ities involving massless fermions remain, but at this stage the absence of higher-order corrections continues to be substantiated.

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[1]S. L. Adler, Phys. Rev. 177, 2426 (1969); J. Schwinger, *ibid.* 82, 664 (1951); see also K. Johnson, Phys. Letters 5, 253 (1963).

[2]S. L. Adler, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser, M. Grisaru, and H. Pendelton (MIT Press, Cambridge, Mass., 1971). This contains a rather complete list of references on the subject. For further discussion and referencing, see R. Jackiw, 1970 Brookhaven Summer School Lectures (unpublished).

[3]S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969).

[4]We thank Professor R. Jackiw for showing us several relevant examples.

[5]E. S. Abers, D. A. Dicus, and V. L. Teplitz, Phys. Rev. D 3, 485 (1971).

[6]This situation has been investigated by Sen [S. Sen, University of Maryland Report No. 70-063 (unpublished)] but with incomplete results.

[7]R. W. Brown, C. -C. Shih, and B.-L. Young, Phys. Rev. 186, 1491 (1969).

[8]W. Pauli and F. Villars, Rev. Mod. Phys. 21, 434 (1949).

[9]See the relevant discussion in Ref. 3.

[10]We use the notation $\int_l \equiv \int d^4l/(2\pi)^4$. Our basic notation and conventions are those of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1965); *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965). In particular, $\epsilon_{0123} = -\epsilon^{0123} = +1$ and $\not p = \gamma^\mu p_\mu$.

[11]By carrying out the u integration in their expression, one obtains a result explicitly proportional to $G(R)$. We thank Professor Adler for pointing this out to us.

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The Asymptotic Action Principle*

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An action principle is derived from the unitarity of the S matrix. Assuming the usual asymptotic behavior of the renormalized interpolating fields, emission-absorption symmetry, and the Bose nature of the S operator, one obtains the connection between spin and statistics and the quantization of the asymptotic fields. It is essential to the proof that the system have a nontrivial interaction, but one does not assume the relativistic invariance of the interacting system.

I. INTRODUCTION

In the work of Lehmann, Symanzik, and Zimmermann[1] (LSZ) and others,[2] the asymptotic condition and some form of causality assumption were used to develop a theory whose solutions included the renormalized S-matrix elements. This approach to field theory has the advantage that no divergences appear at any stage of a calculation. However, this so-called asymptotic field theory is not self-contained as it must borrow the quantization of asymptotic fields from pre-LSZ field theory and it must leave the proof[3] of the connection between spin and statistics to other formulations of relativistic quantum theory. The object of this paper is to complete asymptotic field theory by deriving the field quantization and the spin-statistics connection from assumptions that are plausible in the framework of asymptotic field theory.

In pre-LSZ field theory the quantization is obtained either from the assumed canonical formalism[4] or from Schwinger's quantum action principle.[5] The quantum action principle is a beautiful postulate but two criticisms of it can be made. The first is that it is an arbitrary assumption that the variation of the action should be related to the generators of the variation in the specific way that leads to the desired results. For example, if instead of assuming the action principle in its correct form,

$$\delta W_{12} = F_1 - F_2 ,$$

we assumed instead

$$\delta W_{12} = F_2 - F_1 ,$$

we would have an equally beautiful but now incorrect postulate. Thus, the origin of the exact form of the action principle is mysterious. The second criticism is that when the variations of the fields do not commute with the fields themselves (and they do not always commute if the transformation is a coordinate transformation), the variation of the action is not simply related to derivatives of the Lagrangian with respect to the fields; for example,

$$\delta(\phi^3) = \delta\phi\phi^2 + \phi\delta\phi\phi + \phi^2\delta\phi \neq \delta\phi\frac{\partial(\phi^3)}{\partial\phi}.$$

These problems are solved in the context of asymptotic field theory by relating the generators of transformations to the variation of the S operator. The origin of this relation – called the asymptotic action principle – is in the unitarity of the S operator. The variation of operators is well defined and is simply connected to the functional derivatives of the operators with respect to the asymptotic fields.

We treat only the case of scalar fields explicitly; results are stated for spinor fields and the generalization to other fields is straightforward.

