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Closed Linear Algebra of Currents and Baryon Interpolating Fields*

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A closed linear algebra involving vector and axial-vector currents and baryon interpolating fields (BIF) is constructed. In particular, the equal-time anticommutator of two BIF is assumed to be the most general expression linear in currents allowed by covariance and the discrete symmetries. The algebra is closed via the various Jacobi identities. Whenever possible we proceed in analogy with successful current-algebra concepts. The basic BIF considered here has spin $\frac{3}{2}$ and is assumed to transform linearly under chiral SU(3). Its divergence can be used as a BIF for spin-$\frac{1}{2}$ baryons. A Lagrangian model combining the massive Yang-Mills gauge field ϕ^μ and the Rarita-Schwinger field Ψ^ν is constructed. The canonical commutation relations of such a model imply that the equal-time anticommutator $\{\Psi^0, \bar{\Psi}^\mu\}$ contains a term proportional to ϕ^μ. In a limiting case only this term remains. "Soft-baryon" theorems are derived from the algebra. In particular the s-wave scattering of "soft nucleons" from a target is calculated and agrees with the current-algebra soft-meson s-wave scattering lengths. Weinberg-like sum rules resulting from the algebra are also derived.

I. INTRODUCTION

The vector and axial-vector currents deduced from the study of weak and electromagnetic interactions have been successfully utilized in the study of low-energy mesonic processes.[1] The use of these currents in mesonic processes follows from two basic assumptions: (1) The currents and their divergences may be used as "smooth" meson interpolating fields (MIF); (2) the equal-time commutation relations involving at least one time component of a current form a closed linear algebra. Assumption (2) is strong, yet does not alone give rise to the chiral algebra. In fact, assuming a linear closed algebra, i.e., $[Q_a^5, Q_b^5] = \chi i\epsilon_{abc}Q_c$, forces the choices $\chi^2 = 1$ or 0. The Adler-Weisberger relation, for example, picks out $\chi = +1$. Alternatively, one might arrive at the linear algebra via an ansatz of universality or by studying field-theory models. Once these assumptions have been imposed upon a hadronic amplitude, various approximations have been employed in order to confront meson-meson or meson-baryon data.

Given a set of baryon interpolating fields (BIF) with assumptions analogous to (1) and (2), it is possible to employ similar schemes to confront low-energy baryonic processes. Unfortunately, however, a suitable set of BIF is not suggested by weak or electromagnetic processes and therefore, to pursue the problem further, we shall proceed whenever possible by analogy with successful current-algebra concepts. Thus just as spin-0 MIF are given by the divergence of spin-1 MIF we shall assume that spin-$\frac{1}{2}$ BIF are given by the divergence of spin-$\frac{3}{2}$ BIF. In Sec. II we define and consider general properties of such objects. In Sec. III we construct the most general linear algebra involving the vector and axial-vector currents and the spin-$\frac{3}{2}$ BIF. The commutation relations of the BIF with the vector and axial-vector currents is determined by the transformation properties of the BIF under the chiral SU(2) or chiral SU(3) group. In the present discussion we assume that the spin-$\frac{3}{2}$ BIF transform as *linear* representations. To complete the analogy we introduce anticommutation relations for the BIF. In the present work we examine the simplest possible model, viz., we assume that equal-time anticommutators involving at least one time component of the spin-$\frac{3}{2}$ BIF are the most general expressions linear in the vector and axial-vector currents. Of course the equal-time commutation as well as the anticommutation relations are subject to the requirements of covariance and C, P, and T invariance. The algebra is closed via implementation of the various Jacobi identities satisfied by commutators and anticommutators. Finally, remaining parameters are eliminated by the freedom to fix over-all scales (Jacobi identities are invariant with respect to change of scale). In actual calculations commutation relations involving divergences of BIF and MIF appear, e.g., the σ term in ordinary current algebra, and some such terms are examined.

Other than simplicity, is there any other reason to suspect that the anticommutation relations of BIF should be linear in currents? Straightforward composite models for the BIF, e.g., in terms of canonical quark fields, invariably yield models quadratic in currents.[2] Such models are at best terribly cumbersome and lead one into the difficulties associated with such singular products. The ansatz of linearity might indeed be reasonable if we imagine the currents to be fundamental operators of the theory and that low-energy phenomena can be adequately described by the leading term in a power-series expansion. Furthermore, as shown in Sec. IV, it is not difficult to write down a Lagrangian theory that yields in an appropriate limit the linear algebra we have constructed. This Lagrangian is a combination of the Yang-Mills[3] gauge-field Lagrangian plus a slightly modified Rarita-Schwinger[4] spin-$\frac{3}{2}$ Lagrangian. The appropriate limit has $g_0 \to 0$ (the gauge-field coupling constant), $m_0 \to 0$ (gauge-field mass), and $M_0 \to 0$ (matter-field mass) with g_0^2/m_0^2 and M_0/m_0 fixed. In Sec. V various applications are suggested. Exact "soft-nucleon" theorems are derived, s-wave scattering lengths for "soft nucleons" are given and compare favorably with those of physical nucleons in certain channels, e.g., $N\pi$ and NK, and finally Weinberg-like sum rules[5] are derived. In Sec. VI a discussion of results and a prognosis of future progress in representing low-energy hadronic process within an algebraic framework is made. In the Appendixes are collected some relations satisfied by spin-1 (spin-$\frac{3}{2}$) commutators (anticommutators).

II. SPINOR-VECTOR FIELDS

The spinor-vector field $\Psi^\mu_\alpha(x)$ with Lorentz index μ and Dirac index α transforms under Lorentz transformations as the direct product of a vector field with a Dirac field.[6] Such an object has 16 components and can be covariantly decomposed, for example, as

$$\Psi^\mu = \hat{\Psi}^\mu + \tfrac{1}{3}(\gamma^\mu \partial\cdot\gamma - \partial^\mu)\Psi_1 - \tfrac{4}{3}(\tfrac{1}{4}\gamma^\mu\partial\cdot\gamma - \partial^\mu)\partial^{-2}\Psi_2, \tag{2.1}$$

where $\hat{\Psi}^\mu$ is transverse to both γ_μ and ∂_μ and hence has the required number of independent components (eight) to describe a Dirac spin-$\frac{3}{2}$ particle. $\partial_\mu\Psi^\mu = \Psi_2$ and $\gamma_\mu\Psi^\mu = \Psi_1$ are both Dirac fields and may be used as interpolating fields for spin $\frac{1}{2}$. The single spin-$\frac{3}{2}$ particle to vacuum matrix element is

$$\langle 0|\Psi^\mu|\alpha(q,\lambda)\rangle = \left(\frac{M_\alpha}{(2\pi)^3E}\right)^{1/2} u^\mu(q,\lambda) g_\alpha \epsilon, \tag{2.2}$$

where relative to Ψ^μ, α is defined to have natural (unnatural) parity if it has the same (opposite) parity as Ψ^μ and ϵ is equal to one (γ^5). g_α is the wave-function renormalization. The vector-spinor $u^\mu(q,\lambda)$ with polarization λ is transverse to q_μ and γ_μ and satisfies the Dirac equation $(q\cdot\gamma - M_\alpha)u^\mu(q,\lambda) = 0$ and

$$\sum_\lambda u^\mu(q,\lambda)\bar{u}^\nu(q,\lambda) = -\frac{M_\alpha + q\cdot\gamma}{2M_\alpha}\left[g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{3} - \frac{1}{3M_\alpha^{\,2}}(\gamma^\nu q^\mu q\cdot\gamma + q\cdot\gamma\gamma^\mu q^\nu)\right]. \tag{2.3}$$

As mentioned above, Ψ^μ contains two independent spin-$\frac{1}{2}$ components. The vacuum to spin-$\frac{1}{2}$ particle matrix element is

$$\langle 0|\partial_\mu\Psi^\mu|\beta(q,\lambda)\rangle = \left(\frac{M_\beta}{(2\pi)^3E}\right)^{1/2} u(q,\lambda) g_\beta M_\beta \epsilon, \tag{2.4}$$

where β is normal (abnormal) if it has opposite (same) parity as Ψ^μ and ϵ is one (γ^5). Under the discrete transformations we have

$$\mathcal{P}\Psi^\mu(\vec{x},x^0)\mathcal{P}^{-1} = \eta_P\gamma^0\Psi_\mu(-\vec{x},x^0), \tag{2.5a}$$

$$\mathcal{C}\Psi^\mu(x)\mathcal{C}^{-1} = \eta_C\bar{\Psi}^\mu(x)C, \tag{2.5b}$$

and

$$\mathcal{T}\Psi^\mu(x)\mathcal{T}^{-1} = \eta_T T\Psi_\mu(\vec{x},-x^0), \tag{2.5c}$$

where $|\eta_i|^2 = 1$, $C = i\gamma^2\gamma^0$, and $T = i\gamma^1\gamma^3$, following the conventions in Bjorken and Drell.[7] For later reference we examine the spectral functions and the Lehmann-Källén representation[8] for spinor-vectors.

