligned} \tag{3.8}$$

However, the SU(3) extensions of the π-N and π-Ξ couplings take different forms, because $\bar{N}i\gamma_5\vec{\tau}\cdot\vec{\pi}N$ corresponds to

$$\mathrm{Tr}(\bar{B}i\gamma_5\lambda_i P_i B),$$

while $\bar{\Xi}i\gamma_5\vec{\tau}\cdot\vec{\pi}\Xi$ corresponds to

$$-\mathrm{Tr}(\bar{B}i\gamma_5 B\lambda_i P_i),$$

in which γ_5 and λ_i appear on different sides of B.

In order to carry out the SU(3) generalization of (3.3) and (3.8), it is necessary to ensure that the Dirac and SU(3) matrices occur on different sides of B for terms involving Ξ. For this purpose, we shall introduce commutators and anticommutators with a semicolon as defined by (1.3), which also implies that

$$\begin{aligned}&\mathrm{Tr}(\bar{B}[O;B])=\mathrm{Tr}([\bar{B};O]B),\\&\mathrm{Tr}(\bar{B}\{O;B\})=\mathrm{Tr}(\{\bar{B};O\}B).\end{aligned} \tag{3.9}$$

It is then possible to write the appropriate P-B couplings, containing both (3.3) and (3.8), as

$$\begin{aligned}L_{PB}=-(1/2f)\,\mathrm{Tr}\big(&f\bar{B}\gamma_\mu[(U^{1/2}\partial_\mu U^{-1/2}+U^{-1/2}\partial_\mu U^{1/2});B]\\&+g_F\bar{B}\gamma_\mu[(U^{1/2}\partial_\mu U^{-1/2}-U^{-1/2}\partial_\mu U^{1/2});B]+g_D\bar{B}\gamma_\mu\{(U^{1/2}\partial_\mu U^{-1/2}-U^{-1/2}\partial_\mu U^{1/2});B\}\big),\end{aligned} \tag{3.10}$$

where g_F and g_D are the so-called F and D coupling constants

$$g_F=\tfrac{1}{2}(g_N+g_\Xi),\quad g_D=\tfrac{1}{2}(g_N-g_\Xi). \tag{3.11}$$

Similarly, the current (3.7), when combined with the corresponding current $\vec{J}_{\mu5}(\Xi)$, leads to the SU(3) generalization

$$J_{\mu 5,i}(B)=\tfrac{1}{2}\,\mathrm{Tr}\big(f\overline{B}i\gamma_\mu\gamma_5[(U^{1/2}\lambda_i U^{-1/2}-U^{-1/2}\lambda_i U^{1/2});B]+g_F\overline{B}i\gamma_\mu\gamma_5[(U^{1/2}\lambda_i U^{-1/2}+U^{-1/2}\lambda_i U^{1/2});B]+g_D\overline{B}i\gamma_\mu\gamma_5\{(U^{1/2}\lambda_i U^{-1/2}+U^{-1/2}\lambda_i U^{1/2});B\}\big). \tag{3.12}$$

Thus, the total Lagrangian density of the P-B system can be expressed as

$$L_{P+B}=L_P+L_B+L_{PB}, \tag{3.13}$$

where

$$L_B=-\mathrm{Tr}(\overline{B}\gamma_\mu\partial_\mu B+m_B\overline{B}B), \tag{3.14}$$

and L_P and L_{PB} are given by (2.11) and (3.10). Moreover, the SU(3) generalization of (3.4) gives the P-field equation

$$(\Box^2-m_P{}^2)P_i=\partial_\mu J_{\mu 5,i} \tag{3.15}$$

with

$$J_{\mu 5,i}=J_{\mu 5,i}(P)+J_{\mu 5,i}(B), \tag{3.16}$$

where $J_{\mu 5,i}(P)$ and $J_{\mu 5,i}(B)$ are given by (2.14) and (3.12). As shown in Appendix B, it can be verified by direct calculations that the field equation (3.15) is obtainable from the Lagrangian density (3.13).

IV. V-P-B SYSTEM

By postulating that the source function in the ρ-field equation have a vanishing divergence, it was shown in Ref. 2 that the Lagrangian density of the ρ field is given by

$$L_\rho=-\tfrac{1}{4}\vec{\rho}_{\mu\nu}{}^2-\tfrac{1}{2}m_\rho{}^2\vec{\rho}_\mu{}^2 \tag{4.1}$$

with

$$\vec{\rho}_{\mu\nu}=\partial_\mu\vec{\rho}_\nu-\partial_\nu\vec{\rho}_\mu+g_\rho\vec{\rho}_\mu\times\vec{\rho}_\nu, \tag{4.2}$$

and the interaction of the ρ field can be introduced into the Lagrangian density of any isofield ψ by replacing $\partial_\mu\psi$ by the covariant derivative $D_\mu\psi$. Putting

$$\begin{aligned}\rho_\mu&=\vec{\tau}\cdot\vec{\rho}_\mu/\sqrt{2},\\ \rho_{\mu\nu}&=\vec{\tau}\cdot\vec{\rho}_{\mu\nu}/\sqrt{2}=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu-(i/\sqrt{2})g_\rho[\rho_\mu,\rho_\nu],\end{aligned} \tag{4.3}$$

the Lagrangian density (4.1) can also be expressed as

$$L_\rho=-\mathrm{Tr}(\tfrac{1}{4}\rho_{\mu\nu}{}^2+\tfrac{1}{2}m_\rho{}^2\rho_\mu{}^2), \tag{4.4}$$

where we have used the relations

$$\mathrm{Tr}(\tau_i\tau_j)=2\delta_{ij},\quad [\tau_i,\tau_j]=2i\epsilon_{ijk}\tau_k. \tag{4.5}$$

The SU(3) extension of (4.4) to the Lagrangian density of the vector-meson nonet with SU(3) symmetry can be obtained by replacing $\tau_i\rho_{\mu,i}$ by $\lambda_i V_{\mu,i}$, which gives

$$L_V=-\mathrm{Tr}(\tfrac{1}{4}V_{\mu\nu}{}^2+\tfrac{1}{2}m_V{}^2V_\mu{}^2), \tag{4.6}$$

where

$$\begin{aligned}V_\mu&=\lambda_i V_{\mu,i}/\sqrt{2},\\ V_{\mu\nu}&=\lambda_i V_{\mu\nu,i}/\sqrt{2}\\ &=\partial_\mu V_\nu-\partial_\nu V_\mu-(i/\sqrt{2})g_V[V_\mu,V_\nu],\end{aligned} \tag{4.7}$$

and it is useful to remember that

$$\mathrm{Tr}(\lambda_i\lambda_j)=2\delta_{ij},\quad [\lambda_i,\lambda_j]=2if_{ijk}\lambda_k. \tag{4.8}$$