Consider an infinitesimal unitary transformation

$$U = 1 + iF_{\rm in}, \quad F_{\rm in}^\dagger = F_{\rm in} \tag{1}$$

which generates a change in the fields

$$\delta\phi_{\rm in} = i[F_{\rm in}, \phi_{\rm in}] . \tag{2}$$

Since the S operator can be written as a sum of products of in-fields, $F_{\rm in}$ also generates a variation of S, i.e.,

$$\delta S = i[F_{\rm in}, S] . \tag{3}$$

If we define

$$F_{\rm out} = S^\dagger F_{\rm in} S , \tag{4}$$

then we have the *asymptotic action principle*

$$S^\dagger \delta S = i(F_{\rm out} - F_{\rm in}), \tag{5}$$

which is a consequence of the unitarity of the S matrix. This principle is similar to Schwinger's quantum action principle[5] and can be used in the same way. Note one difference however: The quantum action principle was a postulate while the asymptotic action principle follows from unitarity.

The variation, δS, occurring in the asymptotic action principle is the change in S brought about by a change in $\phi_{\rm in}$. It might, therefore, be more appropriately called $\delta S_{\rm in}$ as a change in $\phi_{\rm out}$ will generally produce a different change in S, namely, $\delta S_{\rm out}$. Expressing S as a function of $\phi_{\rm out}$'s via

$$\phi_{\rm out} = S^\dagger \phi_{\rm in} S \tag{6}$$

and varying $\phi_{\rm out}$, we have

$$\delta S_{\rm out} = i[F_{\rm out}, S] , \tag{7}$$

which leads to an alternate form for the asymptotic action principle, i.e.,

$$(\delta S_{\rm out}) S^\dagger = i(F_{\rm out} - F_{\rm in}) . \tag{8}$$

The two variations of S are, of course, related by

$$\delta S_{\rm out} = S^\dagger (\delta S_{\rm in}) S . \tag{9}$$

In order to get useful results from the asymptotic action principle, some further assumptions are necessary. We collect and discuss our assumptions in Sec. II. Section III is devoted to a proof of the spin-statistics connection and in Sec. IV the generators are identified and the quantization of asymptotic fields is obtained.

II. BASIC ASSUMPTIONS

A. Unitarity

$$S^\dagger S = SS^\dagger = 1 . \tag{10}$$

B. Absence of Parastatistics

We assume that identical-particle states are either symmetric or antisymmetric.

C. Einstein Emission-Absorption Symmetry

In a relativistic quantum theory the production and absorption amplitudes must be related in a specific way[6] known as the substitution law.[7] Apart from a phase this law says that the amplitude for the production of a particle is obtained from the corresponding amplitude for antiparticle absorption simply by replacing the negative-energy antiparticle wave function by the positive-energy particle wave function. An obvious consequence of the substitution law is that the rate of particle production is obtained from the rate of antiparticle absorption by substituting the positive-energy particle wave function for the negative-energy antiparticle wave function.

An accurate field-theoretic statement of the substitution law involves the specification of relative signs of various amplitudes. These signs have their origin in the statistics satisfied by the particles involved. As one of the purposes of this paper is to establish the spin-statistics connection, it would be circular reasoning to assume the relative signs associated with the substitution law. We therefore assume only that the rates of particle emission and antiparticle absorption are related by the wave function substitution. This is a simple generalization of Einstein's assumption of the equality of the probabilities of absorption and induced emission of radiation.[8]

If we consider the S operator instead of its matrix elements, the assumption of emission-absorption symmetry amounts to S being a functional only of the combination $\phi_{\text{in}}^{(+)}+\eta\phi_{\text{in}}^{(-)}$ with $|\eta|=1$. Here $\phi_{\text{in}}^{(+)}$ is the absorption operator for particles and $\phi_{\text{in}}^{(-)}$ is the creation operator for antiparticles. As the relative phase of particle and antiparticle wave functions is not measurable, one can set $\eta=1$ without loss of generality. (The possibility that η might depend on the position of the field operator in a product is ruled out by the requirement that emission and absorption rates are related by substitution.)

The repeated application of emission-absorption symmetry shows that particles and antiparticles must satisfy the same statistics. Furthermore, the transition amplitude must exhibit symmetry (or antisymmetry) in the entire set of initial particles and final antiparticles and also, of course, in the set of initial antiparticles and final particles. For example, the amplitude must be symmetric or antisymmetric upon the interchange of an initial particle wave function and a final antiparticle wave function.