The spectral function is

$$\rho^{\mu\nu}(q) \equiv (2\pi)^3\sum\delta^4(p_n - q)\langle 0|\Psi^\mu(0)|n\rangle\langle n|\bar{\Psi}^\nu(0)|0\rangle, \tag{2.6}$$

where $\bar{\Psi}^\nu \equiv \Psi^{\nu\dagger}\gamma^0$. From Lorentz covariance, and C, P, and T invariance, one deduces that

$$\rho^{\mu\nu}(q)=g^{\mu\nu}_{3/2}(q)(\rho_1+q\cdot\gamma\rho_2)+\left(\frac{4}{3}\right)^2\left(q^\mu-\frac{\gamma^\mu q\cdot\gamma}{4}\right)(\rho_3+q\cdot\gamma\rho_4)\left(q^\nu-\frac{q\cdot\gamma\gamma^\nu}{4}\right)\frac{1}{q^2}$$
$$+\left(\frac{1}{3}\right)^2(q^\mu-\gamma^\mu q\cdot\gamma)(\rho_5+q\cdot\gamma\rho_6)(q^\nu-q\cdot\gamma\gamma^\nu)\frac{1}{q^2}-\frac{4}{9}(q^\mu-\gamma^\mu q\cdot\gamma)(\rho_8+q\cdot\gamma\rho_7)\left(q^\nu-\frac{q\cdot\gamma\gamma^\nu}{4}\right)\frac{1}{q^2}$$
$$-\frac{4}{9}\left(q^\mu-\frac{\gamma^\mu q\cdot\gamma}{4}\right)(\rho_{10}+q\cdot\gamma\rho_9)(q^\nu-q\cdot\gamma\gamma^\nu)\frac{1}{q^2}, \tag{2.7}$$

where

$$g^{\mu\nu}_{3/2}(q)\equiv g^{\mu\nu}-\frac{\gamma^\mu\gamma^\nu}{3}-\frac{1}{3q^2}(q^\mu\gamma^\nu q\cdot\gamma+q\cdot\gamma q^\nu\gamma^\mu) \tag{2.8}$$

is transverse to q^μ and γ^μ and commutes with $q\cdot\gamma$. $\rho_i=\rho_i(q^2)$, ρ_j ($j=1,\ldots,6$) are real and $\rho^*_{7(8)}=\rho_{9(10)}$. It is not difficult to show that ρ_1 and ρ_2 receive contributions exclusively from spin-$\frac{3}{2}$ states in (2.6), ρ_3 and ρ_4 (ρ_5 and ρ_6) receive contributions from spin-$\frac{1}{2}$ states that couple exclusively to $\partial_\mu\Psi^\mu$ ($\gamma_\mu\Psi^\mu$) and ρ_{7-10} receive contributions from states that couple to both spin-$\frac{1}{2}$ components. For the present we shall assume that $\gamma_\mu\Psi^\mu$ does *not* couple strongly to low-lying baryon states and we shall neglect ρ_{5-10}. Had we assumed that $\gamma_\mu\Psi^\mu=0$, it would follow that $\rho_{5-10}\equiv0$. From Eqs. (2.2)–(2.4) it is a straightforward matter to show that the single-particle states make the following contributions to the various spectral functions: Spin-$\frac{3}{2}$ particles with the same parity as Ψ^μ make contributions of the form

$$\rho_1(q^2)+q\cdot\gamma\rho_2(q^2)=-g_\alpha{}^2(M_\alpha+q\cdot\gamma)\delta(q^2-M_\alpha{}^2), \tag{2.9}$$

whereas opposite-parity spin-$\frac{3}{2}$ states contribute with the opposite sign to $\rho_1(q^2)$. Similarly single-particle spin-$\frac{1}{2}$ states with same parity as $\partial_\mu\Psi^\mu$ contribute as

$$\rho_3(q^2)+q\cdot\gamma\rho_4(q^2)=g_\beta{}^2(M_\beta+q\cdot\gamma)\delta(q^2-M_\beta{}^2), \tag{2.10}$$

whereas opposite-parity states contribute with opposite sign to $\rho_3(q^2)$.

Given Eq. (2.7) and assuming that no subtractions are necessary, we obtain in a standard manner the time-ordered product

$$\int dx\,e^{-iq\cdot x}\langle0|T\Psi^\mu(x)\overline{\Psi}^\nu(0)|0\rangle=-i[P^{\mu\nu}(q)+\overline{P}^{\mu\nu}(q)-S^{\mu\nu}(q)], \tag{2.11}$$

where the covariant spin-$\frac{3}{2}$ propagator is

$$P^{\mu\nu}(q)=\int_0^\infty g^{\mu\nu}_{3/2}(q)\bigg|_{q^2=\mu^2}\frac{\rho_1(\mu^2)-q\cdot\gamma\rho_2(\mu^2)}{\mu^2-q^2-i\epsilon}d\mu^2,$$
$$\left(\frac{3}{4}\right)^2\overline{P}^{\mu\nu}=\left(q^\mu-\frac{\gamma^\mu q\cdot\gamma}{4}\right)\int\frac{\rho_3(\mu^2)-q\cdot\gamma\rho_4(\mu^2)}{\mu^2(\mu^2-q^2-i\epsilon)}\left(q^\nu-\frac{q\cdot\gamma\gamma^\nu}{4}\right)d\mu^2+\frac{C_3}{16}\gamma^\mu\gamma^\nu+\frac{C_4}{4}\left(q^\mu\gamma^\nu+q^\nu\gamma^\mu-\frac{\gamma^\mu q\cdot\gamma\gamma^\nu}{4}\right), \tag{2.12}$$

and the seagull or noncovariant term is

$$S^{\mu\nu}(q)=\tfrac{1}{3}(C_1-\tfrac{4}{3}C_3)(\gamma^0\gamma^\mu g^{\nu0}+\gamma^\nu\gamma^0g^{\mu0})+\tfrac{2}{3}(C_2-\tfrac{8}{3}C_4)[g^{\mu0}g^{\nu0}q^\sigma\gamma^\sigma-(q^\nu g^{\mu0}+g^\mu g^{\nu0})\gamma^0],$$

where

$$C_i\equiv\int\frac{d\mu^2}{\mu^2}\rho_i(\mu^2)\quad\text{and}\quad C_i'\equiv\int d\mu^2\rho_i(\mu^2). \tag{2.13}$$

The separation of the time-ordered product into covariant and noncovariant parts is in fact rather arbitrary since the covariant propagator is defined only up to a polynomial. The inverse covariant propagator also has a simple form: With single-particle dominance we have

$$P^{-1}{}_{\mu\nu}(q)=-\frac{1}{g_\alpha{}^2}[(M_\alpha+q\cdot\gamma)(g_{\mu\nu}-\gamma_\mu\gamma_\nu)+q_\mu\gamma_\nu-q_\nu\gamma_\mu]. \tag{2.14}$$

The other time-ordered products we shall encounter are

$$\int dx\,e^{-iq\cdot x}\langle0|T\partial\cdot\Psi(x)\overline{\Psi}^\nu(0)|0\rangle=P^{-\nu}(q)-S^{-\nu}(q), \tag{2.15}$$

where

$$P^{-\nu}=\frac{4}{3}\int\frac{\rho_3(\mu^2)-q\cdot\gamma\rho_4(\mu^2)}{\mu^2-q^2-i\epsilon}d\mu^2\left(q^\nu-\frac{q\cdot\gamma\gamma^\nu}{4}\right)+\frac{C_4'}{3}\gamma^\nu \tag{2.16}$$

and the seagull is

$$S^{-\nu}=\tfrac{4}{3}C_4'g^{\nu 0}\gamma^0. \tag{2.17}$$

Finally, the spin-$\frac{1}{2}$ propagator $-iP(q)$ is given by

$$\int dx\,e^{-iq\cdot x}\langle 0|T\partial\cdot\Psi(x)\partial\cdot\overline{\Psi}(0)|0\rangle=-i\int\frac{\mu^2[\rho_3(\mu^2)-q\cdot\gamma\rho_4(\mu^2)]}{\mu^2-q^2-i\epsilon}d\mu^2. \tag{2.18}$$

Note that all of the above time-ordered products are consistent with Bjorken's[9] observation that under mild assumptions in the limit $\vec{q}$ fixed, $|q^0|\to\infty$, time-ordered products go to zero. Other relations satisfied by the propagators and vacuum expectation values of the various anticommutators may be found in Appendix A.

We shall now introduce a set of BIF that by assumption couple to the low-lying baryons.

