amplitude as defined in Eq. (1) is indicated by an $\mathfrak{M}$ preceding the parentheses. Thus the process is designated by a schematic rapidity plot. [For a discussion of rapidity variables, see R. P. Feynman, in *High Energy Collisions*, Third International Conference held at State University of New York, Stony Brook, 1969, edited by C. N. Yang, J. A. Cole, M. Good, R. Hwa, and J. Lee-Franzini (Gordon and Breach, New York, 1969).] Each vertical line represents a large spacing in rapidity. Inessential labels can be dropped without confusion. Thus $(a{:}c|b)$ becomes $(a{:}c)$ when particle b is understood to remain fixed. For the sake of brevity, the $\mathfrak{M}$ will often be deleted in various relations, but the relations are always for the amplitudes or the asymptotic amplitudes.

[7]The terminology stems from the analog in atomic physics to ionization. It would perhaps be better to call it hadronization. It is awkward to speak of the pionization of the proton, but even more so to talk about the anti-omega-minus-ization.

[8]It is common practice to define the pionization limit as $s\to\infty$ for fixed $p_\perp$ and fixed $p_\parallel$. This corresponds to $|t|/\sqrt{s}$ and $|u|/\sqrt{s}$ fixed. The beauty of Mueller's analysis is that the result, Eq. (9), is valid more generally, that is, no matter how $E_c/\sqrt{s}$ tends to zero. Moreover, to the extent the Regge-pole description is valid, the dependence on $p_\parallel$ is determined explicitly by the secondary trajectories in Eq. (3). See H. D. I. Abarbanel, Phys. Rev. D $\underline{3}$, 2227 (1971).

[9]M. Holder *et al.*, Phys. Letters $\underline{35B}$, 355 (1971).

[10]M.-S. Chen *et al.*, Phys. Rev. Letters $\underline{26}$, 1585 (1971).

[11]This decomposition can easily be generalized to involve a sum over isospins in the $b\bar{b}$ channel. Since most of the interesting applications have already been dealt with in Sec III, we have omitted this generalization.

[12]The precise statement is as follows: As $s\to\infty$, the mean multiplicity of pions of charge c can be written as $\langle n_c\rangle = A_c \ln s + B_c$. The statement is that $2\langle n_0\rangle = \langle n_+\rangle + \langle n_-\rangle$. In fact, the coefficient A_c of the logarithm is determined entirely by the pionization region. Consequently, as discussed in Sec. VI, $A_0 = A_+ = A_-$.

[13]See, for example, L. Caneschi and A. Schwimmer, Phys. Rev. D $\underline{3}$, 1588 (1971).

[14]See the summary in P. Carruthers, *Introduction to Unitary Symmetry* (Interscience, New York, 1966).

[15]C. Michael, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, Berlin, 1970), Vol. 55; B. Kayser (unpublished).

[16]J. V. Allaby *et al.*, Phys. Letters $\underline{30B}$, 500 (1969).

[17]Our notation and conventions for $\overline{SU_3}$ Clebsch-Gordan coefficients are as in J. J. DeSwart, Rev. Mod. Phys. $\underline{35}$, 916 (1963). For a summary, see Chap. 4 of Carruthers, Ref. 14.

[18]This follows from time-reversal invariance applied to the nondiagonal processes $a\bar{c}b \to d\bar{e}f$.

[19]S. Stone *et al.*, Phys. Rev. D (to be published).

[20]This is not to say that most of the pions produced asymptotically are pionization products, since, as Mueller (Ref. 2) showed, to obtain $\ln s$ growth one must include fragmentation products. The interpretation of the growth of multiplicity depends sensitively on the border between fragmentation and pionization.

[21]The relation for the meson case is likely to be broken more than for the baryon case, but perhaps improved agreement will be obtained if the squares of the differential inclusive cross sections are used.

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Radiative Corrections to the $\Sigma \to \Lambda e^+e^-$ Decay*

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At present, nothing is known experimentally about the slope of the dominant form factor in the decay $\Sigma \to \Lambda e^+e^-$. However, it is probably so small that radiative corrections to the decay are an essential complication. We calculate the soft-photon radiative corrections to the Dalitz plot and the spectrum in the mass of the electron-positron pair. The correction to the total decay rate involving hard photons is also evaluated.

