Optical Model for Diffraction Dissociation*

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The "Deck effect" is generalized to include multiple scattering, which should be important in diffractive dissociation on nuclear targets. A simple and intuitively appealing optical (eikonal) representation is given. On the basis of this representation, it is conjectured that form-factor effects should be included as a function of the distance off the energy shell in old-fashioned perturbation theory, at "infinite" momentum, rather than as a function of the distance off the mass shell, as in the usual double-Regge models. Ways to test this conjecture are discussed, and a procedure is given for including absorptive effects in $2 \to 3$ processes.

I. INTRODUCTION

When two hadrons collide at high energy, many final states can be produced. The many possibilities for inelastic reactions result in absorption of the incident waves at impact parameters $\lesssim 1$ F, which gives rise to strong elastic "diffraction" scattering at small momentum transfer. The distortion of the incident waves caused by absorption also includes the mixing in of inelastic states to which the incident particles are coupled, giving rise to production of these states via "diffractive dissociation."[1] Diffractive dissociation can involve no change in the internal quantum numbers, except for spin and parity. Like elastic scattering, it is expected to be large and approximately energy-independent at high energy.

To illustrate these ideas from the point of view to be taken in this paper, consider a deuteron scattering from a nucleus.[2] The incident deuteron can be thought of as a superposition of neutron-proton plane waves, with amplitudes given by its wave function. This superposition is spherically symmetric in the deuteron rest frame, if one neglects the D-state admixture. Scattering from the target will remove a cylindrical piece from the superposition, so that the final state will have finite overlaps with deuteron states at nonzero momentum transfer (diffraction scattering), and with np continuum states (diffractive dissociation). Because the scattering destroys the spherical symmetry of the superposition, it is clear that some np states with nonzero angular momentum will be produced.

Diffractive dissociation produces low-mass enhancements in processes such as $\pi A \to \rho\pi A$, $KA \to K^*(890)\pi A$, $pA \to n\pi^+ A$, where the target particle A may be a proton, or a nucleus which remains in its ground state. Previous calculations of these processes have been based on the "Deck effect" diagram of Fig. 1(a).[3,4] It has become customary to employ a Reggeized form for the pion propagator, and to identify the elastic-scattering vertex with "Pomeranchukon" exchange, resulting in a double-Regge interpretation [Fig. 1(b)].[5,6] This procedure stands on shaky ground, since one is interested in small $\pi\rho$ invariant masses, rather than in the kinematic region in which the double-Regge model is most plausible. Furthermore, there are good reasons to believe that the Pomeranchukon, and perhaps the pion as well, are not ordinary Regge poles.

In the double-Regge model, only the π is allowed to scatter on the target. Aside from form-factor effects, the amplitude for the ρ to scatter instead

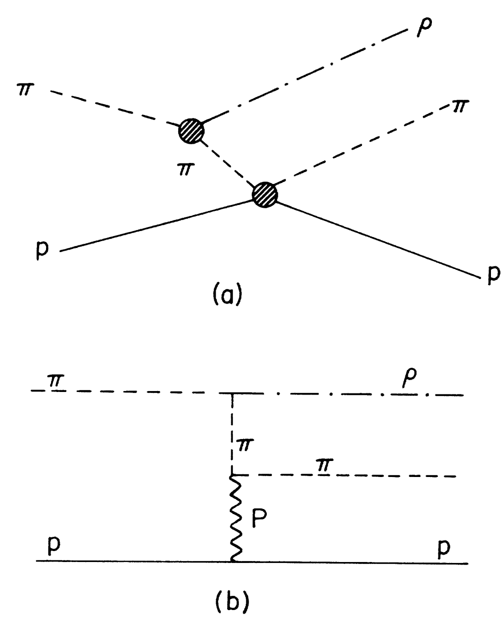

FIG. 1. (a) Deck-effect diagram for $\pi^\pm p \to \rho^0\pi^\pm p$; (b) The same diagram, with off-shell elastic scattering represented by Pomeranchuk exchange (wavy line).

of the π is just as large,[4] and with reasonable form factors, its contribution remains significant, particularly in the kinematic region where the π is forward in the $\pi\rho$ rest frame (ρ slow in lab). Moreover, if the target is a large nucleus, both π and ρ are likely to scatter on at least one nucleon. In this paper, I develop a procedure for including such effects. The procedure has a simple and intuitively appealing interpretation in the "infinite momentum" optical (eikonal) limit.