The S operator will be proportional to sums of products of in-fields. We call these the normal products and determine their properties in accordance with the principle of emission-absorption symmetry. We can allow the normal products to exhibit two symmetries:

$$:\cdots\phi_{\text{in}}(x)\phi_{\text{in}}(x')\cdots:=-\alpha:\cdots\phi_{\text{in}}(x')\phi_{\text{in}}(x)\cdots:,\tag{11}$$

$$:\cdots\phi_{\text{in}}(x)\phi_{\text{in}}^{\dagger}(y)\cdots:=-\beta:\cdots\phi_{\text{in}}^{\dagger}(y)\phi_{\text{in}}(x)\cdots:,$$

where each of α and β can be either + or −. The symmetry of the normal product must be the same as that of the state vectors, for if they were opposite, all matrix elements would vanish.[9] The assumed absence of parastatistics then rules out the possibility that α and β might depend on the position of the fields in a normal product. The state-vector symmetry translates into the commutation rules

$$[\phi_{\text{in}}^{(\pm)},\phi_{\text{in}}^{(\pm)}]_{\alpha}=[\phi_{\text{in}}^{(\pm)},\phi_{\text{in}}^{\dagger(\pm)}]_{\beta}=0\tag{12}$$

because an n-particle state vector is constructed by the application of n creation operators to the vacuum state. The vacuum is defined to be the state that satisfies

$$\phi_{\text{in}}^{(+)}|0\rangle=\phi_{\text{in}}^{\dagger(+)}|0\rangle=0\tag{13}$$

and that is invariant under all coordinate transformations. Nothing is known at this stage about the commutator or anticommutator of $\phi^{(\pm)}$ with $\phi_{\text{in}}^{(\mp)}$ or $\phi_{\text{in}}^{\dagger(\mp)}$.

As emphasized by Dell'Antonio,[10] the value of α determines the statistics of the particles while it is the value of β that was determined by most early attempts at proving the connection between spin and statistics.[11] Recent proofs[3] determine α.

One completes the definition of the normal product by saying that it is obtained from the ordinary product by writing each ϕ_{in} as $\phi_{\text{in}}^{(+)}+\phi_{\text{in}}^{(-)}$ and then rearranging each term so that all absorption operators lie to the right of all creation operators and then multiplying each term by $(-\alpha)^N(-\beta)^M$, where N is the total number of interchanges of ϕ_{in} with ϕ_{in} and of $\phi_{\text{in}}^{\dagger}$ with $\phi_{\text{in}}^{\dagger}$ and M is the number of interchanges of ϕ_{in} with $\phi_{\text{in}}^{\dagger}$ in the rearrangement.

It is convenient to introduce the shorthand notation $:1\cdots n/1\cdots m:$ to stand for the awkward expression $:\phi_{\text{in}}(x_1)\cdots\phi_{\text{in}}(x_n)\,\phi_{\text{in}}^{\dagger}(y_1)\cdots\phi_{\text{in}}^{\dagger}(y_m):$. We also write functional derivatives $\delta/\delta\phi_{\text{in}}(Z)$ and $\delta/\delta\phi_{\text{in}}^{\dagger}(Z)$ as $\delta/\delta Z$ and $\delta/\delta Z^{\dagger}$, respectively.

The functional derivatives of normal products are defined as

$$\begin{aligned}\frac{\delta}{\delta Z}:1\cdots n/1\cdots m:&=\sum_{i=1}^{n}\delta(Z-x_i)(-\alpha)^{i+1}:1\cdots\Lambda_i\cdots n/1\cdots m:,\\ \frac{\delta}{\delta Z^{\dagger}}:1\cdots n/1\cdots m:&=\sum_{i=1}^{m}\delta(Z-y_i)(-\beta)^{n}(-\alpha)^{i+1}:1\cdots n/1\cdots\Lambda_i\cdots m:,\end{aligned}\tag{14}$$

where Λ_i indicates that the ith term is absent.

