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Low-Energy Theorem for $\gamma+\gamma\rightarrow\pi+\pi+\pi$

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We use the hypothesis of the partially conserved axial-vector current (PCAC) to show that the matrix elements for $\gamma+\gamma\rightarrow\pi^0+\pi^0+\pi^0$ and $\gamma+\gamma\rightarrow\pi^0+\pi^++\pi^-$ vanish in the soft-π^0 limit. This, combined with photon gauge invariance, implies low-energy theorems relating these matrix elements to the matrix elements for $\gamma+\gamma\rightarrow\pi^0$ and $\gamma\rightarrow\pi^0+\pi^++\pi^-$. Since the magnitude of the former is determined by the π^0 lifetime, while the ratio of the latter to the former is determined in a model-independent way by isospin and low-energy-theorem arguments, a model-independent prediction for the $\gamma+\gamma\rightarrow\pi+\pi+\pi$ amplitude can be given. Our results agree with those of Aviv, Hari Dass, and Sawyer in the neutral case, but not in the charged case. We give a diagrammatic and effective-Lagrangian interpretation of our formulas which explains the discrepancy.

The reaction $\gamma+\gamma\rightarrow\pi+\pi+\pi$ is of interest, both because it may be observable in electron-positron colliding-beam experiments,[1] and because it is relevant to theoretical unitarity calculations[2] of a lower bound on the decay rate of $K_L^0\rightarrow\mu^+\mu^-$ In recent papers, Aviv, Hari Dass, and Sawyer[3] and Yao[4] have applied effective-Lagrangian methods to calculate the matrix elements for the neutral and charged cases of $\gamma+\gamma\rightarrow\pi+\pi+\pi$. The fact that Refs. 3 and 4 are in disagreement has prompted us to repeat the calculation by standard current-algebra–PCAC methods.[5] Our results agree with Ref. 3 (but not with Ref. 4) in the neutral case $\gamma+\gamma\rightarrow\pi^0+\pi^0+\pi^0$, and disagree with *both* Refs. 3 and 4 in the more interesting charged case $\gamma+\gamma\rightarrow\pi^0+\pi^++\pi^-$. After briefly discussing our method and results, we explain the reasons for our disagreement with the earlier calculations.

We begin with the simple, but powerful observation that the matrix elements

$$\mathfrak{M}^{0+-}\equiv\mathfrak{M}(\gamma(k_1)+\gamma(k_2)\rightarrow\pi^0(q_0)+\pi^+(q_+)+\pi^-(q_-))$$

and

$$\mathfrak{M}^{000}\equiv\mathfrak{M}(\gamma(k_1)+\gamma(k_2)\rightarrow\pi^0(q_0)+\pi^0(q_0')+\pi^0(q_0''))$$

vanish in the single-soft-π^0 limit $q_0\rightarrow0$, with the remaining two pions held on the mass shell. To see this, we follow the standard PCAC procedure[6] of writing the reduction formula describing $\mathfrak{M}^{0+-}$ or $\mathfrak{M}^{000}$ with the π^0 off the mass shell, and then replacing the π^0 field by the divergence of the axial-vector current $(M_\pi^2f)^{-1}\partial_\lambda\mathfrak{F}_3^{5\lambda}$. [The normalization constant f is given by $f\approx f_\pi/(\sqrt{2}M_\pi^2)\approx0.68M_\pi$, with f_π the charged-pion decay amplitude.] Because the corresponding axial charge F_3^5 commutes with the electromagnetic current, *no* equal-time commutator terms are picked up when the derivative ∂_λ is brought outside the T product in the reduction formula. Integration by parts then makes the derivative act on the π^0 wave function, producing a factor $q_{0\lambda}$. Thus both $\mathfrak{M}^{0+-}$ and $\mathfrak{M}^{000}$ are proportional to q_0, and since they contain no pole terms which become singular as $q_0\rightarrow0$, they vanish in this limit. Note that this argument is unaltered by the divergence anomaly[7] in $\partial_\lambda\mathfrak{F}_3^{5\lambda}$, since when $\mathfrak{F}_3^{5\lambda}$ is the only axial-vector current present, its divergence anomaly vanishes when the associated four-momentum q_0 vanishes.[8,9]

