Study of $p\pi^+\pi^-$ Isobars Produced in the $pp\pi^+\pi^-$ Final State by pp Interactions at 16 GeV/c

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We have observed enhancements at around 1450 and 1700 MeV in the $p\pi^+\pi^-$ mass spectrum for the $pp\pi^+\pi^-$ final state produced in pp interactions at 16 GeV/c. Our results show that the 1700-MeV effect does not have a dominant $\Delta^{++}\pi^-$ decay mode. An attempt has been made to determine the spin assignments. We favor $J=\frac{1}{2}$ for the 1450-MeV enhancement, assuming that this decays via the uncorrelated $p\pi^+\pi^-$ mode. For the 1700-MeV enhancement, we require contributions from one or more of the spin-parity series $\frac{3}{2}^-$, $\frac{5}{2}^+$, $\frac{7}{2}^-$ in accord with the proposed Gribov-Morrison parity rule.

I. INTRODUCTION

In recent years there have been numerous studies of the low-mass $p\pi^+\pi^-$ spectrum produced in four-body final states by pion, kaon, and proton beams over a wide range of energies.[1] Most authors agree that two enhancements are present at around 1450 and 1700 MeV. The identification of the one at 1450 MeV with the $P_{11}(1470)$ resonance reported in the phase-shift analyses[2] is still uncertain and several isobars could be contributing at 1700 MeV.

Identification of the decay modes of these resonances is equally complex. Because the 1450-MeV enhancement is just above the $\Delta^{++}(1236)\pi^-$ threshold, it is extremely difficult to separate this mode from the uncorrelated $p\pi^+\pi^-$ decay. There seems to be agreement[3,4] that its spin is consistent with $J=\frac{1}{2}$, though some admixture of $J=\frac{3}{2}$, possibly from the tail of the $D_{13}(1520)$, is likely. In the 1700-MeV region some authors[5] see no evidence for any $\Delta^{++}\pi^-$ decay in their data, while others[6] put the branching ratio at around 70%. Plano[1] has pointed out that experiments with beam momenta less than 10 GeV/c are mainly consistent with no $\Delta^{++}\pi^-$ decay, whereas those with beam momenta above 10 GeV/c show a substantial branching ratio. One can go further with this distinction and observe that these latter experiments[3,4,6] agree that a large $J=\frac{5}{2}$ (and probably $\frac{5}{2}^+$) component is present, suggestive of the $F_{15}(1688)$; whereas at low momentum[5] only $J=\frac{1}{2}$ is required, suggestive of either the $S_{11}(1700)$ or $P_{11}(1780)$. Observations[5,6] concerning a ΛK^+ decay mode of these enhancements are also in accord with this hypothesis.

II. THE ENHANCEMENTS

In this paper we present data for the reaction

$$pp \to pp\pi^+\pi^-$$

at 16.10 GeV/c. We have a total of 2817 events in this channel corresponding to a cross section of $1.67^{+0.12}_{-0.05}$ mb. Experimental details will be reported elsewhere.[7]

The $p\pi^+\pi^-$ mass spectrum with two combinations per event is shown in Fig. 1. Clear peaks centered at 1470 and 1750 MeV can be seen. The shaded histogram was obtained by plotting only that $p\pi^+\pi^-$ combination whose proton has the greater four-momentum transfer from its associated incident proton.[8] This, of course, amounts to choosing the more "peripheral" of the two possible $p\pi^+\pi^-$ combinations and reduces background from the "wrong" choice of proton.

The enhancements are mainly produced at very low $|t|$, consistent with a diffractive process. We have fitted the differential distributions in t' to a form

$$\frac{d\sigma}{dt'} = Ae^{-at'},$$

where $t' = |t - t_{\min}|$.

The slopes a obtained in the 1450- and 1700-MeV regions are 11.2 ± 0.8 and 7.6 ± 0.6 GeV^{-2}, respectively.

The $p\pi^+\pi^-$ mass distribution may be parametrized as the sum of two Breit-Wigner resonances plus a background. However, the shape and magnitude of the latter is uncertain, and so we quote errors that encompass the resonance parameters

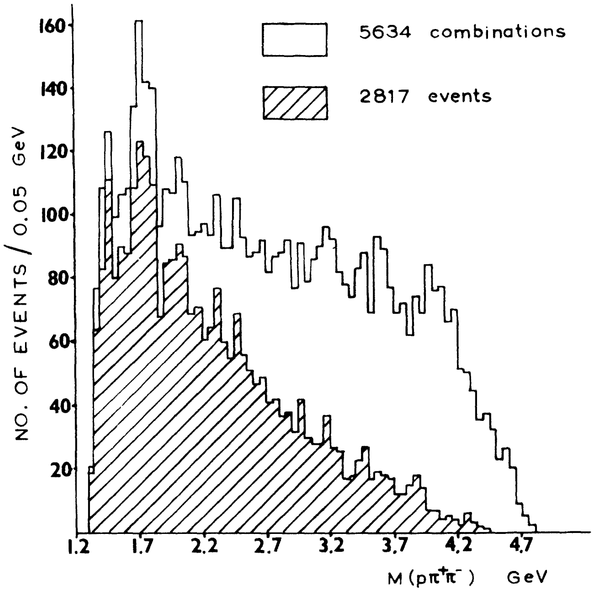

FIG. 1. The $p\pi^+\pi^-$ mass spectrum, two combinations per event. The shaded histogram corresponds to plotting the more peripheral of the two $p\pi^+\pi^-$ combinations.

obtained for different choices of background. The results are

$$M_1 = 1425 \pm 25 \text{ MeV}, \quad \Gamma_1 = 125 \pm 25 \text{ MeV},$$

$$M_2 = 1720 \pm 20 \text{ MeV}, \quad \Gamma_2 = 120 \pm 40 \text{ MeV}.$$

These values are reasonably consistent with the results of other authors,[1] but we cannot claim that either enhancement corresponds to a definite resonant state.