III. COMMUTATION AND ANTICOMMUTATION RELATIONS

We introduce a set of octet and decuplet BIF that can couple to any of the low-lying baryons with $J^P=\frac{1}{2}^\pm$, $\frac{3}{2}^\pm$. Let the vector-spinor octet BIF be ${}_\pm N_a^\mu(x)$ and the vector-spinor decuplet BIF be ${}_\pm\Delta_a^\mu(x)$ with parity equal to ±1.

A. Commutation Relations

The transformation of these spherical SU(3) tensor BIF under the SU(3) vector densities is

$$[V_a^0(x),{}_\pm N_b^\mu(y)]\delta(x^0-y^0)=\sqrt{3}\begin{pmatrix}8&8&8_2\\a&c&b\end{pmatrix}{}_\pm N_c^\mu(x)\delta^4(x-y) \tag{3.1}$$

and

$$[V_a^0(x),{}_\pm\Delta_b^\mu(y)]\delta(x^0-y^0)=\sqrt{6}\begin{pmatrix}8&10&10\\a&c&b\end{pmatrix}{}_\pm\Delta_c^\mu(x)\delta^4(x-y) \tag{3.2}$$

up to possible terms that integrate to zero. Such possible Schwinger terms[10] will not be considered here. We follow the notation and convention of DeSwart[11]; in particular, we use a spherical basis and the SU(3) Clebsch-Gordan coefficients. Let us recall the form of the current-algebra commutators in a spherical basis:

$$[J_a^0(x),J_b^\mu(y)]\delta(x^0-y^0)=\sqrt{3}\begin{pmatrix}8&8&8_2\\b&a&c\end{pmatrix}J_c^\mu(x)\delta^4(x-y), \tag{3.3}$$

where J_b^μ is the appropriate vector or axial-vector current. In accord with our fundamental requirement that the equal-time (anti) commutators be linear, we assume that N^μ and Δ^μ transform linearly under chiral SU(3)×SU(3). Since $\partial_\mu A^\mu\neq0$ this does *not* imply that ∂N and $\partial\Delta$ transform linearly under chiral transformations.[12] Thus the transformation of the BIF under the axial-vector densities is

$$[A_a^0(x),{}_\pm N_b^\mu(y)]\delta(x^0-y^0)=\alpha_\pm^i\begin{pmatrix}8&8&8_i\\c&a&b\end{pmatrix}{}_\mp N_c^\mu(x)\delta^4(x-y)+\beta_\pm\begin{pmatrix}8&10&8\\a&c&b\end{pmatrix}{}_\mp\Delta_c^\mu(x)\delta^4(x-y) \tag{3.4}$$

and

$$[A_a^0(x),{}_\pm\Delta_b^\mu(y)]\delta(x^0-y^0)=\phi_\pm\begin{pmatrix}8&10&10\\a&c&b\end{pmatrix}{}_\mp\Delta_c^\mu(x)\delta^4(x-y)+\delta_\pm\begin{pmatrix}8&8&10\\a&c&b\end{pmatrix}{}_\mp N_c^\mu(x)\delta^4(x-y). \tag{3.5}$$

Note that the axial-vector current changes the parity and that we do *not* restrict ourselves to the case ${}_-N^\mu\propto\gamma^5{}_+N^\mu$.[13] The constants α, β, ϕ, and δ are determined via the Jacobi identities between two axial-vector currents and a BIF,

$$[X,[Y,Z]_+]_-\equiv[Y,[X,Z]_-]_++[[X,Y]_-,Z]_+, \tag{3.6a}$$

$$[X,[Y,Z]_-]_-\equiv[Y,[X,Z]_-]_-+[[X,Y]_-,Z]_-. \tag{3.6b}$$

In fact one obtains two independent solutions.

Solution 1:

$$\alpha_{2\pm}=-\sqrt{3},\quad\phi_\pm=\sqrt{6},\quad\alpha_{1\pm}=0=\beta_\pm=\delta_\pm. \tag{3.7}$$

Solution 2:

$$\alpha_{1\pm} = -(\tfrac{5}{3})^{1/2}, \quad \alpha_{2\pm} = -(\tfrac{4}{3})^{1/2}, \quad \beta_{\pm} = -5(\tfrac{5}{6})^{1/2}, \quad \phi_{\pm} = (\tfrac{2}{3})^{1/2}, \quad \text{and} \quad \delta_{\pm} = -\tfrac{4}{15}\sqrt{6}. \tag{3.8}$$

The freedom to rescale the fields has been used since Eq. (3.6), for example, is invariant under the transformation $X \to rX$ and we have not yet specified the scale of the BIF. Solution 1 states that N^μ and Δ^μ each transform irreducibly under chiral SU(3) and correspond to the (1, 8) and (1, 10) representations, respectively. Had we chosen the opposite signs in Eq. (3.7), e.g., let ${}_+N \to (-1)_+N$, we would have obtained the (8, 1) and (10, 1) representations, respectively. Our choice of sign is motivated by the sign of the axial-vector coupling constant g_A/g_V. In solution 2 N^μ and Δ^μ transform irreducibly under chiral SU(3), i.e., as (8, 10). Had we restricted ourselves to the form ${}_-N^\mu = \xi\gamma^5\, {}_+N^\mu(x)$, Eq. (3.4) with Eq. (3.7) or (3.8) would imply that $\xi^2 = +1$. If we limit ourselves to the chiral SU(2) subalgebra, Eq. (2.1) reduces to

$$[A_a^0(x), {}_\pm N_b^\mu(y)]\delta(x^0 - y^0) = \frac{1}{2}\left(\frac{3}{\sqrt{5}}\,\alpha_{1\pm} + \alpha_{2\pm}\right)_{1/2} C^{1/2}{}_c{}^1{}_a{}^{1/2}{}_b\, {}_\mp N_c^\mu(x)\delta^4(x-y) - \frac{2}{\sqrt{5}}\beta_\pm C^1{}_a{}^{3/2}{}_c{}^{1/2}{}_b\, {}_\mp\Delta_c^\mu(x)\delta^4(x-y), \tag{3.9}$$

where the SU(2) Clebsch-Gordan coefficients have been introduced. Notice that *if* we assume that $\partial_\mu N^\mu$ is a free nucleon field operator that also transforms *linearly* under chiral SU(3) then we would find $g_A/g_V = (-1/\sqrt{3})[\alpha_2 + (3/\sqrt{5})\alpha_1]$. In particular for solution 2 we would obtain $g_A/g_V = +\frac{5}{3}$, the SU(6) result.[14] But in general the situation is *not* expected to be so simple and important corrections will arise due to coupling to other states and the possible nonlinear transformation property. In accord with our linearity ansatz the equal-time commutators of $V_a^\mu(x)$ and $A_a^\mu(x)$ with ${}_\pm N_b^0(y)$ and ${}_\pm\Delta_b^0(y)$ are the same as Eqs. (3.1) and (3.2) and Eqs. (3.4) and (3.5), respectively. For example

$$[J_a^\mu(x), {}_\pm N_b^0(y)]\delta(x^0 - y^0) = [J_a^0(x), {}_\pm N_b^\mu(y)]\delta(x^0 - y^0), \tag{3.10}$$

where J^μ is equal to V^μ or A^μ.

Attempts to use equal-time commutation relations between axial-vector currents and fields within the framework of sidewise dispersion relations,[15] etc.,[16] invariably choose nucleon fields that transform irreducibly as (1, 8) under chiral transformation as suggested, say, by the σ model.[17] However, as indicated above, it may indeed be more interesting from the point of view of understanding and improving SU(6) results to consider solution 2. This question shall be pursued further. One might ask why we do not consider higher-dimensional representations. The answer is simply that baryons with isospin greater than $\frac{3}{2}$ have not been observed and thus we do not admit such higher-isospin BIF (no "exotic baryons").

B. Anticommutation Relations

In order to complete the analogy with current algebra, a model for the equal-time anticommutation relations between BIF involving at least one time component must be considered. As before, our basic ansatz shall be to write down the most general model linear in currents allowed by covariance and the discrete symmetries. The remaining arbitrariness will be eliminated by closing the algebra via the Jacobi identity Eq. (3.6a). Let us consider one such anticommutator in detail:

$$\{{}_+N_a^0(x), {}_+\bar{N}_b^\mu(y)\}\,\delta(x^0 - y^0) = \begin{pmatrix} 8 & 8 & 8_n \\ a & c & b \end{pmatrix}[(\omega_n^0 g^\mu{}_\nu + \omega_n^1 \sigma^\mu{}_\nu)V_c^\nu(x) + i(\omega_n^2 g^\mu{}_\nu + \omega_n^3\sigma^\mu{}_\nu)\gamma^5 A_c^\nu(x)]\delta^4(x-y), \tag{3.11}$$

where ω_n^j are constants, $\sigma^{\mu\nu} = \frac{1}{2}i[\gamma^\mu, \gamma^\nu]$, and linearity has been invoked. If we take the adjoint of Eq. (3.11) and set the result equal to Eq. (3.11) for $\mu = 0$, we find that ω^j are real numbers. The constraints due to parity have already been imposed. Charge-conjugation invariance implies that $\omega_n^{1,2} \equiv 0$. Had we introduced second-class axial-vector currents, then a term such as ω_n^2 would have remained. T invariance implies that $\omega_n^{1,2}$ are imaginary and $\omega_n^{0,4}$ are real and hence yields no new information in this case. The results obtained by imposing C, P, and T invariance on all the other anticommutators is recorded in Appendix B. Note that the relative discrete transformation properties of the BIF are fixed by Eqs. (3.4) and (3.5).