In addition to the soft-π^0 limit which we have just derived, we know that $\mathfrak{M}^{0+-}$ and $\mathfrak{M}^{000}$ must be gauge-invariant. That is, they are bilinear forms in ϵ_1 and ϵ_2 (the polarization vectors of the two photons) and vanish when either ϵ_1 is replaced by k_1 or ϵ_2 is replaced by k_2. We can now invoke the standard lore of current-algebra low-energy theorems,[5] which tells us that since we know three independent pieces of information about the low-energy behavior of $\mathfrak{M}^{0+-}$ and $\mathfrak{M}^{000}$ (the $q_0 \to 0$ limit, gauge invariance for photon 1, and gauge invariance for photon 2), we can determine $\mathfrak{M}^{0+-}$ and $\mathfrak{M}^{000}$ from their pion-pole diagrams up to an error of order $O(q_0 k_1 k_2)$ at least.[10] In particular, the terms in $\mathfrak{M}^{0+-}$ and $\mathfrak{M}^{000}$ *quadratic* in the momenta k_1, k_2, q_0, $q_+(q_0')$, and $q_-(q_0'')$ are completely determined. The relevant pion-pole diagrams are illustrated in Fig. 1. The pion-pion scattering amplitudes which appear are evaluated from the current-algebra expression[11,12]

$$\mathfrak{M}(\pi^a \to \pi^b(q_b)+\pi^c(q_c)+\pi^d(q_d)) = if^{-2}\{\delta_{bc}\delta_{ad}[(q_b+q_c)^2-M_\pi^2]+\delta_{bd}\delta_{ac}[(q_b+q_d)^2-M_\pi^2]+\delta_{cd}\delta_{ab}[(q_c+q_d)^2-M_\pi^2]$$
$$-x[(q_b+q_c)^2+(q_b+q_d)^2+(q_c+q_d)^2-3M_\pi^2](\delta_{bc}\delta_{ad}+\delta_{bd}\delta_{ac}+\delta_{cd}\delta_{ab})\}\,, \qquad (1a)$$

where x is a parameter proportional to the isotensor component of the "σ term" and is related to the $I=0$ pion-pion S-wave scattering length a_0 by

$$a_0 = (7/32\pi)f^{-2}M_\pi(1-\tfrac{5}{7}x)\,. \qquad (1b)$$

The $\gamma+\gamma\to\pi^0$ and $\gamma\to\pi^0+\pi^++\pi^-$ amplitudes are expressed in terms of coupling constants F^π and $F^{3\pi}$ defined by

$$\mathfrak{M}(\gamma(k_1)+\gamma(k_2)\to\pi^0) = ik_1^\alpha k_2^\beta \epsilon_1^\gamma \epsilon_2^\delta \epsilon_{\alpha\beta\gamma\delta}F^\pi\,,$$
$$\mathfrak{M}(\gamma(k_1)\to\pi^0+\pi^+(q_+)+\pi^-(q_-)) = ik_1^\alpha \epsilon_1^\beta q_+^\gamma q_-^\delta \epsilon_{\alpha\beta\gamma\delta}F^{3\pi}\,. \qquad (2)$$