Moreover, the SU(3) extension of

$$D_\mu\pi_i=\partial_\mu\pi_i+g_\rho\epsilon_{ijk}\rho_{\mu,j}\pi_k \tag{4.9}$$

takes the form

$$D_\mu P_i=\partial_\mu P_i+g_V f_{ijk}V_{\mu,j}P_k \tag{4.10}$$

or

$$D_\mu P=\partial_\mu P-(i/\sqrt{2})g_V[V_\mu,P], \tag{4.11}$$

and similarly

$$D_\mu B_i=\partial_\mu B_i+g_V f_{ijk}V_{\mu,j}B_k \tag{4.12}$$

or

$$D_\mu B=\partial_\mu B-(i/\sqrt{2})g_V[V_\mu,B]. \tag{4.13}$$

Thus, the Lagrangian density for the V-P-B system can be taken as

$$L_{V+P+B}=L_V+L'_{P+B}=L_V+L'_P+L'_B+L'_{PB}, \tag{4.14}$$

where L_V is given by (4.6), while L'_P, L'_B, and L'_{PB} are obtained from (2.11), (3.14), and (3.10) by replacing $\partial_\mu P$ by $D_\mu P$ and $\partial_\mu B$ by $D_\mu B$, so that

$$L'_P=-(1/16f^2)\,\mathrm{Tr}(D_\mu U D_\mu U^{-1})+L(m_P), \tag{4.15}$$

$$L'_B=-\mathrm{Tr}(\overline{B}\gamma_\mu D_\mu B+m_B\overline{B}B), \tag{4.16}$$

$$L'_{PB}=-(1/2f)\,\mathrm{Tr}\big(f\overline{B}\gamma_\mu[(U^{1/2}D_\mu U^{-1/2}+U^{-1/2}D_\mu U^{1/2});B]+g_F\overline{B}\gamma_\mu[(U^{1/2}D_\mu U^{-1/2}-U^{-1/2}D_\mu U^{1/2});B]+g_D\overline{B}\gamma_\mu\{(U^{1/2}D_\mu U^{-1/2}-U^{-1/2}D_\mu U^{1/2});B\}\big). \tag{4.17}$$

Since

$$\frac{\partial(\mathrm{Tr}\,V_\alpha{}^2)}{\partial V_{\mu,i}}=\sqrt{2}\,\mathrm{Tr}(V_\mu\lambda_i)=2V_{\mu,i},$$

$$\frac{\partial(\mathrm{Tr}\,V_{\alpha\beta}{}^2)}{\partial V_{\mu,i}}=2ig_V\,\mathrm{Tr}(V_{\mu\nu}[V_\nu,\lambda_i])=4g_V f_{ijk}V_{\nu,j}V_{\mu\nu,k},$$

$$\frac{\partial(\mathrm{Tr}\,V_{\alpha\beta}^2)}{\partial(\partial_\nu V_{\mu,i})}=-2\sqrt{2}\,\mathrm{Tr}(V_{\mu\nu}\lambda_i)=-4V_{\mu\nu,i}\,,$$

the V-field equation resulting from (4.14) is

$$\partial_\nu(\partial_\nu V_{\mu,i}-\partial_\mu V_{\nu,i})-m_V^2V_{\mu,i}=g_Vf_{ijk}\partial_\nu(V_{\mu,j}V_{\nu,k})-g_Vf_{ijk}V_{\mu\nu,j}V_{\nu,k}-\frac{\partial L'_{P+B}}{\partial V_{\mu,i}}\,. \tag{4.18}$$

Moreover, the invariance of (4.14) under the infinitesimal SU(3) transformations

$$\delta V_{\nu,j}=c_if_{ijk}V_{\nu,k}\,,\quad \delta P_j=c_if_{ijk}P_k\,,\quad \delta B_j=c_if_{ijk}B_k\,, \tag{4.19}$$

gives the conserved current

$$S_{\mu,i}=-g_Vf_{ijk}\left(\frac{\partial L_V}{\partial(\partial_\mu V_{\nu,j})}V_{\nu,k}+\frac{\partial L'_{P+B}}{\partial(\partial_\mu P_j)}P_k+\frac{\partial L'_{P+B}}{\partial(\partial_\mu B_j)}B_k\right). \tag{4.20}$$

But L'_{P+B} involves $V_{\mu,i}$ only in the form (4.10) and (4.12), and consequently

$$\frac{\partial L'_{P+B}}{\partial V_{\mu,i}}=-g_Vf_{ijk}\left(\frac{\partial L'_{P+B}}{\partial(\partial_\mu P_j)}P_k+\frac{\partial L'_{P+B}}{\partial(\partial_\mu B_j)}B_k\right), \tag{4.21}$$

which enables us to express (4.20) as

$$S_{\mu,i}=g_Vf_{ijk}V_{\mu\nu,j}V_{\nu\,k}+\frac{\partial L'_{P+B}}{\partial V_{\mu,i}}\,. \tag{4.22}$$

According to (4.18) and (4.22), the V-field equation can be expressed as

$$\partial_\nu(\partial_\nu V_{\mu,i}-\partial_\mu V_{\nu,i})-m_V^2V_{\mu,i}=-J_{\mu,i}\,, \tag{4.23}$$

where the source function

$$J_{\mu,i}=S_{\mu,i}-g_Vf_{ijk}\partial_\nu(V_{\mu,j}V_{\nu,k}) \tag{4.24}$$

satisfies the divergence condition

$$\partial_\mu J_{\mu,i}=\partial_\mu S_{\mu,i}=0\,. \tag{4.25}$$

It is also possible to put (4.18) in the alternative form

$$D_\nu V_{\nu\mu,i}-m_V^2V_{\mu,i}=-\mathcal{J}_{\mu,i}\,, \tag{4.26}$$

where

$$\mathcal{J}_{\mu,i}=\frac{\partial L'_{P+B}}{\partial V_{\mu,i}}\,. \tag{4.27}$$