The next step is to close the algebra via the Jacobi identity Eq. (3.6a) between the axial-vector current and the time components of two BIF. The procedure is straightforward but tedious and we record the results in the following manner:

$$\delta(x^0 - y^0)\{{}_\pm N_a^0(x), {}_\pm \bar{N}_b^\mu(y)\} = \begin{pmatrix} 8 & 8 & 8_i \\ a & c & b \end{pmatrix} \begin{matrix} \omega_i V_c^\mu(x) \\ \phi_i A_c^\mu(x) \end{matrix}\, \delta^4(x-y), \tag{3.12}$$

where the choice ++ (+−) or −− (−+) in the left-hand side yields the term proportional to V_c^μ (A_c^μ). Similarly,

$$\delta(x^0-y^0)\{_\pm\Delta_a^0(x),\ _\pm\bar{\Delta}_b^\mu(y)\}=\begin{pmatrix}10 & 8 & 10\\ a & c & b\end{pmatrix}\begin{matrix}\theta V_c^\mu(x)\\ \kappa A_c^\mu(x)\end{matrix}\delta^4(x-y) \tag{3.13}$$

and

$$\delta(x^0-y^0)\{_\pm N_a^0(x),\ _\pm\bar{\Delta}_b^\mu(y)\}=\begin{pmatrix}8 & 8 & 10\\ a & c & b\end{pmatrix}\begin{matrix}\sigma V_c^\mu(x)\\ \tau A_c^\mu(x)\end{matrix}\delta^4(x-y). \tag{3.14}$$

The parameters $\omega_i, \ldots, \tau$ are fixed by the choice of how the BIF transform under chiral SU(3), i.e., Eq. (3.7) or (3.8). Corresponding to Eq. (3.7) we find solution 1:

$$\omega_i=\phi_i\,,\quad \kappa=\theta,\quad \text{and}\quad \sigma=\tau. \tag{3.15}$$

However, we may fix the scale such that $_\pm N \to {_\pm N'} = (|\omega_2|/\sqrt{3})^{1/2}\, _\pm N$ and $_\pm\Delta \to {_\pm\Delta'} = (|\theta|/\sqrt{6})^{1/2}\, _\pm\Delta$. Note that N and Δ may be *independently* rescaled in solution 1 since *each* transforms irreducibly under chiral SU(3). Thus after dropping the primes, the result implied by solution 1 may be read from Eqs. (3.12)–(3.14) by making the following substitutions.

Solution 1:

$$\begin{aligned}&\omega_i=\phi_i\to\sqrt{3}\ \omega_i/|\omega_2|,\\ &\kappa=\theta\to\sqrt{6}\ \theta/|\theta|,\\ &\text{and}\\ &\tau=\sigma.\end{aligned} \tag{3.16}$$

Thus the algebra, without any further input, is defined up to two parameters whose origin is clear: $\omega_1/|\omega_2|$ is just a "d" to "f" ratio which cannot be determined from within the algebra and σ determines the relative "size" of the two irreducible chiral SU(3) tensor operators.

Corresponding to solution 2 [Eq. (3.8)], we find

$$\begin{aligned}&\begin{pmatrix}\omega_1\\ \phi_1\end{pmatrix}=-\frac{25}{4\sqrt{10}}\begin{pmatrix}\sigma\\ \tau\end{pmatrix},\\ &\begin{pmatrix}\omega_2\\ \phi_2\end{pmatrix}=-\frac{5}{4\sqrt{2}}\begin{pmatrix}2\sigma+3\tau\\ 2\tau+3\sigma\end{pmatrix},\\ &\text{and}\\ &\begin{pmatrix}\theta\\ \kappa\end{pmatrix}=-\frac{1}{5}\begin{pmatrix}\sigma+3\tau\\ \tau+3\sigma\end{pmatrix}.\end{aligned} \tag{3.17}$$

However, we may make a convenient *over-all* change in scale:

$$\begin{pmatrix}N'\\ \Delta'\end{pmatrix}\to\begin{pmatrix}N'\\ \Delta'\end{pmatrix}=\left[\frac{5}{4}\left(\frac{3}{2}\right)^{1/2}\frac{|\tau|}{\tau}\right]^{1/2}\begin{pmatrix}N\\ \Delta\end{pmatrix}.$$

Note that we cannot rescale N and Δ separately without modifying their chiral SU(3) transformation properties. Thus, after dropping the primes, the result implied by solution 2 may be read from Eqs. (3.12)–(3.14) by making the following substitutions.

Solution 2:

$$\begin{aligned}&\begin{pmatrix}\omega_1\\ \phi_1\end{pmatrix}\to-\frac{1}{|\tau|}\left(\frac{5}{3}\right)^{1/2}\begin{pmatrix}\sigma\\ \tau\end{pmatrix},\\ &\begin{pmatrix}\omega_2\\ \phi_2\end{pmatrix}\to\frac{-1}{\sqrt{3}|\tau|}\begin{pmatrix}2\sigma+3\tau\\ 2\tau+3\sigma\end{pmatrix},\\ &\begin{pmatrix}\theta\\ \kappa\end{pmatrix}\to-\frac{4}{25}\left(\frac{2}{3}\right)^{1/2}\frac{1}{|\tau|}\begin{pmatrix}\phi+3\tau\\ \tau+3\sigma\end{pmatrix},\\ &\text{and}\\ &\begin{pmatrix}\sigma\\ \tau\end{pmatrix}\to\frac{1}{|\tau|}\frac{4}{5}\left(\frac{2}{3}\right)^{1/2}\begin{pmatrix}\sigma\\ \tau\end{pmatrix}.\end{aligned} \tag{3.18}$$

Thus for solution 2 we have to determine one parameter, σ, and the sign of τ. In the limit of exact SU(2)×SU(2) symmetry we shall see that $\sigma=0$ in *both* solutions 1 and 2. In the limit of an SU(3)-symmetric vertex function we shall see that $\omega_1=0$ in solution 1 and σ is again equal to zero in solution 2. The undetermined signs in Eqs. (3.16) and (3.18) can probably be fixed by some compactness assumption similar to current algebra, where $[Q_a^5, Q_b^5]=+i\epsilon_{abc}Q_c$ yields a compact algebra, whereas a minus sign leads to a noncompact algebra. However, just as for example the Adler-Weisberger relation chooses the plus sign, here too we shall find at least two tests and a Lagrangian model which imply in Eq. (3.16) that $\omega_2<0$ and in Eq. (3.18) that $\tau>0$.

If we had considered only the Jacobi identity Eq. (3.6a) between the *time* component of the axial-vector current and time components of two BIF, we would have not been able to eliminate terms of the form $\sigma^\mu{}_\nu\gamma^5 J_c^\nu(x)$ from the right-hand side of Eqs. (3.12)–(3.14). In fact such terms would appear with coefficients having precisely the same structure as in Eq. (3.15) or Eq. (3.17). However, if we impose the Jacobi identity between a *space* component of a current and the time components of two BIF we can exclude all such additional terms. For example, consider the Jacobi identity between V^j, $_+N^0$, and $_+\bar{N}^0$. The left-hand side of Eq. (3.6a) gives rise to a term of the form $\omega_n^3\gamma^5\sigma^{0k}[V^j, A^k]$, whereas the right-hand side gives rise to a term of the form $\omega_n^3\gamma^5\sigma^{j\nu}A_\nu$. The Dirac structure of these two terms is at variance and therefore $\omega_n^3\equiv 0$.

