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Helicity Structure of the P and P' Contributions to Two-Body Scattering Amplitudes*

Y. Zarmi†
California Institute of Technology, Pasadena, California 91109
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We study various tests for the determination of the helicity structure of the P and P' contributions to two-body scattering amplitudes. Most types of existing data are shown to be incapable of distinguishing between s- and t-channel helicity conservation. Duality tests probably indicate s-channel helicity conservation for the Pomeranchuk contribution, but are inconclusive as far as the P' is concerned. The only direct evidence for s-channel helicity conservation by the Pomeranchuk contribution is the measurement of the R parameter in the π^-p elastic scattering. The conclusion about the P' depends on what is assumed about the Pomeranchuk contribution.

I. INTRODUCTION

The helicity structure of the contributions of the P and P' to two-body hadronic scattering amplitudes has been treated in several papers.[1-4] In this paper we present various quantitative tests of the helicity structure of the P and P' contributions. We define the P' contributions as that part of the high-energy amplitude which has t-channel isospin $I_t=0$, and is dual to the s-u crossing-symmetric combination of low-energy resonances; that is to say, not the "pure" P' trajectory, but with its accompanying cuts included (the absorbed pole, or a dual absorptive model as proposed lately by Harari[5]).

The Pomeranchuk, or diffraction, contribution is dual to the low-energy background.[6-8] We shall assume that for $-1 \leqslant t \leqslant 0$ (GeV/c)^2 its real part may be neglected. Our main purpose is to confront the conjecture of s-channel helicity conservation with the possibility of t-channel helicity conservation, as it seems that the status of existing data is not conclusive on that issue. In Sec. II we treat the Pomeranchuk contribution. We first discuss existing tests, and then proceed to examine some further experimental features. In Sec. III we study the P' contribution. We find that most tests are not sufficiently sensitive for determination of the helicity structure of the P'. We conclude, in Sec. IV, with several remarks concerning the possibilities of distinguishing s- from t-channel helicity conservation.

II. THE HELICITY STRUCTURE OF THE POMERANCHUK CONTRIBUTION

A. Studying Differential Cross Sections

1. πN Elastic Scattering

It is well known that, if one assumes s-channel helicity conservation in πN, one obtains the relation $A' \cong \nu B/(1-t/4m^2)$. But the t-channel helicity amplitudes are obtained from A' and B by multiplying the latter by some kinematic factors. In fact, the relative contribution of the t-channel helicity amplitudes to the cross section is[9]

$$\frac{|F^t_{-+}|^2}{|F^t_{++}|^2}=\frac{(t/4m^2)[s-(m+E_{\rm lab})^2/(1-t/4m^2)]\,|B|^2}{(1-t/4m^2)\,|A'|^2}, \tag{1}$$

where F^t_{++} and F^t_{-+} are the t-channel helicity amplitudes, t is the momentum transfer squared, m is the nucleon mass, $E_{\rm lab}$ is the pion energy in the laboratory frame, and s is the invariant mass squared of the πN system.

Assuming $A' \cong \nu B/(1-t/4m^2)$, for high energies and small t, Eq. (1) gives

$$\frac{|F^t_{-+}|^2}{|F^t_{++}|^2} \cong -\frac{t}{4m^2}\,. \tag{2}$$

This ratio is plotted in Fig. 1 (solid line).

On the other hand, let us now check the s-channel helicity amplitudes. Again assuming $A' \cong \nu B/(1-t/4m^2)$ we obtain

$$\frac{|F^s_{-+}|^2}{|F^s_{++}|^2} \cong -\frac{tm^2}{s(4q^2+t)}\,, \tag{3}$$

where q is the c.m. momentum. We plot this ratio for $p_{\rm lab} = 3$ GeV/c in Fig. 1 (dashed line). It is evident that already at this energy the calculated ratios prefer s- over t-channel helicity conservation. But one has to go to t beyond ~ -0.6 (GeV/c)^2 in order to get an appreciable t-channel helicity-flip contribution to the cross section. Thus, for smaller t values, while the t-channel flip amplitude is quite large (up to ~45% of the nonflip amplitude), the cross section can be well accounted for by the t-channel nonflip amplitude only. At $|t| > 0.6$ (GeV/c)^2 the distinction between s- and t-channel helicity conservation may, in principle, be easier; but there the Pomeranchuk contribution is small,

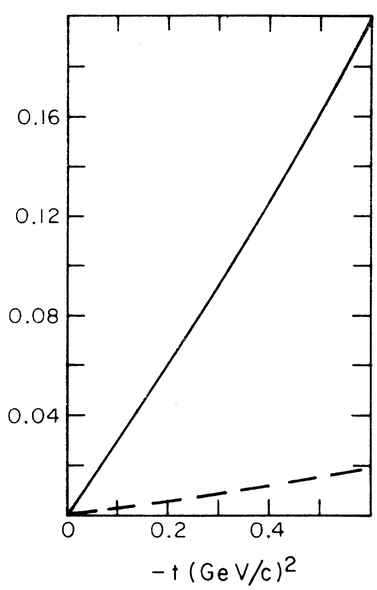

FIG. 1. Comparison between the asymptotic form of the ratio $|F^t_{-+}|^2/|F^t_{++}|^2$ (solid line) and the ratio $|F^s_{-+}|^2/|F^s_{++}|^2$ at 3 GeV/c (dashed line) in πN scattering, under the assumption $A' \cong \nu B/(1-t/4m^2)$.

other trajectories complicate the picture, and the data are not as good as they are at low t values.

2. pp Elastic Scattering

Let us assume that the Pomeranchuk contribution strictly conserves s-channel helicities, i.e., all amplitudes which do not conserve s-channel helicities at each vertex vanish identically,

$$F_{++;--} = F_{+-;-+} = F_{++;+-} = 0\,. \tag{4}$$

Using the crossing matrix, and the natural-parity feature of the Pomeranchuk contribution, one finds that all t-channel amplitudes can be expressed in terms of one amplitude,

$$\begin{aligned} G_{00} &= G_{++;++} = G_{++;--} \equiv G\,,\\ G_{11} &= G_{+-;+-} = G_{+-;-+} = -G\cot^2\chi\,,\\ G_{01} &= G_{++;+-} = G\cot\chi\,, \end{aligned} \tag{5}$$

where χ is the p-p crossing angle,

$$\cot\chi = \frac{1}{2m}\left(\frac{st}{u}\right)^{1/2}. \tag{6}$$

At high energies and small t, $\cot\chi \cong \sqrt{-t}/2m$. For $t = -0.6$ (GeV/c)^2,

$$G_{11} \cong -0.17G\,,$$

$$G_{01} \cong 0.41G\,.$$

Inserting this into the expression for the differential cross section, one finds that at high energies the t-channel helicity-nonconserving amplitudes account for only ~26% of the cross section. For smaller t values, they contribute even less. Again, it is difficult to tell s- from t-channel helicity conservation by merely studying differential cross sections.