I. INTRODUCTION

The decay $\Sigma \to \Lambda e^+e^-$ has been the subject of several theoretical and experimental papers. Until recently the main interest in this decay was to check the relative parity of the Σ and Λ particles. Theoretical analysis was given for both even and odd relative parities, and the details can be found in the papers of Feldman and Fulton,[1] Gatto,[2] Feinberg,[3] Byers and Burkhardt,[4] Michel and Rouhaninejad,[5] and Evans.[6] Experiments by Courant *et al.*[7] and by Alff *et al.*[8] established the relative parity to be even. However, there has been no attempt to obtain detailed information on the form factors involved in the decay.

In 1965 Bernstein, Feinberg, and Lee[9] suggested that this decay is particularly suited to testing time-reversal invariance of the electromagnetic interactions. The average value of the Λ polarization along the normal to the decay plane should be zero if the interaction conserves time-reversal invariance. Experiments reported by Glasser *et al.*[10]

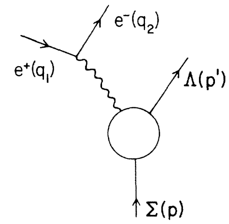

FIG. 1. The basic diagram for the differential decay rate. The wavy line denotes the photon while the solid lines denote hadrons and leptons.

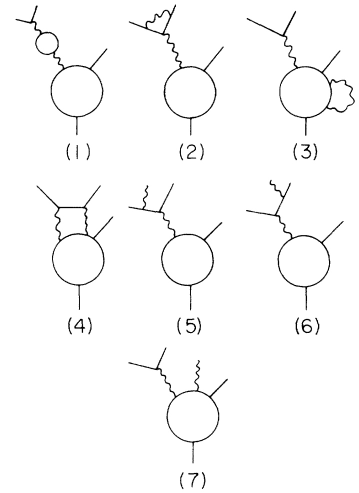

FIG. 2. The radiative corrections to the differential decay rate.

and by Baggett *et al.*[11] have been consistent with no time-reversal violation and give polarizations of 0.020 ± 0.020 and 0.03 ± 0.06, respectively.

Future work to measure the slope of the form factors or to measure the polarization more accurately clearly requires a detailed knowledge of the radiative corrections to this decay. Such second-order effects have not been calculated for this particular decay and that is the reason for reporting them here. The general problem of radiative corrections to decays $A \rightarrow B e^+ e^-$, where A and B are hadrons, has been examined by Lautrup and Smith,[12] so we can make this paper relatively brief. We do, however, outline the general assumptions and give some details regarding the matrix element.

Our calculation is based upon a set of one-photon-exchange diagrams. The basic process, depicted in Fig. 1, depends only upon the transition $\Sigma \rightarrow \Lambda\gamma$ with an off-mass-shell photon. Radiative corrections to this diagram are drawn in Fig. 2. The graphs which involve corrections to the electron and photon lines, namely, (1), (2), (5), and (6), can be calculated exactly. However, graph (4) involves two-photon-exchange contributions which bring in additional complications. Such graphs have been examined by Brown[13] for the case of electron-proton scattering. He showed that they do not lead to mass singularities as the mass of the electron is set equal to zero, and we are therefore justified in dropping the contribution of (4) relative to that of (1), (2), (5), and (6). Graph (7) is dominated by bremsstrahlung from the Σ or Λ magnetic moments and, because the masses of these particles are large, will be much smaller than the corresponding bremsstrahlung contributions from (5) and (6). Graph (3) is a radiative correction to the form factors at the $\Sigma\Lambda\gamma$ vertex and will therefore not influence a determination of the *slope* of the form factors. Thus, the calculation reported in Ref. 12 can be directly applied to the decay $\Sigma \rightarrow \Lambda e^+ e^-$ after we have defined the vertex with an off-mass-shell photon.

In Sec. II we set up some preliminary notation and discuss the general form of the matrix elements for $\Sigma \rightarrow \Lambda\gamma$ and $\Sigma \rightarrow \Lambda e^+ e^-$. The radiative corrections to the total decay rate for $\Sigma \rightarrow \Lambda e^+ e^-$ are given in Sec. III. This calculation is valid for hard photons. The general expression for the differential radiative correction is very complicated but has a particularly simple form in the region where the emitted photon is soft and the mass of the exchanged photon is large compared to the electron mass. Fortunately these conditions are valid in the region of experimental interest, so we are able to give the correction to almost all of the Dalitz plot. However, in view of our approximation we have no check that the total integrated corrections correspond to the value given in Sec. III. These points are discussed in Sec. IV.