The introduction of BIF with opposite parity is crucial for the construction of a nontrivial linear algebra. Had we resorted to the device[13] of changing the parity via γ^5, i.e., setting $_-N=\xi\gamma_5 N_+$ [where $\xi^2=1$ as required by Eq. (3.4)], the only solution to the Jacobi identities would have been the trivial one with the right-hand side of Eqs. (3.12)–(3.14) identically zero. Proof: Equation (3.12) states that the anticommutators of the positive-parity BIF are the same as that of the negative-parity BIF. But since γ^0 and γ^5 anticommute, ${}_{-}\bar{N} = -\xi\, {}_{+}\bar{N}\gamma^5$, implying that $\{{}_{-}N^0, {}_{-}\bar{N}^\mu\} = -\{{}_{+}N^0, {}_{+}\bar{N}^\mu\}$, which would then require that $\omega_i \equiv 0$, Q.E.D. Thus, having been forced to introduce opposite-parity BIF, we shall later exploit this situation and assume that the low-lying baryons couple exclusively to the BIF with the same parity, e.g., $\Delta(1230)$ couples to ${}_{+}\Delta^\mu$ and $\Delta(1670)$ couples to ${}_{-}\Delta^\mu$.

Finally, for future reference, let us write the anticommutator of the nonstrange BIF in Eq. (3.12):

$$\{{}_{-}N_a^0(x), {}_{-}\bar{N}_b^\mu(y)\}\,\delta(x^0-y^0) = \frac{1}{2}\left(\omega_2 + \frac{3}{\sqrt{5}}\omega_1\right) C^{1/2\;\,1\;\,1/2}_{a\;\;\,c\;\;\,b} V_c^\mu(x)\delta^4(x-y) + \frac{1}{2}\left(\omega_2 - \frac{1}{\sqrt{5}}\omega_1\right) C^{1/2\;\,0\;\,1/2}_{a\;\;\,0\;\;\,b} V_8^\mu(x)\delta^4(x-y)$$

$$= \frac{1}{\sqrt{12}}\left[\left(\omega_2 + \frac{3}{\sqrt{5}}\omega_1\right)(\tau_c)_{ba} V_c^\mu(x) + \sqrt{3}\left(\omega_2 - \frac{1}{\sqrt{5}}\omega_1\right)\delta_{ba} V_8^\mu(x)\right]\delta^4(x-y), \qquad (3.19)$$

where

$$\tau_0 = \tau_3,\ \tau_{\pm 1} = \mp(\tfrac{1}{2})^{1/2}(\tau_1 \pm i\tau_2),$$

and $\tau_{1,2,3}$ are the Pauli Cartesian spin matrices (recall that we are using a spherical basis and $\sum_{\text{Cartesian}} \tau_i\tau_i = \sum_{\text{spher}} \tau_a^T\tau_a$).

C. Commutators Involving Divergences

In actual calculations where for example Ward-identity techniques are employed, one encounters (anti) commutators between *time* components of currents or BIF with the divergence of currents or BIF. The original notorious such example is the σ term encountered in ordinary current algebra[1]:

$$[A_a^0(x), \partial\cdot A_b(y)]\delta(x^0-y^0) \equiv \sigma_{ab}(x)\delta^4(x-y) \qquad (3.20)$$

(in a Hermitian basis). The only model-independent statement available concerning such a term is that for SU(2)×SU(2) it has no $I=1$ term. Otherwise, one falls back on various models such as the σ model[17] which suggest that $I=2$ is also absent. To consider such σ type commutators we avail ourselves of the following rather general identity. Given

$$\delta(x^0-y^0)[X^0(x), Y^\mu(y)]_\pm = \delta(x^0-y^0)[X^\mu(x), Y^0(y)]_\pm = Z^\mu(x)\delta^4(x-y),$$

one may readily show that up to terms that integrate to zero,

$$[X^0(x), \partial\cdot Y(y)]_\pm\delta(x^0-y^0) + [\partial\cdot X(x), Y^0(y)]_\pm\delta(x^0-y^0) = \partial\cdot Z(x)\delta^4(x-y). \qquad (3.21)$$

For example, if X and Y are axial-vector currents, then Z is the conserved vector current [for chiral SU(2)] and Eq. (3.21) implies the standard result that there is no $I=1$ σ term. Similarly, if we apply Eq. (3.21) to Eq. (3.12), we obtain (we restrict ourselves to nonstrange BIF throughout the remainder of this section) from the anticommutators proportional to the conserved vector current

$$\delta(x^0-y^0)\{{}_{\pm}N_a^0(x), {}_{\pm}\partial\cdot\bar{N}_b(y)\} + \delta(x^0-y^0)\{{}_{\pm}\partial\cdot N_a(x), {}_{\pm}\bar{N}_b^0(y)\} = 0, \qquad (3.22)$$

from which we deduce that the "σ term" defined by

$$\{{}_{\pm}N_a^0(x), {}_{\pm}\partial\cdot\bar{N}_b(y)\}\,\delta(x^0-y^0) \equiv i\,{}_{\pm}\Sigma_{ab}^N(x)\delta^4(x-y) \qquad (3.23)$$

[where ++ (− −) on the left-hand side corresponds to + (−) on the right-hand side] has no $I=1$ piece and is Hermitian, i.e.,

$${}_{\pm}\Sigma_{ab}^N(x) = \delta_{ab}\,{}_{\pm}\Sigma^N(x). \qquad (3.24)$$

In the limit of SU(3) symmetry, i.e., $\partial_\mu V_a^\mu = 0$, or in the limit of exact chiral SU(2), i.e., $\partial_\mu A_a^\mu = 0$, $a = 1, 2$, or 3, one arrives at similar conclusions for the other components of Eq. (3.12). Equations (3.13) and (3.21) imply

$$\{{}_{\pm}\Delta_a^0(x), {}_{\pm}\partial\cdot\bar{\Delta}_b(y)\}\,\delta(x^0-y^0) = i\delta_{ab}\,{}_{\pm}\Sigma^\Delta(x)\delta^4(x-y), \qquad (3.25)$$

where we have omitted possible isospin-two and isospin-three terms on the right-hand side of Eq. (3.25). We shall not now pursue further the various requirements imposed upon these and other "Σ terms" by the Jacobi identities, Eq. (3.6a). Suffice it to say that in practice we shall attempt to minimize the importance of such terms wherever possible. In current-algebra calculations one also attempts to do away with such terms when consistent with other general constraints. This is not always possible, e.g., in Weinberg's calculation of π-π scattering lengths[18] the σ term plays an important role.

Let us briefly turn our attention to Eqs. (3.4) and (3.5). Equation (3.21) implies that

$$[A_a^0(x), {}_\pm\partial\cdot N_b(y)]\delta(x^0-y^0)+[\partial\cdot A_a(x), {}_\pm N_b^0(y)]\delta(x^0-y^0)$$
$$=\alpha^i\begin{pmatrix}8&8&8_i\\c&a&b\end{pmatrix}{}_\mp\partial\cdot N_c(x)\delta^4(x-y)+\beta\begin{pmatrix}8&10&8\\a&c&b\end{pmatrix}{}_\mp\partial\cdot\Delta_c(x)\delta^4(x-y). \quad (3.26)$$

If $\partial\cdot A_a=0$ [e.g., exact chiral SU(3) symmetry] the above equation would imply that $\partial\cdot N$ also transformed linearly; however this is not the case and the second term on the left-hand side of Eq. (3.25) is in general nonvanishing.

IV. A LAGRANGIAN MODEL

We have constructed a closed algebra consistent with our ansatz of linearity and the general constraints due to covariance and the discrete symmetries. Is it possible to reproduce this algebra from some simple Lagrangian model? The first kind of model one might consider is a quark model. The construction of currents from quark fields satisfying canonical anticommutation relations, $J^\mu\,\alpha\bar q\gamma^\mu q$, yields the current algebra. If one attempts to construct the BIF in terms of three quark fields it is not difficult to see that in general the equal-time anticommutators will be quadratic in currents, i.e., the canonical anticommutation relations for the quark fields will remove one quark and one antiquark field leaving two quark and two antiquark fields.[2] Similarly, any naive composite model for the BIF leads to anticommutation relations quadratic in currents, e.g., BIF $\propto$ nucleon field$\times$current. In any event, such results are suspect due to the highly singular nature of products of so many fields. Thus we are led to study models that involve spin-$\frac{3}{2}$ fields. The simplest such model one might consider combines a Yang-Mills[3] gauge field with a Rarita-Schwinger[4] spin-$\frac{3}{2}$ field. To simplify the notation somewhat we shall restrict the discussion to SU(2) Hermitian gauge fields ϕ_a^μ and nonstrange vector-spinor fields Ψ_σ^μ. Consider the following Lagrangian:

$$\mathcal{L}=-\tfrac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}+\tfrac{1}{2}m_0{}^2\phi_a^\mu\phi_{a\mu}+\bar G_\tau^{\mu\nu}D_{\tau\sigma\mu}\Psi_{\sigma\nu}-M_0\bar\Psi_\tau^\mu\Psi_{\tau\mu}, \quad (4.1)$$

where

$$F_a^{\mu\nu}(x)=\partial^\mu\phi_a^\nu(x)-\partial^\nu\phi_a^\mu(x)-\frac{g_0}{2}\left[\epsilon_{abc}\phi_b^\mu(x)\phi_c^\nu(x)-i\frac{M_0}{m_0{}^2}(\tau_a)_{\tau\sigma}\Psi_\tau^\mu(x)\Psi_\sigma^\nu(x)-(\mu\leftrightarrow\nu)\right], \quad (4.2)$$

$$\bar G_\tau^{\mu\nu}(x)=i\bar\Psi_{\tau\lambda}(x)(g^{\lambda\nu}\gamma^\mu-g^{\lambda\mu}\gamma^\nu-g^{\mu\nu}\gamma^\lambda+\gamma^\lambda\gamma^\mu\gamma^\nu), \quad (4.3)$$

and D_ν is the covariant derivative with

$$D_{\tau\sigma}^\nu=\partial^\nu\delta_{\tau\sigma}+g_0\tfrac{1}{2}i(\tau_a)_{\tau\sigma}\phi_a^\nu(x). \quad (4.4)$$

