*Work supported in part by the U. S. Atomic Energy Commission (Report No. NYO-2262-TA-249).

[1]L. W. Jones, A. E. Bussian, G. D. DeMeester, B. W. Loo, D. E. Lyon, Jr., P. V. Ramana Murthy, R. F. Roth, J. G. Learned, F. E. Mills, D. D. Reeder, K. N. Erickson, and Bruce Cork, Phys. Rev. Letters 25, 1679 (1970).

[2]H. M. Fried and T. K. Gaisser, Phys. Rev. 179, 1491 (1969).

[3]T. K. Gaisser, Phys. Rev. D 2, 1337 (1970).

[4]H. M. Fried and K. Raman, Phys. Rev. D 3, 269 (1971).

[5]H. M. Fried and T. K. Gaisser, Phys. Rev. D 3, 224 (1971); H. M. Fried and Hector Moreno, Phys. Rev. Letters 25, 625 (1970).

[6]A. M. Shapiro, private communication about a proposed proportional-hybrid-system experiment at NAL.

[7]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969); in *High Energy Collisions*, Third International Conference held at State University of New York, Stony Brook, 1969, edited by C. N. Yang, J. A. Cole, M. Good, R. Hwa, and J. Lee-Franzini (Gordon and Breach, New York, 1969), p. 237.

[8]D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962); G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968).

[9]K. Symanzik, J. Math. Phys. 1, 249 (1960).

[10]This is true for those soft terms corresponding to linkages between different proton legs; the self-linkage terms are related to (but differ from) the wave-function renormalization constants of the model, and require a separate discussion.

[11]Direct summation of the SVM diagrams by neglecting all k^2 dependence in nucleon propagators makes explicit the relation between the exact energy-momentum-conserving δ function and the coordinate dependence in $(i\sum_{l,m}\int f_l \Delta_{(+)} f'_m)^n$. Explicitly summing over all emitted SVM's gives

$$P_n \propto \frac{1}{n!}\left[\frac{g^2}{(2\pi)^3}\right]^n \prod_{i=1}^{n} \int \frac{d^3k_i}{2\omega_i}\left(\frac{P_{3\mu}}{P_3\cdot k_i}-\frac{P_{1\mu}}{P_1\cdot k_i}+\frac{P_{4\mu}}{P_4\cdot k_i}-\frac{P_{2\mu}}{P_2\cdot k_i}\right)^2 \times |M_{\rm el}|^2\,\delta^4\left(P_1+P_2-P_3-P_4-\sum_{j=1}^{n}k_j\right),$$

where $M_{\rm el}$ is the amplitude for $P_1+P_2 \to P_3+P_4$, including all soft exchanges, and the δ function comes from the phase-space factors for the emitted SVM's. Writing an exponential representation for the δ function puts the expression in a factorized form in which an $e^{ik_i\cdot x}$ is associated with each k_i factor. The replacement $e^{ik_i\cdot x} \to 1$ required to decouple the SVM emission thus ruins the energy-momentum conservation. The subsequent (molecular-field type of) approximation may be simply defined starting from this explicit form for P_n.

[12]Let i represent the six independent variables in $p_1p_2 \to p_3p_4X$. $P_{n(i)}$ is the probability of producing n vector mesons when p_3 and p_4 go into the ith phase-space bin. $P_{(i)} = \sum_{n=0}^{\infty} P_{n(i)}$ is the total (inclusive) probability for p_3 and p_4 to go into the ith phase-space bin. Thus the average differential multiplicity has the experimental meaning $\langle \nu_i \rangle = \langle n_i \rangle / P_{(i)}$, where $\langle n_i \rangle$ is the observed average vector-meson multiplicity in the ith phase-space bin and $\sum_{{\rm all}\, i} P_{(i)} = 1$.

[13]P. V. Ramana Murthy, ANL Report No. ANL/HEP 6909, 1968 (unpublished).

[14]Don M. Tow, Phys. Rev. D 2, 154 (1970).

PHYSICAL REVIEW D VOLUME 4, NUMBER 11 1 DECEMBER 1971

Nonanalytic Behavior of the Σ Term in π-N Scattering*

Heinz Pagels
The Rockefeller University, New York, New York 10021

and

W. J. Pardee
The University of Washington, Seattle, Washington 98105
(Received 9 August 1971)

The Σ term in the πN scattering amplitude is commonly evaluated at squared nucleon momentum transfer $t = 2\mu^2$, where μ is the pion mass. Because of the nonanalytic nature of perturbations about the chiral SU(2)×SU(2) limit, $\Sigma(2\mu^2)$ differs from $\Sigma(0)$ by a term linear in μ. We calculate the difference term exactly to $O(\mu)$ and find

$$\Sigma(2\mu^2) - \Sigma(0) = \frac{3}{8\pi}\left(\frac{g_A}{2F_\pi}\right)^2 \frac{\mu}{F_\pi} + O(\mu^2 \ln\mu^2)\,.$$

This represents a 14-MeV correction to the value of $\mu^2 F_\pi \Sigma(2\mu^2)$.