It then follows from (4.26) and (4.23) that

$$D_\mu\mathcal{J}_{\mu,i}=m_V^2D_\mu V_{\mu,i}=m_V^2\partial_\mu V_{\mu,i}=\partial_\mu J_{\mu,i}\,, \tag{4.28}$$

and thus (4.25) yields the covariant divergence condition

$$D_\mu\mathcal{J}_{\mu,i}=0\,. \tag{4.29}$$

By following the same reasoning as given in Ref. 2 to obtain the pion-field equation in the presence of the ρ field, it can be shown that the P-field equation for the V-P-B system is the same as in the absence of the V field except that now D_μ appears in place of ∂_μ. Thus, the P-field equation, obtained by replacing ∂_μ by D_μ in (3.15), is

$$(D_\mu^2-m_P^2)P_i=D_\mu J'_{\mu5,i} \tag{4.30}$$

with

$$J'_{\mu5,i}=J'_{\mu5,i}(P)+J_{\mu5,i}(B)\,, \tag{4.31}$$

where, corresponding to (2.14),

$$J'_{\mu5,i}(P)=D_\mu P_i+(1/8f)\,\mathrm{Tr}[i\gamma_5\lambda_i(U^{-1}D_\mu U-UD_\mu U^{-1})]\,, \tag{4.32}$$

while $J_{\mu5,i}(B)$ is given by (3.12).

In the above treatment we have considered only the simplest couplings for the V field. As in the case of the ρ field, it is possible to introduce additional couplings involving $V_{\mu\nu}$ without violating the divergence condition for the V field. However, it is not known at present whether such couplings are fundamental or manifestations of the higher-order effects generated by the simpler couplings.

V. SU(3)-SYMMETRY BREAKING

We shall introduce the SU(3)-symmetry breaking in the Lagrangian density through the mass matrix,[6] and for simplicity we shall expand the Lagrangian density in powers of the coupling constants, which will be sufficient to bring out the essential features arising from the symmetry breaking.

A. Expansion of L'_P, L'_{PB}, and $J'_{\mu5,i}$

Let us put

$$U=1+2\sum_{n=1}^{\infty}a_n(\sqrt{2}\,if\gamma_5P)^n\,, \tag{5.1}$$

where the coefficients a_n are real, and $a_1=1$. It can then be inferred from the unitary property of U that

$$U=1+2(\sqrt{2}\,if\gamma_5P)+2(\sqrt{2}\,if\gamma_5P)^2+2a_3(\sqrt{2}\,if\gamma_5P)^3+O(f^4)\,, \tag{5.2}$$

where a_3 is a model-dependent parameter.[7]
Substitution of (5.2) into (4.15) gives

$$L'_P=-\tfrac{1}{2}\,\mathrm{Tr}[(D_\mu P)^2+4(1-2a_3)f^2P^2(D_\mu P)^2+4(1-a_3)f^2P(D_\mu P)P(D_\mu P)+\cdots]+L(m_P)\,, \tag{5.3}$$

while, substituting (5.2) into (2.12) and carrying out the necessary integration, we find

$$L(m_P) = -\tfrac{1}{2} m_P{}^2\,\mathrm{Tr}[P^2 + (2-3a_3) f^2 P^4 + \cdots\,], \tag{5.4}$$

where the dots denote terms of orders higher than 2 in the coupling constants. Further, according to (5.2),

$$U^{1/2} D_\mu U^{-1/2} = -\sqrt{2}\, i f \gamma_5 D_\mu P - f^2[D_\mu P, P] + \cdots,$$
$$U^{-1/2} D_\mu U^{1/2} = \sqrt{2}\, i f \gamma_5 D_\mu P - f^2[D_\mu P, P] + \cdots,$$

so that it is possible to expand (4.17) as

$$L'_{PB} = \mathrm{Tr}(\sqrt{2}\, g_F \bar{B} i\gamma_\mu\gamma_5 [D_\mu P, B] + \sqrt{2}\, g_D \bar{B} i \gamma_\mu \gamma_5 \{D_\mu P, B\} + f^2 \bar{B}\gamma_\mu[[D_\mu P, P], B] + \cdots\,). \tag{5.5}$$

Similarly, it follows from (4.31), (4.32), and (3.12) that

$$J'_{\mu 5, i} = J'_{\mu 5, i}(P) + J_{\mu 5, i}(B),$$
$$J'_{\mu 5, i}(P) = \sqrt{2} f^2\,\mathrm{Tr}[a_3\{P^2, D_\mu P\}\lambda_i + (a_3 - 2) P (D_\mu P) P \lambda_i] + \cdots, \tag{5.6}$$
$$J_{\mu 5, i}(B) = g_F\,\mathrm{Tr}(\bar{B} i\gamma_\mu\gamma_5[\lambda_i, B]) + g_D\,\mathrm{Tr}(\bar{B} i \gamma_\mu\gamma_5\{\lambda_i, B\}) + \sqrt{2} f^2\,\mathrm{Tr}(\bar{B}\gamma_\mu[[\lambda_i, P], B]) + \cdots.$$

The above results can be expressed in the matrix form as indicated in Appendix C, which gives

$$\begin{aligned} J'_{\mu 5} &= J'_{\mu 5}(P) + J_{\mu 5}(B),\\ J'_{\mu 5}(P) &= 2a_3 f^2\{P^2, D_\mu P\} + 2(a_3 - 2) f^2 P (D_\mu P) P + \cdots,\\ J_{\mu 5}(B) &= -\sqrt{2}\, g_F[\bar{B} i\gamma_\mu\gamma_5; B] + \sqrt{2}\, g_D\{\bar{B} i\gamma_\mu\gamma_5; B\} + 2 f^2[[\bar{B}\gamma_\mu; B], P] + \cdots, \end{aligned} \tag{5.7}$$

where, in accordance with (1.3),

$$[\bar{B} i\gamma_\mu\gamma_5; B] = \tfrac{1}{2}(\bar{B}_i i\gamma_\mu\gamma_5 B_j)[\lambda_i, \lambda_j], \text{ etc.}$$

B. Symmetry Breaking Through Mass Matrix

It was proposed in Ref. 6 that in the multispinor hadron formalism the role of the mass parameter be replaced by the mass matrix

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m+\Delta \end{pmatrix}, \tag{5.8}$$

which leads to mass splittings for each of the fields P, V_μ, and B.