The coupling constant F^π is related to the π^0 lifetime by[13]

$$\tau_{\pi^0}^{-1} = (M_\pi^3/64\pi)(F^\pi)^2\,. \qquad (3)$$

Comparison with experiment gives $|F_\pi| = (\alpha/\pi)(0.66\pm0.08M_\pi)^{-1}$, with α the fine-structure constant. While the coupling constant $F^{3\pi}$ has not been measured, both the theory of PCAC anomalies[14] and model-independent isospin and low-energy-theorem arguments (see below) predict

$$eF^{3\pi} = f^{-2}F^\pi, \quad e=(4\pi\alpha)^{1/2}\,. \qquad (4)$$

Combining Eqs. (1) and (2) with the appropriate propagators to form the pion-pole diagrams, and adding the unique second-degree polynomial which guarantees gauge invariance and vanishing of the matrix elements as $q_0\to 0$, we get the following predictions for $\mathfrak{M}^{0+-}$ and $\mathfrak{M}^{000}$:

$$\mathfrak{M}^{000} = (1-3x)\overline{\mathfrak{M}}(q_0,q_0',q_0'')\,,$$
$$\overline{\mathfrak{M}}(q_0,q_0',q_0'') = if^{-2}F^\pi k_1^\alpha k_2^\beta \epsilon_1^\gamma\epsilon_2^\delta\epsilon_{\alpha\beta\gamma\delta}\left(1-\frac{(q_0+q_0')^2+(q_0+q_0'')^2+(q_0'+q_0'')^2-3M_\pi^2}{(q_0+q_0'+q_0'')^2-M_\pi^2}\right)$$
$$= if^{-2}F^\pi k_1^\alpha k_2^\beta\epsilon_1^\gamma\epsilon_2^\delta\epsilon_{\alpha\beta\gamma\delta}\left(\frac{-M_\pi^2}{(k_1+k_2)^2-M_\pi^2}\right) \quad \text{(when three final pions are on mass shell)}\,, \qquad (5a)$$

$$\mathfrak{M}^{0+-} = if^{-2}F^\pi k_1^\alpha k_2^\beta\epsilon_1^\gamma\epsilon_2^\delta\epsilon_{\alpha\beta\gamma\delta}\left(1-\frac{(q_++q_-)^2-M_\pi^2}{(q_0+q_++q_-)^2-M_\pi^2}\right) - x\overline{\mathfrak{M}}(q_0,q_+,q_-)$$
$$-ieF^{3\pi}\epsilon_1^\gamma\epsilon_2^\delta\left[\left(\frac{(2q_+-k_2)_\delta}{k_2^2-2q_+\cdot k_2}k_1^\alpha(q_+-k_2)^\sigma q_-^\tau - \frac{(2q_--k_2)_\delta}{k_2^2-2q_-\cdot k_2}k_1^\alpha q_+^\sigma(q_--k_2)^\tau\right)\epsilon_{\alpha\gamma\sigma\tau}\right.$$
$$\left.+(k_1\to k_2,\gamma\to\delta)+(k_1-k_2)^\alpha q_0^\tau\epsilon_{\alpha\gamma\delta\tau}\right]. \qquad (5b)$$

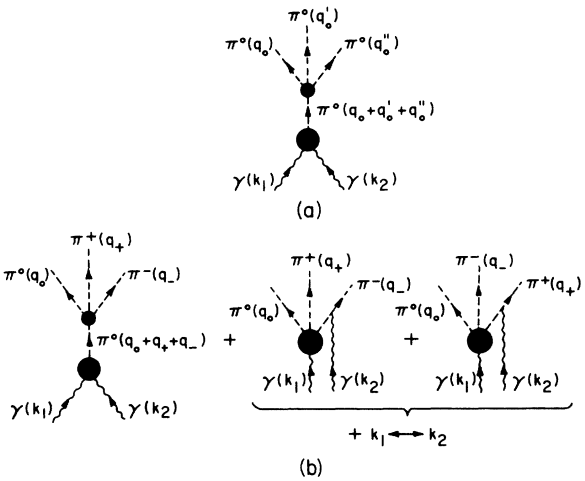

FIG. 1. Pion-pole diagrams for (a) the neutral and (b) the charged cases.