Note that $F^{\mu\nu}$ and $G^{\mu\nu}$ are antisymmetric. If we turn off the interaction between the gauge field and the vector-spinor field the Lagrangian reduces to

$$\mathcal{L}_{\rm YM}+\mathcal{L}_{\rm RS}-M_0\bar\Psi_\tau\cdot\gamma\gamma\cdot\Psi_\tau.$$

The inclusion of the final term allows great simplification of the equations of motion in the interacting case. However, in order to describe a *free* spin-$\frac{3}{2}$ particle, one would have to introduce a subsidiary condition $\gamma_\mu\Psi^\mu=0$. Recall that the equations of motion deduced from $\mathcal{L}_{\rm RS}$ are $\gamma_\mu\Psi^\mu=0$, $\partial_\mu\Psi^\mu=0$, and $(-i\gamma\cdot\partial+M_0)\Psi^\mu=0$. Let us return to the interacting case. The momenta canonically conjugate to ϕ_a^ν and Ψ_τ^ν are given by

$$\frac{\partial\mathcal{L}}{\partial\partial^0\phi_a^\nu}=F_{a\nu0} \quad (4.5)$$

and

$$\frac{\partial\mathcal{L}}{\partial\partial^0\Psi_\tau^\nu}=\bar G_{\tau0\nu}, \quad (4.6)$$

respectively. Since $F^{\mu\nu}$ and $\bar G^{\mu\nu}$ are antisymmetric, ϕ^0 and Ψ^0 have no canonically conjugate variables. The canonical commutation rules are

$$[F_{aj0}(x), \phi_b^k(y)]\delta(x^0-y^0)=-i\delta_{jk}\delta_{ab}\delta^4(x-y), \quad (4.7a)$$

$$[\phi_a^i(x), \phi_b^j(y)]\delta(x^0-y^0)=0, \quad (4.7b)$$

$$\{\Psi_{\sigma j}(x), \bar G_\tau^{0k}(y)\}\,\delta(x^0-y^0)=+i\delta_{jk}\delta_{\tau\sigma}\delta^4(x-y)$$

or (4.8a)

$$\{G_\tau^{0k}(x), \bar\Psi_{\sigma j}(x)\}\,\delta(x^0-y^0)=-i\delta_{jk}\delta_{\tau\sigma}\delta^4(x-y),$$

and

$$\{\Psi_\sigma^i(x), \Psi_\tau^j(y)\}\,\delta(x^0-y^0)=0. \quad (4.8b)$$

The equations of motion are[19]

$$m_0{}^2\phi_a^\nu=+\partial_\mu F_a^{\nu\mu}+\tfrac{1}{2}g_0\epsilon_{abc}\{F_b^{\nu\mu}, \phi_{c\mu}\}-ig_0\bar G^{\nu\mu}\tfrac{1}{2}\tau_a\Psi_\mu$$

and

$$-M_0\bar\Psi_\sigma^\nu=\bar G_\tau^{\mu\nu}D_{\tau\sigma\mu}^\dagger+i\frac{g_0M_0}{m_0{}^2}(\tfrac{1}{2}\tau_a)_{\tau\sigma}F_a^{\mu\nu}\bar\Psi_{\tau\mu} \quad (4.9)$$

or

$$-M_0\Psi_\sigma^\nu=D_{\sigma\tau\mu}G_\tau^{\mu\nu}-i\frac{g_0M_0}{m_0{}^2}(\tfrac{1}{2}\tau_a)_{\sigma\tau}\Psi_{\tau\mu}F_a^{\mu\nu}. \quad (4.10)$$

The interesting and useful feature of the above equations of motion is that both ϕ_a^0 and Ψ_σ^0 are given in terms of the canonical variables, i.e., ϕ_a^k, F_a^{0j}, G_τ^{0l}, and $\overline{\Psi}_\tau^k$. Therefore, given the canonical commutation relations it is a straightforward exercise to calculate commutators involving ϕ_a^0 and Ψ_σ^0. Since the matter fields have been coupled via the covariant derivative, the field-algebra commutators are the same as those calculated by Lee, Weinberg, and Zumino,[19] i.e.,

$$[\phi_a^0(x), \phi_b^\mu(y)]\delta(x^0-y^0) = i\frac{g_0}{m_0^2}\epsilon_{abc}\phi_c^\mu(x)\delta^4(x-y) - \frac{i}{m_0^2}\delta_{ab}g^{\mu k}\partial^k\delta^4(x-y). \tag{4.11}$$

Furthermore, one can calculate the other commutation relations that involve at least one time component:

$$[\phi_a^0(x), \Psi_\sigma^\mu(y)]\delta(x^0-y^0) = \frac{-g_0}{m_0^2}(\tfrac{1}{2}\tau_a)_{\sigma\tau}\Psi_\tau^\mu(x)\delta^4(x-y) \tag{4.12}$$

and

$$[\phi_a^\mu(x), \Psi_\sigma^0(y)]\delta(x^0-y^0) = [\phi_a^0(x), \Psi_\sigma^\mu(y)]\delta(x^0-y^0). \tag{4.13}$$

Equation (4.12) is expected; it states the transformation property of the matter field Ψ^μ under the non-Abelian gauge transformation. Note that no Schwinger terms appear. The derivation of Eq. (4.13) involved the identity $G^{0j} = +\gamma^0\sigma^{ij}\Psi^j$ combined with the canonical anticommutator to yield

$$\{G^{0j}(x), \overline{G}^{0k}(y)\}\delta(x^0-y^0) = -i\gamma^0\sigma^{kj}\delta^4(x-y), \tag{4.14a}$$

and for later reference we note that

$$\{\Psi^j(x), \overline{\Psi}^k(y)\}\delta(x^0-y^0) = \gamma^k\tfrac{1}{2}\gamma^j\gamma^0\delta^4(x-y) \tag{4.14b}$$

and

$$\overline{G}^{0j}\Psi_j = \overline{\Psi}^j G^{0j}.$$

Finally, we calculate the anticommutators between the spin-$\frac{3}{2}$ fields:

$$-M_0\{\Psi_\sigma^0(x), \overline{\Psi}_\tau^k(y)\}\delta(x^0-y^0) = -g_0(\tfrac{1}{2}\tau_a)_{\sigma\tau}\phi_a^k(x)\delta^4(x-y) + i\partial^k\delta^4(x-y)\delta_{\sigma\tau} + i\frac{g_0M_0}{m_0^2}(\tfrac{1}{2}\tau_a)_{\sigma\tau}\tfrac{1}{2}\gamma^k\gamma^j\gamma^0F^{j0}(x)\delta^4(x-y) \tag{4.15}$$

and

$$M_0^2\{\Psi_\sigma^0(x), \overline{\Psi}_\tau^0(y)\}\delta(x^0-y^0) = g_0M_0(\tfrac{1}{2}\tau_a)_{\sigma\tau}\phi_a^0(x)\delta^4(x-y) + i\frac{3}{4}\delta_{\sigma\tau}\frac{g_0^2M_0}{m_0^2}\overline{G}_\alpha^{0j}(x)\Psi_{j\alpha}(x)\delta^4(x-y) + \left(\frac{g_0^2M_0^2}{m_0^4}(\tfrac{1}{2}\tau_a\tfrac{1}{2}\tau_b)_{\sigma\tau}F_a^{j0}(x)F_b^{k0}(x)\gamma_k\gamma_j - \tfrac{1}{2}g_0(\tfrac{1}{2}\tau_a)_{\sigma\tau}[\partial^j\phi_a^k(x)\tfrac{1}{2}g_0\epsilon_{abc}\phi_b^j(x)\phi_c^k(x)]\sigma_{jk}\right)\gamma^0\delta^4(x-y). \tag{4.16}$$

