Exchange-Current Radiation Model for Inclusive Spectra and Elastic Scattering*

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A semiclassical, geometrical model for high-energy hadron collisions is proposed, which incorporates multiparticle unitarity. The model leads to asymptotically constant elastic and total cross sections (fixed Pomeranchuk pole at $J = 1$), together with logarithmically increasing average multiplicities. A specific phenomenological form of the hadronic current distribution, first used by Heckman, is found to give very good agreement with single-particle inclusive spectra with no adjustable parameters. Correlation functions are not present in this model.

I. INTRODUCTION

In high-energy hadron collisions, the properties of elastic scattering and multiple-hadron production are connected by unitarity. Models for elastic scattering have been suggested in which the Pomeranchuk singularity[1] is taken as a simple moving pole, a fixed pole, or a cut in the angular momentum variable appropriate to the crossed t-channel reactions.[2] These various assumptions lead to characteristic energy dependences of total cross sections (σ_T) and integrated elastic cross sections (σ_E). At the same time, various models for multiple production have been proposed which lead to characteristic energy dependences of average production multiplicity and secondary-particle-momentum spectra. These properties are contained in single-particle inclusive cross sections[3] $\omega(k)d^3\sigma/d^3k = F(k, s)$. The inelastic-unitarity contributions usually determine the energy dependence of σ_T and σ_E.

Examples of models including unitarity previously discussed extensively in the literature are (1) multiperipheral models,[4] (2) fixed-pole Pomeranchukon schemes,[5] and (3) bremsstrahlung or semiclassical models.[6] Other types of scattering or production models, such as thermodynamic (statistical), pure geometrical, or dual-resonance types, either do not include unitarity or have internal-consistency problems, at their present stage of development.

In the three classes of models considered, one finds no example which allows a strictly geometrical behavior for elastic and total cross sections; i.e., both σ_T and σ_E are constant at asymptotic energies, while accommodating an asymptotically growing average multiplicity such as $\bar{n} \sim \ln s$ which is strongly indicated by cosmic-ray data.[7]

Multiperipheral models which include Pomeranchukon-induced contributions in production reactions, as is necessary in principle if the Pomeranchukon (P) is a factorizable pole, can be of two types: (1A) The intercept $\alpha_P(0)$ of the P trajectory $\alpha_P(t)$ is exactly unity, leading to constant σ_T; or (1B) the intercept $\alpha_P(0)$ is less than unity, leading to the asymptotic vanishing of σ_T. In the former case, investigated extensively by Gribov, Migdal, and co-workers,[8] consistency with unitarity is arranged through the vanishing of the 3-Pomeranchukon vertex when momentum transfers are zero. Because P is a moving singularity (a combination of poles and cuts) σ_E asymptotically decreases to zero as an inverse logarithm of the squared energy variable (s). In the latter case, as discussed by Chew, Goldberger, and Abarbanel,[9] both σ_E and σ_T asymptotically decrease with a small power of s, and the 3-P vertex is not zero. These models predict a logarithmic growth of average multiplicity, as well as many other features of the inclusive spectra which are consistent with production data. A very slow decrease of σ_T or σ_E may not be measurable at available energies, and these models have provided valuable guides for interpretation of a wide range of experimental data; but if one desires a model with strictly constant cross sections, and with no shrinkage effect in elastic scattering, these models must ultimately be abandoned.

The self-generation of a fixed pole P has been suggested as a viable alternative by several authors.[5,10,11] It has been demonstrated that models based on s-channel unitarity, in which the P induces "diffraction-dissociation" particle production, can yield a satisfactory theory of asymptotic elastic scattering. However, the production multiplicity in these models appears asymptotically to approach a constant. Although not conclusively excluded by present data, this possibility is not suggested by the available information from all sources, and would be a surprising phenomenon if true.

Models for production based on bremsstrahlung (semiclassical) pictures,[12-14] although intuitively appealing, have never been formulated in such a way to consistently accommodate unitarity, except for one case which (as in the fixed-pole schemes) includes a nonincreasing average multiplicity.[15] Thus, one cannot see the constraints between production and scattering, which must be imposed implicitly in a manner not specified by published work in this field.

Feynman has suggested[16] that many of the characteristic features of high-energy hadron collisions can be understood in terms of a semiclassical current-radiation picture similar to that used in bremsstrahlung models, but he has not made specific proposals for the distribution of hadronic currents in such processes. Many of his suggestions are incorporated in the model to be discussed below.

We will propose a specific model for high-energy hadron collisions which has the following properties:

(1) The Pomeranchukon is a fixed pole with $\alpha(0)=1$; thus, σ_T and σ_E are asymptotically constant.

(2) Inclusive particle spectra have the scaling properties[17, 18] suggested by multiperipheral models and "limiting fragmentation."[19]

(3) Average production multiplicities grow logarithmically with s.

(4) The most important regions in production spectra can be calculated from a knowledge of elastic scattering amplitudes.

As incidental features of the model, we indicate how some of the fixed-pole-P "bootstrap" theories can be consistent with such a model, and argue that a fixed-pole P should not factorize.