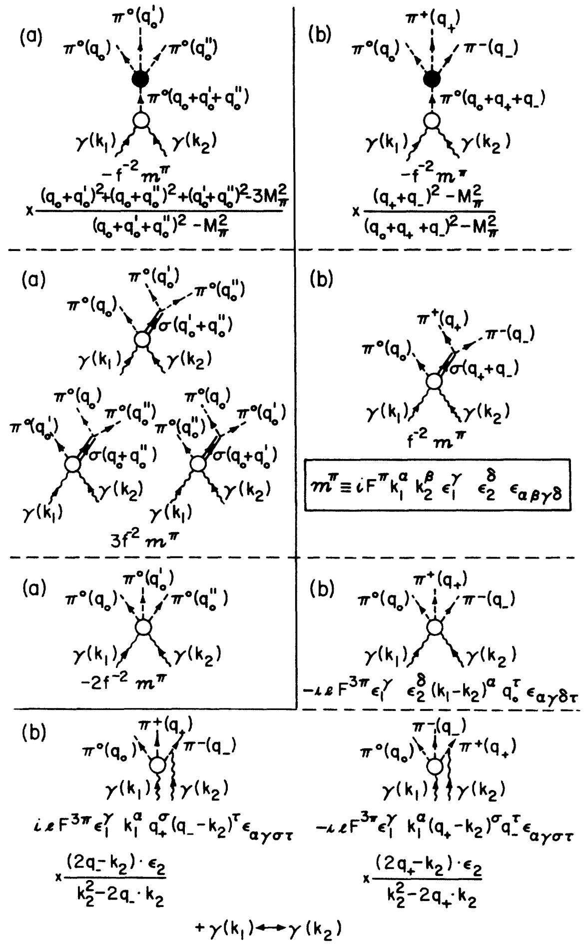

FIG. 2. Lowest-order diagrams contributing to (a) $\mathfrak{M}^{000}$ and (b) $\mathfrak{M}^{0+-}$ in the Gell-Mann – Lévy σ model. The single solid line propagating around each loop denotes the nucleon. In this order of perturbation theory, $f^{-1}=g_r/M_N$, with g_r the pion-nucleon coupling constant and with M_N the nucleon mass. {The large black dot at the four-pion vertices denotes the pion-pion scattering amplitude of Eq. (1). To lowest order in perturbation theory, this arises as the sum of a direct four-pion interaction [coming from the term $(\vec{\pi}\cdot\vec{\pi})^2$ in the σ-model Lagrangian] and of pole terms involving isoscalar σ mesons exchanged between pairs of pions.}

These equations are our basic results.[15]

Our expression for $\mathfrak{M}^{000}$ in Eq. (5a) agrees with that given by Aviv *et al.* We disagree with the result for $\mathfrak{M}^{000}$ quoted by Yao, who has (through an apparent algebraic error) replaced $-M_\pi^2$ in Eq. (5a) by $-4M_\pi^2$. In the case of strictly massless pions, our on-shell result for $\mathfrak{M}^{000}$ is the simple statement that the terms in the matrix element quadratic in the external momenta vanish.[16] This result can be immediately generalized to the reaction $\gamma+\gamma\to n\pi^0$, as follows: The PCAC argument given above tells us that in the limit when any one π^0 has zero four-momentum, with the other $n-1$ π^0's on the mass shell, the matrix element $\mathfrak{M}(\gamma+\gamma\to n\pi^0)$ must vanish. In addition, gauge invariance implies that $\mathfrak{M}$ must vanish when either of the photon four-momenta, k_1 or k_2, vanishes. Taking four-momentum conservation into account, this gives us $n+2-1=n+1$ independent conditions on the low-energy behavior of $\mathfrak{M}$. Since for massless, neutral pions the pion-pole diagrams (tree diagrams) sum to a constant, independent of pion four-momenta, the $n+1$ conditions can be satisfied only if $\mathfrak{M}(\gamma+\gamma\to n\pi^0)$ vanishes[17] up to terms which are at least of order $(\text{momentum})^{n+1}$.