III. BRANCHING RATIOS

We show in Fig. 2 the $p\pi^+\pi^-$ mass spectrum when the mass of the $p\pi^+$ is required to lie in the $\Delta^{++}(1236)$ band defined by

$$1.15 \leq M(p\pi^+) < 1.30 \text{ GeV}.$$

Over half our events (1473) have at least one $p\pi^+$ combination satisfying this condition and 29 events have both.

The branching ratio r of a resonance into the $\Delta^{++}\pi^-$ state is defined as

$$r = \frac{N^{*+} \to \Delta^{++}\pi^-}{N^{*+} \to p\pi^+\pi^- \text{ (all modes)}}.$$

In the 1450-MeV region, the value of r, estimated by comparing the number of events above background in each of Figs. 1 and 2, is consistent with 100%. However, this result is not sufficient to imply an intermediate Δ^{++} state, because nearly all events are kinematically constrained to have the $p\pi^+$ mass inside the Δ^{++} mass band.

The second peak in Fig. 2 is centered about a

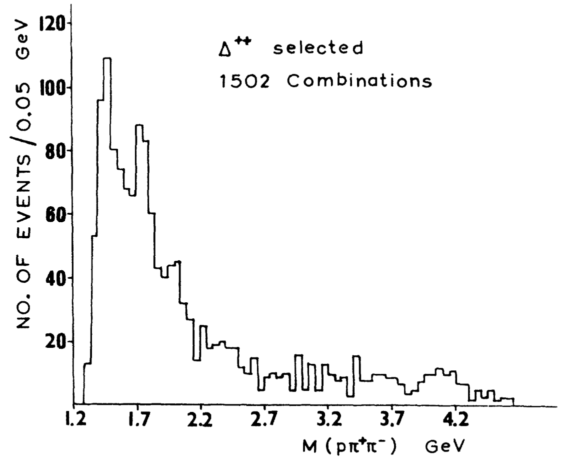

FIG. 2. The $p\pi^+\pi^-$ mass spectrum where the $p\pi^+$ mass lies in the Δ^{++} region $1.15 \leq M(p\pi^+) < 1.30$ GeV.

mass of 1750 MeV, and has Breit-Wigner parameters

$$M_2 = 1755 \pm 10 \text{ MeV}, \quad \Gamma_2 = 80 \pm 30 \text{ MeV}$$

(again the errors reflect the uncertainty in the choice of background).

There is thus some evidence that the second enhancement occurs at a slightly higher mass in Fig. 2 than in the total $p\pi^+\pi^-$ spectrum of Fig. 1. Assuming that both peaks do in fact represent the same effect, we have attempted to obtain the branching ratio of the 1700-MeV enhancement by analyzing the $p\pi^+\pi^-$ Dalitz plot using a method proposed by Barnes *et al.*[5] The total $p\pi^+\pi^-$ mass distribution was divided into intervals of 75 MeV and each bin analyzed independently.

The amplitude for the decay of the $p\pi^+\pi^-$ system is written

$$|M|^2 = \frac{x|A_\Delta|^2}{\sum_i \int |A_\Delta^i|^2 d\phi^i} + \frac{1-x}{\sum_i \int d\phi^i}, \tag{1}$$

where $\int d\phi^i$ signifies integration over the $p\pi^+\pi^-$ Dalitz plot for the ith event and the summation is over the events in the given mass interval; x is the fraction of events decaying via $\Delta^{++}\pi^-$.

The matrix element for the subsequent Δ^{++} decay is written

$$|A_\Delta|^2 = |F_{\text{BW}}|^2 \tfrac{1}{4}[(1+b) + 3(1-b)\cos^2\theta], \tag{2}$$

where $|F_{\text{BW}}|^2$ is a P-wave Breit-Wigner term, θ is the angle between the outgoing proton and Δ^{++} rest system, and b is a parameter to adjust the Δ^{++} decay distribution. We have neglected any possible decay into $\Delta^0\pi^+$, which is suppressed by a factor 9 to 1, on the assumption that the low-mass $p\pi^+\pi^-$ system has isospin $\frac{1}{2}$, consistent with a diffractive production mechanism. The fitting was performed with the maximum-likelihood technique, where the