This model has been developed by including important parts of several other works, primarily those of Feynman,[16] Heckman,[13] Gundzik,[15] and Steinhoff and the present author,[10] together with clues from the extensive literature on multiperipheral Pomeranchukon theories beginning with Chew

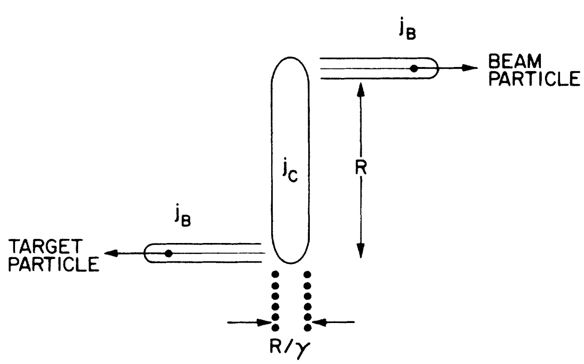

FIG. 1. Sketch of spatial distributions of exchange currents in collisions.

and Pignotti.[20] For simplicity, we will (1) consider only proton-proton collisions; (2) ignore spin, parity, isospin, and other quantum numbers, and similar other details; (3) assume the inelastic processes which dominate the aspects of the problem we are concerned with can be considered as production (emission) of secondary spinless particles ("pions," which might better be considered s-wave $\pi\pi$ pairs); and, consequently, (4) neglect any resonance or baryon pair production compared to this dominant process. Thus, we can theoretically identify a "leading particle" in almost every collision and separate it in principle from produced pions.

II. FRAMEWORK OF MODEL

In a high-energy proton-proton (pp) collision, we will consider the basic interaction process as an *exchange*, generating a *hadronic exchange-current distribution* in the collision. Such a current, coupled to a pion field, will radiate secondary pions. Other types of quanta (e.g., K-meson pairs) can also be radiated, with a different coupling; but we assume pion emission dominates, as suggested by experimental data. The spatial distribution of the current will be one of the determining factors in the S-matrix element connecting the initial pp state with any final state containing pp plus secondary pions.

Following Heckman's work,[13] we formulate this picture with a factorized ansatz for matrix elements determining production of n secondary pions in pp collisions:

$$A_n(p_1'p_2';k_1\cdots k_n;p_1p_2) = g_\pi{}^n J(k_1)\cdots J(k_n)A_0(p_1'p_2';p_1p_2)\,. \tag{1}$$

Here, p_1, p_2 are the incoming protons' 4-momenta; p_1', p_2' are the outgoing protons' momenta; k_i is the momentum of the ith outgoing pion; g_π is the coupling constant of the pion field to the exchange current; $J(k)$ is the Fourier transform of the exchange-current distribution $j(x)$; and A_0 is a "bare" exchange amplitude. In the last identification, we depart from Heckman who assumed A_0 was essentially an elastic scattering amplitude.

Such matrix elements correspond to classical radiation, and the total pion field is in a "coherent state" after the collision, as discussed by Gundzik[15] (who, however, also assumed A_0 to be quasielastic). Although not explicitly indicated above, the current distribution J depends on the parameters of the exchange collision (the arguments of A_0), which may be taken to be $s=(p_1+p_2)^2$, and $\vec{p}_\perp$

and p_z, components of the 3-momentum of one of the outgoing protons in the center-of-mass frame. Since in our model we will immediately assume J is approximately independent of p_z, we write J as an explicit function of $\vec{k}$, with s and $p_\perp$ as parametric variables, $J(\vec{k}; s, p_\perp)$.

For an explicit model of the current distribution, we adopt Heckman's *ad hoc* assumption for the $p_\perp$-independent part, J_C, of J which will dominate secondary-particle production. However, we also introduce another contribution, J_B, which will describe emission processes concentrated along the trajectories of the protons. We assume J_B is small, depends on $p_\perp$, and describes "break-up," excitation of the protons, or "shaking-off" of pions from the trajectories of the "leading" protons. By contrast, J_C will generate pions which are peaked in momentum around the center-of-mass (c.m.) momentum. Specifically, $J_C(\vec{k}; s)$ is to be proportional to the Fourier transform of

$$j_C(\vec{b}, z) = \gamma^{-1}\frac{\partial}{\partial z}|\varphi(\vec{b}, z)|^2, \tag{2}$$

where φ is the eikonal pp wave function; it is obtained from an optical potential $V(\vec{b}, z; s)$ which is assumed Lorentz-contracted in the c.m. system, and whose b dependence is determined by the pp elastic scattering amplitude.[13] Here, γ is the Lorentz-contraction factor $(\sqrt{s}/2M_p)$. Explicitly,

$$j_C(b, z) = \operatorname{Im} V(\vec{b}, z)\exp\left[-2\operatorname{Im}\int_{-\infty}^{z} V(\vec{b}, z')dz'\right]. \tag{3}$$

Motivation for (2) is rather *ad hoc*; one might interpret it as a proportionality of current strength to the gradient of a hadronic density along the direction of motion. We will return later to the phenomenological applications of this expression. We find at this point, however, that J_C satisfies a *scaling relation:* It depends on s only through the ratio $2k_z/\sqrt{s} = x$, where $\vec{k}_\perp$ and k_z are defined in the c.m. frame,

$$J_C(\vec{k}_\perp, k_z; s) = J_C(\vec{k}_\perp, x). \tag{4}$$

This is a consequence of the Lorentz-contraction effect assumed in the center-of-mass system. Qualitatively, the assumption (3) leads to a strong damping in $\vec{k}_\perp^2$ and a Gaussian behavior in x. The total current J is now $J_C + J_B$.