II. THE DECAYS $\Sigma \rightarrow \Lambda\gamma$ and $\Sigma \rightarrow \gamma e^+ e^-$

For convenience the kinematics of the general process $\Sigma \rightarrow \Lambda e^+ e^-$ is given in Fig. 1. We use the notation m_Σ, m_Λ, and m to denote the masses of the Σ, Λ, and electron, respectively. We take the case of positive relative parity between the Σ and Λ hyperons. It is also convenient to introduce the following combinations of masses:

$$\Delta = m_\Sigma - m_\Lambda\,, \quad M = \tfrac{1}{2}(m_\Sigma + m_\Lambda)\,, \quad \text{and} \quad \rho = \frac{m_\Sigma + m_\Lambda}{m_\Sigma - m_\Lambda} > 1\,.$$

Our notation for dimensionless parameters used in the text is summarized in Table I. The most general form of the hadronic vertex satisfying gauge invariance is

$$\langle \Lambda(p')|J_\mu|\Sigma(p)\rangle = eM_\mu = e\bar{u}_\Lambda(p')\left[\left(\gamma_\mu\frac{k^2}{M^2} - k_\mu\frac{\Delta}{M^2}\right)F_1(k^2) + i\frac{\sigma_{\mu\nu}}{M}k^\nu F_2(k^2)\right]u_\Sigma(p)\,, \tag{2.1}$$

where the dimensionless form factors are denoted by $F_1(k^2)$ and $F_2(k^2)$, and $k=p-p'$. These form factors are relatively real, as we assume time-reversal invariance to be valid. If we form the tensor

$$M_{\mu\nu}(p',p) = \sum_{\text{spins of }\Lambda,\Sigma}\langle\Lambda|J_\mu|\Sigma\rangle\langle\Lambda|J_\nu|\Sigma\rangle^*\,,$$

then, in terms of kinematic singularity-free amplitudes M_1 and M_2,

$$M_{\mu\nu}(p',p) = (k_\mu k_\nu - k^2 g_{\mu\nu})M_1(k^2) - [g_{\mu\nu}(p\cdot k)^2 - (k_\mu p_\nu + p_\mu k_\nu)p\cdot k + p_\mu p_\nu k^2]M_2(k^2)\,. \tag{2.2}$$

M_1 and M_2 are functions of the squared photon momentum k^2. It is more convenient however, to decompose $M_{\mu\nu}$ into transverse and longitudinal parts, defined by

$$M_T(k^2) = k^2 M_1(k^2) + (p\cdot k)^2 M_2(k^2)\,,$$

$$M_L(k^2) = k^2 M_1(k^2) + k^2 m_\Sigma^{\,2} M_2(k^2)\,.$$

After some algebra, using the projection operators in Ref. 12, we find

$$\begin{aligned} M_T(k^2) = {}& -\frac{k^2}{M^2}[4m_\Sigma m_\Lambda + 2(3m_\Sigma^{\,2} - m_\Lambda^{\,2} + k^2)]F_2^{\,2}(k^2) + \frac{8k^2}{M^3}[m_\Sigma k^2 - \Delta(m_\Sigma^{\,2} + k^2 - m_\Lambda^{\,2})]F_1(k^2)F_2(k^2) \\ & + \frac{2k^2}{M^4}[k^2(3m_\Sigma + m_\Lambda)\Delta + (m_\Sigma^{\,2} - m_\Lambda^{\,2})^2]F_1^{\,2}(k^2) + \frac{2}{M^2}(m_\Sigma^{\,2} + k^2 - m_\Lambda^{\,2})^2[F_2^{\,2}(k^2) - (k^2/M^2)F_1^{\,2}(k^2)], \end{aligned} \tag{2.3}$$

$$\begin{aligned} M_L(k^2) = {}& -\frac{k^2}{M^2}[4m_\Sigma m_\Lambda + 2(3m_\Sigma^{\,2} - m_\Lambda^{\,2} + k^2)]F_2^{\,2}(k^2) + \frac{8k^2}{M^3}[m_\Sigma k^2 - \Delta(m_\Sigma^{\,2} + k^2 - m_\Lambda^{\,2})]F_1(k^2)F_2(k^2) \\ & + \frac{2k^2}{M^4}[k^2(3m_\Sigma + m_\Lambda)\Delta + (m_\Sigma^{\,2} - m_\Lambda^{\,2})^2]F_1^{\,2}(k^2) + \frac{8k^2}{M^2}m_\Sigma^{\,2}[F_2^{\,2}(k^2) - (k^2/M^2)F_1^{\,2}(k^2)]\,. \end{aligned} \tag{2.4}$$