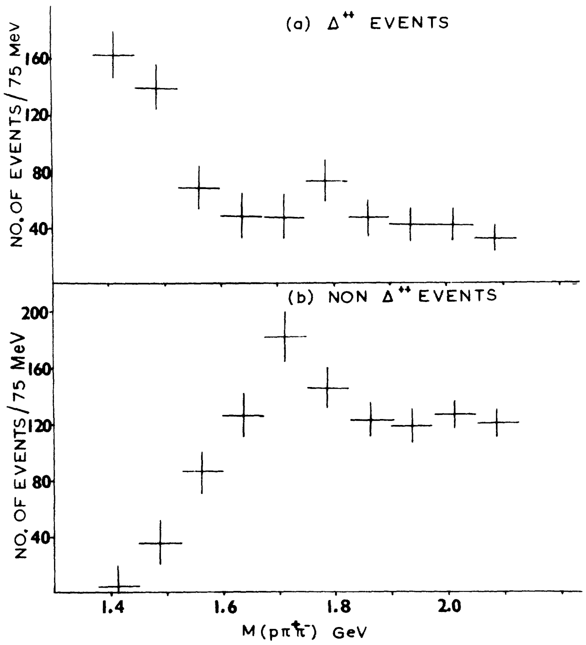

FIG. 3. (a) Number of Δ^{++} events as a function of the $p\pi^+\pi^-$ mass. (b) Number of non-Δ^{++} events. These numbers are derived from a maximum-likelihood fit using the decay amplitude of Eqs. (1) and (2) for the $p\pi^+\pi^-$ system.

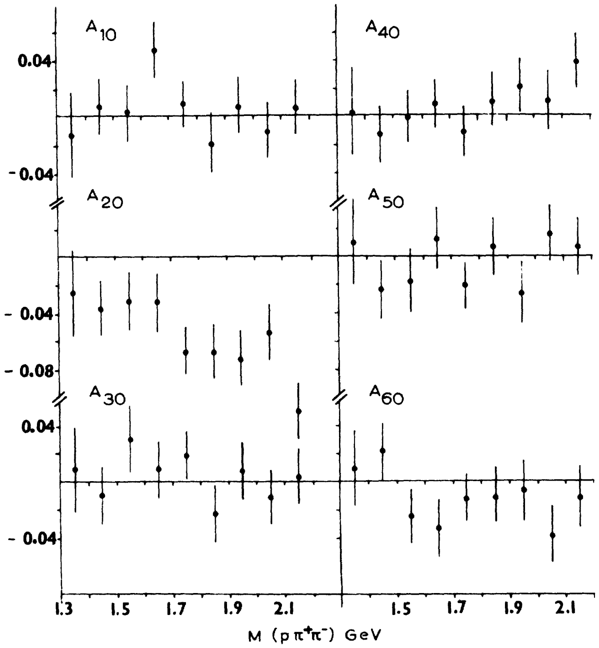

FIG. 4. Moments A_{lm}, as defined in Eq. (3), for the distribution in the normal to the decay plane.

likelihood function for the kth event is written

$$L_k = \frac{|M^k|^2}{\int |M^k|^2 \, d\phi^k} \, .$$

The function to be maximized is then $\sum_k \ln L_k$.

The numbers of Δ^{++} and non-Δ^{++} events in each mass bin, estimated from the parameter x, are shown in Fig. 3. There is evidence for a peak above background in the region of 1700 MeV for both the Δ^{++} and the uncorrelated $p\pi^+\pi^-$ decays. Again, the peak is seen at higher mass for events decaying into $\Delta^{++}\pi^-$ than for $p\pi^+\pi^-$ events.

From Fig. 3 we obtain a branching ratio, r, for the decay of the 1700-MeV enhancement of (35 ± 20)%, where the error encompasses statistics and the uncertainty in the background.

IV. ANGULAR DISTRIBUTIONS

Figure 1 shows that the resonance features of the interaction are shown most clearly by considering only the more peripheral of the two $p\pi^+\pi^-$ combinations. The analysis of this section will therefore be concerned only with that combination.

The angular distribution of the normal to the $p\pi^+\pi^-$ decay plane in the $p\pi^+\pi^-$ rest frame is expanded in a series of spherical harmonics, using the usual Gottfried-Jackson[9] set of axes.

We write

$$I(\beta,\Phi) = \sum_{l,m} A_{lm} Y_l^m(\beta,\Phi) \, , \qquad (3)$$

where the z axis is defined as the associated incoming proton direction and the y axis is the normal to the plane containing the other proton and its associated incident proton. The angles β and Φ are then the polar and azimuthal angles of the normal to the $p\pi^+\pi^-$ plane.

The mass spectrum was divided into intervals of 100 MeV and the coefficients A_{lm} evaluated in each bin. All the A_{lm} with $m \neq 0$ are consistent with zero. The imaginary moments are required to be zero by parity conservation, but the fact that the real moments are consistent with zero is evidence for spin-zero exchange.

We show in Fig. 4 the moments with $m=0$, up to $l=6$, as a function of $p\pi^+\pi^-$ mass. For the entire mass region considered the $l=2$ moment is negative. Hence, even in the 1400-MeV region there is some evidence that spin $J \geq \frac{3}{2}$ is contributing, but above 1700 MeV this is certainly required. Over the range of mass values considered all other moments appear to be small and consistent with zero.