As concerns J_B, we will assume (1) J_B is strongly damped in $\vec{k}_\perp$, (2) J_B is negligible except when $|k_z|$ is within a finite interval of $(k_z)_{\max} = \sqrt{s}$, and (3) integrals of J_B^2 can be neglected compared to integrals involving the product $J_C J_B$ or J_C^2. In Fig. 1 we qualitatively indicate the proposed spatial current distributions, j_C and j_B.

From the ansatz (1), we can compute, in terms of J and A_0, all production cross sections multiplicity, and all inelastic unitarity contributions; hence, we can determine pp elastic scattering. We will make assumptions necessary to obtain constant σ_T and σ_E, yet growing multiplicity. The remaining essential ingredient is the energy dependence of A_0 Guided partially by Feynman's arguments,[16] for the moment we assume specifically that for $p_z/p_{z\,\max} = x_p$ held at an s-independent constant

$$A_0(s, \vec{p}_\perp, p_z) \underset{s\to\infty}{\sim} a_0(\vec{p}_\perp^{\,2}) s^{\,1/2}, \tag{5}$$

independent of x_p in that configuration, as long as $1 - x_p$ is not very small.

III. GENERAL FEATURES OF PARTICLE PRODUCTION IN MODEL

Integrating $|A_n|^2$ over the phase space for n particles, and taking into account the properties of J and A_0, we obtain

$$\sigma_n \underset{s\to\infty}{\sim} \frac{[\bar{n}(s)]^n}{n!}\sigma_0(s), \tag{6}$$

where, keeping the terms dominant for $s\to\infty$,

$$\bar{n}(s) \cong g^2\int_{\Omega(s)}\frac{d^3k}{\omega(k)}|J_C(\vec{k}, s)|^2. \tag{7}$$

The denominator $n!$ arises[20] from combinatoric arguments; and $\Omega(s)$ is the available phase space for one emitted pion. Using the scaling property,[18] and strong damping in k, we can evaluate the asymptotic s dependence,

$$\int_\Omega \frac{d^3k}{\omega(k)}|J_C|^2 \to \int_{-\sqrt{s}}^{\sqrt{s}}\frac{dk_z}{k_z}\int_0^\infty d\vec{p}_\perp^{\,2}|J_C(\vec{p}_\perp^{\,2}, x)|^2 = h\ln s. \tag{8}$$

The essential property used here is the existence[16,18] of a nonzero limit for $|J_C|^2$ as $x\to 0$. From (6), we identify $\bar{n}$ as the *mean* pion multiplicity. We can now calculate the sum of (leading logarithmic terms in) σ_n and thus obtain the asymptotic form of the inelastic cross section $\sigma_I = \sigma_T - \sigma_E$,

$$\sigma_I = \sum_{n=0}^{\infty}\frac{(\bar{n})^n}{n!}\sigma_0 = e^{\bar{n}(s)}\sigma_0(s). \tag{9}$$

Since $\bar{n}(s)\to g_0^2\ln s$, where $g_0^2 = g^2 h$, we obtain

$$\sigma_I(s) \sim s^{g_0^2 - 1}. \tag{10}$$

This can be constant, as desired, only when $g_0^2 = 1$. This constraint fixes the normalization of the hadronic current, given a pion coupling constant. Such a condition gives a reasonably good prediction for charged multiplicity,[13,16,20] and enables the phenomenological current form to be used without adjustable constants. Our assumption (5) thus replaces the soft-pion normalization used by Heckman.[13]

We can now examine some detailed features of particle production in this model.

(1) Consider an ensemble of collisions with given $p_\perp$. We can compute the differential cross section for production of any number of particles by integrating $|A_n|^2$ over the mesons' phase space. If $|p_z - p_{z\,\max}|$ is large enough, i.e., x_p not close to unity, we can carry out these integrals over all phase space, sum over all n, and obtain

$$\frac{d\sigma_I}{dp_\perp^2} = \frac{d\sigma_0}{dp_\perp^2}\exp\left[\int \frac{d^3k}{\omega(k)}|J(\vec{k}; s, p_\perp)|^2\right]$$
$$\equiv \frac{d\sigma_0}{dp_\perp^2} e^{\bar{n}_1(s, p_\perp)}. \qquad (11)$$

If we compute the mean multiplicity of emitted mesons under these conditions, we obtain $\bar{n}_1(s, p_\perp)$; i.e., at fixed $p_\perp^2$, the multiplicity distribution is Poisson with a mean value $\bar{n}_1$. Such general features have been emphasized in semiclassical models by Kastrup and co-workers.[14]

Note that whatever form we assume for $J_B(p_\perp)$, we obtain $d\sigma_I/dp_\perp^2$ which is independent of p_z, and s, in this "deep-inelastic" region. The first seems to agree with the data,[21] although the s dependence for such a region has not been checked.