B. Duality Tests

Using duality arguments, one expects to detect trends of the helicity structure of specific exchanges already at energies in the 1 – 2 GeV region, where phase-shift analyses enable the detailed study of amplitudes.

Starting from the Harari-Freund conjecture,[6] Harari and Zarmi have calculated the low-energy background in πN amplitudes. They found that, while both $A'^{(+)}$ and $B^{(+)}$ have significant background contributions superimposed on the resonance contribution,[8] the $I_t = 0$ s-channel helicity-flip amplitudes are accounted for by resonances only[4] (at least for small t values). The s-channel nonflip amplitude requires a large background contribution. This implies that for the Pomeranchuk contribution (being dual to the low-energy background), s-channel helicity conservation is preferred over t-channel helicity conservation. They also used finite-energy sum rules (FESR) in order to calculate the Pomeranchuk residue functions from the low-energy background.[8] The results were consistent with

$$A'_P \cong \frac{\nu B_P}{1 - t/4m^2}\,.$$

Again, this implies s-channel helicity conservation at high energies. These conclusions depend on the values of the resonance parameters and on the specific resonance parametrization they used.

Less model-dependent FESR calculations have been performed by Barger and Phillips,[10] Phillips and Ringland,[11] and Dass and Michael.[12] All these authors performed FESR calculations on $A'^{(+)}$ and $B^{(+)}$ as constructed from phase-shift data. They therefore dealt with the sum of the P and P' contributions.

Barger and Phillips[10] used the 1967 CERN πN phase-shift data[13] as input in a continuous-moment sum-rule calculation, coupled with a fit to high-energy data. They found that the FESR were consistent with the assumption $A' \cong \nu B$, both for the P and the P'. However, it should be noted that while the $A'^{(+)}$ FESR were consistent with this assumption for all the moments, the $B^{(+)}$ FESR agreed with it in the higher moments, and the agreement in the lower moments was slightly worse. For example, at $t=0$ there is a discrepancy of ~25% between the low-moment FESR and the high-energy parametrization of $B^{(+)}$ (which assumes $A'^{(+)} \cong \nu B$). This may be interpreted in the following two ways:

(i) Higher moments are less reliable. They enhance the contribution of high-mass resonances as compared to low-mass ones; the latter being, generally, better known. This is especially important in $B^{(+)}$, where the resonances add up destructively. If we, therefore, trust the lowest moments only, we do not obtain $A' \cong \nu B$ for $P+P'$. Therefore, either one (or both) may break s-channel helicity conservation. If, for example, one assumes s-channel helicity conservation for the Pomeranchuk contribution, one finds that for small t $A'_{P'} \cong \frac{1}{2}\nu B_P$

(ii) The $B^{(+)}$ amplitude contains some low-lying contribution (P'', or cuts) which shows up mainly in the low moments, and does not conserve s-channel helicities. Thus, the higher moments, which are presumably dominated by $P+P'$, may be consistent with the $A' \cong \nu B$ assumption.

Phillips and Ringland[11] use the latest Saclay πN phase-shift data[14] as input for a FESR calculation. They find that, for the $I_t=0$ amplitudes, s-channel helicity is conserved. For example, at the cutoff energy of their calculation (2.8 GeV – the highest energy included in the Saclay analysis) and $t=-0.2$ $(\text{GeV}/c)^2$ (near that t value the s-channel helicity-flip amplitude is maximal), they have

$$\frac{F_{+-}^{s(I_t=0)}}{F_{++}^{s(I_t=0)}} \cong 0.28\,.$$

But, using their results, one finds that t-channel helicity conservation is even better.[15] For example, at $t=-0.2$ $(\text{GeV}/c)^2$ and $E_{\text{lab}}=2.8$ GeV, we find

$$\frac{F_{+-}^{t(I_t=0)}}{F_{++}^{t(I_t=0)}} \cong -0.035\,.$$

Since they do not separate the P from the P' contribution, one cannot determine whether either or both is the origin of this problem. We thus see that the conclusions depend on the particular phase-shift solution one adopts.

Dass and Michael[12] have done a continuous-moment calculation using KN and $\bar{K}N$ phase-shift data. Their results are consistent with $A'^{(+)} \cong \nu B^{(+)}$.

We shall return to the results of Ref. 10 and 12 in Sec. III where we study the P' contribution alone.

A model-independent test which does not involve FESR is presented in Ref. 4, which studies the Argand plots of the $I_t=0$ s-channel helicity partial waves. The helicity-flip waves, $F_{-+}^{J(I_t=0)}$, seem to have no background. Their plots show "clean" circles. The helicity-nonflip waves, on the other hand, show a significant, predominantly imaginary background superimposed on the "clean" circles.

In order to rule out t-channel helicity conservation, one would like to study analogous quantities for $A'^{(+)}$ and $B^{(+)}$. Instead of $A'^{(+)}$ we use $A^{(+)}$ for the following reasons: (i) $A^{(+)}$ and $B^{(+)}$ have similar expansions in terms of derivatives of Legendre polynomials. (ii) If the Pomeranchuk contribution conserves s-channel helicities, we expect the imaginary part of $A^{(+)}$ to be dominated by resonances, while $\text{Im}B^{(+)}$ should show significant background contributions.

From Refs. 4 and 8 it is clear that if we compare the total amplitudes (calculated from the CERN phase-shift data) to the resonance contributions (as parametrized there), the result will be consistent with the conjecture stated above. Since we want a model-independent test, we study the phase-shift data only.