The Birmingham-Glasgow-Oxford collaboration[3] has pointed out that more information is to be gained by looking at the decay angular distribution in a coordinate system originally devised for single-pion production reactions at low energy.[10] In the $p\pi^+\pi^-$ center-of-mass system, the z axis is

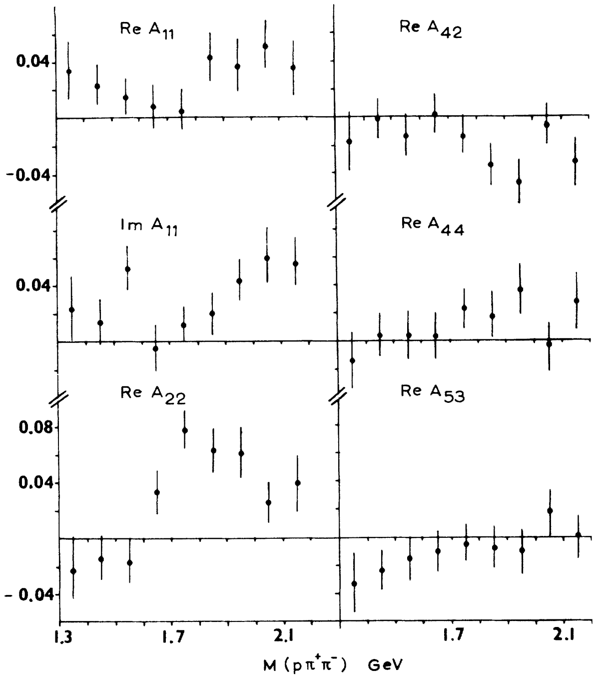

FIG. 5. Moments of $Y_l^m(\beta,\psi)$ with $m \neq 0$ for the distribution in the normal to the decay plane. See text for details of the coordinate system used.

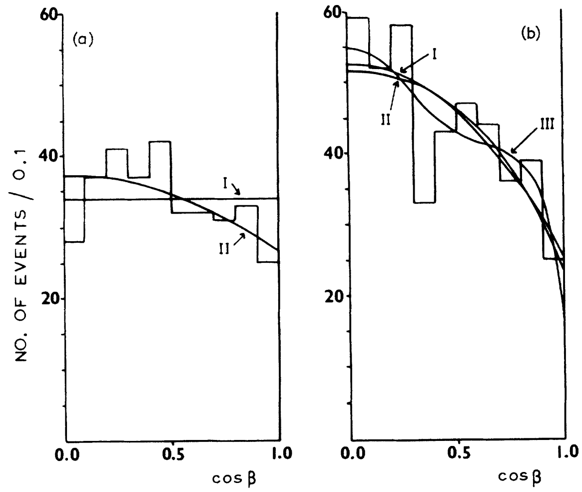

FIG. 6. Folded polar angular distributions of the normal to the $p\pi^+\pi^-$ decay plane. (a) 1450-MeV region. I and II are the predicted curves from the fits to $J=\frac{1}{2}$ and $J=\frac{3}{2}$. (b) 1700-MeV region. I, II, and III are the predicted curves from the fits to $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. The predicted curves are derived from Eq. (4) and given in Ref. 4 for values of J up to $\frac{5}{2}$.

taken as normal to the decay plane and the y axis is along the π^- direction in this plane. The incoming proton associated with the $p\pi^+\pi^-$ system then defines the polar and azimuthal angles β and ψ.

The mass spectrum has again been divided into intervals of 100 MeV and the decay angular distributions expanded in terms of the real and imaginary parts of the moments of $Y_l^m(\beta,\psi)$ in each bin. The $m=0$ moments are of course the same as for the Gottfried-Jackson system, since the polar angles β are identical. We show in Fig. 5 several of the moments with $m\neq 0$. Those not shown are consistent with zero over the whole mass range.

Although above 1800 MeV the real and imaginary parts of Y_1^1 depart significantly from zero, thus requiring two states of opposite parity, in the mass region of interest the results are not so clear-cut, and there is no strong evidence that this is required. Similarly, it is only above 1800 MeV that the moments calculated for Y_4^m may imply that spin $J\geq\frac{5}{2}$ is required.

In conclusion, it appears that the angular distribution of the low-mass (i.e., $\lesssim 1800$ MeV) $p\pi^+\pi^-$ system can be well described by moments with l values not greater than two.

V. SPIN-PARITY ANALYSIS

A. Decay-Plane Normal

We have attempted to determine the spin-parity of the enhancements at around 1450 and 1700 MeV, assuming that these are definite resonance states. Possible interference effects have been neglected in the following analysis.

The two mass regions of interest have been defined by the following selections:

"1450" region: $1.35 \leq M(p\pi^+\pi^-) < 1.55$ GeV,

"1700" region: $1.60 \leq M(p\pi^+\pi^-) < 1.80$ GeV.

For each region we have looked at the polar angular distribution of the normal to the decay plane in the $p\pi^+\pi^-$ center-of-mass system. The z axis is again the associated initial proton direction.

Following Berman and Jacob[11] the decay distribution of an object of spin J can be written

$$I(\theta)=2\pi \sum_{M\geq 0}^{J} \sum_{m>0}^{J} \rho_{mm} Z_{mm}^{JM+}(\beta) R_M^+ , \qquad (4)$$

where β is the polar angle of the decay normal and the R_M^+ are related to the phenomenological decay amplitudes, F_M, summed over final-state proton helicity and integrated over the decay Dalitz plot. ρ_{mm} is the spin-density matrix for the spin-J state and the Z functions are defined by

$$Z_{mm}^{JM+}(\beta)=[d_{mM}^J(\beta)]^2+[d_{m\,-M}^J(\beta)]^2, \qquad (5)$$

where the d's are the usual rotation matrices. It should be noted that the significance of this analysis is not affected by any possible $\Delta^{++}\pi^-$ intermediate state.