(2) Consider experiments which measure the outgoing flux of mesons with a given momentum $\vec{k}$ in the c.m. system, where $|\vec{k}|$ is small compared to $\sqrt{s}$. Then we compute the *inclusive* single-particle spectrum, by extracting one factor of $|J|^2$ from each $|A_n|^2$ and summing over all other mesons. The result[13] (ignoring J_B in this region) is

$$\omega(k)\frac{d^3\sigma}{d^3k} = |J_C(\vec{k}, s)|^2\sigma_I(s). \qquad (12)$$

The square of the current J_C thus is directly proportional to the inclusive single-particle spectrum in the regions not close to the meson's phase-space boundary. The choice (3) for J immediately provides a phenomenological theory for the inclusive spectrum, even at nonasymptotic energies. We will discuss the agreement with experimental data later. The expression (12) shows that discussions of scaling, possible dependence on quantum numbers of incoming channel,[22] and other properties of inclusive spectra can be transferred to J directly.

(3) Multiple-particle inclusive meson-production cross sections factorize completely in this model, if phase-space constraints are ignored. Thus, they trivially obey scaling laws, since the single-particle spectrum does. All correlation functions then vanish, except for phase-space boundary effects.

(4) Scaling for inclusive baryon spectra, and the effects from phase-space constraints, can be discussed by considering the limitation created on the c.m. energy of emitted mesons by specifying a small value of $|p_z - p_{z\,\max}|$. The maximum available c.m. energy for a pion is $k_{z\,\max} = \sqrt{s} - p_z$; in terms of the baryon's x this becomes

$$k_{z\,\max} = \sqrt{s}\,(1-x).$$

The squared missing mass M^2 is related to x by $1 - x = M^2/s$. Taking this value as the upper limit in integrals over $|J|^2$, we obtain for the inclusive cross section for a leading baryon, summing over all secondary mesons,

$$\omega(p)\frac{d^3\sigma}{dp_z d^2p_\perp} = \omega(p)\frac{d^3\sigma_0(s, p_\perp^2, p_z)}{dp_z dp_\perp^2} e^{\bar{n}_2(s,x) + 2f(p_\perp^2)}, \qquad (13)$$

where

$$\bar{n}_2(s, x) = \int_{-\sqrt{s}(1-x)}^{+\sqrt{s}(1-x)} \frac{d^3k}{\omega(k)}|J_C(\vec{k}; s)|^2, \qquad (14)$$

and, if we assume $J_C \gg J_B$,

$$f(p_\perp^2) = \int \frac{d^3k}{\omega(k)} J_C(\vec{k}; s) J_B(\vec{k}; s, p_\perp^2). \qquad (15)$$

In (15) we assume the currents are real for simplicity; we consistently ignore J_B^2 compared to $J_C J_B$; and we assume f becomes independent of s as $s \to \infty$. This is consistent with a scaling behavior for J_C, if J_B is strongly damped around $k_z = k_{z\,\max}$, independent of s. If we thus assume J_C is the dominant feature, we can evaluate $\bar{n}_2$ for asymptotically large s, but fixed x, using the scaling of J_C. We obtain

$$\bar{n}_2(s, x) \to g_0^2[\ln s + 2\ln(1-x)]$$

and finally, with $\omega(p) \cong p_{\text{lab}} \cong s$ and $g_0^2 = 1$,

$$\omega(p)\frac{d^3\sigma}{dp_z d^2p_\perp} = \frac{s^2 d^3\sigma_0(s, p_\perp^2, p_z)}{dp_z dp_\perp^2} e^{2f(p_\perp^2)}(1-x)^{2g_0^2}. \qquad (16)$$

This expression shows the scaling of the inclusive baryon spectrum, provided $s\sigma_0$ scales. Thus, we assume further that asymptotically

$$\frac{s^2 d^3\sigma_0}{dp_z dp_\perp^2} = F_0(x, p_\perp^2) \quad \text{independent of } s, \qquad (17)$$

where F_0 is presumed to vary slowly with x if x is not close to unity. If this is forced to be consistent with the usual x behavior at the phase-space boundary $(1-x)^{1-2\alpha_1(p_\perp^2)}$ as discussed by Peccei and Pignotti,[23] it is necessary to assume that in the limit $x \to 1$,

$$F_0(x, p_\perp^2) \cong F_1(p_\perp^2)(1-x)^{1-2\alpha_1(p_\perp^2)-2g_0^2}. \qquad (18)$$

This implies a form of the cross section for production of a few particles of small momentum in the center-of-mass system in the limit of small inelasticity (x close to 1).

$$\omega(p)\frac{d^3\sigma_n}{dp_z dp_\perp{}^2}\cong\frac{[\bar{n}_2(s,x)]^n}{n!}(1-x)^{-1-2\alpha_1(p_\perp{}^2)}F_2(p_\perp{}^2)s^{-1}, \tag{19}$$

where $F_2 = F_1 e^{2f(p_\perp{}^2)}$.