The eikonal picture (derived in Secs. II and III) leads one naturally to conjecture that off-shell effects be accounted for by a cutoff in the distance off the energy shell, in old-fashioned perturbation theory, rather than in the distance off the mass shell, as is done in the double-Regge model (Sec. IV). This conjecture is supported by a calculation involving a nonrelativistic system, and ways to test it experimentally are suggested.

The question of resonance effects in diffractively produced $\pi\rho$, πK^*, πN systems is an interesting one, but will not be addressed here. The possibility still appears open that Deck effects can by themselves explain the observed mass distributions, even in the case of πN,[6,7] where resonances with appropriate quantum numbers for diffractive production are known to exist from phase-shift analysis. It has been suggested that the Deck effects might perhaps be related to resonance production in some extended form of "duality."[8]

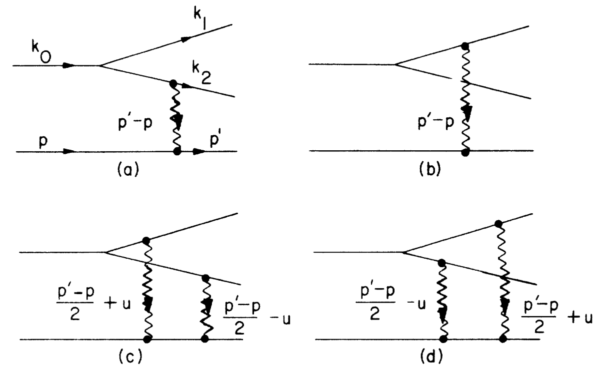

FIG. 2. Single- and double-scattering diagrams for diffractive dissociation. The wavy line represents elastic scattering.

II. SINGLE AND DOUBLE SCATTERING AT HIGH ENERGY

In this section, I derive the amplitude for a $2\to 3$ diffraction-dissociation process, corresponding to the single- and double-scattering diagrams shown in Fig. 2. Effects due to form factors, and nonzero spins, are ignored until Secs. III and IV.

The four-momenta, to leading order in the energy, in the over-all c.m. frame are given by

$$\begin{aligned}
k_0 &= (\vec{0},\, k,\, k+m_0^2/2k),\\
p &= (\vec{0},\, -k,\, k+M^2/2k),\\
k_1 &= (\tfrac{1}{2}\vec{\Delta}+\vec{q},\, xk',\, xk'+[(\tfrac{1}{2}\vec{\Delta}+\vec{q})^2+m_1^2]/2kx),\\
k_2 &= (\tfrac{1}{2}\vec{\Delta}-\vec{q},\, (1-x)k',\, (1-x)k'+[(\tfrac{1}{2}\vec{\Delta}-\vec{q})^2+m_2^2]/2k(1-x)),\\
p' &= (-\vec{\Delta},\, -k',\, k'+(\vec{\Delta}^2+M^2)/2k),
\end{aligned} \tag{1}$$

where the first component is the transverse momentum vector, the second is the longitudinal momentum, and the third is the energy. The longitudinal-momentum fraction x lies in $0<x<1$. Useful invariants are

$$\begin{aligned}
s &\equiv (k_0+p)^2 = 4k^2,\\
t &\equiv (p'-p)^2 = -\vec{\Delta}^2,\\
m^{*2} &\equiv (k_1+k_2)^2 = [(\tfrac{1}{2}\vec{\Delta}+\vec{q})^2+m_1^2]/x+[(\tfrac{1}{2}\vec{\Delta}-\vec{q})+m_2^2]/(1-x)-\Delta^2,\\
k'-k &\equiv (m_0^2-m^{*2}-2\vec{\Delta}^2)/4k.
\end{aligned} \tag{2}$$

In (1) and (2), and in what follows, terms which become unimportant as $s\to\infty$ have been dropped. The target particle has been assumed for simplicity to be relativistic in the c.m. frame, but this is not essential. One could also use the lab frame.