Let us first consider the P field for which, according to Ref. 6, the mass term

$$-\tfrac{1}{2} m_P{}^2\,\mathrm{Tr}(P^2)$$

should be replaced by

$$-\mathrm{Tr}(M^2P^2 + MPMP).$$

However, since $L(m_P)$ does not possess the above simple form except in a special model, we first express it as

$$L(m_P) = -\tfrac{1}{2} m_P{}^2\,\mathrm{Tr}(\mathcal{P}^2), \tag{5.9}$$

where $\mathcal{P}$ is a model-dependent function of P, and then replace it by

$$\bar{L}(m_P) = -\mathrm{Tr}(M^2\mathcal{P}^2 + M\mathcal{P}M\mathcal{P}). \tag{5.10}$$

This procedure implies that by a redefinition of the P field the Lagrangian density can be expressed in a form such that the coupling terms are SU(3)-invariant, which agrees with the viewpoint of Ref. 6. It also follows on comparing (5.4) and (5.9) that

$$\mathcal{P} = P + (1 - \tfrac{3}{2} a_3) f^2 P^3 + \cdots, \tag{5.11}$$

so that (5.10) can be expanded as

$$\bar{L}(m_P) = -\mathrm{Tr}[(M^2P^2 + MPMP) + (2 - 3a_3) f^2 (M^2 P^4 + MPMP^3) + \cdots\,]. \tag{5.12}$$

The symmetry breaking for the V_μ and B fields is quite straightforward. In accordance with the results of Ref. 6, we replace

$$L(m_V) = -\tfrac{1}{2} m_V{}^2\,\mathrm{Tr}(V_\mu{}^2) \tag{5.13}$$

by

$$\bar{L}(m_V) = -\mathrm{Tr}(M^2 V_\mu{}^2 + M V_\mu M V_\mu), \tag{5.14}$$

and

$$L(m_B) = -m_B\,\mathrm{Tr}(\bar{B}B) \tag{5.15}$$

by

$$\bar{L}(m_B) = -\mathrm{Tr}(\bar{B}MB - \bar{B}BM) - \mathrm{Tr}(M)\,\mathrm{Tr}(\bar{B}B). \tag{5.16}$$

The parameter Δ in (5.8) not only leads to intrinsic mass splittings but also gives rise to SU(3)-dependent self-energies. The effect of self-energies on mass splittings has been fully discussed in Ref. 6, and will not be considered here. Also note that in the SU(3)-symmetry limit, which corresponds to $\Delta = 0$, the mass matrix (5.8) becomes a multiple of the unit matrix, and $m_P = m_V = 2m$, $m_B = 3m$.

It will be convenient here to use the matrix form for the derivation of the field equations from the Lagrangian density. The matrix form, of course, can be converted into the component form or vice versa as explained in Appendix C. The baryon field equations with symmetry breaking, obtained with the help of (4.16), (5.16), and (5.5), are

$$\gamma_\mu D_\mu B + [M, B] + \mathrm{Tr}(M) B = \sqrt{2}\, g_F[D_\mu P, i\gamma_\mu\gamma_5 B] + \sqrt{2}\, g_D\{D_\mu P, i\gamma_\mu\gamma_5 B\} + f^2[[D_\mu P, P], \gamma_\mu B] + \cdots, \tag{5.17}$$

$$D_\mu \bar{B}\gamma_\mu - [\bar{B}, M] - \mathrm{Tr}(M)\bar{B} = -\sqrt{2}\, g_F[\bar{B} i\gamma_\mu\gamma_5, D_\mu P] - \sqrt{2}\, g_D\{\bar{B} i\gamma_\mu\gamma_5, D_\mu P\} - f^2[\bar{B}\gamma_\mu, [D_\mu P, P]] + \cdots,$$

and we shall now discuss the field equations for the P and V fields.

C. P-Field Equation

The P-field equation with symmetry breaking, resulting from (5.3), (5.12), and (5.5), is found to be

$$\begin{aligned} D_\mu{}^2 P - \{M,\{M,P\}\} = D_\mu(&-\sqrt{2}\,g_F[\bar{B} i\gamma_\mu\gamma_5; B] + \sqrt{2}\,g_D\{\bar{B} i\gamma_\mu\gamma_5; B\} + f^2[[\bar{B}\gamma_\mu; B], P] \\ &+ 2(2a_3-1)f^2\{P^2, D_\mu P\} + 4(a_3-1)f^2 P(D_\mu P)P) \\ &+ f^2[[\bar{B}\gamma_\mu; B], D_\mu P] + 2(1-2a_3)f^2\{P, (D_\mu P)^2\} + 4(1-a_3)f^2(D_\mu P)P(D_\mu P) \\ &+ (1-\tfrac{3}{2}a_3)f^2(\{P^2,\{M,\{M,P\}\}\} + P\{M,\{M,P\}\}P + \{M,\{M,P^3\}\}) + \cdots , \end{aligned} \tag{5.18}$$

which can be expressed in terms of $J'_{\mu5}$, given by (5.7), as

$$\begin{aligned} D_\mu{}^2 P - \{M,\{M,P\}\} = D_\mu J'_{\mu5} + D_\mu(&-f^2[[\bar{B}\gamma_\mu; B], P] + 2(a_3-1)f^2\{P^2, D_\mu P\} + 2a_3 f^2 P(D_\mu P)P) \\ &+ f^2[[\bar{B}\gamma_\mu; B], D_\mu P] + 2(1-2a_3)f^2\{P, (D_\mu P)^2\} + 4(1-a_3)f^2(D_\mu P)P(D_\mu P) \\ &+ (1-\tfrac{3}{2}a_3)f^2(\{P^2,\{M,\{M,P\}\}\} + P\{M,\{M,P\}\}P + \{M,\{M,P^3\}\}) + \cdots . \end{aligned} \tag{5.19}$$

After some simplification, (5.19) becomes

$$\begin{aligned} D_\mu{}^2 P - \{M,\{M,P\}\} = D_\mu J'_{\mu5} &- f^2[[D_\mu\bar{B}\gamma_\mu; B], P] - f^2[[\bar{B}; \gamma_\mu D_\mu B], P] + 2(a_3-1)f^2\{P^2, D_\mu{}^2 P\} + 2a_3 f^2 P(D_\mu{}^2 P)P \\ &+ (1-\tfrac{3}{2}a_3)f^2(\{P^2,\{M,\{M,P\}\}\} + P\{M,\{M,P\}\}P + \{M,\{M,P^3\}\}) + \cdots , \end{aligned} \tag{5.20}$$

where the f^2 terms can be further simplified by using the field equations for $\bar{B}$, B, and P, which gives

$$\begin{aligned} D_\mu{}^2 P - \{M,\{M,P\}\} = D_\mu J'_{\mu5} &- f^2[P,[M,[\bar{B};B]]] - \tfrac{1}{4}(3-\tfrac{1}{2}a_3)f^2[P,[P,\{M,\{M,P\}\}]] \\ &- \tfrac{1}{4}(1-\tfrac{3}{2}a_3)f^2\{P,\{P,\{M,\{M,P\}\}\}\} + (1-\tfrac{3}{2}a_3)f^2\{M,\{M,P^3\}\} + \cdots . \end{aligned} \tag{5.21}$$

It is interesting to verify that in the SU(3)-symmetry limit, when M becomes a multiple of the unit matrix, the right-hand side of (5.21) reduces to $D_\mu J'_{\mu5}$.