The expansion of $A^{(+)}$ and $B^{(+)}$ is given by

$$A^{(+)}(s,t) = \sum_{l\geq 1} \frac{P_l{}'(\cos\theta)}{l(l+1)}\, A_l^{(+)}(s)\,,$$
$$B^{(+)}(s,t) = \sum_{l\geq 1} \frac{P_l{}'(\cos\theta)}{l(l+1)}\, B_l^{(+)}(s)\,. \tag{7}$$

Since at $\theta=0$ $P_l{}'(1)=\frac{1}{2}l(l+1)$, we define the expansion coefficients by

$$A_l^{(+)}(s) = 2\pi l(l+1)\left[\frac{\sqrt{s}+m}{E+m}\left(f^{(+)}_{(l-1)_+} - f^{(+)}_{(l-1)_-}\right) - \frac{\sqrt{s}-m}{E-m}\left(f^{(+)}_{l_-} - f^{(+)}_{l_+}\right)\right],$$
$$B_l^{(+)}(s) = 2\pi l(l+1)\left[\frac{1}{E+M}\left(f^{(+)}_{(l-1)_+} - f^{(+)}_{(l+1)_-}\right) + \frac{1}{E-M}\left(f^{(+)}_{l_-} - f^{(+)}_{l_+}\right)\right]. \tag{8}$$

$E=(s+m^2-\mu^2)/2\sqrt{s}$ is the nucleon energy in the c.m. frame. $f_l^{(+)}$ are the $I_t=0$ combinations of the ordinary partial waves,

$$f^{(+)}_{l_\pm} = \tfrac{1}{3}\left(f^{(I_s=1/2)}_{l_\pm} + 2f^{(I_s=3/2)}_{l_\pm}\right).$$

We use the CERN experimentally fitted phase-shift data.[16] Figure 2 shows the Argand plots of $2qA_l^{(+)}$, and Fig. 3 shows the Argand plots of $2qE_{\text{lab}}B_l^{(+)}$. q is the c.m. momentum. We multiply $B_l^{(+)}$ by E_{lab} for two reasons: (i) The quantities we want to compare are $A^{(+)}$ and $\nu B^{(+)}$ (B has one power of ν less than A). (ii) We want to compensate partially for the distortion in $B_l^{(+)}$ due to the kinematic factors multiplying the $f_{l_\pm}$.

From the figures one sees that even after this correction is included it is not clear that $A_l^{(+)}$ are dominated by resonances, or that $B_l^{(+)}$ have a significant background contribution. One might explain the lack of success by arguing that $A_l^{(+)}$ and $B_l^{(+)}$ are complicated combinations of the partial waves, distorted by kinematic factors. However, on the partial-wave level one may state the following qualitative conjecture: At sufficiently high energies ($\sqrt{s} \gg m$, $E\cong\frac{1}{2}\sqrt{s}$),

$$A_l^{(+)} \cong 4\pi l(l+1)\left(f^{(+)}_{(l-1)_+} - f^{(+)}_{(l+1)_-} - f^{(+)}_{l_-} + f^{(+)}_{l_+}\right),$$
$$B_l^{(+)} \cong \frac{4\pi l(l+1)}{\sqrt{s}}\left(f^{(+)}_{(l-1)_+} - f^{(+)}_{(l+1)_-} + f^{(+)}_{l_-} - f^{(+)}_{l_+}\right).$$

If we define $J=l-\frac{1}{2}$, then

$$A_l^{(+)} \cong 8\pi l(l+1)\left(F_{-+}^{J(+)} + F_{-+}^{J+1\,(+)}\right),$$
$$B_l^{(+)} \cong \frac{8\pi l(l+1)}{\sqrt{s}}\left(F_{++}^{J(+)} - F_{++}^{J+1\,(+)}\right), \tag{9}$$

where $F_{++}^{J(+)}$ and $F_{-+}^{J(+)}$ are the s-channel helicity partial waves with $I_t=0$.

If we *conjecture* that what has been found in Ref. 4, namely, that $F_{-+}^{J(+)}$ have no background while $F_{++}^{J(+)}$ show significant imaginary background, is true at energies above the phase-shift-analysis region, we arrive at the following conclusion: $A_l^{(+)}$, being a combination of s-channel helicity-flip partial waves, do not have any background (i.e., Pomeranchuk) contribution. Consequently, the Pomeranchuk contribution contributes only to $B_l^{(+)}$, and hence $A'_P \cong \nu B_P$, i.e., s-channel and *not* t-

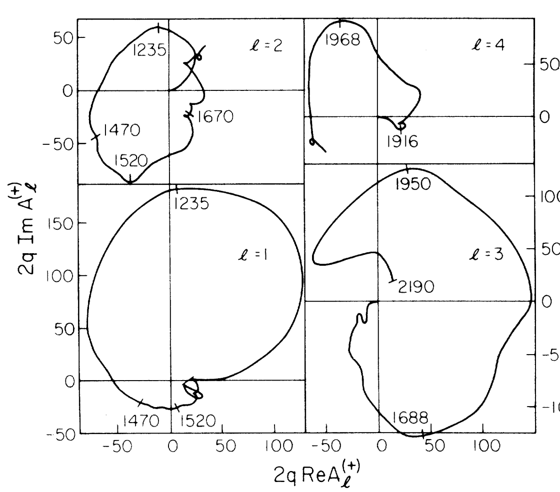

FIG. 2. Argand plots of $A_l^{(+)}$ for πN scattering, calculated from the CERN phase-shift data (Ref. 16). If the Pomeranchuk contribution conserves s-channel helicities, it is expected to decouple from $A^{(+)}$, and $A_l^{(+)}$ should be resonance-dominated. Except for the $l=3$ coefficient, there does not seem to be much background. The distortion of the different contributions due to kinematic factors make the test inconclusive, and the picture is not as simple as it is in Ref. 4.

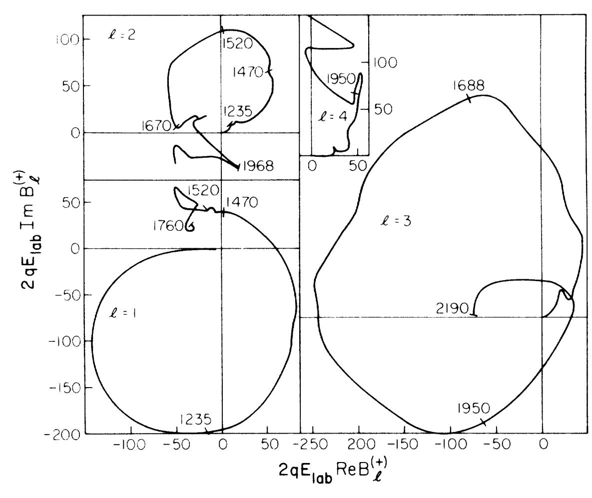

FIG. 3. Argand plots of $E_{\text{lab}}B_l^{(+)}$ for πN scattering, calculated from the CERN phase-shift data (Ref. 16). If the Pomeranchuk contribution conserves s-channel helicities, it should contribute to $B^{(+)}$, and $B_l^{(+)}$ should exhibit a significant imaginary background superimposed on the resonance circles. If it conserves t-channel helicities, the Pomeranchuk contribution should decouple from $B^{(+)}$, and $B_l^{(+)}$ should show "clean" circles. Except for the $l=3$ coefficient, some nonresonant imaginary part seems to develop at the higher energies, but due to kinematic distortion the test is inconclusive.

channel helicity conservation.