Denote the elastic scattering amplitude for particle i by $f_i(t)$, normalized so $\mathrm{Im} f_i(0)$ is the total cross section, which is assumed for simplicity to be energy-independent. In the high-energy limit, with no form factors or spin, and the coupling constant set equal to one, the amplitude corresponding to Fig. 2(a) is

$$M^{(a)}=\frac{-i(k_2+p')^2f_2(t)}{(k_0-k_1)^2-m_2{}^2}$$

$$=\frac{isf_2(t)}{[(\frac{1}{2}\vec{\Delta}+\vec{q})^2+m_1{}^2]/x+[(\frac{1}{2}\vec{\Delta}+\vec{q})^2+m_2{}^2]/(1-x)-m_0{}^2}. \tag{3}$$

Similarly,

$$M^{(b)}=\frac{isf_1(t)}{[(\frac{1}{2}\vec{\Delta}-\vec{q})^2+m_1{}^2]/x+[(\frac{1}{2}\vec{\Delta}-\vec{q})^2+m_2{}^2]/(1-x)-m_0{}^2}. \tag{4}$$

The normalization is such that

$$d\sigma=(512\pi^5 s)^{-1}\int\frac{d^3k_1d^3k_2d^3p'}{E_1E_2E'}\delta^{(4)}(k_0+p-k_1-k_2-p')|M|^2. \tag{5}$$

The propagators in Figs. 2(a) and 2(b) lead to energy denominators in (3) and (4) which are best interpreted in old-fashioned perturbation theory wherein all particles are on the mass shell $[E=(\vec{p}^2+m^2)^{1/2}]$, and momentum, but not energy, is conserved at vertices. For instance, in Fig. 2(a), the energy difference between intermediate and initial states is

$$\Delta E=\frac{(m_{\text{int}}{}^2-m_0{}^2)}{2k}, \tag{6}$$

where

$$m_{\text{int}}{}^2=\frac{(\frac{1}{2}\vec{\Delta}+\vec{q})+m_1{}^2}{x}+\frac{(\frac{1}{2}\vec{\Delta}+\vec{q})^2+m_2{}^2}{1-x} \tag{7}$$

is the invariant mass squared of the 1-2 system in the intermediate state, calculated with these particles on the mass shell.

The double-scattering terms involve integration over a loop momentum,

$$M^{(c+d)}=-s^2x(1-x)(2\pi)^{-4}\int d^4u f_1([\tfrac{1}{2}(p'-p)+u]^2)f_2([\tfrac{1}{2}(p'-p)-u]^2)$$
$$\times\frac{1}{[k_1+\frac{1}{2}(p'-p)+u]^2-m_1{}^2+i\epsilon}\frac{1}{[k_2+\frac{1}{2}(p'-p)-u]^2-m_2{}^2+i\epsilon}$$
$$\times\left(\frac{1}{[\frac{1}{2}(p'+p)+u]^2-M^2+i\epsilon}+\frac{1}{[\frac{1}{2}(p'+p)-u]^2-M^2+i\epsilon}\right). \tag{8}$$

Possible off-shell effects in the elastic amplitudes are neglected here. This is a self-consistent approximation, in that it will lead to the appearance of δ functions which limit the off-shell distance. Similarly it is assumed that the transverse momenta remain finite as $s\to\infty$. Analogous to previous work with the eikonal model,[9] let $u_\pm=u_0\pm u_z$. Then in the high-energy limit, the term in the large round parentheses in (8) becomes

$$\frac{1}{2k}\left(\frac{1}{u_++O(1/\sqrt{s})+i\epsilon}+\frac{1}{-u_++O(1/\sqrt{s})+i\epsilon}\right)=\frac{-2\pi i\delta(u_+)}{\sqrt{s}}. \tag{9}$$