D. V-Field Equation

The Lagrangian density of the V-P-B system can be obtained by adding (4.6), (4.16), (5.3), and (5.5), and inserting the mass matrix in accordance with (5.12), (5.14), and (5.16). The resulting V-field equation with the symmetry breaking is given by

$$D_\nu V_{\nu\mu} - \{M,\{M, V_\mu\}\} = -\mathcal{J}_\mu , \tag{5.22}$$

where

$$\mathcal{J}_\mu = -(ig_V/\sqrt{2})([D_\mu P, P] + [\bar{B}\gamma_\mu; B] + \sqrt{2}g_F[[\bar{B}i\gamma_\mu\gamma_5; B], P] - \sqrt{2}g_D[\{\bar{B}i\gamma_\mu\gamma_5; B\}, P]) + \cdots . \tag{5.23}$$

Further, according to (5.18),

$$\begin{aligned} D_\mu{}^2 P = \{M,\{M,P\}\} - \sqrt{2}g_F D_\mu[\bar{B}i\gamma_\mu\gamma_5; B] \\ + \sqrt{2}g_D D_\mu\{\bar{B}i\gamma_\mu\gamma_5; B\} + O(f^2), \end{aligned} \tag{5.24}$$

while it follows from (5.17) that

$$\begin{aligned} D_\mu[\bar{B}\gamma_\mu; B] &= [D_\mu\bar{B}\gamma_\mu; B] + [\bar{B}; \gamma_\mu D_\mu B] \\ &= [[\bar{B}; B], M] - \sqrt{2}g_F[[\bar{B}i\gamma_\mu\gamma_5; B], D_\mu P] \\ &\quad + \sqrt{2}g_D[\{\bar{B}i\gamma_\mu\gamma_5; B\}, D_\mu P] + O(f^2) . \end{aligned} \tag{5.25}$$

By using (5.24) and (5.25), the covariant divergence of (5.23) can be reduced to

$$D_\mu \mathcal{J}_\mu = (ig_V/\sqrt{2})([M,[\bar{B};B]] + [P,\{M,\{M,P\}\}]) + \cdots , \tag{5.26}$$

where terms of the second order in the coupling constants on the right-hand side of (5.26) vanish because of mutual cancellations.

Note that the right-hand side of (5.26) vanishes altogether when M becomes a multiple of the unit matrix in the SU(3)-symmetry limit.

VI. BROKEN DIVERGENCE CONDITIONS

Although it is well known[8] that the SU(3)-symmetry breaking leads to broken divergence conditions for the vector and axial-vector currents, our formalism enables us to obtain these divergence conditions in an explicit form. The essential results have already been obtained in Sec. V, and we shall only add a few remarks.

By using the relation

$$[P,\{M,\{M,P\}\}]=-[M,\{P,\{M,P\}\}],$$

the covariant divergence of the vector current $\mathcal{J}_\mu$, given by (5.26), can be expressed as

$$D_\mu \mathcal{J}_\mu = (ig_V/\sqrt{2})[M,([\bar{B};B]-\{P,\{M,P\}\})]+\cdots. \tag{6.1}$$

Moreover, by defining the axial-vector current $\mathcal{J}_{\mu 5}$ as

$$\mathcal{J}_{\mu 5} = J'_{\mu 5} - D_\mu P, \tag{6.2}$$

it is possible to put (5.21) in the form

$$\begin{aligned} D_\mu \mathcal{J}_{\mu 5} = &-\{M,\{M,P\}\}+f^2[P,[M,[\bar{B};B]]] \\ &+\tfrac{1}{4}(3-\tfrac{1}{2}a_3)f^2[P,[P,\{M,\{M,P\}\}]] \\ &+\tfrac{1}{4}(1-\tfrac{3}{2}a_3)f^2\{P,\{P,\{M,\{M,P\}\}\}\} \\ &-(1-\tfrac{3}{2}a_3)f^2\{M,\{M,P^3\}\}+\cdots. \end{aligned} \tag{6.3}$$

Confining ourselves to terms of up to second order in the coupling constants, we can write (6.1) as

$$D_\mu \mathcal{J}_\mu = (ig_V/\sqrt{2})[M,u], \tag{6.4}$$

where

$$u=[\bar{B};B]-\{P,\{M,P\}\}, \tag{6.5}$$

and, according to (5.8),

$$M=(\tfrac{1}{6})^{1/2}(3m+\Delta)\lambda_0-(\tfrac{1}{3})^{1/2}\Delta\lambda_8. \tag{6.6}$$

The matrix relation (6.4), with the use of (6.6), gives in the component form

$$D_\mu \mathcal{J}_{\mu,i} = (\tfrac{2}{3})^{1/2} g_V \Delta f_{i8k} u_k, \tag{6.7}$$

which shows that $D_\mu \mathcal{J}_{\mu,i}$ is nonvanishing only for $i=4,5,6,7$.

It is, of course, also possible to express (6.3) in the component form, but this only yields a nonvanishing and complicated result for each component of $D_\mu \mathcal{J}_{\mu 5,i}$.

APPENDIX A: DERIVATION OF PSEUDOSCALAR-MESON FIELD EQUATION

The manipulations used in Ref. 1 for the derivation of the pion-field equation in the form (2.8) become extremely cumbersome when the τ_i matrices are replaced by the λ_i matrices; and therefore we shall follow an alternative approach here. Let us introduce a field variable ϕ_i that is a function of P_i and satisfies the condition $\phi_i = P_i$ for $f=0$. We shall show that it is possible to find $\delta\phi_i$ such that the variation of the Lagrangian density (2.11) with respect to ϕ_i directly gives the field equation (2.13), i.e.,

$$\frac{\partial L_P}{\partial \phi_i}-\partial_\nu\left(\frac{\partial L_P}{\partial(\partial_\nu \phi_i)}\right)\equiv(\Box^2-m_P{}^2)P_i-\partial_\mu J_{\mu 5,i}(P). \tag{A1}$$