Another low-energy test which, coupled with duality arguments, should be expected to help in determining the helicity structure of the $P+P'$ contribution is the study of the quantity

$$\Delta\sigma = \frac{d\sigma}{d\Omega}(\pi^- p) - \frac{d\sigma}{d\Omega}(\pi^+ p),$$

which can be written in two forms:

$$\begin{aligned}\Delta\sigma &= \Delta|F^s_{++}|^2 + \Delta|F^s_{-+}|^2 \\ &= \Delta|F^t_{++}|^2 + \Delta|F^t_{-+}|^2 .\end{aligned} \tag{10}$$

The s (t) superscripts denote s- (t-) channel helicity amplitudes. Each of the terms can be expressed as

$$\begin{aligned}\Delta|F^s_{\lambda\mu}|^2 &= |F^s_{\lambda\mu}(\pi^- p)|^2 - |F^s_{\lambda\mu}(\pi^+ p)|^2 \\ &= 4\,\mathrm{Re}(F^{s(+)*}_{\lambda\mu} F^{s(-)}_{\lambda\mu}),\end{aligned} \tag{11}$$

and a similar expression for the t-channel quantities,

$$F^{(\pm)} = \tfrac{1}{2}[F(\pi^- p) \pm F(\pi^+ p)].$$

From the differential cross section of πN charge-exchange scattering, and from FESR calculations,[17] it is well known that the $I_t = 1$ (ρ-exchange) contribution is predominantly in the (s- or t-channel) helicity-flip amplitude. One, therefore, expects the $I_t = 0$ flip (s- or t-channel) amplitudes to be enhanced in $\Delta|F^{s(t)}_{-+}|^2$, while the $I_t = 0$ nonflip (s- or t-channel) amplitudes are expected to be damped in $\Delta|F^{s(t)}_{++}|^2$. Thus, a comparison of the four quantities $\Delta|F^s_{++}|^2$, $\Delta|F^s_{-+}|^2$, $\Delta|F^t_{++}|^2$, and $\Delta|F^t_{-+}|^2$, as reconstructed from phase-shift data, may teach us something about the helicity structure of the $I_t = 0$ contribution.

We use the CERN experimentally fitted phase-shift data.[16] In Fig. 4 we show these four quantities for three representative energies: $E_{\rm lab} = 1.040$, 1.512, and 2.015 GeV, and -1 (GeV/c)$^2 \le t \le 0$. Figure 4(a) shows the s-channel quantities, and Fig. 4(b) shows the corresponding t-channel quantities. Both show a clear predominance of the $I_t = 0$ helicity-nonflip amplitude over the helicity-flip amplitude. It is also evident that there is no clear-cut distinction between s- and t-channel helicity conservation. This is further shown in Fig. 5, where we present $\Delta|F^s_{-+}|^2$ and $\Delta|F^t_{-+}|^2$ as functions of energy at $t = -0.1$ (GeV/c)^2 (where these quantities

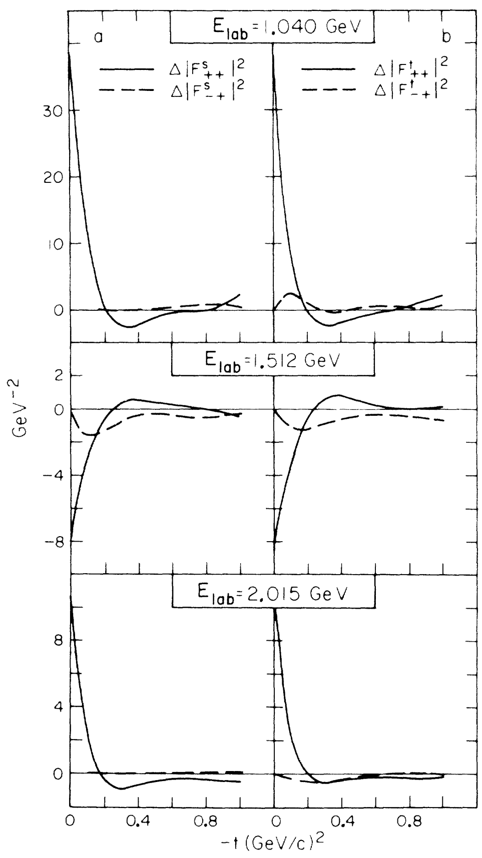

FIG. 4. The t dependence of various contributions to $d\sigma/dt(\pi^- p) - d\sigma/dt(\pi^+ p)$, calculated from the CERN phase-shift data (Ref. 16), at $E_{\rm lab} = 1.040$, 1.512, and 2.015 GeV. (a) s-channel helicity-nonflip (solid line) and -flip (dashed line) contributions; (b) corresponding t-channel quantities. Both figures indicate that, at small t, the (s- or t-channel) helicity-nonflip contribution is much larger than the helicity-flip contribution, but there is no indication that s-channel helicity conservation is better than t-channel helicity conservation, or vice versa.

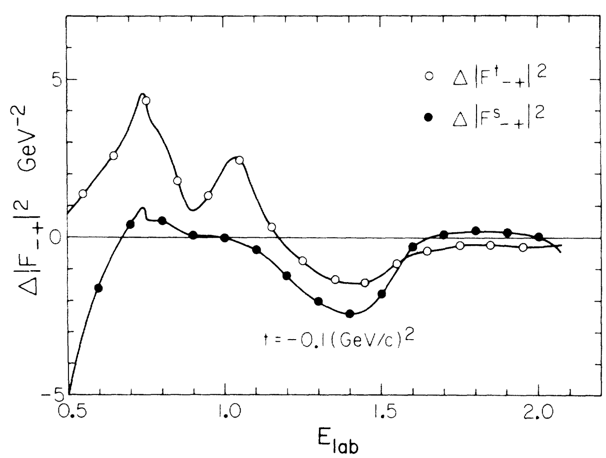

FIG. 5. Comparison between the contributions of the s- and t-channel helicity-flip contributions, $\Delta|F^s_{-+}|^2$ and $\Delta|F^t_{-+}|^2$, to $d\sigma/dt(\pi^- p) - d\sigma/dt(\pi^+ p)$ at $t = -0.1$ (GeV/c)^2 and $0.5 \le E_{\rm lab} \le 2$ GeV. The quantities were calculated from the CERN phase-shift data (Ref. 16). No preference for s-channel over t-channel helicity conservation is indicated.

have a maximum). Had one type of helicity conservation been preferred, one would have hoped to see a consistent relation between the sizes of the two quantities (e.g., $\Delta|F^s_{-+}|^2 > \Delta|F^t_{-+}|^2$ over most of the energy region, if s-channel helicity conservation is preferred).