TABLE I. Summary of fits to the theoretical expressions derived from Eq. (4) for the polar angular distribution of the normal to the $p\pi^+\pi^-$ decay plane for events in the 1450-MeV region.

J	χ^2 prob (%)	ρ_{11}	No. of free parameters
$\frac{1}{2}$	51.5	$\frac{1}{2}$	0
$\frac{3}{2}$	70.4	$0.49^{+0.00}_{-0.06}$	2

TABLE II. Summary of fits to the theoretical expressions derived from Eq. (4) for the polar angular distribution of the normal to the $p\pi^+\pi^-$ decay plane for events in the 1700-MeV region.

J	χ^2 prob (%)	ρ_{11}	No. of free parameters
$\frac{3}{2}$	16.5	$0.5^{+0.00}_{-0.06}$	2
$\frac{5}{2}$	11.0	$\frac{1}{2}$	2
$\frac{7}{2}$	19.4	$\frac{1}{2}$	3

These expressions are explicitly written out in the Appendix to the paper of Rhode *et al.*,[4] up to $J=\frac{5}{2}$, although the authors place some restrictions on the values of the ρ_{mm}.

The folded distribution of the polar angle β of the decay normal for the 1450-MeV region is shown in Fig. 6(a). We have fitted this distribution to the theoretical expressions for spin $J=\frac{1}{2}$ and $J=\frac{3}{2}$ and the resultant curves are also shown in Fig. 6(a). The fitting for $J=\frac{3}{2}$ was performed with the maximum-likelihood technique and the χ^2 for the best fit evaluated. The results for the 1450-MeV effect are summarized in Table I. Clearly the uniform distribution required if $J=\frac{1}{2}$ is perfectly adequate to describe our data.

We next consider the 1700-MeV region. Evidently from the moments analysis in Sec. IV spin $J\geq\frac{3}{2}$ is required, although of course there may be some spin $J=\frac{1}{2}$ contributing. In Fig. 6(b) we show the folded polar angular distribution in the normal to the decay plane. Fits have been performed assuming $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. Following Rhode *et al.* we have imposed the restriction $\rho_{mm}=0$ for $m\geq\frac{3}{2}$ and $J\geq\frac{5}{2}$. This serves to reduce the number of free parameters in the fits and is justified on two grounds. Firstly, the $p\pi^+\pi^-$ system is produced extremely peripherally, so the $p\pi^+\pi^-$ direction lies very close to the incident direction (the quantization axis) and the maximum spin projection is $\pm\frac{3}{2}$; this allows us to set $\rho_{mm}=0$ for $m\geq\frac{5}{2}$. Secondly, we have shown that the production of the 1700-MeV enhancement is consistent with spin-zero exchange and thus ρ_{11} is expected to dominate over ρ_{33}.

The results of the fits are summarized in Table II and the predicted curves are shown in Fig. 6(b). All three spin assignments provide acceptable fits, though in no case does the χ^2 probability exceed 20%.

B. Sequential Decay

We next turn to a discussion of the sequential decay process,

$$N^{*+}\to\Delta^{++}\pi^-,\quad \Delta^{++}\to p\pi^+\pi^-,$$

where the symbol N^{*+} refers either to the 1450-MeV or the 1700-MeV enhancement.

Analysis of the angular distribution of such a double decay provides a more powerful means of determining the spin of the N^{*+}. These distributions are based upon the four angles θ, ϕ and θ', ϕ'. θ and ϕ are the polar and azimuthal angles of the Δ^{++} momentum in the Gottfried-Jackson frame. θ' and ϕ' are the polar and azimuthal angles of the decay proton in the Δ^{++} rest frame. These angles are defined by taking for the z' axis the Δ^{++} direction in the N^{*+} rest frame, and choosing $\hat{y}'=\vec{z}'\times\vec{z}/|\vec{z}'\times\vec{z}|$.

We show in Fig. 7 the polar angular distribution of the Δ^{++} for the 1450- and 1700-MeV regions. Events were chosen according to the Δ^{++} mass selection defined in Sec. III.

A strong asymmetry is present in these distributions, corresponding to the Δ^{++} traveling predominantly in the incident proton direction. Such an asymmetry is inconsistent with the decay of a pure $p\pi^+\pi^-$ resonance state.

We saw previously that by rejecting what were considered as "wrong" $p\pi^+\pi^-$ combinations on a

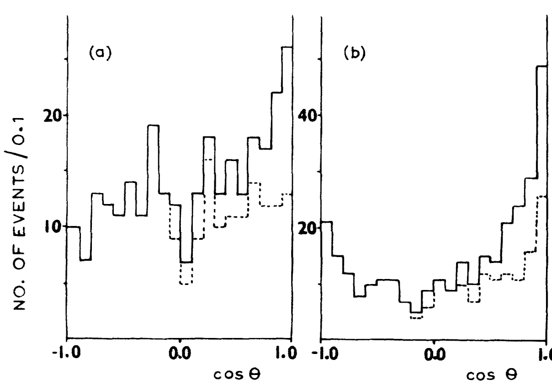

FIG. 7. Distributions in the polar angle θ of the Δ^{++} direction measured in the Gottfried-Jackson frame. Dashed histograms correspond to plotting only those events where the $p\pi^+\pi^-$ combination is the more peripheral of the two. (a) 1450-MeV region. (b) 1700-MeV region.