According to (2.11), we have

$$\frac{\partial L_P}{\partial \phi_i}-\partial_\nu\left(\frac{\partial L_P}{\partial(\partial_\nu \phi_i)}\right)=-\frac{1}{16f^2}\mathrm{Tr}\left[\frac{\partial(\partial_\mu U)}{\partial \phi_i}(\partial_\mu U^{-1})+(\partial_\mu U)\frac{\partial(\partial_\mu U^{-1})}{\partial\phi_i}-\partial_\nu\left(\frac{\partial(\partial_\mu U)}{\partial(\partial_\nu\phi_i)}(\partial_\mu U^{-1})+(\partial_\mu U)\frac{\partial(\partial_\mu U^{-1})}{\partial(\partial_\nu \phi_i)}\right)\right]+\frac{\partial L(m_P)}{\partial\phi_i}, \tag{A2}$$

where, in view of (2.12),

$$\frac{\partial L(m_P)}{\partial \phi_i}=\frac{m_P{}^2}{4f}\mathrm{Tr}\left(i\gamma_5\lambda_j P_j U^{-1}\frac{\partial U}{\partial\phi_i}\right). \tag{A3}$$

Since

$$\partial_\mu U=\left(\frac{\partial U}{\partial \phi_j}\right)\partial_\mu\phi_j,$$

it follows that

$$\frac{\partial(\partial_\mu U)}{\partial\phi_i}=\partial_\mu\left(\frac{\partial U}{\partial\phi_i}\right),\quad \frac{\partial(\partial_\mu U)}{\partial(\partial_\nu\phi_i)}=\frac{\partial U}{\partial\phi_i}\delta_{\mu\nu}, \tag{A4}$$

which enables us to express (A2) as

$$\frac{\partial L_P}{\partial\phi_i}-\partial_\nu\left(\frac{\partial L_P}{\partial(\partial_\nu\phi_i)}\right)=\frac{1}{16f^2}\mathrm{Tr}\left(\frac{\partial U}{\partial\phi_i}(\Box^2U^{-1})+\frac{\partial U^{-1}}{\partial\phi_i}(\Box^2 U)\right)+\frac{\partial L(m_P)}{\partial\phi_i}. \tag{A5}$$

On the other hand, (2.14) gives

$$(\Box^2-m_P{}^2)P_i-\partial_\mu J_{\mu 5,i}(P)=-(1/8f)\,\mathrm{Tr}[i\gamma_5\lambda_i(U^{-1}\Box^2U+\partial_\mu U^{-1}\partial_\mu U-\partial_\mu U\partial_\mu U^{-1}-U\Box^2U^{-1})]-m_P{}^2P_i,$$

where the trace can be simplified by noting that

$$\Box^2(U^{-1}U)=\Box^2(UU^{-1})=0\,,$$

from which it follows that

$$\partial_\mu U^{-1}\partial_\mu U=-\tfrac{1}{2}\left[(\Box^2U^{-1})\,U+U^{-1}(\Box^2U)\right]\,,$$
$$\partial_\mu U\partial_\mu U^{-1}=-\tfrac{1}{2}\left[(\Box^2U)\,U^{-1}+U(\Box^2U^{-1})\right]\,,$$

and thus

$$(\Box^2-m_P{}^2)P_i-\partial_\mu J_{\mu 5,\,i}(P)=(1/16f)\mathrm{Tr}\left[i\gamma_5(U\lambda_i+\lambda_iU)(\Box^2U^{-1})-i\gamma_5(U^{-1}\lambda_i+\lambda_iU^{-1})(\Box^2U)\right]-m_P{}^2P_i\,. \tag{A6}$$

A comparison of (A5) and (A6) shows that the condition (A1) will be fulfilled, if

$$\frac{\partial L(m_P)}{\partial\phi_i}=-m_P{}^2P_i \tag{A7}$$

and

$$\frac{\partial U}{\partial\phi_i}=if\gamma_5(U\lambda_i+\lambda_iU)\,, \tag{A8}$$

which also implies the Hermitian-conjugate relation

$$\frac{\partial U^{-1}}{\partial\phi_i}=-if\gamma_5(U^{-1}\lambda_i+\lambda_iU^{-1})\,. \tag{A9}$$

We further observe that (A7) can be obtained by substituting (A8) into (A3) and remembering that $\mathrm{Tr}(\lambda_i\lambda_j)=2\delta_{ij}$. Thus, the only relation to be satisfied by ϕ_i is given by (A8), which can also be expressed as

$$\delta U=\sqrt{2}\;if\gamma_5(U\delta\phi+\delta\phi U)\,, \tag{A10}$$

where $\delta\phi=\lambda_i\delta\phi_i/\sqrt{2}$.

In order to establish the existence of $\delta\phi$ satisfying (A10), let us put

$$U=\frac{1+if\gamma_5K}{1-if\gamma_5K}\,, \tag{A11}$$

where the unitary property of U simply requires that K be Hermitian. According to (A11),

$$(1-if\gamma_5K)U(1-if\gamma_5K)=1+f^2K^2\,,$$

so that

$$\begin{aligned}(1-if\gamma_5K)\delta U(1-if\gamma_5K)=&(if\gamma_5\delta K)U(1-if\gamma_5K)\\&+(1-if\gamma_5K)U(if\gamma_5\delta K)\\&+f^2(\delta KK+K\delta K)\,,\end{aligned}$$

which becomes, on using (A11) again,

$$(1-if\gamma_5K)\delta U(1-if\gamma_5K)=2if\gamma_5\delta K\,. \tag{A12}$$

Moreover, according to (A10),

$$\begin{aligned}(1-if\gamma_5K)\delta U(1-if\gamma_5K)=&\sqrt{2}\;if\gamma_5(1-if\gamma_5K)\\&\times(U\delta\phi+\delta\phi U)(1-if\gamma_5K)\,,\end{aligned}$$

which gives, on using (A11),

$$(1-if\gamma_5K)\delta U(1-if\gamma_5K)=2\sqrt{2}\;if\gamma_5(\delta\phi+f^2K\delta\phi K)\,. \tag{A13}$$

It follows from (A12) and (A13) that

$$\delta K=\sqrt{2}(\delta\phi+f^2K\delta\phi K)\,, \tag{A14}$$

and thus it is possible to determine $\delta\phi$ in terms of K as

$$\begin{aligned}\sqrt{2}\delta\phi&=\delta K-\sqrt{2}f^2K\delta\phi K\\&=\delta K-f^2K\delta KK+\cdots+(-1)^nf^{2n}K^n\delta KK^n+\cdots\,.\end{aligned} \tag{A15}$$

Note that

$$K=(1/f)\tan(\sqrt{2}fP) \tag{A16}$$

for model A,

$$K=\frac{1-(1-8f^2P^2)^{1/2}}{2\sqrt{2}\,f^2P} \tag{A17}$$

for model B, and

$$K=\sqrt{2}P \tag{A18}$$

for model C.