3. Spin Parameters in πN Scattering

The spin parameters we want to study are

$$P = \frac{2\,\mathrm{Im}F^*_{++}F_{-+}}{d\sigma/d\Omega},$$
$$Y = \frac{|F_{++}|^2 - |F_{-+}|^2}{d\sigma/d\Omega}, \qquad (12)$$
$$Z = \frac{2\,\mathrm{Re}F^*_{++}F_{-+}}{d\sigma/d\Omega}.$$

P is the polarization and Y and Z are related to the Wolfenstein parameters A and R.[18]

The polarization cannot tell us much about the helicity structure of an individual exchange, since it necessarily involves the interference between different exchanges. Therefore, one has to make several assumptions in order to obtain information about the helicity structure of a specific exchange. Such an approach will be used in Sec. III, where we study the P' contribution.

The quantity Y does not distinguish between s- and t-channel helicity conservation. In both cases $Y \cong 1$. Z, on the other hand, will show different features in each case. F_{++} is dominated by the Pomeranchuk contribution, whose real part is assumed negligible for -1 $(\mathrm{GeV}/c)^2 < t \leq 0$. Therefore,

$$Z \cong -\frac{2|F_{++}|\mathrm{Im}F_{-+}}{d\sigma/d\Omega}. \qquad (13)$$

(i) *t dependence. If the Pomeranchuk contribution conserves s-channel helicities,* then F_{-+} only contains the contributions of "ordinary" exchanges, or in s-channel language, the peripheral part which is dual to resonances. The imaginary part of such an amplitude is expected to have a zero near $t \cong -0.5$ $(\mathrm{GeV}/c)^2$.[5] *We therefore expect Z to have a zero near* $t \cong -0.5$ $(\mathrm{GeV}/c)^2$. On the other hand, *if P conserves t-channel helicities, we do not expect such a zero.*

(ii) *Energy dependence.* Expressing Z in terms of invariant amplitudes, one finds that at high energies,

$$Z = \frac{m\sqrt{-t}}{16\pi^2 s}\,\frac{|A|^2 + \frac{1}{2}s|B|^2 + (s/2m)\,\mathrm{Re}(AB^*)}{d\sigma/d\Omega}. \qquad (14)$$

s-channel helicity conservation implies $A_P \cong 0$. Therefore,

$$Z \sim s^{\alpha_R - \alpha_P},$$

where α_P is the Pomeranchuk-trajectory function and α_R is a typical ordinary Regge-trajectory function. Thus, at high energies Z should tend to zero. t-channel helicity conservation means $B_P \cong 0$. This implies that

$$Z \cong \frac{\sqrt{-t}}{m(1 - t/4m^2)} > 0.$$

We see that Z can serve for testing the helicity structure of the Pomeranchuk contribution. The existing data for the A and R parameters are those of 6-GeV/c π^-p scattering.[19] They are shown, together with the R values predicted by t-channel helicity conservation, in Fig. 6. Since, at high energies and small t, $A \cong Y$ and $R \cong Z$, we can study A and R directly.

The data show $A \cong 1$, which means either s- or t-channel helicity conservation. R is negative and seems to approach zero as t approaches ~ -0.5 $(\mathrm{GeV}/c)^2$. The measured values differ in sign and magnitude from the values predicted if t-channel helicity is assumed for the Pomeranchuk contribution. Thus, s-channel helicity conservation seems to be preferred.

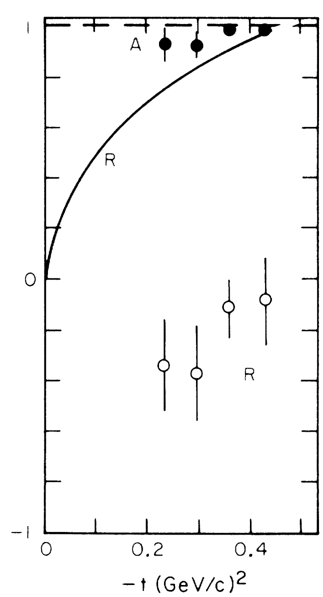

FIG. 6. The A and R parameters measured in 6-GeV/c π^-p elastic scattering. $A \cong 1$ is consistent with s- and t-channel helicity conservation. R is negative, small, and seems to approach zero near $t \cong -0.5$ $(\mathrm{GeV}/c)^2$. Comparing this to the large positive values predicted if t-channel helicity conservation is assumed (solid line) we see that s-channel helicity conservation is preferred for the Pomeranchuk contribution.

III. THE HELICITY STRUCTURE OF THE P'

The arguments for s-channel helicity conservation by the P' are numerous. We refer the reader to the good summary presented in Refs. 3 and 20. We shall study in this section tests involving duality arguments and spin-parameter measurements.

A. Duality Tests

Gilman et al.[7] and Harari and Zarmi[8] have calculated the P' residue functions in πN and $\bar{K}N$ scattering by saturating FESR with resonances. The result obtained in Refs. 7 and 8 is that the P' contribution does not satisfy the relation $A' \cong \nu B$, derived from s-channel helicity conservation. A similar result was obtained by Dass and Michael.[12] Using resonance saturation they find P' residue functions such that near $t \cong 0$, at the cutoff energy of their calculation,

$$1.5 < \frac{\nu B_{P'}}{A'_{P'}} < 3 .$$

We have repeated the calculations performed in Refs. 7 and 8, using the resonance parameters listed in Table I. They were taken from Ref. 21. We saturate the appropriate FESR by the resonance contributions, and obtain the P' residue functions under the assumption that it is a single Regge trajectory with $\alpha_{P'}(t) = 0.5 + t$. Clearly, a crude calculation of this kind cannot distinguish a pole from its accompanying cut, due to the approximately equal $\alpha(t)$ the two have for small t. One may also assume a dual absorptive model, as suggested by Harari.[5]

The sum rules involved are

$$\begin{aligned} S_{2n+1} &= \frac{1}{N^{2n}} \int_0^N \nu^{2n} \mathrm{Im} A'^{(+)}_{\mathrm{Res}}(\nu, t) d\nu \\ &= \frac{\beta^{A'}_{P'} N^{\alpha_{P'}(t)}}{\alpha_{P'}(t) + 2n + 2} , \\ S_{2n} &= \frac{1}{N^{2n}} \int_0^N \nu^{2n} \mathrm{Im} B^{(+)}_{\mathrm{Res}}(\nu, t) d\nu \\ &= \frac{\beta^{A'}_{P'} N^{\alpha_{P'}(t)}}{\alpha_{P'}(t) + 2n} , \quad n = 0, 1, 2, 3 . \end{aligned} \tag{16}$$

N, the cutoff energy, is given by $N = N_0 + t/4m$. N_0 is the highest E_{lab} value included in the integral; we use three different values: $N_0 = 1$ GeV (I), 1.35

TABLE I. The πN resonance parameters used in the calculation of the P' residue functions. These are the average values presented in Ref. 21. Replacing them by different sets of parameters or excluding some of the less-established resonances did not change our results by any significant amount.