It was recently observed by Li and Pagels[1] that, as a consequence of long-range forces introduced as the pion mass μ vanishes in the chiral limit, many amplitudes are not analytic about zero in μ^2 and naive expansion in the symmetry-breaking parameter fails. We find this occurs for the nucleon matrix element of the current-algebra Σ term, which we define (crossing one nucleon) by

$$\bar{v}(-\vec{p}', -s)\,\mu^2 F_\pi \Sigma(t) u(\vec{p}, s) = \tfrac{1}{3}\sum_{a=1}^{3} \langle 0 | [X_a, [X_a, H'(0)]] | \bar{N}(-p', -s) N(p, s)\rangle . \tag{1}$$

The axial charge X_a has the current-algebra isospin normalization, $t=(p'-p)^2$, and $H'(x)$ is the $SU(2)\times SU(2)$-breaking Hamiltonian density.

It is possible to show that $\Sigma(2\mu^2)$ is equal to a particular combination of on-shell pion-nucleon amplitudes at an unphysical point (which can be obtained from phase shifts) plus terms which are $O(\mu^2)$.[2] However, $\Sigma(0)$ is of perhaps greater theoretical interest, since it is what enters discussions on the dimensions of field operators and the matrix element of the energy-momentum tensor between states of a single nucleon at rest. It will be shown that the two-pion intermediate state gives a contribution

$$\Sigma(2\mu^2) - \Sigma(0) = \left(\frac{g_A}{2F_\pi}\right)^2 \frac{3\mu}{8\pi F_\pi} + O(\mu^2 \ln\mu^2) . \tag{2}$$

The pion decay constant $F_\pi \cong 93$ MeV and $g_A \cong 1.25$. No other state contributes a term larger than $O(\mu^2 \ln\mu^2)$. Note that the difference is $O(\mu)$, not $O(\mu^2)$, and that, in this case, the scale for μ is $\sim F_\pi$. The expression (2) is the leading term in an asymptotic series obtained from the dispersion relation for $\Sigma(t)$.

In order to establish Eq. (2), we insert and sum over two-pion intermediate states in the expression

$$\bar{v}(-\vec{p}')\,\mathrm{Im}\Sigma(t) u(\vec{p}) = \frac{1}{6\mu^2 F_\pi}\langle 0 | \sum_{a=1}^{3} [X_a, [X_a, H'(0)]] T^\dagger | \bar{N}(-p') N(p)\rangle . \tag{3}$$

The leading term in the matrix element of the double commutator between the vacuum and $|\pi^c \pi^d\rangle$ is $\mu^2\delta^{cd}$. The only important contribution to $T^\dagger_{\pi N}$ is the nucleon pseudovector Born term, which is exact in the chiral $SU(2)\times SU(2)$ limit. All other terms yield less singular contributions. This gives, for t near the unphysical threshold at $4\mu^2(-0)$,

$$\mathrm{Im}\Sigma(t) = \frac{6M}{(4M^2-t)^{1/2}} \frac{\theta(t-4\mu^2)}{8\pi F_\pi \sqrt{t}} (t-2\mu^2)\left(\frac{g_A}{2F_\pi}\right)^2 \times \tan^{-1}\left[\frac{(4M^2-t)^{1/2}(t-4\mu^2)^{1/2}}{t-2\mu^2}\right]. \tag{4}$$

From a once-subtracted dispersion relation we then obtain

$$\Sigma(2\mu^2) - \Sigma(0) \cong \frac{2\mu^2}{\pi}\int_{4\mu^2}^{4M^2} dt' \frac{\mathrm{Im}\Sigma(t')}{t'(t'-2\mu^2)} \equiv \frac{3\mu^2}{4\pi^2 F_\pi}\left(\frac{g_A}{2F_\pi}\right)^2 \frac{1}{\mu} J\left(\frac{\mu}{2M}\right). \tag{5}$$

The dimensionless integral $J(z)$ can be approximated by the first term of its asymptotic expansion for small z,

$$\lim_{z\to 0} J(z) = \lim_{z\to 0}\int_4^{z^{-2}} dx \frac{\tan^{-1}[(1-z^2x)^{1/2}(x-4)^{1/2}/(z(x-2))]}{x\sqrt{x}(1-z^2x)^{1/2}} = \frac{\pi}{2} . \tag{6}$$

It is not hard to obtain $J(z) = \frac{1}{2}\pi - z\ln z^2 + \cdots$, but we do not retain the nonleading terms, for there are other contributions of the same order. This establishes Eq. (2). Numerically, we have

$$\mu^2 F_\pi[\Sigma(2\mu^2) - \Sigma(0)] \cong 14 \text{ MeV} . \tag{7}$$

This should be compared with the value for $\mu^2 F_\pi \Sigma(2\mu^2)$. Cheng and Dashen[3] obtain

$$\mu^2 F_\pi \Sigma(2\mu^2) \cong 105 \pm 10 \text{ MeV} . \tag{8}$$

From the estimate of Höhler, Jakob, and Strauss[4] with $a^+_{0+} = 0$, one finds

$$\mu^2 F_\pi \Sigma(2\mu^2) \cong 58 \pm 13 \text{ MeV} , \tag{9}$$

although they point out that $\mu^2 F_\pi \Sigma(2\mu^2)$ can, in fact, be much smaller than (9) if a^+_{0+} is negative. Our correction (7) represents a 13% effect for the estimate (8) and a 25% effect for the estimate (9).