Although $\bar{n}_2$ will not necessarily be asymptotic in $\ln(s)$ in experimentally accessible regimes, such a form may be used to compare multiplicity distributions at various values of $p_\perp{}^2$ for fixed s and x. One sees the Poisson distribution with mean $\bar{n}_2$, which is independent of $p_\perp{}^2$, but dependent on x.

At fixed M^2, as $s\to\infty$, the form (19) exhibits a moving-Regge-pole structure in the s and $p_\perp{}^2$ dependence. Thus, it is consistent with a physical charge-exchange-amplitude interpretation with suitable α_1. Only when x is near the phase-space boundary is this relevant; for smaller x the Regge-pole behavior is replaced by a fixed pole, relevant to high-multiplicity production at $l=\frac{1}{2}$.

IV. MULTIPARTICLE UNITARITY AND HIGH-ENERGY ELASTIC SCATTERING

To explore the consequences of the model for σ_E, we consider nonforward elastic scattering and saturate the unitarity condition with the pion production amplitudes A_n of our model, together with the elastic-unitarity contribution from the elastic amplitude A_E. We obtain

$$\operatorname{Im}A_E(s,t)=\iint\frac{dt'dt''}{\Delta^{1/2}(t,t',t'')}A_E^*(s,t')A_E(s,t'')$$
$$+\sum_{n=0}^{\infty}\frac{g^{2n}}{n!}\iint\frac{dt'dt''}{\Delta^{1/2}(t,t',t'')}A_0^*(s,t')A_0(s,t'')\prod_{i=1}^{n}\int\frac{d^3k}{\omega(k)}J^*(\vec{k};s,t')J(\vec{k};s,t''), \tag{20}$$

where Δ is the standard triangle function resulting from a change of integration variable from $\cos\theta$ to squared momentum transfers t, in the high-s limit; the integrals in t', t'' are carried out over regions where $\Delta>0$. [We have written here t instead of $p_\perp{}^2$; it does not refer to $(p_1-p_1')^2$.]

The sum over n can be performed using coherent-state formalism[15]; neglecting phase-space boundary effects, and dropping terms of order $s^{-1/2}$, we obtain

$$\operatorname{Im}A_E(s,t)=(\operatorname{Im}A_E)_0+\iint\frac{dt'dt''}{\Delta^{1/2}}A_0^*A_0G(s,t',t''), \tag{21}$$

where

$$G(s,t',t'')=\exp\left[\frac{g^2}{2}\int\frac{d^3k}{\omega(k)}|J^*(\vec{k};s,t')+J(\vec{k};s,t'')|^2\right], \tag{22}$$

and $(\operatorname{Im}A_E)_0$ is the elastic-unitarity contribution, the first term on the right in expression (20). For simplicity, we will assume J is real henceforth. Following previous assumptions on J, we ignore $J_B{}^2$ and carry out integrals on $J_C{}^2$ and J_CJ_B, obtaining

$$G(s,t',t'')\cong e^{\bar{n}(s)}e^{f(t')+f(t'')},$$

where f is given by the integral (17). Taking the limit $s\to\infty$, using assumption (5), we obtain

$$\operatorname{Im}A_E(s,t)=(\operatorname{Im}A_E)_0+c\iint\frac{dt'dt''a_0^*(t')a_0(t'')}{\Delta^{1/2}(t,t',t'')}h(t')h(t''), \tag{23}$$

where $h(t)=e^{f(t)}$ and c is a constant. Now we see the assumption of a fixed pole for A_E, i.e., $A_E(s,t)\to ia_E(t)$, is compatible with our model for particle production, since both sides of Eq. (23) are independent of s in that case. This conclusion would not be affected if we included $J_B{}^2$ terms, although the kernel would be more complicated, but it depends on f (or more complicated t dependences) being independent of s asymptotically. This holds trivially if J_B vanishes as $s\to\infty$; but that is not necessarily a desirable assumption.

If in fact a nonvanishing $f(t)$ is proposed which satisfies our criteria, we can go a bit further and suggest a connection between the expression (23) and some self-consistent theories of the Pomeranchukon which yield fixed-pole solutions.[10,11] A possible hypothesis is the identification of the t dependence of a_0 with the t dependence of a_E. Such an hypothesis would incorporate the "droplet model" point of view[24] which regards the t distribution in two-body exchange processes as given by geometrical (energy-independent) considerations. In our case, it immediately yields a quadratic integral equation for $a_E(t)$; ignoring the elastic-unitarity term for simplicity, we obtain

$$a_E(t) = c \iint \frac{dt'dt''}{\Delta^{1/2}} [a_E(t')h(t')][a_E(t'')h(t'')]. \quad (24)$$

This can be transformed into the form proposed, e.g., by Steinhoff and the present author,[10] for the residue function of the Pomeranchukon, by writing

$$\beta(t) = a_E(t)h(t)$$

and $\lambda U^2(t) = ce^{-f(t)}$; then we obtain

$$\beta(t) = U^2(t) \iint \frac{dt'dt''}{\Delta^{1/2}} \beta(t')\beta(t''),$$

which is Eq. (17) of Ref. 10. The relationship of the physical assumptions involved in the two approaches remains to be clarified. However, it is tempting to speculate on the possible calculation of $U(t)$ through pion-exchange diagrams as in Ref. 10. Through solution of the quadratic integral equation above, one could then obtain a_E and a_0. Since the high-energy limit of J_G is determined by $a_E(t)$, this would determine essentially all observable cross sections, providing a complete theory for asymptotically high energies, except for production near phase-space boundaries.