D. Asymptotic Behavior of Interpolating Fields

The renormalized interpolating field may be defined[2] by giving its source

$$(\Box-m^2)\phi(x)=iS^{\dagger}\delta S/\delta x^{\dagger}\tag{15}$$

and by demanding that it satisfies the following asymptotic conditions:

$$\lim_{\sigma\to+\infty,-\infty}\int_\sigma d\sigma_\mu \psi^{(-)}(x)\overleftrightarrow{\partial}_\mu\phi(x) = \int d\sigma_\mu\psi^{(-)}(x)\overleftrightarrow{\partial}_\mu\phi_{\text{out},\text{in}}(x)\,, \tag{16a}$$

$$\lim_{\sigma\to+\infty,-\infty}\int_\sigma d\sigma_\mu \phi(x)\overleftrightarrow{\partial}_\mu\psi^{(+)}(x) = \int d\sigma_\mu\phi_{\text{out},\text{in}}(x)\overleftrightarrow{\partial}_\mu\psi^{(+)}(x)\,. \tag{16b}$$

These asymptotic conditions hold also for $\phi^\dagger(x)$. Here ψ stands for any of ϕ_{in}, ϕ_{out}, $\phi^\dagger_{\text{in}}$, $\phi^\dagger_{\text{out}}$ and the limits are understood in the usual weak sense.

These new asymptotic conditions are quite likely weaker than the LSZ asymptotic conditions as they are shown in Appendix A to be consequences of the LSZ form of the asymptotic condition, but a satisfactory proof of the converse has not yet been found.

E. Conservation of Statistics

The S operator is assumed to be a Bose-Einstein operator. In other words, if S is expanded as a sum of normal products each normal product shall contain an even number of fields that anticommute with any given field. Specifically, if a term contains n ϕ_{in}'s and m $\phi^\dagger_{\text{in}}$'s, then one must have

$$(-\alpha)^n(-\beta)^m = (-\alpha)^m(-\beta)^n = 1\,. \tag{17}$$

This assumption means that the number of fermions is conserved modulo 2, or more generally if there are two types of fermions that are relatively Bose, then the numbers of each are separately conserved modulo 2. In the work of Lu and Olive,[3] the conservation of statistics is shown to be a consequence of their assumption of cluster decomposition. Therefore, the assumption of conservation of statistics replaces the postulates of cluster decomposition which is used in most other proofs of the connection between spin and statistics. In Lagrangian theories these results follow from the need for H to be a Bose operator in order that $[H, \phi]$ be calculable from the canonical commutation or anticommutation rules.

III. CONNECTION BETWEEN SPIN AND STATISTICS

The variation of the S operator depends on the variation of the fields. The precise relationship between these variations is obtained in Appendix B. The specific result for a Bose-Einstein operator is

$$S^\dagger\delta S = \int d^4\xi\left[\delta\phi^{(-)}_{\text{out}}(\xi)S^\dagger\frac{\delta S}{\delta\xi} + \delta\phi^{\dagger(-)}_{\text{out}}(\xi)S^\dagger\frac{\delta S}{\delta\xi^\dagger} - \alpha S^\dagger\frac{\delta S}{\delta\xi}\delta\phi^{(+)}_{\text{in}}(\xi) - \alpha S^\dagger\frac{\delta S}{\delta\xi^\dagger}\delta\phi^{\dagger(+)}_{\text{in}}(\xi)\right]. \tag{18}$$

From the definition of the interpolating field [Eq. (15)], one has

$$S^\dagger\delta S = -i\int d^4\xi[\delta\phi^{(-)}_{\text{out}}(\Box - m^2)\phi^\dagger + \delta\phi^{\dagger(-)}_{\text{out}}(\Box - m^2)\phi - \alpha(\Box - m^2)\phi^\dagger\delta\phi^{(+)}_{\text{in}} - \alpha(\Box - m^2)\phi\delta\phi^{\dagger(+)}_{\text{in}}]\,. \tag{19}$$

One integrates by parts and employs the asymptotic condition in the form of Eq. (16) to get

$$\begin{aligned} S^\dagger\delta S = -i\int d\sigma_\mu[&\delta\phi^{(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi^\dagger_{\text{out}} + \delta\phi^{\dagger(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi_{\text{out}} - \delta\phi^{(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi^\dagger_{\text{in}} - \delta\phi^{\dagger(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi_{\text{in}} \\ &- \alpha\phi^\dagger_{\text{in}}\overleftrightarrow{\partial}_\mu\delta\phi^{(+)}_{\text{in}} + \alpha\phi^\dagger_{\text{out}}\overleftrightarrow{\partial}_\mu\delta\phi^{(+)}_{\text{in}} - \alpha\phi_{\text{in}}\overleftrightarrow{\partial}_\mu\delta\phi^{\dagger(+)}_{\text{in}} + \alpha\phi_{\text{out}}\overleftrightarrow{\partial}_\mu\delta\phi^{\dagger(+)}_{\text{in}}]\,. \end{aligned} \tag{20}$$