The decay rates for $\Sigma\to\Lambda\gamma$ and $\Sigma\to\Lambda e^+e^-$ are now simple functions of M_T and M_L. For a massive photon we have

$$\Gamma_0(k^2) = \frac{\alpha}{8m_\Sigma^{\,3}}[(m_\Sigma + m_\Lambda)^2 - k^2]^{1/2}[(m_\Sigma - m_\Lambda)^2 - k^2]^{1/2}[2M_T(k^2) + M_L(k^2)]$$

or, using the notation $x = k^2/\Delta^2$,

$$\Gamma_0(x) = \frac{\alpha\Delta^2}{8m_\Sigma^{\,3}}[(1-x)(\rho^2 - x)]^{1/2}[2M_T(x) + M_L(x)]\,. \tag{2.5}$$

When the photon is on its mass shell, then $M_L(k^2)=0$ and we find

$$\Gamma_0(0) = 4\alpha M(\Delta/m_\Sigma)^3 F_2^{\,2}(0)\,. \tag{2.6}$$

The differential decay rate for $\Sigma\to\Lambda e^+e^-$, in terms of x and the energy partition between the electrons in the Σ rest system,

$$y = \frac{E_1 - E_2}{|\vec{q}_1 - \vec{q}_2|}\,,$$

is given by the well-known expression[14] ($r = 2m/\Delta$)

$$\frac{d^2\Gamma(\Sigma\to\Lambda e^+e^-)}{dx\,dy} = \frac{\alpha^2}{32\pi}\frac{\Delta^2}{m_\Sigma^{\,3}}\frac{[(1-x)(\rho^2-x)]^{1/2}}{x}\left[\left(1 + y^2 + \frac{r^2}{x}\right)M_T + (1-y^2)M_L\right]. \tag{2.7}$$

The expressions (2.3) and (2.4), when substituted into (2.7), can be checked against the expression given by Baggett.[11] In the limit where we drop all terms proportional to the electron mass and neglect all terms of order $(\Delta/M)^2$, we reproduce the result of Evans.[6] [Note that Eq. (4) of this reference contains misprints. The correct formula is given in Ref. 8.]

III. RADIATIVE CORRECTION TO THE TOTAL DECAY RATE FOR $\Sigma\to\Lambda e^+e^-$

In Ref. 12 we showed that the radiative correction to the total decay rate for $A\to Be^+e^-$ can be simply obtained from the properties of the vacuum-polarization tensor. The general formula, through terms in α^2, is given by

$$\frac{\Gamma(\Sigma\to\Lambda e^+e^-)}{\Gamma(\Sigma\to\Lambda\gamma)}=\frac{\alpha}{\pi}\left[\frac{2}{3}\ln\left(\frac{\Delta}{m}\right)-\frac{5}{9}+\frac{1}{3}I_1\right]+\left(\frac{\alpha}{\pi}\right)^2\left[\frac{4}{9}\ln^2\left(\frac{\Delta}{m}\right)+\left(\frac{4}{9}I_1-\frac{13}{54}\right)\ln\left(\frac{\Delta}{m}\right)+\zeta(3)-\frac{\pi^2}{27}+\frac{65}{648}-\frac{13}{108}I_1-\frac{2}{9}I_2\right], \quad (3.1)$$

where $\zeta(3)=1.202\,056\,9$ is the Riemann ζ function,

$$I_1=\int_0^1\frac{dx}{x}[K(x)-1], \quad (3.2)$$

$$I_2=\int_0^1\frac{dx}{x}\ln\left(\frac{1}{x}\right)[K(x)-1], \quad (3.3)$$

and $K(x)=\Gamma_2(x)/\Gamma_2(0)$. Hence, from Eqs. (2.5), (2.6), (2.3), and (2.4) we can determine the integrals I_1 and I_2, and thereby the expression (3.1). In deriving (3.1) we neglected terms $\alpha(m/\Delta)^2$ relative to α, and $\alpha^2(m/\Delta)$ relative to α^2. In our case $m/\Delta=\frac{1}{154}$, so the approximation should be very good. To proceed further we must parametrize the form factors $F_1(k^2)$ and $F_2(k^2)$ so that we can carry out the integrations.