The remaining two propagators can be combined to give

$$\frac{1}{sx(1-x)}\frac{1}{u_-+(m_0{}^2-m^{*2}-\vec{\Delta}^2)/2\sqrt{s}+[(\vec{q}+\frac{1}{2}\vec{\Delta})^2-(\vec{q}+\vec{u}_\perp)^2]/x\sqrt{s}+O(1/s)+i\epsilon}$$
$$\times\frac{1}{-u_-+(m_0{}^2-m^{*2}-\vec{\Delta}^2)/2\sqrt{s}+[(\vec{q}-\frac{1}{2}\vec{\Delta})^2-(\vec{q}+\vec{u}_\perp)^2]/(1-x)\sqrt{s}+O(1/s)+i\epsilon}$$
$$=\frac{2\pi i\delta(u_-)}{x(1-x)\sqrt{s}}\{[(\vec{q}+\vec{u}_\perp)^2+m_1{}^2]/x+[(\vec{q}+\vec{u}_\perp)^2+m_2{}^2]/(1-x)-m_0{}^2\}^{-1}. \tag{10}$$

Letting $\vec{v}=\vec{q}+\vec{u}_\perp$, and using the δ functions, we are left with the *two*-dimensional integral

$$M^{(c+d)}=\frac{-s}{8\pi^2}\int d\vec{v}f_1(-(\tfrac{1}{2}\vec{\Delta}+\vec{q}-\vec{v})^2)f_2(-(\tfrac{1}{2}\vec{\Delta}-\vec{q}+\vec{v})^2)\frac{1}{(\vec{v}^2+m_1{}^2)/x+(\vec{v}^2+m_2{}^2)/(1-x)-m_0{}^2}. \tag{11}$$

Note that only one energy denominator remains, which, as in the case of single scattering, is equal to $m_{\text{int}}^2 - m_0^2 = \sqrt{s}\,\Delta E$, where ΔE is the difference in energy before and after dissociation.

The amplitude $M = M^{(a)} + M^{(b)} + M^{(c+d)}$ contains all multiple-scattering effects, since f_1, f_2 are full elastic amplitudes. If one represents these amplitudes as sums of exchanges of a fictitious spin-1 particle, including crossed exchanges in the eikonal limit as done in Ref. 9, then the similar sum of all possible exchanges to the two-body system leads to the same M found here.

III. EIKONAL PICTURE

Let us express the elastic amplitudes in an impact-parameter representation:

$$f_j(-\vec{\Delta}^2) = -2i \int d\vec{b}\, e^{i\vec{b}\cdot\vec{\Delta}}(\exp[i\chi_j(\vec{b})] - 1). \tag{12}$$

Then

$$M = \frac{s}{2\pi^2}\int \frac{d\vec{v}}{m_{\text{int}}^2 - m_0^2}\int d\vec{b}_1 \exp[i\vec{b}_1\cdot(\tfrac{1}{2}\vec{\Delta} + \vec{q} - \vec{v})]\int d\vec{b}_2 \exp[i\vec{b}_2\cdot(\tfrac{1}{2}\vec{\Delta} - \vec{q} + \vec{v})]\{\exp[i\chi_1(\vec{b}_1) + i\chi_2(\vec{b}_2)] - 1\}, \tag{13}$$

where

$$m_{\text{int}}^2 = \frac{\vec{v}^2 + m_1^2}{x} + \frac{\vec{v}^2 + m_2^2}{1-x} \tag{14}$$

is the squared invariant mass of the 1-2 system after dissociation, calculated with these particles on the mass shell. This simple form is obtained by applying a Fourier transform to the single-scattering terms, after which the curly bracket in (13) results from adding the double-scattering term $\{\exp[i\chi_1(\vec{b}_1)] - 1\} \times\{\exp[i\chi_2(\vec{b}_2)] - 1\}$ to the two single-scattering terms $\{\exp[i\chi_1(\vec{b}_1)] - 1\} + \{\exp[i\chi_2(\vec{b}_2)] - 1\}$. In the result (13), the phase shifts $\chi_1(\vec{b}_1)$, $\chi_2(\vec{b}_2)$ to the two-body system simply add, as in the Glauber theory for scattering of nonrelativistic systems.[10]

The simple dependence on the off-energy-shell distance,

$$m_{\text{int}}^2 - m_0^2 = \sqrt{s}\,\Delta E,$$

suggests that form-factor effects be included by means of a damping in this variable, rather than by damping the off-shell Feynman propagators. This suggestion will be pursued in Sec. IV. Even with such off-shell effects, it is easy to generalize (13) to allow dissociation into more than two particles.