Since all integrals of the form $\int_\sigma d\sigma_\mu\phi_A\overleftrightarrow{\partial}_\mu\phi^\dagger_B$, with A and B standing for in or out, are independent of σ, they are invariant under coordinate transformations:

$$\delta\int_\sigma d\sigma_\mu\phi_A\overleftrightarrow{\partial}_\mu\phi^\dagger_B = 0\,. \tag{21}$$

When we further note that

$$\int d\sigma_\mu\phi^{(\pm)}_A\overleftrightarrow{\partial}_\mu\phi^{\dagger(\pm)}_B = 0\,, \tag{22}$$

we have

$$\begin{aligned} S^\dagger\delta S = -i\int d\sigma_\mu[&\delta\phi^{(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi^{\dagger(+)}_{\text{out}} + \delta\phi^{\dagger(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi^{(+)}_{\text{out}} + \alpha\delta\phi^{(-)}_{\text{in}}\overleftrightarrow{\partial}_\mu\phi^{\dagger(+)}_{\text{in}} + \alpha\delta\phi^{\dagger(-)}_{\text{in}}\overleftrightarrow{\partial}_\mu\phi^{(+)}_{\text{in}} \\ &- (1+\alpha)\delta\phi^{(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi^{\dagger(+)}_{\text{in}} - (1+\alpha)\delta\phi^{\dagger(-)}_{\text{out}}\overleftrightarrow{\partial}_\mu\phi^{(+)}_{\text{in}}]\,. \end{aligned} \tag{23}$$

It is now easy to see that the unitarity of S implies the correct connection between spin and statistics.

Since $S^\dagger \delta S$ is anti-Hermitian when S is unitary, the integral in (23) must be Hermitian. Whenever $\phi_{\rm out} \neq \phi_{\rm in}$, i.e., whenever there is interaction, the last two terms are not Hermitian. Thus, we must choose $\alpha = -1$ and the spin-zero particles satisfy Bose-Einstein statistics.

Note that the value of β has not yet been fixed. It is determined in Sec. IV.

For spinor fields satisfying the Dirac equation, the result corresponding to (23) is

$$S^\dagger \delta S = -\int d\sigma_\mu [\delta\overline{\phi}_{\rm out}^{(-)} \gamma_\mu \phi_{\rm out}^{(+)} - \alpha \delta\overline{\phi}_{\rm in}^{(-)} \gamma_\mu \phi_{\rm in}^{(+)} - (1-\alpha)\delta\overline{\phi}_{\rm out}^{(-)} \gamma_\mu \phi_{\rm in}^{(+)} + \delta\phi_{\rm out}^{(-)T}(\overline{\phi}_{\rm out}^{(+)}\gamma_\mu)^T - \alpha\delta\phi_{\rm in}^{(-)T}(\overline{\phi}_{\rm in}^{(+)}\gamma_\mu)^T - (1-\alpha)\delta\phi_{\rm out}^{(-)T}(\overline{\phi}_{\rm in}^{(+)}\gamma_\mu)^T]. \quad (24)$$

Here the integral must be anti-Hermitian and this would not be so in the presence of interaction if the terms proportional to $(1-\alpha)$ were present. Thus $\alpha = +1$ and the spinor particles satisfy Fermi-Dirac statistics.

Using the methods of Rarita and Schwinger,[12] one can generalize these results to particles of arbitrary spin in a very straightforward way.

IV. QUANTIZATION OF THE ASYMPTOTIC FIELDS

With Bose-Einstein statistics for the scalar field, we have

$$F_{\rm out} - F_{\rm in} = -\int d\sigma_\mu [\delta\phi_{\rm out}^{(-)}\overleftrightarrow{\partial}_\mu \phi_{\rm out}^{\dagger(+)} + \delta\phi_{\rm out}^{\dagger(-)}\overleftrightarrow{\partial}_\mu \phi_{\rm out}^{(+)} - \delta\phi_{\rm in}^{(-)}\overleftrightarrow{\partial}_\mu \phi_{\rm in}^{\dagger(+)} - \delta\phi_{\rm in}^{\dagger(-)}\overleftrightarrow{\partial}_\mu \phi_{\rm in}^{(+)}]. \quad (25)$$

Thus, $F_{\rm in}$ and $F_{\rm out}$ are determined up to the addition of an Hermitian operator that commutes with S. This arbitrariness reflects the possibility of unitarily transforming both the in and out states simultaneously. Such a transformation has no physical significance as it is merely a change of basis.