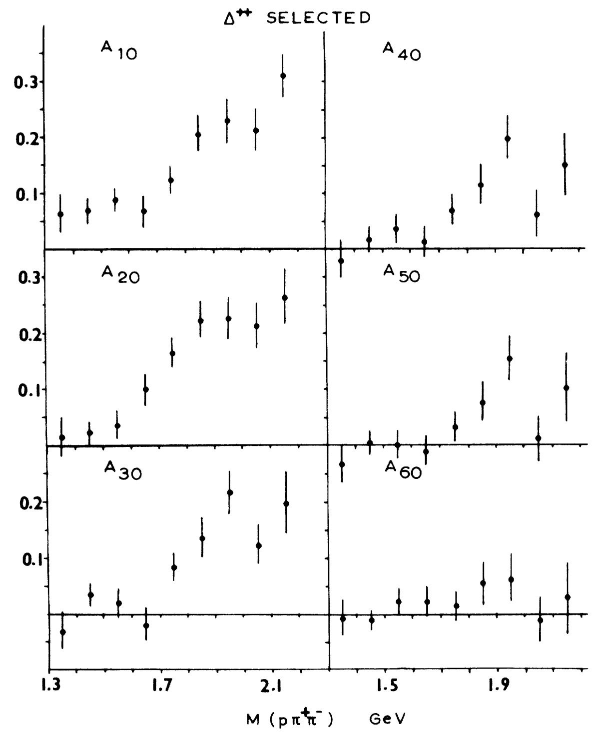

FIG. 8. Moments A_{lm} for the angular distribution of the Δ^{++} expanded in terms of $Y_l^m(\theta, \phi)$, where θ and ϕ are polar and azimuthal angles measured in the Gottfried-Jackson frame.

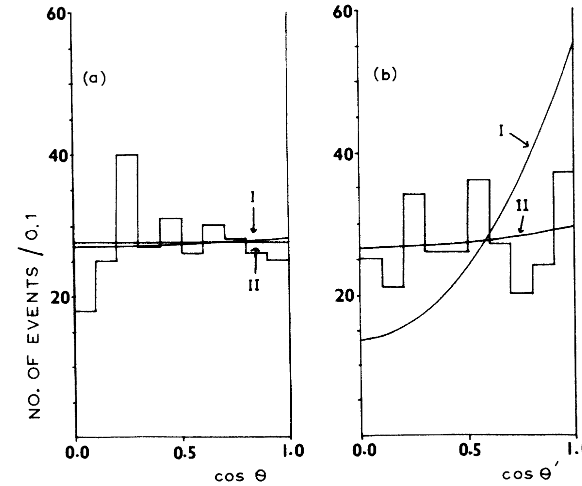

FIG. 9. Distributions in $\cos\theta$ and $\cos\theta'$ (see text) for the 1450-MeV region. I and II are the predicted curves for $J=\frac{1}{2}$ and $\frac{3}{2}$, obtained from Eqs. (6) and (7).

peripheral basis, the low-mass structure was virtually unaffected. If we use the same basis for rejecting "wrong" events from Fig. 7, the resulting distributions become more symmetric. These are shown as dashed histograms in Fig. 7 and only those events contributing to these symmetric distributions are used in the following analysis.

We have again performed a moments analysis for the decay $N^* \to \Delta^{++}\pi^-$. The angular distribution of the Δ^{++} has been expanded in terms of the sperical harmonics $Y_l^m(\theta, \phi)$, and the moments A_{lm} are shown as a function of $p\pi^+\pi^-$ mass in Fig. 8. Only the $m=0$ moments are shown, all others being consistent with zero. It will be seen that there are significant contributions from a greater number of the Y_l^0 than was the case in Sec. IV, when we examined the distribution of the decay normal.

Both the $l=2$ and $l=4$ moments are significant above 1700 MeV. The $l=6$ moment is hardly present, but as Willman *et al.*[6] have pointed out, if $J^P=\frac{7}{2}^-$ and only the orbital angular momentum wave with $l=2$ contribute to the decay, then the coefficient of Y_6^0 may vanish. The $l=1$ moment is strong throughout the entire region and the $l=3$ moment also starts to contribute above 1700 MeV. This suggests that interference between different angular momentum states may be present, but in the following analysis this will be neglected.

The theoretical expressions[11] for the distributions in θ and θ' for spin J are given by

$$I(\theta)=\sum_{\lambda>0}^{3/2}\sum_{m>0}^{J}\rho_{mm}Z_{mm}^{J,\lambda+}(\theta)|F_\lambda|^2 \qquad (6)$$

and

$$I'(\theta')=\sum_{m>0}^{3/2}\rho'_{mm}Z_{mm}^{3/2,1/2+}(\theta'), \qquad (7)$$

where ρ'_{mm} is understood to mean $\int\rho'_{mm}(\theta)d(\cos\theta)$.

TABLE III. Summary of joint fits to the theoretical expressions derived from Eqs. (6) and (7) relating to the sequential decay $N^* \to \Delta^{++}\pi^-$, $\Delta^{++} \to p\pi^+$ for events in the 1700-MeV region. (a) χ^2 probability for parent decay. (b) χ^2 probability for daughter decay. (c) Joint fit.