V. AN ALTERNATIVE HYPOTHESIS FOR A_0 AND $\bar{n}(s)$

We have described the simplest possible set of assumptions providing a combined theory of multiple production and elastic scattering in which σ_E and σ_T are asymptotically constant while accommodating a logarithmic growth of multiplicity. However, it is possible to consider a slightly different set of hypotheses for A_0 and J, which have an advantage of increased theoretical unity, at the expense of tolerating less specific properties for J.

We had introduced the fixed pole at $l = \frac{1}{2}$ in A_0 in a completely *ad hoc* way. There is, however, a natural theoretical model which leads to a fixed singularity at $l = \frac{1}{2}$ in exchange amplitudes: the hybrid model.[25] In this model, one begins with a moving Regge pole for exchange amplitudes; but after iterations of a fixed Pomeranchuk singularity with this exchange pole, one obtains a fixed cut at $l = \frac{1}{2}$. In fact, a strong result appears: The $p_\perp{}^2$ dependence of the exchange amplitude, as a consequence of these iterations, is identical to the $p_\perp{}^2$ dependence of the Pomeranchukon terms $s \to \infty$, except for a tiny neighborhood of $p_\perp{}^2 = 0$.

We have seen that such a situation enables us to construct a bootstrap theory of the $p_\perp{}^2$ dependence of A_0. The replacement of our fixed-pole assumption for A_0 by the hybrid-model fixed cut, however, requires a slight modification in our assumptions concerning J.

We now assume $\sigma_0(s) \sim s^{-1}(\ln s)^{-2}$; this requires, if we want $\sigma_E \sim$ const, that $\bar{n}(s) \sim \text{const} + \ln s + 2\ln(\ln s)$ as $s \to \infty$. We obtained the logarithmic (second) term from scaling behavior of J_C. The $\ln(\ln s)$ term must be obtained from contributions which are less important asymptotically than the $x = 0$ region of J_C; possibly it could come from J_B. We have not pursued this approach sufficiently as of this writing to propose a specific form in J which would generate the $\ln(\ln s)$ term. Experimentally, this term in $\bar{n}$ would probably not be distinguished from the constant term.

Such a set of assumptions provides a theoretical framework which is highly unified, with intrinsic bootstrap conditions determining A_E in terms of J_B and the usual Regge-pole exchanges.

VI. COMPARISON OF PHENOMENOLOGICAL MODEL WITH DATA

We take assumption (3) to determine inclusive pion spectra in pp collisions, and also K^+p collisions, since in such cases a leading particle can be identified. These are expected to be "asymptotic" at a reasonably low energy, in our model, since the imaginary parts of the appropriate optical potentials, which are related to total cross sections, appear energy-independent above 3 GeV. To make rough comparisons with data we first consider, at small $k_\perp{}^2$, only the lowest-order terms in ImV (ignoring "multiple-scattering" terms) and assume a Gaussian form for ImV, as did Heckman,[13] who kept higher-order terms. His fits to $pp \to \pi^+$ spectra are qualitatively very good, except at large c.m. angles, and we do not consider further here the data he discussed. Recently, there have been published several K^+p and π^+p inclusive spectra from bubble chambers, and a few points from CERN Intersecting Storage Ring, which confirm scaling in pp collisions. We, therefore, will compare only with the K^+p and π^+p data, in detail.

A Gaussian form for ImV, after normalization of the current is specified, contains only one parameter R^2, determined by elastic scattering data; for K^+p, R^2 (at 10–15 GeV) is of order 8 GeV^{-2}; for pp, R^2 is about 12 GeV^{-2} at high energy.

We obtain from (3), normalizing to unity at $k_\perp{}^2 = 0$, and dropping higher powers of V,

$$|J(k_\perp{}^2, x)|^2 \cong \exp(-R^2k_\perp{}^2)\exp(-R^2M^2x^2), \quad (25)$$

where M is the proton mass. (We will retain the proton mass here when examining K^+ and π^+ data, even though one might guess a geometric mean should be used; this point requires further investigation, since a geometric mean would yield much poorer agreement with experiment.) We will concentrate on a comparison of x distributions, for x^2 not close to unity.

The comparison of this model with experimental pion-production data should be carried out in a manner chosen to maximize independence of the precise assumptions on the nature of the primary emitted unit, e.g., ϵ meson, which decays into observed pions. The decay process will strongly affect the $k_\perp^2$ dependence at $k_\perp^2 \lesssim 1$ GeV^2, but should not change the qualitative behavior for large $k_\perp^2$. On the other hand, the limiting x distribution will not be affected by a decay process (with finite Q value) as an intermediate step; hence, our emphasis on x distributions, in what follows.