Theoretical arguments have been given by Feldman and Fulton[1] and by Evans[6] to justify neglecting F_1 relative to F_2 in the expression for the matrix element. We expect that the size of F_1 is probably of the same order of magnitude as the size of the *slope* of F_2. Hence, in a Taylor-series expansion we retain only the terms

$$F_2(x)=F_2(0)(1+ax), \quad (3.4)$$

$$F_1(x)=bF_2(0). \quad (3.5)$$

The quantity $F_2(0)$ therefore cancels when we form the branching ratio $\Gamma(\Sigma\to\Lambda e^+e^-)/\Gamma(\Sigma\to\Lambda\gamma)$. The small quantities a and b are now regarded as parameters to be obtained from experiment. In the expression for the branching ratio we retain only the first-order terms in a and b. Clearly, to distinguish a from b experimentally we need measurements of the differential decay rate. Having chosen this parametrization of the form factors, we can go ahead and calculate the integrals I_1 and I_2. First we find new expressions for M_T and M_L as functions of x, i.e.,

$$M_T(x)=8\Delta^2F_2{}^2(0)\left[(1-x)(1+2ax)+4b\frac{x}{\rho^2}\left(x-\frac{x}{\rho}-2\right)\right], \quad (3.6)$$

$$M_L(x)=8\Delta^2F_2{}^2(0)\left[\frac{x}{\rho^2}(1-x)(1+2ax)+4b\frac{x}{\rho^2}\left(x-\frac{x}{\rho}-2\right)\right]. \quad (3.7)$$

Remembering that $\rho^{-2}=4\Delta^2/M^2$, we see that M_L is in general very small. Similarly the term proportional to b in M_T will be very small. To a very good approximation we could retain only the first term on the right-hand side of Eq. (3.6). However, to have more accurate results we have retained all terms in the subsequent calculations. The expression for $K(x)$ is now

$$K(x)=\frac{[(1-x)(\rho^2-x)]^{1/2}}{\rho}\left[(1-x)\left(1+\frac{x}{2\rho^2}\right)+2ax(1-x)\left(1+\frac{x}{2\rho^2}\right)+\frac{6bx}{\rho^2}\left(x-\frac{x}{\rho}-2\right)\right]. \quad (3.8)$$

The integrals I_1 and I_2 can now be integrated analytically, but the algebra became so tedious that we resorted to computer evaluation, finding

$$I_1=-1.2804+0.8000a-0.0072b, \quad (3.9)$$

$$I_2=-1.3967+1.3439a-0.0102b. \quad (3.10)$$

Hence, if we retain the terms in (3.1) proportional to α, αa, αb, and α^2, the branching ratio becomes

$$\frac{\Gamma(\Sigma\to\Lambda e^+e^-)}{\Gamma(\Sigma\to\Lambda\gamma)}=(5.532+0.627a-0.006b)\times10^{-3}. \quad (3.11)$$

Without any radiative correction this expression would have the value

$$\frac{\Gamma(\Sigma\to\Lambda e^+e^-)}{\Gamma(\Sigma\to\Lambda\gamma)}=(5.486+0.620a-0.006b)\times10^{-3}, \quad (3.12)$$

so the radiative correction is approximately 1%. To a very good approximation we are therefore

TABLE I. Dimensionless parameters used in the text.

$\Delta=m_\Sigma-m_\Lambda$	$M=\frac{1}{2}(m_\Sigma+m_\Lambda)$
$\rho=\dfrac{m_\Sigma+m_\Lambda}{m_\Sigma-m_\Lambda}$	$r=\dfrac{2m}{\Delta}$
$x=\dfrac{(q_1+q_2)^2}{\Delta^2}=\dfrac{k^2}{\Delta^2}$	$y=\dfrac{E_1-E_2}{\lvert\vec{q}_1-\vec{q}_2\rvert}$
$\sigma=\dfrac{\rho+x}{[(1-x)(\rho^2-x)]^{1/2}}$	$\beta^2=1-\dfrac{r^2}{x}$

justified in neglecting b completely. In fact, higher terms in the Taylor-series expansion for $F_2(x)$ will probably be of the same order of magnitude. When $a=b=0$ the basic branching ratio in (3.12) is in agreement with the value $\frac{1}{185}$ obtained by Evans.[6] For comparison we also list here the value of the branching ratio obtained by numerical double integration over the Dalitz plot with no radiative correction:

$$\frac{\Gamma(\Sigma\rightarrow\Lambda e^+e^-)}{\Gamma(\Sigma\rightarrow\Lambda\gamma)}=(5.494+0.619a-0.006b)\times 10^{-3}\,, \tag{3.13}$$

which agrees almost identically with (3.12).