Our result for $\mathfrak{M}^{0+-}$ in Eq. (5b) disagrees with the formulas quoted by Aviv *et al.* and by Yao, both of which overlook the class of pole diagrams proportional to $F^{3\pi}$. The formula of Aviv *et al.* also has the 1 in the large round parentheses multiplying F^π replaced by $\frac{1}{3}$. In order to better understand this latter discrepancy, it is helpful to have a diagrammatic interpretation of the various terms in Eq. (5b). This is given in Fig. 2, which illustrates the lowest-order perturbation-theory contributions to $\mathfrak{M}^{000}$ and $\mathfrak{M}^{0+-}$ in the Gell-Mann–Lévy σ model.[18] The first and fourth rows give just the lowest-order contributions to the pole diagrams of Fig. 1. The σ-pole diagrams in the second row can clearly be represented as matrix elements of the effective Lagrangian

$$i\mathfrak{L}^{\sigma}_{\text{eff}}=\tfrac{1}{16}if^{-2}F^{\pi}F^{\alpha\beta}F^{\gamma\delta}\epsilon_{\alpha\beta\gamma\delta}\pi^0\vec{\pi}\cdot\vec{\pi}\,, \tag{6}$$

with $F^{\alpha\beta}$ the electromagnetic field-strength tensor. As a check, we note that $\pi^0\vec{\pi}\cdot\vec{\pi}=(\pi^0)^3+2\pi^0\pi^+\pi^-$, and since the matrix element of $(\pi^0)^3$ has a Bose symmetry factor of 6, the contributions of Eq. (6) to $\mathfrak{M}^{000}$ and to $\mathfrak{M}^{0+-}$ are in the correct ratio of 3:1.

Let us turn next to the five-point functions in the third row. Aviv *et al.* assume that these are represented by the same effective-Lagrangian structure as in Eq. (6). If this were so, a five-point contribution of $-2f^{-2}\,\mathfrak{M}^{\pi}$ to $\mathfrak{M}^{000}$ would imply a corresponding contribution of $-\frac{2}{3}f^{-2}\,\mathfrak{M}^{\pi}$ to $\mathfrak{M}^{0+-}$, which would then combine with the σ-pole diagram to give a total nonpole contribution of $\frac{1}{3}f^{-2}\,\mathfrak{M}^{\pi}$. This is the origin of the $\frac{1}{3}$ in the formula of Aviv *et al.* In actual fact, however, we find that the five-point diagrams are not described by Eq. (6), but rather by the effective Lagrangian

$$i\mathfrak{L}_{\text{eff}}^{5\text{-pt}} = -\tfrac{1}{2}\,i\,e\,F^{3\pi}(\partial^{\alpha}A^{\gamma})A^{\delta}\epsilon_{\alpha\gamma\delta\tau}(\partial^{\tau}\pi^{0})\vec{\pi}\cdot\vec{\pi}\,. \qquad (7)$$

Equation (7) still couples the three final pions through a pure $I=1$ state, as required by G parity. In the charged-pion case, Eq. (7) obviously leads to the five-point contribution listed in the third row of Fig. 2(b). Although not gauge-invariant by itself, this contribution combines with the pole terms in the fourth row of Fig. 2(b) (which are also not by themselves gauge-invariant) to give a gauge-invariant sum. In the neutral case, using the fact that the matrix element of $\partial^{\delta}\pi^{0}(\pi^{0})^{2}$ is $2i(q_0+q_0'+q_0'')=2i(k_1+k_2)$ and using Eq. (4) to eliminate $F^{3\pi}$ in terms of F^{π}, we find that Eq. (7) just gives the gauge-invariant contribution $-2f^{-2}\,\mathfrak{M}^{\pi}$, as required.[19] Finally, we note that while Yao obtains the correct value of 1 for the constant term in the large round parentheses multiplying F^{π}, he gets this by using an incorrect effective Lagrangian, which does not respect the $\Delta I=1$ rule, to generalize from the neutral to the charged case. The moral is that effective Lagrangians must be handled with caution. When ambiguities arise as to the form of the effective Lagrangian, they must be resolved by reference back to the basic current-algebra relations, which the effective Lagrangian is supposed to represent.[20]

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[1]S. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. D 4, 1532 (1971).