Since the vacuum must be invariant, we have

$$F_{\rm in} = -\int d\sigma_\mu [\delta\phi_{\rm in}^{(-)}\overleftrightarrow{\partial}_\mu \phi_{\rm in}^{\dagger(+)} + \delta\phi_{\rm in}^{\dagger(-)}\overleftrightarrow{\partial}_\mu \phi_{\rm in}^{(+)}] \quad (26)$$

and a similar expression for $F_{\rm out}$. These generators determine the quantization of the fields in the usual way, i.e.,

$$\delta\phi_{\rm in}^{(-)}(x) = i[F_{\rm in}, \phi_{\rm in}^{(-)}(x)] = -i\int d\sigma_\mu \{\delta\phi_{\rm in}^{(-)}(y)\overleftrightarrow{\partial}_\mu [\phi_{\rm in}^{\dagger(+)}(y), \phi_{\rm in}^{(-)}(x)]_- + \delta\phi_{\rm in}^{\dagger(-)}\overleftrightarrow{\partial}_\mu [\phi_{\rm in}^{(+)}(y), \phi_{\rm in}^{(-)}(x)]_\beta\}. \quad (27)$$

Here we used Eq. (12). Since the Green's function for the Klein-Gordon equation is unique and since $\delta\phi_{\rm in}^{(-)}$ and $\delta\phi_{\rm in}^{\dagger(-)}$ are independent, we have

$$[\phi_{\rm in}^{\dagger(+)}(y), \phi_{\rm in}^{(-)}(x)]_- = -i\Delta_+(y-x), \quad (28)$$

$$[\phi_{\rm in}^{(+)}(y), \phi_{\rm in}^{(-)}(x)]_\beta = 0. \quad (29)$$

From (28) we see that $\beta = -1$; thus we have

$$[\phi_{\rm in}^{(+)}(y), \phi_{\rm in}^{(-)}(x)]_- = 0, \quad (30)$$

which completes the quantization of the fields. Finally, with $\beta = -1$, one can rewrite (26) as

$$F_{\rm in} = -\int d\sigma_\mu : \delta\phi_{\rm in}\overleftrightarrow{\partial}_\mu \phi_{\rm in}^\dagger : = \int d\sigma_\mu : \phi_{\rm in}\overleftrightarrow{\partial}_\mu \delta\phi_{\rm in}^\dagger : . \quad (31)$$

V. DISCUSSION

It has been shown that the spin-statistics connection and the quantization of the asymptotic fields are consequences of unitarity, asymptotic conditions, emission-absorption symmetry, and conservation of statistics. The simplicity of the proof resulted from dealing mainly with the asymptotic fields. It must be emphasized, however, that it is essential to the proof that there is an interaction in the nonasymptotic region. Indeed, no connection between spin and statistics can be obtained by these methods in the free-field case ($S = 1$).

It should also be emphasized that no assumption was made about the relativistic invariance of the S matrix. Thus, the proof is valid whether or not the interactions are such that the usual conservation laws hold. Other proofs[3,10] require invariance of the interacting system.

APPENDIX A: THE ASYMPTOTIC CONDITION

Here we prove that the new asymptotic conditions used in this paper are consequences of the LSZ asymptotic conditions. Consider a matrix element of $\phi_{\rm in}^{(-)}\overleftrightarrow{\partial}_\mu\phi$ between states $\langle(\alpha)|$ and $|(\beta)\rangle$ with $|(\alpha)\rangle \equiv |\alpha_1\cdots\alpha_n\ \mathrm{in}\rangle$. Using

$$\phi_{\rm in}^{(+)}|\alpha_1\cdots\alpha_n\mathrm{in}\rangle=\sum_{i=1}^{n} f_{\alpha i}\,|\alpha_1\cdots\Lambda_i\cdots\alpha_n\mathrm{in}\rangle\,, \tag{A1}$$

where $f_{\alpha i}$ is the wave function of the state $|\alpha_i\rangle$, we have