To begin such a comparison, it is useful to recall the analysis of Bali, Brown, Peccei, and Pignotti[18] on scaling of single-pion spectra in a compilation of data on pp collisions. They found x dependence was fitted very well by Gaussian forms, with the coefficient of x^2 for π^- production of the order 10–11 GeV^{-2}; and the $k_\perp$ dependence was independent of energy, dropping sharply. They used a linear exponential in $k_\perp$, but the data can also be fitted by e^{-15k^2} for small $k_\perp^2$ as was done in some of the original experimental work[26] at 12.2 GeV/c. Thus, qualitative properties in pp pion spectra are accounted for in our model at small $k_\perp^2$. We will discuss later the behavior at large $k_\perp^2$, which involves consideration of the nonlinear ("multiple-scattering") terms in the expression (3).

Continuing with other data, we compare in Fig. 2 the k_z distribution results of a 12.7-GeV/c K^+p experiment reported by the Rochester group.[27] In Fig. 3, 7-GeV/c π^+p data are shown, with the same Gaussian function in x for comparison. Agreement is good in both cases. We have compared the same x distribution function with data published by Ko and Lander[28] from a K^+p exposure at 11.8 GeV/c. Again, for small $k_\perp^2$, agreement is reasonably good, confirming scaling, except for a few points. Other data[29] on x distributions in $pp \to \pi^-$ +anything at 28.5 GeV/c, and $\pi^\pm p$ interactions[30] at 18.5 GeV/c, confirm a universal Gaussian shape and normalization proportional to σ_T (which is noticeably different in pp and K^+p collisions).

Going beyond the lowest-order terms in ImV, we can ask for the large-$k_\perp$ behavior of J. An examination of the function (3), with Gaussian V, will show a series of alternating terms with successively shorter range in b^2, similar to multiple-scattering series, e.g., as in the hybrid model.[25] The large-$k_\perp$ behavior will, therefore, be qualitatively similar to that found in elastic scattering, which is predicted asymptotically to be $\sim\exp(-ak_\perp)$ (up to more slowly varying terms in $k_\perp$) for sufficiently large $k_\perp$; a should have the same value as in elastic scattering. The x dependence will be rather weak at large $k_\perp$.

The factorization of approximation (25) is not maintained when higher-order terms are kept. One finds qualitatively that the mean value of $k_\perp$

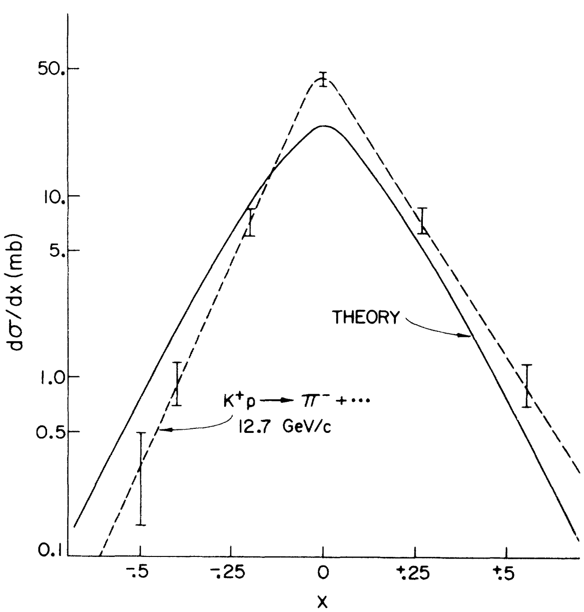

FIG. 2. Comparison of predicted inclusive x distributions with data of Ref. 27 on K^+p collisions at 12 GeV/c.

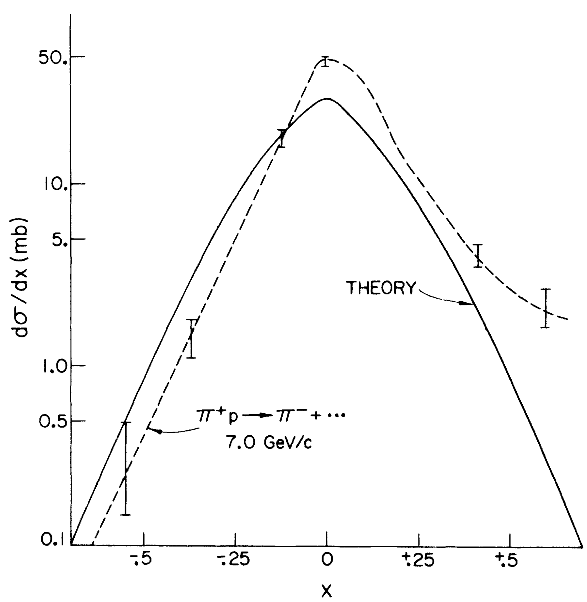

FIG. 3. Comparison of predicted inclusive spectra with π^+p data at 7 GeV/c from Ref. 27.

varies with x, but not a great amount, becoming slightly smaller as $x \to 0$.

Without a detailed model for energy dependence and phases of V, we cannot predict what J should look like in much greater detail. However, at the special point $x=0$ we can do better; one may perform an integration by parts in the Fourier transform of expression (3) and relate $J_C(\vec{k}_\perp^2, 0)$ directly to the elastic scattering amplitude, and, thus, establish a more direct connection with the $k_\perp$ distribution in a region ($x \to 0$) of large cross section.