The correction obtained by including the Σ-term contribution to $T_{\pi N}$ in the absorptive part, although possessing similar formal problems, is numerically small and contributes $O(\mu^2)$ to $\Sigma(0)$. This seems to be "accidental" in the sense that the scale for μ is again F_π, and $(\mu/F_\pi)^2 \cong 2$.

The Cheng and Dashen evaluation of $\Sigma(2\mu^2)$ depends upon the assumption that their limit of the πN amplitude gives $\mu^2 F_\pi^{-1}\Sigma(=O(\mu^2))$ plus a remainder which is $O(\mu^4)$. More precisely,[2] for $t<4\mu^2$, $\mu^2 F_\pi^{-1}\Sigma(t)$ can be obtained from the experimental πN data with an error that is given by $q'_{\mu'} q_\mu A^{\mu'\mu}(q, q', p)$, where q, q' are initial and final pion momenta. The tensor $A^{\mu'\mu}$ is largely unknown, but has no pion or nucleon poles. One might worry that a phenomenon analogous to that demonstrated above could lead to a remainder of order μ^3, perhaps spoiling Cheng and Dashen's evaluation. However, study of the dispersion relation for the remainder shows that the two-pion intermediate state can produce such behavior only in the coefficient of $g^{\mu\mu'}$; but $q'_{\mu'}q_\mu g^{\mu'\mu} = \frac{1}{2}(2\mu^2 - t)$ vanishes identically at $t=2\mu^2$. Indeed, the fact that this point is special was emphasized in Ref. 2.

*Research supported in part by the U. S. Atomic Energy Commission.

[1]Ling-Fong Li and Heinz Pagels, Phys. Rev. Letters 26, 1204 (1971).

[2]Lowell S. Brown, W. J. Pardee, and R. D. Peccei, Phys. Rev. D 4, 2801 (1971). The uncertainty in $\Sigma(2\mu^2)$ as obtained from the on-shell pion-nucleon amplitude is, strictly speaking, $O(\mu^2 \ln \mu^2)$. The important fact in this development is the absence of any term which is of $O(\mu)$.

[3]T. P. Cheng and Roger Dashen, Phys. Rev. Letters 26, 594 (1971).

[4]A. G. Höhler, H. P. Jakob, and R. Strauss, Phys. Letters 35B, 445 (1971).

PHYSICAL REVIEW D VOLUME 4, NUMBER 11 1 DECEMBER 1971

Consequences of Internal Symmetries for Inclusive Processes*

Robert N. Cahn†
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720
and
Martin B. Einhorn
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 26 August 1971)

Consequences of C invariance, isospin invariance, and unitary symmetry for limiting distributions are derived. The same symmetries are used to isolate the nonscaling contributions. Many of the predictions are susceptible to direct experimental test. The relations based on unitary symmetry are expected to be violated and thus furnish extensive challenges to symmetry-breaking models. The relations based on isospin and C invariance are expected to be exact (asymptotically) at high energies.

I. INTRODUCTION

The optical theorem for two-body scattering [Fig. 1(a)] enables one to understand many features of total cross sections, even without a detailed knowledge of how they are built up by multiparticle states. A corresponding optical theorem for three-body scattering is related to the cross section for the production of a single particle of definite momentum, along with anything else – what has been called inclusive production [see Fig. 1(b)]. This application of the three-body optical[1] theorem is not straightforward, involving both crossing symmetry and careful analytic continuation. Still, the result is a conceptual and theoretical simplification similar to the one achieved through the two-body optical theorem.

Many general features of single-particle inclusive-production experiments can be understood in terms of the three-body forward scattering amplitude. From this point of view, Mueller[2] was able to show that the existence of limiting distributions in those parts of phase space referred to as the fragmentation and pionization regions is a general consequence of the dominance of Pomeranchukon exchange. These results, which will be described in Sec. II, had been established much earlier in the context of the multiperipheral model.[3] These would also appear to be satisfied in dual-resonance models,[4] although the representation of the Pomeranchukon in such models is unsettled.

Charge conjugation and isospin invariance provide useful relations among two-body total cross sections. Furthermore, these invariances allow one to eliminate or isolate crossed-channel amplitudes with certain quantum numbers, by selecting appropriate linear combinations of reactions. This is particularly useful when a Regge description is being employed. It is the purpose of this paper to extend such considerations to single-particle inclusive-production cross sections.[5] We shall also consider some consequences of SU_3 symmetry.

The outline of the paper is as follows: In Sec. II, we briefly review previous results for inclusive re-