IV. RADIATIVE CORRECTIONS TO THE DALITZ PLOT AND THE PHOTON SPECTRUM

The $\Sigma\rightarrow\Lambda e^+e^-$ Dalitz plot in terms of the variables x and y is bounded by

$$1\geqslant x\geqslant r^2=\left(\frac{2m}{\Delta}\right)^2,$$

$$\beta\geqslant y\geqslant -\beta\,,$$

where

$$\beta^2=1-\frac{4m^2}{k^2}=1-\frac{r^2}{x}\,.$$

To order α^2 the expression for the matrix element is given by Eq. (2.7). We now assume, as in Ref. 12, that the soft-photon higher-order corrections are given by the graphs (1), (2), (5), and (6) of Fig. 2. The radiative correction to the Dalitz plot is thus defined by

$$\frac{d^2\Gamma^{\mathrm{rad}}}{dx\,dy}=\delta(x,y)\frac{d^2\Gamma}{dx\,dy}\,, \tag{4.1}$$

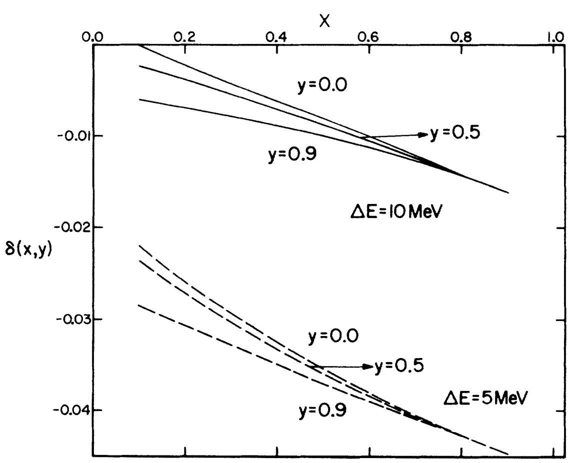

FIG. 3. $\delta(x,y)$ as a function of x for fixed values of y. The solid lines are for $\Delta E=10$ MeV and the dashed ones for $\Delta E=5$ MeV.

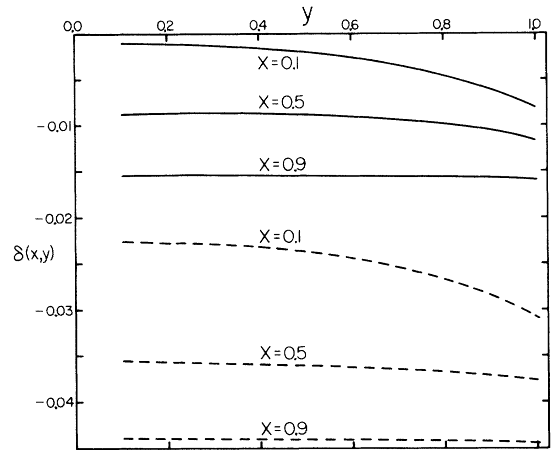

FIG. 4. $\delta(x,y)$ as a function of y for fixed values of x. The solid lines are for $\Delta E=10$ MeV and the dashed ones for $\Delta E=5$ MeV.

where, from Ref. 12,

$$\delta(x,y)=\frac{\alpha}{\pi}\left[-\left(\ln\frac{\Delta^2x}{m^2}-1\right)\left(\ln\frac{\Delta^2(\sigma^2-y^2)x}{4\Delta E^2(\sigma^2-1)}-\frac{13}{6}\right)-\frac{17}{18}-\tfrac{1}{2}\ln^2\!\left(\frac{\sigma+y}{\sigma-y}\right)+\mathrm{Li}_2\!\left(\frac{1-y^2}{\sigma^2-y^2}\right)-\frac{\pi^2}{6}\right]. \tag{4.2}$$

The quantity ΔE is the photon cutoff energy, and σ is defined by the expression

$$\sigma=(\rho+x)[(1-x)(\rho^2-x)]^{-1/2}\,. \tag{4.3}$$

The function denoted by $\mathrm{Li}_2(x)$ is the dilogarithmic function as defined by Lewin.[15] We have plotted $\delta(x,y)$ for two values of the photon cutoff energy, $\Delta E=5$ MeV and 10 MeV, in Fig. 3. The curves are drawn for various values of y. Because Eq. (4.2) is only valid for $x>>r^2$, we cannot extrapolate the