APPENDIX B: DERIVATION OF PSEUDOSCALAR-MESON FIELD EQUATION WITH BARYON COUPLING

Since we have already established the relation (A1) in Appendix A, it is now sufficient to show that

$$\frac{\partial L_{PB}}{\partial\phi_i}-\partial_\nu\left(\frac{\partial L_{PB}}{\partial(\partial_\nu\phi_i)}\right)\equiv-\partial_\mu J_{\mu5,\,i}(B)\,, \tag{B1}$$

where L_{PB} and $J_{\mu5,i}(B)$, given by (3.10) and (3.12), can be written in a compact form as

$$\begin{aligned}L_{PB}=-(1/2f)\,\mathrm{Tr}(f\bar{B}\gamma_\mu[U'_\mu;B]+g_F\bar{B}\gamma_\mu[U_\mu;B]\\+g_D\bar{B}\gamma_\mu\{U_\mu;B\})\,,\end{aligned} \tag{B2}$$

$$\begin{aligned}J_{\mu5,\,i}(B)=\tfrac{1}{2}\,\mathrm{Tr}(f\bar{B}i\gamma_\mu\gamma_5[\Lambda'_i;B]+g_F\bar{B}i\gamma_\mu\gamma_5[\Lambda_i;B]\\+g_D\bar{B}i\gamma_\mu\gamma_5\{\Lambda_i;B\})\,,\end{aligned} \tag{B3}$$

with

$$
\begin{aligned}
U_\mu &= U^{1/2}\partial_\mu U^{-1/2} - U^{-1/2}\partial_\mu U^{1/2},\\
U'_\mu &= U^{1/2}\partial_\mu U^{-1/2} + U^{-1/2}\partial_\mu U^{1/2},\\
\Lambda_i &= U^{1/2}\lambda_i U^{-1/2} + U^{-1/2}\lambda_i U^{1/2},\\
\Lambda'_i &= U^{1/2}\lambda_i U^{-1/2} - U^{-1/2}\lambda_i U^{1/2}.
\end{aligned}
\tag{B4}
$$

In view of (3.9), it is possible to express (B2) and (B3) in an alternative form involving the commutators and anticommutators of $\bar{B}$ instead of B.

Note that U is a function of $i\gamma_5\lambda_i P_i$, and therefore γ_5 commutes with U_μ, U'_μ, Λ_i, and Λ'_i, while

$$
\begin{aligned}
&\gamma_\mu U_\nu = -U_\nu\gamma_\mu, \quad \gamma_\mu U'_\nu = U'_\nu\gamma_\mu,\\
&\gamma_\mu\Lambda_i = \Lambda_i\gamma_\mu, \quad \gamma_\mu\Lambda'_i = -\Lambda'_i\gamma_\mu.
\end{aligned}
\tag{B5}
$$

It is also useful to remember that relations of the form (A4) hold not only for U but also for $U^{\pm 1/2}$.

According to (B2), we have

$$
\begin{aligned}
\frac{\partial L_{PB}}{\partial\phi_i} - \partial_\nu\left(\frac{\partial L_{PB}}{\partial(\partial_\nu\phi_i)}\right) = -(1/2f)\,\mathrm{Tr}\Bigg(&f\bar{B}\gamma_\mu\left[\frac{\partial U'_\mu}{\partial\phi_i};B\right] + g_F\bar{B}\gamma_\mu\left[\frac{\partial U_\mu}{\partial\phi_i};B\right] + g_D\bar{B}\gamma_\mu\left\{\frac{\partial U_\mu}{\partial\phi_i};B\right\}\\
&- \partial_\mu\left(f\bar{B}\gamma_\mu[\Phi'_i;B] + g_F\bar{B}\gamma_\mu[\Phi_i;B] + g_D\bar{B}\gamma_\mu\{\Phi_i;B\}\right)\Bigg),
\end{aligned}
\tag{B6}
$$

where we have taken into account the fact that

$$
\frac{\partial U_\mu}{\partial(\partial_\nu\phi_i)} = \delta_{\mu\nu}\Phi_i, \quad \frac{\partial U'_\mu}{\partial(\partial_\nu\phi_i)} = \delta_{\mu\nu}\Phi'_i,
\tag{B7}
$$

with

$$
\begin{aligned}
\Phi_i &= U^{1/2}\frac{\partial U^{-1/2}}{\partial\phi_i} - U^{-1/2}\frac{\partial U^{1/2}}{\partial\phi_i},\\
\Phi'_i &= U^{1/2}\frac{\partial U^{-1/2}}{\partial\phi_i} + U^{-1/2}\frac{\partial U^{1/2}}{\partial\phi_i},
\end{aligned}
\tag{B8}
$$

which also implies

$$
\gamma_\mu\Phi_i = -\Phi_i\gamma_\mu, \quad \gamma_\mu\Phi'_i = \Phi'_i\gamma_\mu.
\tag{B9}
$$

We now express (B6) as

$$
\begin{aligned}
\frac{\partial L_{PB}}{\partial\phi_i} - \partial_\nu\left(\frac{\partial L_{PB}}{\partial(\partial_\nu\phi_i)}\right) = (1/2f)\,\mathrm{Tr}\Bigg(&f(\partial_\mu\bar{B}\gamma_\mu)[\Phi'_i;B] + g_F(\partial_\mu\bar{B}\gamma_\mu)[\Phi_i;B] + g_D(\partial_\mu\bar{B}\gamma_\mu)\{\Phi_i;B\} + f\bar{B}[\Phi'_i;(\gamma_\mu\partial_\mu B)]\\
&- g_F\bar{B}[\Phi_i;(\gamma_\mu\partial_\mu B)] - g_D\bar{B}\{\Phi_i;(\gamma_\mu\partial_\mu B)\} + f\bar{B}\gamma_\mu\left[\left(\partial_\mu\Phi'_i - \frac{\partial U'_\mu}{\partial\phi_i}\right);B\right]\\
&+ g_F\bar{B}\gamma_\mu\left[\left(\partial_\mu\Phi_i - \frac{\partial U_\mu}{\partial\phi_i}\right);B\right] + g_D\bar{B}\gamma_\mu\left\{\left(\partial_\mu\Phi_i - \frac{\partial U_\mu}{\partial\phi_i}\right);B\right\}\Bigg),
\end{aligned}
\tag{B10}
$$