$$\begin{aligned}\lim_{\sigma\to+\infty,-\infty}\int_\sigma d\sigma_\mu\langle(\alpha)|\phi_{\rm in}^{(-)}\overleftrightarrow{\partial}_\mu\phi\,|(\beta)\rangle&=\sum_{i=1}^{n}\lim_{\sigma\to+\infty,-\infty}\int_\sigma d\sigma_\mu\langle \mathrm{in}\alpha_1\cdots\Lambda_i\cdots\alpha_n|f_{\alpha i}^{*}\overleftrightarrow{\partial}_\mu\phi\,|(\beta)\rangle\\&=\sum_{i=1}^{n}\int d\sigma_\mu\langle \mathrm{in}\alpha_1\cdots\Lambda_i\cdots\alpha_n|f_{\alpha i}^{*}\overleftrightarrow{\partial}_\mu\phi_{\rm out\,,in}\,|(\beta)\rangle\\&=\int d\sigma_\mu\langle(\alpha)|\phi_{\rm in}^{(-)}\overleftrightarrow{\partial}_\mu\phi_{\rm out\,,in}\,|(\beta)\rangle\,.\end{aligned} \tag{A2}$$

Here we used the LSZ form of the asymptotic condition in going from the second line to the third. To prove the asymptotic conditions that involve $\phi_{\rm out}$, one uses states $|(\alpha)\rangle$ and $|(\beta)\rangle$ that are out states and the proof follows the same lines as above. Thus, our asympototic condition is a consequence of the LSZ asymptotic condition.

In the above proof Hermitian scalar fields were used for simplicity. It is clear, however, that the result does not depend on either the spin or Hermitian nature of the fields.

APPENDIX B: OPERATOR VARIATIONS

The operators occurring in field theory are sums of operators of the form

$$A_{nm}\equiv\int d^4x_1\cdots d^4x_n d^4y_1\cdots d^4y_m\ a_{nm}(x_1\cdots x_n y_1\cdots y_m){:}\,1\cdots n/1\cdots m\,{:}\,. \tag{B1}$$

Here a_{nm} are generalized functions of their arguments. Since all variables are integrated over, it follows from (11) that one can take a a_{nm} to have the symmetry properties

$$\begin{aligned}&a_{nm}(\cdots x_i x_j\cdots)=-\alpha a_{nm}(\cdots x_j x_i\cdots)\,,\\&a_{nm}(\cdots y_i y_j\cdots)=-\alpha a_{nm}(\cdots y_j y_i\cdots)\end{aligned} \tag{B2}$$

without any loss of generality.

Consider the change induced in A_{nm} when all in-fields undergo the infinitesimal variation

$$\phi_{\rm in}\to\phi_{\rm in}+\delta\phi_{\rm in}\,. \tag{B3}$$

We shall prove by induction that the variation of A_{nm} is given by

$$\delta A_{nm}=\int d^4\xi\left[\delta\phi_{\rm in}^{(-)}(\xi)\frac{\delta A_{nm}}{\delta\xi}+\delta\phi_{\rm in}^{\dagger(-)}(\xi)\frac{\delta A_{nm}}{\delta\xi^\dagger}+(-\alpha)^{n-1}(-\beta)^m\frac{\delta A_{nm}}{\delta\xi}\delta\phi_{\rm in}^{(+)}(\xi)+(-\beta)^n(-\alpha)^{m-1}\frac{\delta A_{nm}}{\delta\xi^\dagger}\delta\phi_{\rm in}^{\dagger(+)}(\xi)\right]. \tag{B4}$$

This result is clearly true in the trivial cases $n=1$, $m=0$ and $n=0$, $m=1$. Assume it true up to some n and some m and consider $A_{n+1,m}$. Because of Eqs. (B2) and (12) we can write

$$A_{n+1,m}=\int d^4x_1\cdots d^4x_{n+1}d^4y_1\cdots d^4y_m a_{n+1,m}[\phi_{\rm in}^{(-)}(x_1){:}\,2\cdots(n+1)/1\cdots m\,{:}+(-\beta)^m{:}\,1\cdots n/1\cdots m\,{:}\,\phi_{\rm in}^{(+)}(x_{n+1})]\,. \tag{B5}$$

The rest of the proof is straightforward: One varies (B5) and uses the result (B4). After collecting terms, one finds that (B4) extends to $\delta A_{n+1,m}$. A similar method extends (B4) to $\delta A_{n,m+1}$, thus completing the proof.