[2]For a review, see S. B. Treiman, National Accelerator Laboratory Report No. THY-13 (unpublished). See also R. Aviv and R. F. Sawyer, Phys. Rev. D 4, 451 (1971).

[3]R. Aviv, N. D. Hari Dass, and R. F. Sawyer, Phys. Rev. Letters 26, 591 (1971).

[4]Tsu Yao, Phys. Letters 35B, 225 (1971).

[5]S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968), Chaps. 2 and 3.

[6]The reasoning is identical to that used to obtain the soft-π^0 theorem for η decay; see D. G. Sutherland, Phys. Letters 23, 384 (1966), and S. L. Adler, Phys. Rev. Letters 18, 519 (1967) for details.

[7]S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60, 47 (1969); W. A. Bardeen, Phys. Rev. 184, 1848 (1969).

[8]For example, the $\gamma+\gamma\to\pi^0$ matrix element given in Eq. (2) below may be rewritten, by using four-momentum conservation, in the form $\mathfrak{M}(\gamma+\gamma\to\pi^0)=iq_0^{\alpha}k_2^{\beta}\epsilon_1^{\gamma}\epsilon_2^{\delta}\epsilon_{\alpha\beta\gamma\delta}F^{\pi}$, and so vanishes when $q_0=0$. The effect of the PCAC anomaly on this reaction is to make soft-pion calculations give $F^{\pi}\neq 0$.

[9]In fact, the diagrammatic analysis given below in Fig. 2 shows that the soft-π^0 limit of $\mathfrak{M}(\gamma+\gamma\to\pi+\pi+\pi)$ involves only axial-vector Ward identities for ring diagrams which have pseudoscalar (and in some cases scalar) vertices in addition to vector vertices and the axial-vector vertex. These Ward identities are known *not* to have anomalies; see W. A. Bardeen, Ref. 7, and R. W. Brown, C.-C. Shih, and B. L. Young, Phys. Rev. 186, 1491 (1969).

[10]Since the matrix elements in question are even functions of the external four-momenta, the error will actually be of order (momentum)^4.

[11]S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

[12]We use the notation and metric conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), pp. 377–390.

[13]For a discussion of experimental evidence on the sign of F^{π}, see F. J. Gilman, Phys. Rev. 184, 1964 (1969), and S. L. Adler, in *Proceedings of the Third International Conference on High Energy Physics and Nuclear Structure, New York, 1969,* edited by S. Devons (Plenum, New York, 1970), p. 647.

[14]In a renormalizable fermion-triplet model which satisfies PCAC, the anomaly predictions for F^{π} and $F^{3\pi}$ individually are $F^{\pi}\approx-(\alpha/\pi)f^{-1}2\overline{Q}$ and $F^{3\pi}\approx-(e/4\pi^2)\times f^{-3}2\overline{Q}$. The quantity $\overline{Q}$, which is the average charge of the nonstrange triplet particles, drops out in the ratio. See S. L. Adler, Ref. 7; S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969); and R. Aviv and A. Zee (unpublished).

[15]In the large square-bracketed terms in Eq. (5b), we have specialized to the case in which the charged pions are on the mass shell: $q_+{}^2=q_-{}^2=M_{\pi}{}^2$.

[16]However, one cannot conclude that $\gamma+\gamma\to\pi^0+\pi^0+\pi^0$ is suppressed relative to $\gamma+\gamma\to\pi^0+\pi^++\pi^-$. In fact, for all values of the parameter x the *threshold* value of $\mathfrak{M}^{000}$ is *three times larger* than that of $\mathfrak{M}^{0+-}$, as required by the $\Delta I=1$ rule.