We now reexpress Eqs. (4.11)–(4.16) in terms of new rescaled fields defined by

$$\phi_a^\mu(x) = (g_0/m_0^2)V_a^\mu(x) \tag{4.17a}$$

and

$$\Psi_\sigma^\mu(x) = [g_0/m_0(M_0)^{1/2}]N_\sigma^\mu(x). \tag{4.17b}$$

Equation (4.11) has the same form as that of the algebra of currents with an explicit Schwinger term (S.T.):

$$[V_a^0(x), V_b^\mu(y)]\delta(x^0-y^0) = i\epsilon_{abc}V_c^\mu(x)\delta^4(x-y) - i\frac{m_0^2}{g_0^2}\delta_{ab}g^{\mu k}\partial_x^k\delta^4(x-y). \tag{4.18}$$

Equation (4.12) exhibits the transformation property of $N_\sigma^\mu(x)$:

$$[V_a^0(x), N_\sigma^\mu(y)]\delta(x^0-y^0) = -(\tfrac{1}{2}\tau_a)_{\sigma\tau}N_\tau^\mu(x)\delta^4(x-y). \tag{4.19}$$

Finally, the anticommutators of interest are

$$\{N_\sigma^0(x), \overline{N}_\tau^k(y)\}\delta(x^0-y^0) = (\tfrac{1}{2}\tau_a)_{\sigma\tau}V_a^k(x)\delta^4(x-y) - i\delta_{\sigma\tau}\frac{m_0^2}{g_0^2}\partial_x^k\delta^4(x-y) - i\frac{M_0}{m_0^2}(\tfrac{1}{2}\tau_a)_{\sigma\tau}\tfrac{1}{2}\gamma^k\gamma^j\gamma^0\tilde{F}^{j0}(x)\delta^4(x-y) \tag{4.20}$$

and

$$\{N_\sigma^0(x), \bar{N}_\tau^0(y)\}\,\delta(x^0-y^0) = (\tfrac{1}{2}\tau_a)_{\sigma\tau} V_a^0(x)\delta^4(x-y) + i\tfrac{3}{4}\delta_{\sigma\tau}\frac{g_0^2}{m_0^2 M_0}\bar{\tilde{G}}^{0j}(x)N_j(x)\delta^4(x-y)$$
$$+\frac{g_0^2 M_0}{m_0^6}(\tfrac{1}{2}\tau_a\tfrac{1}{2}\tau_b)_{\sigma\tau}\tilde{F}_a^{j0}(x)\tilde{F}_b^{k0}(x)\gamma^k\gamma^j\gamma^0\delta^4(x-y)$$
$$+\frac{i}{2M_0}(\tfrac{1}{2}\tau_a)_{\sigma\tau}\left[\partial^j V_a^k(x) - \frac{g_0^2}{m_0^2}\epsilon_{abc}V_b^j(x)V_c^k(x)\right]\sigma_{jk}\gamma^0\delta^4(x-y), \tag{4.21}$$

where

$$F_a^{\mu\nu}(x) = (g_0/m_0^2)\tilde{F}_a^{\mu\nu}(x), \quad G_\sigma^{\mu\nu}(x) = (g_0/m_0^2\sqrt{M_0})\tilde{G}_\sigma^{\mu\nu}(x) \tag{4.22}$$

and

$$\tilde{F}_a^{\mu\nu}(x) = \partial^\mu V_a^\nu(x) - \partial^\nu V_a^\mu(x) - \frac{g_0^2}{2m_0^2}\{[\epsilon_{abc}V_b^\mu(x)V_c^\nu(x) - i\bar{N}^\mu(x)\tau_a N^\nu(x)] - (\mu \leftrightarrow \nu)\}. \tag{4.23}$$

Thus our primary objective has been achieved and we have exhibited a model in which the anticommutators Eq. (4.20) and Eq. (4.21) contain a term linear in the vector field. This does not occur in standard composite models. It is not difficult to find limits in which most of the other terms in Eqs. (4.20) and (4.21) disappear or become c numbers. For example, consider the limit (similar to the limit studied by Bardakci *et al.*[20])

$$m_0 \to 0, \quad g_0 \to 0, \quad \text{and} \quad M_0 \to 0$$

with

$$C^{-1} = g_0^2/m_0^2 \quad \text{and} \quad d = m_0/M_0 \tag{4.24}$$

fixed. In terms of the new variables the canonical commutation relation Eq. (4.7a) becomes

$$[\tilde{F}_{aj0}(x), V_b^k(y)]\,\delta(x^0-y^0) = -iCm_0^2\delta_{jk}\delta_{ab}\delta^4(x-y). \tag{4.25}$$

In this limit[21] $\tilde{F}_a^{0k}(x) \to 0$ like m_0^2 and we assume that the piece of $\tilde{F}_a^{jk}$ given by

$$\left\{\partial^j V_a^k(x) - \partial^k V_a^j(x) - \frac{1}{2C}\epsilon_{abc}[V_b^j(x)V_c^k(x) - V_b^k(x)V_c^j(x)]\right\}$$

also goes like m_0^2 in the same limit. It is then straightforward to take the limit of Eqs. (4.20) and (4.21) and obtain the desired result:

$$\{N_\sigma^0(x), \bar{N}_\tau^\mu(y)\}\,\delta(x^0-y^0) = (\tfrac{1}{2}\tau_a)_{\sigma\tau} V_a^\mu(x)\delta^4(x-y) + \text{S.T.} + (I=0 \text{ term}). \tag{4.26}$$

Our basic intention in this section has been to exhibit a Lagrangian whose canonical commutation rules imply a structure similar to the one we hypothesized. We shall not pursue here the difficult questions associated with such a Lagrangian field theory. However, we do feel that the Lagrangian merits further study in that the equation of motion, Eq. (4.9), is particularly simple and allows one to eliminate Ψ^0 in terms of canonical variables.

V. SUGGESTED APPLICATIONS

Given the algebra of Sec. III, a host of possible applications present themselves. In this section we shall primarily outline several such applications and some of the results. A more detailed implementation of the program will be presented in a separate communication. We shall restrict the discussion here to nonstrange BIF so as not to get entangled in problems associated with the breaking of SU(3).

A. Nucleon Zeros

The first class of theorems is a direct consequence of the ansatz that the interpolating field of, say, the nucleon is proportional to the divergence of a spin-$\frac{3}{2}$ field just as the pion interpolating field is proportional to the divergence of a spin-1 object. This ansatz immediately allows one to state theorems entirely analogous to those of Adler.[22] Namely, a scattering amplitude involving a nucleon vanishes at the *unphysical* point where the four-momentum of the nucleon goes to zero with the other particles on their mass shells. Note that in our case because of conservation of baryon number no additional terms arise from external-line insertions. Clearly, a zero four-momentum nucleon is not a physical object and the utility of such a theorem will lie in its use, for example, as a boundary condition for off-shell models of nucleon amplitudes, or in the treatment of subtractions in ordinary dispersion relations and in sidewise dispersion relations.

B. PCBIF (Partially Conserved BIF)

There may be circumstances in which the value of the amplitude at the "soft-nucleon" point may be extrapolated back to the physical region. Such cases provide examples of what we may call PCBIF (partially conserved baryon interpolating field). One may ask what is the basis for hoping that such extrapolations have anything to do with reality? After all, the nucleon mass is not a small number, whereas the pion mass (140 MeV) is. One might respond as has recently been emphasized, particularly by Dashen,[23] that the success of partial conservation of the axial-vector current (PCAC) does not depend exclusively on the smallness of the pion mass but rather on the nature of the chiral SU(2) symmetry breaking, i.e., extrapolation to physical pions from soft pions is valid provided the neglected terms are of order of the chiral symmetry-breaking parameter. Furthermore, successful results obtained from applying PCAC to kaons are difficult to understand on the basis of the "small" kaon mass (more than *half* that of the nucleon) and most probably will find their explanation in terms of chiral SU(3) symmetry breaking. Other extrapolations in meson masses, e.g., in the ρ meson, have also yielded unexplained surprisingly good results: As an example, we may cite the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation.[24] Note that this relation cannot be derived by current algebra alone. Thus we argue that the magnitude of the nucleon mass is not *a priori* a basis for pessimism. In fact we shall show, for example, that s-wave scattering lengths of soft nucleons *can* be extrapolated successfully to physical nucleons. A rigorous justification for such a procedure, however, will have to await either an understanding of the symmetry (and possible symmetry breaking) embodied by the baryon algebra we have constructed or a dispersion-theoretic estimate of the terms neglected in such an extrapolation.