We note that if A_{nm} is a Bose-Einstein operator, then

$$\delta A_{nm}=\int d^4\xi\left[\delta\phi_{\rm in}^{(-)}(\xi)\frac{\delta A_{nm}}{\delta\xi}+\delta\phi_{\rm in}^{\dagger(-)}(\xi)\frac{\delta A_{nm}}{\delta\xi^\dagger}-\alpha\frac{\delta A_{nm}}{\delta\xi}\delta\phi_{\rm in}^{(+)}(\xi)-\alpha\frac{\delta A_{nm}}{\delta\xi^\dagger}\delta\phi_{\rm in}^{\dagger(+)}(\xi)\right]. \tag{B6}$$

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[1]H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento 1, 205 (1955); 6, 319 (1957).

[2]R. E. Pugh, Ann. Phys. (N. Y.) 23, 335 (1963); J. Math. Phys. 6, 740 (1965); 7, 376 (1966); F. Rohrlich, in *Perspectives in Modern Physics,* edited by R. E. Marshak (Interscience, New York, 1966); F. Rohrlich and J. C. Stoddart, J. Math. Phys. 6, 495 (1965); F. Rohrlich, Phys. Rev. 183, 1359 (1969).

[3]R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics and All That* (Benjamin, New York, 1964); H. P. Stapp, Phys. Rev. 125, 2139 (1962); E. C. Y. Lu and D. Olive, Nuovo Cimento 45A, 205 (1966); M. Froissart and J. R. Taylor, Phys. Rev. 153, 1636 (1967).

[4]See, for example, G. Wentzel, *Quantum Theory of Fields* (Interscience, New York, 1949).

[5]J. Schwinger, Phys. Rev. 82, 914 (1951); 91, 713 (1953); 91, 728 (1953).

[6]R. P. Feynman, Phys. Rev. 76, 749 (1949).

[7]J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley, Reading, Mass., 1959, second printing).

[8]A. Einstein, Phys. Zeits. 18, 121 (1917).

[9]More precisely, a state vector can have any symmetry or none at all; only that part of the state vector that has the same symmetry as the S operator will lead to nonzero S-matrix elements and therefore only that part is observable.

[10]G. F. Dell'Antonio, Ann. Phys. (N. Y.) 16, 153 (1961).

[11]N. Burgoyne, Nuovo Cimento 8, 607 (1958); G. Luders and B. Zumino, Phys. Rev. 110, 1450 (1958); W. Pauli, *ibid.* 58, 716 (1940).

[12]W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).

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Massive Vector Meson in External Fields

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The propagation of classical and quantized vector fields under the influence of external fields constant over wide regions of space-time is investigated. Noncausal propagation is established for suitable external fields. The canonically quantized vector field generates a perturbation expansion for the propagation function which is in agreement with the behavior of the solutions for the classical vector field.

I. INTRODUCTION

The complexity of field equations describing both interacting and free particles with higher spin[1] has generated a spectrum of different viewpoints from which such equations are analyzed.[2-9] We propose to treat particular models involving external fields which are constant in appropriate domains of space-time and are acting on a vector meson.

In Sec. II the uniqueness and existence of solutions as well as the domain of dependence for smooth external fields in two-dimensional space-time are analyzed. We take the obtained results as a justification for discussing the situation of constant external fields. This is done in Sec. III, where a particular external tensor field, as well as the quadrupole coupling, is treated in four dimensions. The tensor field $T_{\mu\nu}=m^2\epsilon\delta_{\mu\nu}$ (m is the mass of the vector meson, ϵ the strength of the tensor field) corresponds to an optical medium[5] with refractive indices $n_{\mathrm{Tr}}=1$ for the transversely polarized modes and $n_L=[(1+\epsilon)/(1-\epsilon)]^{1/2}$ for the longitudinally polarized mode.

Earlier discussions[5,6] used the method of characteristics for the discussion of causality. But there remained some technical questions as to the applicability of this method to the case of higher-spin equations. Thus it is desirable to have a model with the analysis based on an independent method which confirms the previous results by Velo and Zwanziger.[5]

In Sec. IV the canonical field theory is discussed. The correctness of "Feynman rules for any spin"[4] is proven for the tensor model. The complete propagator function can be calculated explicitly. In two-dimensional space-time the propagation function coincides with the unperturbed one in a