Wave	Mass (GeV)	Width (MeV)	Elasticity
S_{11}	1535	118	0.39
S_{11}	1706	256	0.69
P_{11}	1468	244	0.61
P_{11}	1783	350	0.34
P_{13}	1864	335	0.27
D_{13}	1520	120	0.53
D_{13}	2039	274	0.17
D_{15}	1672	142	0.42
F_{15}	1688	127	0.62
F_{17}	1989	238	0.11
G_{17}	2180	299	0.35
S_{31}	1650	151	0.27
P_{31}	1908	325	0.25
P_{33}	1236	120	1.00
P_{33}	1689	267	0.09
P_{33}	2160	260	0.25
D_{33}	1674	240	0.13
D_{35}	1958	356	0.14
F_{35}	1885	273	0.17
F_{37}	1952	202	0.44

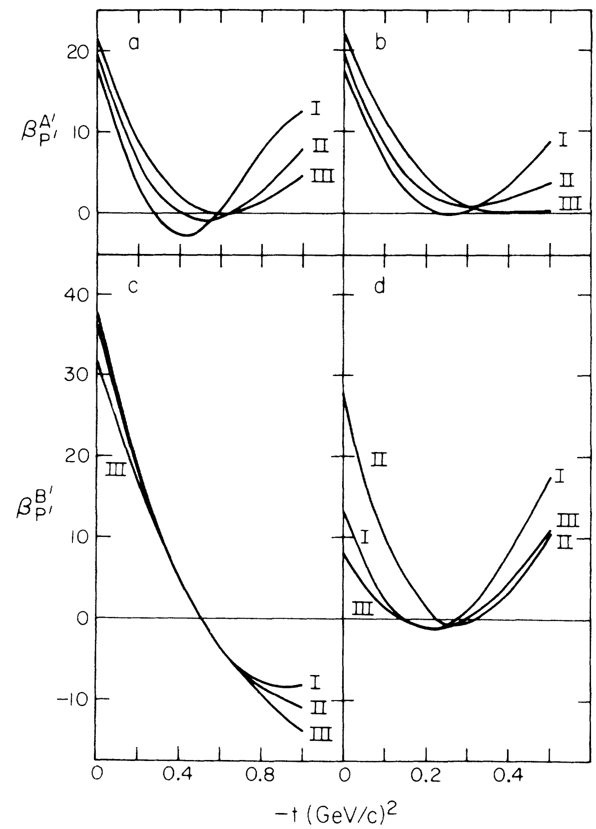

FIG. 7. The P' residue functions $\beta^{A'}_{P'}$, $\beta^{B}_{P'}$, as calculated from FESR, using the resonances listed in Table I. (a) $\beta^{A'}_{P'}$ calculated from S_1; (b) $\beta^{A'}_{P'}$ calculated from S_3; (c) $\beta^{B}_{P'}$ calculated from S_0; (d) $\beta^{B}_{P'}$ calculated from S_2. The cutoff N is given by $N = N_0 + t/4m$, where $N_0 = 1$ GeV (I), 1.35 GeV (II), and 1.70 GeV (III).

GeV (II), and 1.70 GeV (III).

Figures 7(a) and 7(c) show the residue functions obtained from the lowest-moment ($n=0$) FESR: S_1 (for $A'^{(+)}$) and S_0 (for $B^{(+)}$). It is evident that for small t, $\nu B_{P'} \gtrsim 2A'_{P'}$. The results of the $n=1$ FESR are shown in Figs. 7(b) and 7(d). We see that $\beta_{P'}^{A'}$ does not change in a significant manner for -0.5 $(\mathrm{GeV}/c)^2 \lesssim t \lesssim 0$. On the other hand, the resulting $\beta_{P'}^{B}$ is changed to a large extent. This may be attributed to two reasons: (i) The resonance contributions to $B^{(+)}$ tend to cancel each other; therefore, the errors involved in their calculation (due to the specific values of the parameters and the parametrization) become rather important. (ii) As mentioned in Sec. II B, with relation to the work of Barger and Phillips,[10] there may be some nonleading terms in $B^{(+)}$, whose contribution is important mainly in the low-moment FESR, S_0, and not as much in S_2. Figures 7(a) and 7(b) indicate that there is no need to include such a nonleading term in $A'^{(+)}$.

The results obtained from S_2 can be consistent with s-channel helicity conservation [i.e., $\beta_{P'}^{A'} \cong \beta_{P'}^{B}(S_2)$], but are certainly inconclusive.

Trying to study the nature of the possible nonleading term in $B^{(+)}$ is not rewarding due to the sensitivity of such a calculation. If we assume that $\beta_{P'}^{A'}(t) \cong \beta_{P'}^{B}(t) = \beta_{P'}(t)$, then

$$S_0 = \int_0^N \mathrm{Im} B^{(+)}_{\mathrm{Res}}(\nu, t)\, d\nu$$

$$= \beta_{P'}(t) \frac{N^{\alpha_{P'}(t)}}{\alpha_{P'}(t)} + \int_0^N X(\nu, t)\, d\nu ,$$

where $X(\nu, t)$ is the imaginary part of the nonleading term. Using S_1 in order to eliminate $\beta_{P'}(t)$, we obtain

$$\alpha_{P'}(t) \int_0^N X(\nu, t)\, d\nu = \alpha_{P'}(t) S_0 - [\alpha_{P'}(t) + 2] S_1 . \tag{17}$$

We show the right-hand side of Eq. (17) in Fig. 8. We find that it is consistent with an effective, rather flat, $\alpha_X(t) \cong 0$.

We conclude this section by mentioning the following argument.

Dass and Michael[12] and Gilman *et al.*[7] have used resonance saturation in FESR for the P' contribution in KN scattering. Here, as in πN scattering, neither s- nor t-channel helicity conservation is indicated.

On the other hand, Di Vecchia *et al.*[22] calculated the ω residue functions in KN scattering by saturating the appropriate FESR with the known KN resonances. The residue functions they obtained satisfy, near $t \cong 0$ and the cutoff energy of their calculation,

$$\frac{\nu B_\omega}{A'_\omega} \cong 1.4 \pm 0.1 .$$

Moreover, both amplitudes have zeros near $t \cong -0.2$ $(\mathrm{GeV}/c)^2$.