Performing the transverse-momentum integral in (13) yields

$$M = \frac{sx(1-x)}{\pi}\int d\vec{b}_1 d\vec{b}_2 \exp[i\vec{\Delta}\cdot\tfrac{1}{2}(\vec{b}_1 + \vec{b}_2)]\exp[i\vec{q}\cdot(\vec{b}_1 - \vec{b}_2)]K_0(|\vec{b}_2 - \vec{b}_1|\tilde{m})\{\exp[i\chi_1(\vec{b}_1) + i\chi_2(\vec{b}_2)] - 1\}, \tag{15}$$

where

$$\tilde{m} = [(1-x)m_1^2 + xm_2^2 - x(1-x)m_0^2]^{1/2}. \tag{16}$$

Equation (15) is worthy of careful study. The integration variables $\vec{b}_1$ and $\vec{b}_2$ are the impact parameters of particles 1 and 2. One can see that the average impact parameter, $\frac{1}{2}(\vec{b}_1 + \vec{b}_2)$, is conjugate to the total momentum transfer $\vec{\Delta}$, while the relative impact parameter, $\vec{b}_1 - \vec{b}_2$, is conjugate to the transverse momentum difference $\vec{q}$. The modified Bessel function $K_0(|\vec{b}_1 - \vec{b}_2|\tilde{m})$ is the same function which gives the impact-parameter distribution for exchange of a particle mass $\tilde{m}$ between spinless particles in a two-body reaction at high energy. Just as in that case, the large-$|\vec{b}_1 - \vec{b}_2|$ region, where $K_0(z) \approx (\pi/2z)^{1/2}e^{-z}$, corresponds to the pole, at $m_{\text{int}}^2 - m_0^2 = 0$; while the small-$|\vec{b}_1 - \vec{b}_2|$ region is probably modified by form-factor and absorption effects.[11] Absorption in the small-separation region probably accounts for the apparent "conspiracy" of the pion coupling noted by Berger in $pp \to n\pi^+p$,[5] in a way which is analogous to two-body reactions like $np \to pn$, where it produces sharp forward peaks.[11] The scale for the separation of particles 1 and 2 in the dissociation process is set by $1/\tilde{m}$ ($\hbar = c = 1$). For example, in $\gamma A \to \pi^+\pi^- A$, $\tilde{m} = m_\pi$, so the π's are separated by $1/m_\pi$ in the transverse direction for all x. In $nA \to p\pi^- A$, $\tilde{m} = [xm_\pi^2 + (1-x)^2 m_p^2]^{1/2}$, so for $x \approx 1$ (p forward in the $p\pi^-$ rest frame), the separation is $\approx 1/m_\pi$, while for $x \approx 0$, it is only $\approx 1/m_p$. Taking into account the spins of the particles in these examples would not change the picture. For example, in $nA \to p\pi^- A$ there is a helicity-nonflip amplitude $\propto K_0(|\vec{b}_1 - \vec{b}_2|\tilde{m})$, and a helicity-flip amplitude $\propto K_1(|\vec{b}_1 - \vec{b}_2|\tilde{m})$, but K_1 has the same asymptotic behavior as K_0.

It is instructive, and also relevant to diffractive production on a large nucleus, to consider (13) in the special case where the elastic amplitude corresponds to absorption on a "grey disk,"

$$\exp[i\chi_j(\vec{b})]-1=(-\sigma_j/2\pi R^2)\theta(R-|\vec{b}|). \tag{17}$$

If the radius R is large compared to the impact-parameter separation $1/\tilde{m}$, the important contributions to the integral in (13) come from the region $\vec{v}\approx\vec{q}$. The variations of $m_{\rm int}{}^2-m_0{}^2$ from the value at $\vec{v}\approx\vec{q}$ can be neglected, to obtain

$$M\cong(\sigma_1+\sigma_2-\sigma_1\sigma_2/2\pi R^2)[-2s(m^{*2}-m_0{}^2)] \times J_1(|\vec{\Delta}|R)/|\vec{\Delta}|R. \tag{18}$$