J	χ^2 prob (%) (a)	χ^2 prob (%) (b)	χ^2 prob (%) (c)	ρ_{11}	ρ'_{11}	R	No. of free parameters
$\frac{1}{2}$	32.8	0.3×10^{-6}	0.9×10^{-5}	$\frac{1}{2}$	$\frac{1}{2}$	∞	0
$\frac{3}{2}$	24.9	18.7	16.0	0.36 ± 0.05	0.27 ± 0.03	1.2 ± 0.2	2

If $J=\frac{1}{2}$ then there is only one term in the expression for $I(\theta)$ corresponding to $\lambda=\frac{1}{2}$ and $m=\frac{1}{2}$, and only one term in $I'(\theta')$ with $m=\frac{1}{2}$. F_λ is the helicity amplitude for the decay $N^{*+}\rightarrow\Delta^{++}\pi^-$. These formulas are also explicitly written out in the Appendix of the paper by Rhode *et al.*[4] up to $J=\frac{5}{2}$.

Formula (6) can be rewritten as

$$I(\theta)=\sum_{m>0}^{J} R\rho_{mm}Z_{mm}^{J,1/2\,+}(\theta)+\rho_{mm}Z_{mm}^{J,3/2\,+}(\theta), \qquad (8)$$

where

$$R=\frac{|F_{1/2}|^2}{|F_{3/2}|^2}=\frac{\rho'_{11}}{\frac{1}{2}-\rho'_{11}}\,. \qquad (9)$$

The folded polar angular distributions in θ and θ' for the 1450-MeV region are shown in Fig. 9.

These distributions have been fitted jointly with the theoretical expressions for spins $J=\frac{1}{2}$ and $J=\frac{3}{2}$. The results of the joint fits are summarized in Table III, together with the χ^2 probabilities for the individual distributions. The fitted curves are shown in Fig. 9.

For $J=\frac{1}{2}$, the parent decay $N^{*+}\rightarrow\Delta^{++}\pi^-$ is required to be uniform in $\cos\theta$ and the χ^2 probability for such a fit is 32.8%. However, the daughter decay is required to have the form $1+3\cos^2\theta'$ and our data are clearly inconsistent with this. If we assume $J=\frac{3}{2}$, then the results of the joint fit are perfectly acceptable. These results are in agreement with those of Rhode *et al.*[4] who also obtained a very poor fit for the $J=\frac{1}{2}$ assignment. Clearly if the 1450-MeV effect has $J=\frac{1}{2}$, then $\Delta^{++}\pi^-$ cannot be the principal decay mode. As was pointed out in Sec. III, because of the small phase space available for the decay and consequent uncertainty as to whether the $p\pi^+$ system forms a Δ^{++}, it is extremely difficult to come to a firm conclusion on this point.

In Fig. 10 we show the folded polar angular distributions in θ and θ' for the 1700-MeV region. We have fitted these distributions with the theoretical expressions for spins $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$, where we have imposed the condition $\rho_{mm}=0$ for $m\geq\frac{5}{2}$ and $J\geq\frac{5}{2}$. This is consistent with the results of Willman *et al.*[6] who obtained essentially zero values

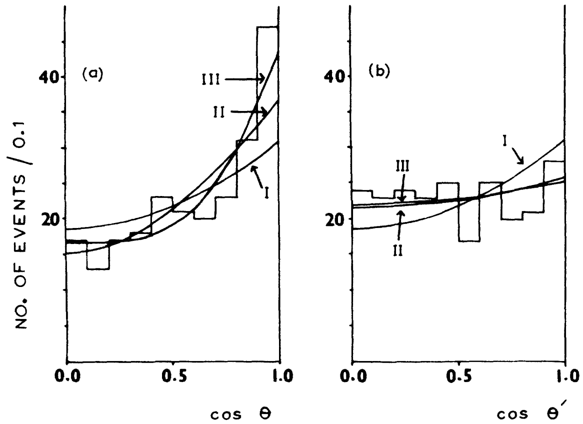

FIG. 10. Distributions in $\cos\theta$ and $\cos\theta'$ (see text) for the 1700-MeV region. I, II, and III are the predicted curves for $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$, obtained from Eqs. (6) and (7).

for these density-matrix elements. The results are presented in Table IV and these show that spin $J=\frac{5}{2}$ or $\frac{7}{2}$ is preferred, but $J=\frac{3}{2}$ cannot be ruled out. The curves resulting from the fits are shown in Fig. 10.

C. Parity

We have seen that the 1450-MeV enhancement gives a good fit to the $J=\frac{1}{2}$ assignment for the uncorrelated $p\pi^+\pi^-$ decay. One is therefore tempted to associate this enhancement with the $P_{11}(1470)$, $J^P=\frac{1}{2}^+$ seen in the phase-shift analyses.[2] However, if the dominant decay mode of this enhancement is $\Delta^{++}\pi^-$, then our data do not support the $J=\frac{1}{2}$ assignment, but good fits are obtained for $J=\frac{3}{2}$. When $R=1$, both a $J=\frac{3}{2}$ parent decay ($N^{*+}\rightarrow\Delta^{++}\pi^-$) and daughter decay ($\Delta^{++}\rightarrow p\pi^+$) should have uniform polar angular distributions. Our data are clearly consistent with this; in fact, we obtain a value for R of 1.2 ± 0.2 and the χ^2 probability for the joint fit for $J=\frac{3}{2}$ is 16.0%. We note that $R=1$ can occur[12] for pure S-wave decay of a $\frac{3}{2}^-$ state or an appropriate mixture of P- and F-wave decay of a $\frac{3}{2}^+$ state, and that lower partial waves should domi-

TABLE IV. Summary of joint fits to the theoretical expressions derived from Eqs. (6) and (7) relating to the sequential decay $N^*\rightarrow\Delta^{++}\pi^-$, $\Delta^{++}\rightarrow p\pi^+$ for events in the 1700-MeV region. (a) χ^2 probability for parent decay. (b) χ^2 probability for daughter decay. (c) Joint fit.