A similar result was obtained by Dass and Michael.[12] Using resonance saturation in KN-scattering FESR, they obtained residue functions which were consistent with $\nu B_\omega / A'_\omega \cong 1$, and both amplitudes have zeros near $t \cong -0.1$ $(\mathrm{GeV}/c)^2$.

These calculations indicate s-channel helicity conservation for the ω trajectory. Due to the exotic nature of the K^+p channel this implies a cancellation between the imaginary parts of the P' and the ω contributions, which, in turn, indicates s-channel helicity conservation for the P'.

B. Spin Parameters

1. πN Scattering

(i) The mirror symmetry in $\pi^{\pm} p$ polarization can serve as a test for the helicity structure of the P'. The symmetry is only gradually reached. At low and intermediate energies the sum of the polarizations of $\pi^- p$ and $\pi^+ p$ does not vanish, but it falls off, as energy is increased, at a rate faster than a Regge model (including P, P', ρ) would predict.[23] This is demonstrated in Fig. 9, where we compare the energy dependence of

$$\Delta = \tfrac{1}{2}[P(\pi^- p) + P(\pi^+ p)]$$

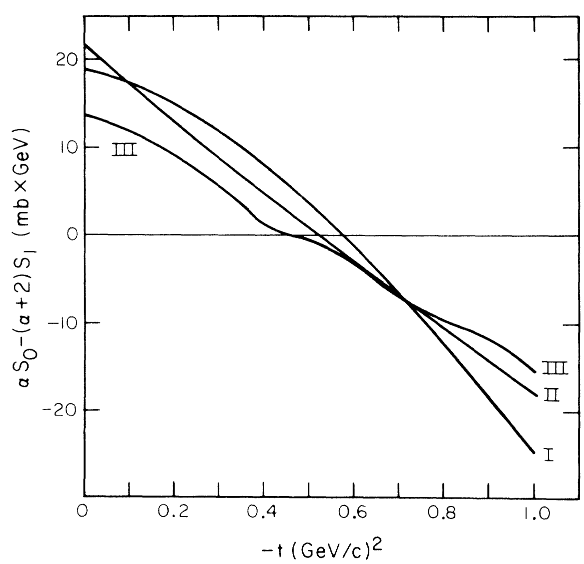

FIG. 8. Extraction of the contribution of the "low-leading" term in $B^{(+)}$ to S_0, under the assumption that the "leading term," the P', conserves s-channel helicities. The "nonleading" term is consistent with a rather flat trajectory function $\alpha_X(t) \cong 0$.

with that of a typical Regge term, at $t=-0.14$ $(\mathrm{GeV}/c)^2$.

The sum of the polarizations involves the crossing-even contributions only, namely,

$$\Delta \cong \frac{\sin\theta q^2}{16\pi^2\sqrt{s}} \frac{\mathrm{Im}(A'_P B^*_{P'} + A'_{P'} B^*_P)}{d\sigma/d\Omega(\pi N)} . \tag{18}$$

The Regge term shown in Fig. 8 is the first of the two in Eq. (18),

$$\frac{\sin\theta q^2}{16\pi^2\sqrt{s}} \frac{\mathrm{Im}(A'_P B^*_{P'})}{d\sigma/d\Omega(\pi N)} .$$

The P and P' parameters were taken from Barger and Phillips,[10] and the data were taken from the sources listed in Ref. 24.

The P-P' interference term falls off as $\sim p_{\mathrm{lab}}^{-0.3}$, while the data points fall off as $\sim p_{\mathrm{lab}}^{-1.5\pm0.5}$.

This implies that the leading term, namely, the right-hand side of Eq. (18), vanishes. As has already been noted,[11] it only implies that the helicity structures of the P and P' are the same. Neglecting the real part of the Pomeranchuk contribution,

$$\frac{\mathrm{Re}B_{P'}}{\mathrm{Re}A'_{P'}} = \frac{B_P}{A'_P} . \tag{19}$$

Thus, if the Pomeranchuk contribution conserves s-channel helicities, so does the P'.

(ii) A study of Eqs. (13) and (14) reveals the following: If *P and P' both conserve s-channel helicities,* then F_{-+} is dominated by the ρ contribution. And, therefore, Eq. (13) predicts *mirror symmetry in R,* $R(\pi^+ p) \cong -R(\pi^- p)$, at least for small t. If, on the other hand, *the Pomeranchuk contribution conserves s-channel helicities, and the P' does not,* $R(\pi^- p)$ and $R(\pi^+ p)$ will probably have the *same sign, at least for small t.*

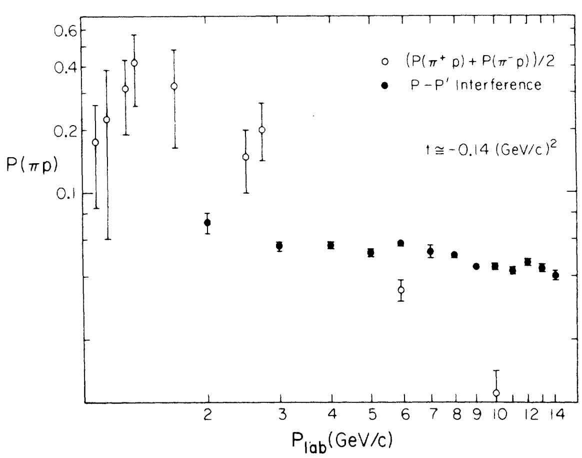

FIG. 9. Comparison of the energy dependence of $\frac{1}{2}[P(\pi^- p) + P(\pi^+ p)]$ (○) and a typical P-P' interference term (●) at $t \cong -0.14$ $(\mathrm{GeV}/c)^2$. The data were taken from Ref. 24. The P-P' term was calculated with the parameters of Ref. 10.

2. Polarization in pp Scattering

pp polarization is not well understood as yet. Odorico, Garcia, and Garcia-Canal[3] have noticed the rapid falloff of the polarization as a function of energy. Their conclusion is that the P' and the ω contributions both conserve s-channel helicities. It is based on the following assumptions: (i) The Pomeranchuk contribution conserves s-channel helicities. (ii) The ρ and A_2 contributions to pp scattering are small.