[17]This generalizes the result of E. S. Abers and S. Fels, Phys. Rev. Letters 26, 1512 (1971).

[18]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960). The σ meson in this model is pure isoscalar, and so the parameter x vanishes. A similar calculation in the σ model has been done independently by T. F. Wong (unpublished).

[19]We emphasize that this consistency check means that Eq. (4) is a model-independent result, since it is required by Eq. (5), together with the fact that the *only* two-derivative 2γ-3π couplings consistent with the $\Delta I=1$ rule are given by Eqs. (6) and (7). For a closely related discussion, see J. Wess and B. Zumino, Phys. Letters (to be published).

[20]After this work was completed, we learned that similar results have been obtained independently by M. V. Terentiev. See M. V. Terentiev, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 14, 140 (1971).

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Multiperipheral Model for Pomeranchukon Couplings

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An intimate connection between the Pomeranchukon and the f and f' Regge trajectories is derived in the context of a specific multiperipheral model, embodying duality. The explicit dependence of the Pomeranchukon couplings on the f (and f') Regge poles clarifies certain features of diffraction scattering data.

Recently[1] we have proposed that the couplings of the Pomeranchukon are proportional to those of the f and f' Regge trajectories, and have verified some consequences of this proposal for total cross-section data and s-channel helicity conservation. We believe that the general form of the diffraction amplitude in which the Pomeranchukon couples through the f and f' is common to many models regardless of the models' detailed J-plane structure. Thus, although experimental data on multiparticle production does not conclusively favor any particular production mechanism, the conclusions of Ref. 1 extend beyond the details of the model used to derive them.[2]

The model of Ref. 1 described the totality of inelastic states as a pair of fireballs with single-Pomeranchukon exchange between them. In this note we discuss an alternative to the two-fireball picture, based on multiperipheral bootstrap ideas. Unlike previous treatments,[3] we include the f' as well as the f trajectory and make use of duality in summing the intermediate states in the unitarity equation. This provides an explicit dependence of Pomeranchukon couplings on the residue functions of the f and f' trajectories.

We illustrate our approximation to a production amplitude in Fig. 1(a). All stable particles in the final state are assumed to come from the decay of resonances which are produced by multiperipheral Pomeranchukon-plus-Reggeon exchange (at this stage we do not need to specify the nature of the exchanged Pomeranchukon). Unitarity is shown in Fig. 1(b). The sum over inelastic states is reduced to a sum over excited resonance intermediate states with integrals over the mass of each resonance $(s_i)^{1/2}$. Using duality[4] the sum over resonances in each leg is replaced by the imaginary part of the leading non-Pomeranchuk Regge pole with vacuum quantum numbers[5] (i.e., the f and/or f' trajectory). Throughout this paper we shall assume that we can neglect the contributions of lower-lying Regge trajectories to the Pomeranchukon singularity. We also restrict ourselves to values of t very close to $t=0$. The resulting expression for the amplitudes in the J plane is illustrated by Fig. 2. Each bubble, $B(J, t)$, in the chain corresponds to double Pomeranchukon-plus-Reggeon exchange, and is assumed to be an SU(3) singlet. (In principle, this should be derived from consistency requirements, but we shall assume it here.) The links correspond to f and f' propagators $[R(J, t)$ and $R'(J, t)]$ which are taken to be that ideal mixture of SU(3) singlet and octet which decouples the f' from pions and nucleons. Thus, if we have exact SU(3) couplings, each internal link has the form $R(J, t) + \frac{1}{2}R'(J, t)$ and the ratio of f to f' in the first and last links is determined by the quantum numbers of the external particles. The J-plane structure corresponding to Fig. 2 may be exhibited by applying the rules of Gribov and Migdal.[6] These rules were not originally derived with reference to summing over inelastic states in the unitarity equation (and care must be taken since some of the lines in Fig. 2 correspond to the imaginary parts of Reg-