In (18), the double-scattering term appears as a typical Glauber-theory-type shadow correction $-\sigma_1\sigma_2/2\pi R^2$. If $\sigma_1=2\pi R^2$, which corresponds to a totally absorbing disk for particle 1, M becomes independent of σ_2, as would be expected in an optical model in this limit, where on the scale of R, 1 and 2 have essentially the same impact parameter. The dependence on $|\vec{\Delta}|=\sqrt{-t}$ is the same as for elastic scattering of a single particle from a grey disk. A picture similar to this one has been given by Bauer for $\gamma A\to\pi^+\pi^-A$.[12]

IV. OFF-ENERGY-SHELL FORM FACTORS

Results like (13) which are derived using elementary Feynman propagators, but without including "all possible diagrams" in a field theory, overestimate amplitudes in which the virtual particles are far off shell, and must be corrected for this by introducing some kind of damping of the off-shell effects. This notion is well known in $2\to2$ reactions, where elementary exchanges predict a grossly too-slow decrease with momentum transfer. In $2\to3$ diffractive dissociation, bare propagators lead to transverse momentum and invariant-mass distributions which are much too broad to agree with experiment, particularly when the extra factors of transverse momentum required by spin are included. Moreover, independently of specific models, if the dissociation process were as strong away from the pole as given by elementary propagators, it would be impossible to understand the observed limiting of transverse momenta in the inclusive reactions $a+b\to c+$ *anything,* in the face of the processes shown in Fig. 3.[13]

The "form factor" which damps out far-off-shell dissociations in (13) might in principle be a function of both x and

$$m_{\rm int}{}^2=\frac{\vec{v}^2+m_1{}^2}{x}+\frac{\vec{v}^2+m_2{}^2}{1-x}.$$

For example, in the double-Regge approach, the

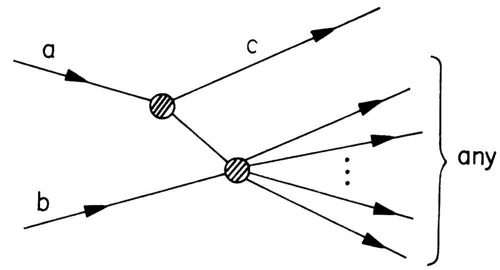

FIG. 3. A diagram which contributes to the inclusive process $a+b\to c+$ anything. Without form-factor effects, it would be unreasonably large.

vertex function for $M^{(a)}$ would depend on $(k_0-k_1)^2-m_2{}^2=-(1-x)(m_{\rm int}{}^2-m_0{}^2)$; for $M^{(b)}$, it would depend on $(k_0-k_2)^2-m_1{}^2=-x(m_{\rm int}{}^2-m_0{}^2)$; while for $M^{(c+d)}$, the situation would be complicated by the presence of two off-mass-shell particles. It seems promising to conjecture, instead, that the "form factor" depends only on $m_{\rm int}{}^2-m_0{}^2$, which is proportional to the distance off the energy shell in old-fashioned perturbation theory, at "infinite" momentum.

I therefore propose to include in (13) a function $F(m_{\rm int}{}^2-m_0{}^2)$, where $F(0)=1$ so as not to alter the residue at the pole, and F becomes small as $m_{\rm int}{}^2-m_0{}^2$ becomes large. Roughly, F might behave like $\exp[-A(m_{\rm int}{}^2-m_0{}^2)]$, where $A\sim1$ GeV^{-2}. This conjecture is motivated by simplicity, and by the idea that the damping results from higher-mass intermediate states. In addition, it is supported by a calculation involving diffraction dissociation of a nonrelativistic system, which is discussed below. The conjecture can be tested experimentally by studying the decay angular distributions; e.g., it predicts that large values of m^* are suppressed even in the region for which $(1-x)(m^{*2}-m_0{}^2)$ is not large, contrary to the double-Regge model.