The second assumption is probably true for the helicity-nonflip amplitudes which can be estimated by studying differential cross sections. We try to estimate the contribution of the ρ and A_2 to the s-channel single-helicity-flip amplitude, $F_{++;+-}$, using the universality of the ρ coupling and the fact that in $K^- p \to \bar{K}^0 n$ and $K^+ n \to K^0 p$ (which get contributions from the approximately degenerate ρ and A_2) the differential cross sections are dominated by the helicity-flip amplitude.[25] We hope to get a reasonable estimate since these two reactions are among those for which a pure Regge model seems to work quite well.[26]

The polarization is given in terms of the s-channel helicity amplitudes by

$$\frac{d\sigma}{d\Omega}(pp)P(pp) = \mathrm{Im}F_{++;+-}(F_{++;++} + F_{++;--} + F_{+-;+-} - F_{+-;-+})^* . \tag{20}$$

We propose a model in which the ρ and A_2 con-

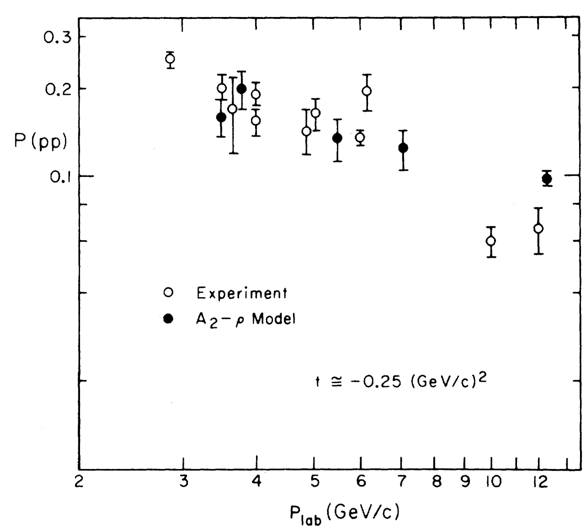

FIG. 10. Comparison of the p-p polarization data (Refs. 24 and 27) (○), and an "A_2-ρ" model (●), at $t \cong -0.25$ $(\mathrm{GeV}/c)^2$. The comparison shows that an important contribution from leading trajectories is not ruled out by data.

tributions dominate the single-flip amplitude, and the Pomeranchuk contribution conserves s-channel helicities. With these assumptions, and using the universality of the ρ coupling, the pp polarization becomes

$$P^{(A_2-\rho)}(pp) \cong \left(\frac{d\sigma/dt(KN_{\rm CEX})}{d\sigma/dt(pp)}\right)^{1/2} \frac{1}{\sin\chi}, \tag{21}$$

where χ is the p-p crossing angle,

$$\sin\chi = \left(\frac{1+t/(s-4m^2)}{1-t/4m^2}\right)^{1/2}.$$

In Fig. 10 we compare the predictions of this model with experimental data at $t \cong -0.25$ (GeV/c)^2.

The polarization data were taken from Benary *et al.*[27] and Dick.[24] The pp differential cross sections were taken from Ref. 27, and the KN charge-exchange (CEX) data from the sources listed in Ref. 28.

The model seems to fit the data quite well. It should be realized that we are not proposing this model as the only explanation of p-p polarization. Even a discrepancy of, say, a factor of -2 between the predictions of the model and the data would be sufficient for our purpose. It is evident that the existing data do not rule out the possibility of an important S^{α_R} contribution to the single-helicity-flip amplitude [where α_R is a trajectory function, e.g., $\alpha_R(t) \cong 0.5+t$]. Therefore, only a detailed study can show whether the P' and ω do, or do not, contribute to the s-channel single-helicity-flip amplitude. The sharp energy dependence of the polarization may be a true feature or a consequence of a nonleading term whose contribution is important only at low energies. The most one can say is that the data can be consistently explained under the assumption of s-channel helicity conservation for the P, P', and ω.

IV. REMARKS AND CONCLUSIONS

The study presented here indicates that most existing phenomenological tests for the helicity structure of the P and P' contributions are too crude, and one certainly needs more accurate data in order to solve this problem.

(i) *The Pomeranchuk contribution.* The various duality tests seem, altogether, to indicate s-channel helicity conservation, but they all depend on the specific phase-shift solution one uses. The FESR calculations of Barger and Phillips[10] and Dass and Michael[12] are consistent with s-channel helicity conservation ($A'_{P+P'} \cong \nu B_{P+P'}$), while the calculation of Phillips and Ringland[11] seems to prefer t-channel helicity conservation. The model-dependent calculation of Harari and Zarmi[8] seems to indicate s-channel helicity conservation. A direct study of partial waves[4] does the same.

Höhler and Strauss[29] have shown that existing data are consistent with the decoupling of the $P+P'$ contribution from $A^{(+)}$. They found that $A^{(+)}$ can probably satisfy an unsubtracted dispersion relation, which implies that its extracted values can be consistently described by a low-lying term (having $\alpha < 0$).

A model-independent test for s-channel helicity conservation by the Pomeranchuk contribution is the measurement of the R parameter in $\pi^- p$ scattering.[19] Further measurements of this quantity are necessary.

(ii) *The* P'. The only direct evidence for s-channel helicity conservation by the P' is the various FESR calculations which do not separate the P from the P'.[10-12] On the other hand, resonance-saturated FESR calculations[7,8,12] contradict this conclusion, at least in the lowest-moment FESR. FESR calculations of the ω residue functions in KN scattering[12,22] indicate s-channel helicity conservation for the ω contribution. This, coupled with the expected cancellation of the imaginary parts of the P' and ω contributions in K^+p scattering, implies s-channel helicity conservation for the P'.

Mirror symmetry in the $\pi^\pm p$ polarization implies s-channel helicity conservation for the P', provided the same is assumed for the Pomeranchuk contribution. If a similar symmetry is found in the R parameter (when it is measured for $\pi^+ p$), then s-channel helicity conservation for the $P+P'$ will be preferred, as t-channel conservation for one, or both, does not predict such a symmetry.

(iii) *The meson vertex.* Throughout this paper, we have only dealt with the baryon vertex. As for the meson vertex, the work of Gilman *et al.*[1] shows that in $\gamma p \to \rho^0 p$ s-channel helicity conservation is preferred. On the other hand, the analysis of the diffractive production of the A_1 in $\pi N \to A_1 N$,[30] seems to prefer t-channel helicity conservation. This may be due to the importance of one-pion exchange.[31,32]

(iv) *Theoretical models.* Hara[31,33] proposes s-channel helicity conservation for the Pomeranchuk contribution at high energies in a model in which the interaction is symmetric under chiral SU(2) $\times$ SU(2), the nucleon transforms linearly under the group transformations and the high-energy scattering is dominated by Regge poles with natural parity.

In the dual-loops model[34] it is shown that the Pomeranchuk contribution is coupled to external particles by an f-dominated form factor. This implies that both have the same helicity structure.

The same conclusion is reached by Carlitz *et al.*[35] in a model which uses duality and unitarity for the construction of the Pomeranchuk contribution.

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