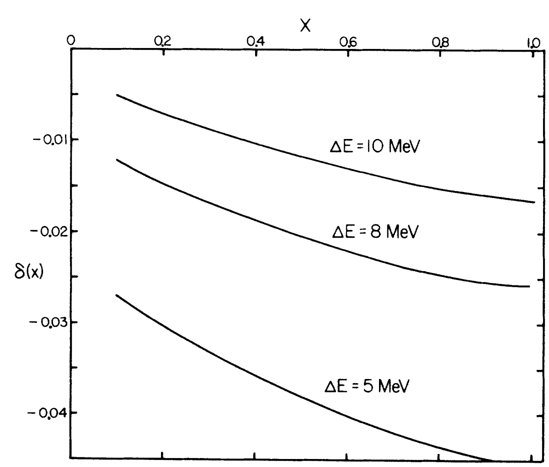

FIG. 5. $\delta(x)$ as a function of x for values of ΔE.

curves to small values of x. Figure 4 gives the results for various values of x.

In a similar fashion we can obtain the radiative corrections to the virtual-photon spectrum. Integration of Eq. (2.7) over the variable y yields

$$\frac{d\Gamma}{dx}=\frac{\alpha}{3\pi}\frac{\beta}{x}\left(1+\frac{r^2}{2x}\right)\Gamma_0(x)\,, \tag{4.4}$$

and we now define the radiative corrections to the spectrum by

$$\frac{d\Gamma^{\rm rad}}{dx}=\delta(x)\frac{d\Gamma}{dx}. \tag{4.5}$$

From Ref. 12 we have immediately an expression for $\delta(x)$, namely,

$$\begin{aligned}\delta(x)&=f_1(x)+\frac{M_L}{2M_T+M_L}f_2(x)\\&=f_1(x)+\left(\frac{x}{2\rho^2+x}\right)f_2(x)\,,\end{aligned} \tag{4.6}$$

because $M_L(x)$ and $M_T(x)$ are given by (3.6) and (3.7) with $a=b=0$. [However, the full expressions for $M_L(x)$ and $M_T(x)$ should be inserted in (4.4) to give the a and b dependence of the spectrum in x.] The functions $f_1(x)$ and $f_2(x)$ are defined by

$$f_1(x)=\frac{\alpha}{\pi}\left[-\left(\ln\frac{\Delta^2x}{m^2}-1\right)\left(\ln\frac{\Delta^2x}{4\Delta E^2}-\frac{13}{6}+\frac{1}{3}f(\sigma)+\sigma\ln\frac{\sigma+1}{\sigma-1}-2\right)-\frac{17}{18}-\frac{\pi^2}{6}+f(\sigma)\right]\,,$$

$$f_2(x)=\frac{\alpha}{\pi}\left(\ln\frac{\Delta^2x}{m^2}-4\right)f(\sigma)\,,$$

where

$$f(\sigma)=1-\tfrac{3}{2}\sigma^2+\tfrac{3}{4}(\sigma^2-1)\sigma\ln\frac{\sigma+1}{\sigma-1}.$$

In Fig. 5 we plot $\delta(x)$ for various values of ΔE. The sign of the soft-photon corrections is always negative for the ranges of ΔE considered here. This does not contradict the fact that the total correction to the decay rate is positive, because we have omitted both the hard-photon corrections to the Dalitz plot and the soft-photon corrections for very small x. Hard photons must be detected experimentally before a value of ΔE can be ascertained. They may come from diagrams which do not contain the simple $\Sigma\to\Lambda\gamma$ coupling. We close by making the obvious comment that experiments which do not measure photons but only detect charged particles need a different radiative correction from the one reported here. Such experiments need hard-photon corrections as well as soft-photon corrections.

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$$\begin{aligned}M_{\mu\nu}(p_A,p_B)={}&(k_\mu k_\nu-g_{\mu\nu}k^2)M_1(k^2)\\&+[g_{\mu\nu}(p_A\cdot k)^2-(k_\mu p_{A\nu}+p_{A\mu}k_\nu)p_A\cdot k\\&+p_{A\mu}p_{A\nu}k^2]M_2(k^2).\end{aligned}$$

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