Diffraction dissociation of a nonrelativistic system, such as a deuteron, which was discussed qualitatively in Sec. I, can be calculated by extending the Glauber theory[10] to include continuum final states. Neglecting final-state interaction effects by approximating the final wave function by its plane-wave part,

$$M\propto\int d\vec{v}\,\phi(\vec{v},q_z)\int d\vec{b}_1d\vec{b}_2\exp[i(\vec{b}_1+\vec{b}_2)\cdot\tfrac{1}{2}\vec{\Delta}] \times\exp[i(\vec{b}_2-\vec{b}_1)\cdot(\vec{q}-\vec{v})]\{\exp[i\chi(\vec{b}_1)+i\chi(\vec{b}_2)]-1\} \tag{19}$$

in the *rest frame* of the deuteron. The final momenta of the constituents are $\frac{1}{2}\vec{\Delta}\pm\vec{q}$, where $\vec{\Delta}$ is purely transverse to leading order in energy. All off-shell effects are included in $\phi(\vec{v},q_z)$, which is the ground-state wave function in momentum space.

For an s state, ϕ is a function of $\vec{v}^2 + q_z^2$ only, which means it is a function of $m_{\text{int}}^2 - m_0^2 = 4(\vec{v}^2 + q_z^2 + mB)$, where m is the nucleon mass and B the binding energy. Note that $m_{\text{int}}^2 - m_0^2$ is Lorentz-invariant, so it is correct to calculate it in the deuteron rest frame. Further, $\phi(\vec{v}, q_z)$ has a pole at $m_{\text{int}}^2 - m_0^2 = 0$, corresponding to the asymptotic behavior $\exp(-\sqrt{mB}\ r)/r$ of the coordinate space wave function. These results support the conjecture made above, that off-shell effects are accounted for by some $F(m_{\text{int}}^2 - m_0^2)$, where $F(0) = 1$. In the relativistic problem, of course, the "wave function" is not known, and there is more than one channel into which dissociation can take place.

V. CONCLUSION

In this paper, I have presented a theory for diffractive dissociation which seems intuitively appealing. The next step, to be undertaken in a forthcoming paper, is to compare this theory with available data for $\pi A \to \rho\pi A$, $NA \to N\pi A$, where A is a proton or nucleus. Hopefully, it will be possible to choose between this theory and the traditional double-Regge one, for example by studying $\pi p \to \rho\pi p$ events for which the ρ has a relatively large transverse momentum, but the π does not, thus favoring the diagram in which the ρ scatters. Or one could study events for which the ρ has a large-momentum fraction x, and attempt to determine whether the form-factor effects vary with $m_{\text{int}}^2 - m_0^2$ as given by the optical picture, or vary with $(k_0 - k_1)^2 - m_2^2 = -(1-x)(m_{\text{int}}^2 - m_0^2)$ as given by double-Regge theory.

One could try to extend this work theoretically by searching for a way to allow for resonances of the diffractively producted pair. An interesting attempt has been made to do this for the reaction $\gamma p \to \pi^+\pi^- p$, by identifying $F(m_{\text{int}}^2 - m_0^2)$ with the $\pi\pi$ spectral function measured in $e^+e^- \to \pi^+\pi^-$.[14]

A second direction to extend this work would be to try to use the picture, in which an incident particle dissociates into low-mass states with an amplitude $\propto F(m_{\text{int}}^2 - m_0^2)/(m_{\text{int}}^2 - m_0^2)$ in old-fashioned perturbation theory, for calculating one-particle inclusive distributions $a + b \to c +$ anything.[13,15] In this regard, the assumption that F is reasonably independent of the particles involved offers a way to relate the observed transverse momentum cutoff to the ratios for $\pi : K : \bar{p}$ production.[16]

The authors of Ref. 4 add to the diagrams of Figs. 1(a) and (b) a contribution in which the incident particle first undergoes elastic scattering, and then dissociates. Some such contribution is probably present, since it is needed to fulfill the physical requirement that there be no diffractive dissociation if the initial and final systems are absorbed equally by the target. Further analysis of this diagram is in progress.

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