which can be simplified by using the baryon field equations

$$
\begin{aligned}
\gamma_\mu\partial_\mu B &= -m_B B - (1/2f)(f\gamma_\mu[U'_\mu;B] + g_F\gamma_\mu[U_\mu;B] + g_D\gamma_\mu\{U_\mu;B\}),\\
\partial_\mu\bar{B}\gamma_\mu &= m_B\bar{B} + (1/2f)(f[\bar{B};U'_\mu]\gamma_\mu - g_F[\bar{B};U_\mu]\gamma_\mu - g_D\{\bar{B};U_\mu\}\gamma_\mu),
\end{aligned}
\tag{B11}
$$

and the relations

$$
\begin{aligned}
\partial_\mu\Phi_i - \frac{\partial U_\mu}{\partial\phi_i} &= \tfrac{1}{2}[\Phi_i,U'_\mu] + \tfrac{1}{2}[\Phi'_i,U_\mu],\\
\partial_\mu\Phi'_i - \frac{\partial U'_\mu}{\partial\phi_i} &= \tfrac{1}{2}[\Phi_i,U_\mu] + \tfrac{1}{2}[\Phi'_i,U'_\mu].
\end{aligned}
\tag{B12}
$$

Thus, (B10) reduces to

$$
\begin{aligned}
\frac{\partial L_{PB}}{\partial\phi_i} - \partial_\nu\left(\frac{\partial L_{PB}}{\partial(\partial_\nu\phi_i)}\right) = (1/4f^2)\,&\mathrm{Tr}\left((f^2 - g_F^2 - g_D^2)\bar{B}\gamma_\mu[[\Phi_i,U_\mu];B] - 2g_F g_D\bar{B}\gamma_\mu\{[\Phi_i,U_\mu];B\}\right)\\
&+ (m_B/f)\,\mathrm{Tr}(g_F\bar{B}[\Phi_i;B] + g_D\bar{B}\{\Phi_i;B\}).
\end{aligned}
\tag{B13}
$$

On the other hand, (B3) gives

$$\partial_\mu J_{\mu 5,\, i}(B) = \tfrac{1}{2}\,\mathrm{Tr}\big(f(\partial_\mu \bar{B}\gamma_\mu)i\gamma_5[\Lambda_i'; B] + g_F(\partial_\mu \bar{B}\gamma_\mu)i\gamma_5[\Lambda_i; B] + g_D(\partial_\mu \bar{B}\gamma_\mu)i\gamma_5\{\Lambda_i; B\}$$
$$+ f\bar{B}i\gamma_5[\Lambda_i'; (\gamma_\mu\partial_\mu B)] - g_F\bar{B}i\gamma_5[\Lambda_i; (\gamma_\mu\partial_\mu B)] - g_D\bar{B}i\gamma_5\{\Lambda_i; (\gamma_\mu\partial_\mu B)\}$$
$$+ f\bar{B}i\gamma_\mu\gamma_5[\partial_\mu\Lambda_i'; B] + g_F\bar{B}i\gamma_\mu\gamma_5[\partial_\mu\Lambda_i; B] + g_D\bar{B}i\gamma_\mu\gamma_5\{\partial_\mu\Lambda_i; B\}\big)\,. \tag{B14}$$

By using the baryon field equations (B11) and the relations

$$\begin{aligned}\partial_\mu\Lambda_i &= \tfrac{1}{2}[\Lambda_i, U_\mu'] + \tfrac{1}{2}[\Lambda_i', U_\mu]\,,\\ \partial_\mu\Lambda_i' &= \tfrac{1}{2}[\Lambda_i, U_\mu] + \tfrac{1}{2}[\Lambda_i', U_\mu']\,,\end{aligned} \tag{B15}$$

we can put (B14) in the form

$$\partial_\mu J_{\mu 5,\, i}(B) = (1/4f)\,\mathrm{Tr}\big((f^2 - g_F{}^2 - g_D{}^2)\bar{B}i\gamma_\mu\gamma_5[[\Lambda_i, U_\mu]; B] - 2g_F g_D\bar{B}i\gamma_\mu\gamma_5\{[\Lambda_i, U_\mu]; B\}\big)$$
$$+ m_B\,\mathrm{Tr}(g_F\bar{B}i\gamma_5[\Lambda_i; B] + g_D\bar{B}i\gamma_5\{\Lambda_i; B\})\,. \tag{B16}$$

It follows from (B13) and (B16) that the relation (B1) will hold if

$$\Phi_i = -if\gamma_5\Lambda_i \tag{B17}$$

or

$$U^{1/2}\frac{\partial U^{-1/2}}{\partial\phi_i} - U^{-1/2}\frac{\partial U^{1/2}}{\partial\phi_i} = -if\gamma_5(U^{1/2}\lambda_i U^{-1/2} + U^{-1/2}\lambda_i U^{1/2})\,, \tag{B18}$$

which is indeed satisfied by virtue of the relation (A8) and the identities

$$U^{1/2}\frac{\partial U^{-1/2}}{\partial\phi_i} + \frac{\partial U^{1/2}}{\partial\phi_i}U^{-1/2} = 0\,, \quad U^{1/2}\frac{\partial U^{1/2}}{\partial\phi_i} + \frac{\partial U^{1/2}}{\partial\phi_i}U^{1/2} = \frac{\partial U}{\partial\phi_i}\,.$$

APPENDIX C: RELATIONSHIP BETWEEN EQUATIONS IN COMPONENT AND MATRIX FORMS

Any 3×3 matrix A can be expressed as

$$A = A_i\lambda_i/\sqrt{2}\,, \tag{C1}$$

which defines A_i, and implies the inverse relation

$$A_i = \mathrm{Tr}(A\lambda_i/\sqrt{2})\,, \tag{C2}$$

in view of the fact that $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$. Substitution of (C2) into (C1) gives

$$A = \mathrm{Tr}(A\lambda_i/\sqrt{2})(\lambda_i/\sqrt{2})\,. \tag{C3}$$

The relation (C2) shows that from an equation in the matrix form we can obtain the corresponding equation in the component form by multiplying by $\lambda_i/\sqrt{2}$ and taking the trace.

On the other hand, an equation in the component form with the index i can be converted into the corresponding equation in the matrix form by multiplying by $\lambda_i/\sqrt{2}$ and using the relations (C1) and (C3).

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