J	χ^2 prob (%) (a)	χ^2 prob (%) (b)	χ^2 prob (%) (c)	ρ_{11}	ρ'_{11}	R	No. of free parameters
$\frac{3}{2}$	16.9	30.8	17.7	$0.50^{+0.00}_{-0.02}$	0.31 ± 0.03	1.6 ± 0.3	2
$\frac{5}{2}$	53.7	80.2	77.4	0.39 ± 0.02	0.28 ± 0.04	1.3 ± 0.3	2
$\frac{7}{2}$	79.2	81.9	91.1	0.36 ± 0.07	0.27 ± 0.03	1.2 ± 0.2	2

TABLE V. Summary of theoretical and experimental values of R in the 1700-MeV region assuming that for a state of definite J^P the lower-angular-momentum state dominates the decay.

J^P	Lowest l value [a]	R (theor)	R (expt)
$\frac{3}{2}^+$	1	$\frac{1}{9}$	1.6 ± 0.3
$\frac{3}{2}^-$	0	1	
$\frac{5}{2}^+$	1	$\frac{3}{2}$	1.3 ± 0.3
$\frac{5}{2}^-$	2	$\frac{1}{6}$	
$\frac{7}{2}^+$	3	$\frac{1}{5}$	1.2 ± 0.2
$\frac{7}{2}^-$	2	$\frac{9}{5}$	

[a] Assumed to be dominant.

nate because of the small decay energy. Figure 8 shows that the $l=1$ moment is nonzero in this region so that one cannot rule out interference between states of opposite parities.

We next consider each of the three spin assignments for the 1700-MeV effect. Assuming only the lower partial wave contributes to the decay, the values of R for $J^P = \frac{3}{2}^+$ and $\frac{3}{2}^-$ are $\frac{1}{9}$ and 1, respectively. The result from the fit (Table IV) is 1.6 ± 0.3 so we favor the negative-parity state. For $J^P = \frac{5}{2}^+$ and $\frac{5}{2}^-$, again assuming the lower l wave dominates, we have $R = \frac{3}{2}$ and $\frac{1}{6}$. Hence we would prefer $J^P = \frac{5}{2}^+$ since our analysis gives a value $R = 1.3 \pm 0.3$. For spin $\frac{7}{2}$ (and lowest partial wave) the predicted values of R are $\frac{1}{5}$ for $\frac{7}{2}^+$ and $\frac{9}{5}$ for $\frac{7}{2}^-$. The fitted value is 1.2 ± 0.2 so $J^P = \frac{7}{2}^-$ is favored. These results are summarized in Table V.

In general, then, with the assumption that the lower partial wave dominates, we require the 1700-MeV enhancement to contain contributions from one or more of the spin-parity series $\frac{3}{2}^-$, $\frac{5}{2}^+$, $\frac{7}{2}^-$. This is in agreement with the proposed Gribov-Morrison parity rule,[13] for reactions proceeding via diffraction dissociation.

VI. SUMMARY AND CONCLUSIONS

We have examined the $p\pi^+\pi^-$ mass spectrum from the $pp\pi^+\pi^-$ final state, produced in 16-GeV/c pp interactions. The highly peripheral nature of the interactions has enabled us to associate the two pions with a particular final-state proton and therefore to make a reasonable analysis of the properties of any possible $p\pi^+\pi^-$ resonance.

We see two enhancements centered about masses of 1470 and 1750 MeV. While no firm conclusions can be drawn about the decay of the former, the effect at 1700 MeV seems not to have a dominant $\Delta^{++}\pi^-$ decay mode. We in fact obtain a figure for the branching ratio of $(35 \pm 20)\%$.

We have also attempted to determine the spin-parity of these enhancements. If the 1450-MeV effect decays into the uncorrelated $p\pi^+\pi^-$ mode then spin-parity $\frac{1}{2}^+$ is perfectly adequate to describe our data. If however, the $\Delta^{++}\pi^-$ mode is dominant, then we favor $J^P = \frac{3}{2}^-$.

For the uncorrelated decay of the 1700-MeV enhancement, spin assignments $\frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$ all give acceptable fits. However, when we examine those events having a $p\pi^+$ combination in the Δ^{++}, we prefer $\frac{5}{2}$ or $\frac{7}{2}$ over $\frac{3}{2}$.

Assuming that the state has a definitive parity and the lower of the two possible orbital angular momentum states dominates the decay, we require the enhancement to belong to the spin-parity series $\frac{3}{2}^-$, $\frac{5}{2}^+$, $\frac{7}{2}^-$. This is consistent with the Gribov-Morrison parity rule[13] for diffractive processes.

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