Specifically, one finds

$$J(\vec{k}_\perp^2, 0) = \int_0^\infty b db\, J_0(bk_\perp)\{1-\exp[-2\,\mathrm{Im}\, i\,\chi(s, b^2)]\}. \tag{26}$$

Here $\chi(s, b^2)$ is the eikonal (phase-shift) function, related to the elastic scattering amplitude by

$$A_E(s,t) = i\int_0^\infty b db\, J_0(b\sqrt{-t})\{1-\exp[i\chi(s,b^2)]\}. \tag{27}$$

This relation may be useful to estimate the location of possible zeros in J, which would appear as "diffraction" minima in the $k_\perp$ distribution (at $x=0$). For example, in $\bar{p}p$ interactions, if one takes a black-disk model for scattering at energies of a few GeV, one expects a minimum in the inclusive pion spectrum around $k_\perp^2 \leq 0.6$ GeV^2 since $\bar{p}p$ elastic scattering has such a minimum. At asymptotic energies, if the fixed-pole model is correct for pp scattering, then accompanying any diffraction zeros in pp elastic scattering, one should find associated zeros in $k_\perp^2$ dependence of the inclusive spectrum of primary emitted units (but not necessarily the pion spectrum).

VII. CONCLUDING THEORETICAL DISCUSSION

We have described a model for high-energy hadron collisions which provides that σ_T and σ_E approach nonzero constants at high energy, while $\bar{n}$ grows logarithmically. The model incorporates multiparticle unitarity. It is convenient for phenomenological studies of inclusive single-particle spectra since it gives explicit predictions in terms of elastic-scattering information. It does not give correlation functions in multiparticle inclusive reactions.

The model is based on geometrical, semiclassical ideas. It assumes a certain nonmeasurable amplitude ("bare" exchange amplitude) A_0 to have a fixed pole at $l=\frac{1}{2}$ for large enough inelasticity. From this, it generates a physical elastic scattering amplitude with a fixed pole at $l=1$. Regge (moving) poles are not an essential part of this model, but can be introduced consistently, together with hybrid-model concepts, for a unified theoretical framework.

If we take the present model seriously in these respects, it suggests that in reactions such as $pp \to pp\pi^+\pi^-$ where the pion pair is slow ($x_\pi \approx 0$) in the center-of-mass system, but x_p is not close to unity, the cross section should drop off with energy (as in multiperipheral models with ρ, ω exchange); but the $p_\perp^2$ distribution of the leading proton should not show a shrinkage effect, if x_p is fixed, since the production is presumably associated with a fixed "exchange current" pole rather than a moving pole.

Since we can obtain elastic scattering with a constant cross section, it should also be possible to obtain, through unitarity sums, a constant cross section for some special inelastic processes which do not involve quantum-number exchange, historically called "diffraction dissociation." These would include $pp \to (p\pi)+p$ and $pp \to (p\pi)+(p\pi)$ in our model, where the invariant (mass)^2 s' of each $(p\pi)$ system is confined to a vanishing fraction of s.

The dependence of such cross sections on s', as s' increases to a large value (but remains small compared to s), has been discussed in terms of a triple-Pomeranchukon-pole vertex in models with a moving Pomeranchukon pole.[9] In our model, large-s' behavior is determined by J_B. We assumed, generally, in our development that J_B was peaked strongly around the momenta of the leading particles. In terms of s', this means there is no constant term as $s' \to \infty$; i.e., no Pomeranchukon contribution to J_B itself. This means the effective triple-Pomeranchukon vertex vanishes; diffraction-dissociation cross sections should drop off fairly rapidly as s' increases.

Finally, without restricting our attention to this specific model, one may consider the more general problem of reconciling constant σ_T and σ_E with a growing multiplicity. It is sufficient, for the latter, to obtain a six-point function for $3 \to 3$ scattering with zero momentum transfers whose value does not vanish[31] at $x=0$ as $s \to \infty$. It would appear that the slope of a Pomeranchukon trajectory is not constrained by this condition, so we could propose a fixed pole if desired. However, the question of factorization immediately arises. If P factorizes, we must include it in multiperipheral chains; diffraction-dissociation channels. This will lead to a conflict with s-channel unitarity as $s \to \infty$. Such a difficulty can be avoided, if the pole moves, by requiring the vertex functions to vanish at zero momentum transfers,[8] since an additional factor of $(\ln s)^{-1}$ will be associated with each internal pp vertex in that case; but if the slope vanishes, such compensating terms will not appear. This leads us to the conclusion that *a fixed-pole Pomeranchukon amplitude must not factorize.*

Such nonfactorization of scattering amplitudes appears naturally in the hybrid model,[29] where hadron-hadron scattering amplitudes are obtained by iterating a "bare" fixed Pomeranchukon pole (optical potential); such a potential term may factorize, but not the amplitude's fixed pole, whose residue contains an infinite series in the bare residue.

Of course, approximate residue factorization may be manifest in some limited range of kinematic variables, such as small $k_\perp^2$. Thus, rough checks of experimental data[32] will not generate conflicts with our point of view.

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