C. Off-Shell Nucleon Scattering Amplitudes

Consider the off-shell scattering amplitude $R_{ab}^{\mu\nu}$ representing the scattering of a BIF, i.e., $N^\mu + \alpha \to N^\nu + \beta$[25]:

$$(2\pi)^4\delta^4(p+k+q+q')R_{ab}^{\mu\nu} = -i\int dxdy\, e^{-ik\cdot x}e^{-ip\cdot y} \times\langle\beta(-q')|TN_a^\mu(x)\bar{N}_b^\nu(y)|\alpha(q)\rangle. \tag{5.1}$$

The Ward identities implied by the baryon algebra are

$$k_\mu R_{ab}^{\mu\nu} = -iR_{ab}^{-\nu} - \mathcal{V}_{ab}^{\nu}, \tag{5.2a}$$

$$p_\nu R_{ab}^{\mu\nu} = -iR_{ab}^{\mu -} + \mathcal{V}_{ab}^{\mu}, \tag{5.2b}$$

$$k_\mu R_{ab}^{\mu -} = -iR_{ab}^{--} - \Sigma_{ab}, \tag{5.2c}$$

and

$$p_\nu R_{ab}^{-\nu} = -iR_{ab}^{--} - \Sigma_{ab}, \tag{5.2d}$$

where a dash(-) appears in place of a Lorentz index whenever the appropriate off-shell amplitude involves the divergence of a BIF, e.g.,

$$R_{ab}^{\mu -} = -i\int dx\, e^{-ik\cdot x}\langle\beta(-q')|TN_a^\mu(x)\partial\cdot\bar{N}_b(0)|\alpha(q)\rangle. \tag{5.3}$$

In addition, we have defined

$$\mathcal{V}_{ab}^{\nu} \equiv \int dx\, e^{-ik\cdot x}\langle\beta(-q')|\{N_a^0(x), \bar{N}_b^\nu(0)\}\,\delta(x^0)|\alpha(q)\rangle$$

$$= \frac{1}{\sqrt{12}}\left[\left(\omega_2 + \frac{3}{\sqrt{5}}\omega_1\right)(\tau_c)_{ba}\langle\beta(-q')|V_c^\mu|\alpha(q)\rangle + \sqrt{3}\left(\omega_2 - \frac{1}{\sqrt{5}}\omega_1\right)\delta_{ab}\langle\beta(-q')|V_8^\mu|\alpha(q)\rangle\right] \tag{5.4}$$

and

$$\Sigma_{ab} \equiv \delta_{ab}\langle\beta(-q')|\Sigma^N|\alpha(q)\rangle. \tag{5.5}$$

The pole structure exhibited by these off-shell amplitudes may be represented as follows:

$$R_{ab}^{--} = -iP(k)L_{ab}^{--}P(-p), \tag{5.6a}$$

$$R_{ab}^{-\nu} = -P(k)[iL_{ab}^{-\lambda}P_\lambda{}^\nu(-p) + \tfrac{4}{3}L_{ab}^{--}\bar{P}(-p)(p^\nu - \tfrac{1}{4}p\cdot\gamma\gamma^\nu)], \tag{5.6b}$$

$$R_{ab}^{\mu -} = -i[P^\mu{}_\lambda(k)L_{ab}^{\lambda} + \tfrac{4}{3}(k^\mu - \tfrac{1}{4}\gamma^\mu k\cdot\gamma)\bar{P}(k)L_{ab}^{--}]P(-p), \tag{5.6c}$$

and

$$iR_{ab}^{\mu\nu} = P^\mu{}_\lambda(k)L_{ab}^{\lambda\sigma}P_\sigma{}^\nu(-p) - i\tfrac{4}{3}[(k^\mu - \tfrac{1}{4}\gamma^\mu k\cdot\gamma)\bar{P}(k)L_{ab}^{-\lambda}P_\lambda{}^\nu(-p) + P^\mu{}_\sigma(k)L_{ab}^{\sigma -}\bar{P}(p)(p^\nu - \tfrac{1}{4}p\cdot\gamma\gamma^\nu)$$
$$-(\tfrac{4}{3})^2(k^\mu - \gamma^\mu\tfrac{1}{4}k\cdot\gamma)\bar{P}(k)L_{ab}^{--}\bar{P}(-p)(p^\nu\tfrac{1}{4}p\cdot\gamma\gamma^\nu)], \tag{5.6d}$$

where we have introduced the covariant spin-$\frac{3}{2}$ propagator $P^{\mu\nu}(q)$ given in Eq. (2.12), the spin-$\frac{1}{2}$ propagator given in Eq. (2.18), and the additional propagator

$$\overline{P}(q) \equiv \int d\mu^2 \frac{[\rho_3(\mu^2) - q \cdot \gamma\rho_4(\mu^2)]}{\mu^2 - q^2 - i\epsilon}. \tag{5.7}$$

The Ward identities Eq. (5.2) may now be reexpressed in terms of the proper off-shell amplitudes L_{ab} (whose momentum dependence has been suppressed). The off-shell nucleon + α → nucleon + β proper amplitude is given by

$$L_{ab}^{--} = -(C_3' - k\cdot\gamma C_4')^{-1}\{k_\mu P^\mu{}_{\mu'}(k)L_{ab}^{\mu'\nu'}P_{\nu'}{}^\nu(-p)p_\nu + i\mathcal{V}_{ab}^\nu p_\nu + (C_3' - k\cdot\gamma C_4')P^{-1}(k)\Sigma_{ab} - \Sigma_{ab}P^{-1}(-p)(C_3' + p\cdot\gamma C_4') + \Sigma_{ab}\}(C_3' + p\cdot\gamma C_4')^{-1}. \tag{5.8}$$

The other two amplitudes are also given in terms of $L_{ab}^{\mu\nu}$:

$$(C_3' - k\cdot\gamma C_4')L_{ab}^{-\nu} = +ik_\mu P^\mu{}_\lambda(k)L_{ab}^{\lambda\nu} - \mathcal{V}_{ab}^\lambda P^{-1}{}_\lambda{}^\nu(p) + i\tfrac{4}{3}\Sigma_{ab}P^{-1}(p)\overline{P}(p)(p^\lambda - \tfrac{1}{4}p\cdot\gamma\gamma^\lambda)P^{-1}{}_\lambda{}^\nu(p) \tag{5.9}$$

and

$$L_{ab}^{\mu-}(C_3' + p\cdot\gamma C_4') = iL_{ab}^{\mu\lambda}P_\lambda{}^\nu(p)p_\nu + P^{-1\mu}{}_\lambda(k)\mathcal{V}_{ab}^\lambda - i\tfrac{4}{3}P^{-1\mu}{}_\lambda(k)(k^\lambda - \tfrac{1}{4}\gamma^\nu k\cdot\gamma)\overline{P}(k)P^{-1}(k)\Sigma_{ab}, \tag{5.10}$$

where C_i' has been defined in Eq. (2.13). The construction of "soft" nucleon theorems follows directly from Eq. (5.8). In the limit $k,p \to 0$ the left-hand side of Eq. (5.8) is the amplitude for the process soft $N+\alpha$ → soft $N+\beta$, whereas the first term on the right-hand side is at least second-order in p and k and we shall assume that the terms involving Σ_{ab} are negligible. Thus to lowest order, the term we shall retain is proportional to $\mathcal{V}_{ab}^\nu p_\nu$, which is usually explicitly known.

A particular choice of states α and β might, for example, be either vacuum or single-particle states. Identical considerations are applicable to different choices of BIF in Eq. (5.1).

D. Nucleon Scattering Lengths

We shall now derive a general expression for the s-wave scattering lengths of a soft nucleon from a target. The derivation is subject to the provision that we may neglect the Σ term that appears in Eq. (5.8). In certain channels, e.g., $N\pi$ and NK, this result gives good agreement with the s-wave scattering lengths of physical nucleons. However, in other channels, e.g., NN and $N\overline{K}$, this result breaks down probably due to the presence of nearby inelastic thresholds. A similar circumstance occurs in the current-algebra calculations. πN and KN scattering lengths for soft mesons will approximate that of physical mesons; however, nucleus or $\overline{K}N$ soft-meson scattering lengths differ greatly from the physical scattering lengths. The explanation is generally sought in terms of the appearance of inelastic thresholds, e.g., in the $\overline{K}N$ channel one must contend with the $\pi\Sigma$ channel below threshold.[26]

Let $\alpha=\beta$ be a single-particle state. The off-shell $\partial\cdot N$ scattering amplitude, L_{ab}^{--}, Eq. (5.8), in the limit $k,p\to 0$ is to lowest order given by

$$L_{ab}^{--} \underset{p,k\to 0}{\sim} \frac{-i}{C_3'^2}P_\nu\mathcal{V}_{ab}^\nu = \frac{-i}{C_3'^2}N_\alpha{}^2\frac{p\cdot q}{\sqrt{3}}\left[\left(\omega_2 + \frac{3}{\sqrt{5}}\omega_1\right)\left(\frac{\tau_c}{2}\right)_{ba}(t_c)_{\beta\alpha} + \frac{3}{4}\left(\omega_2 - \frac{1}{\sqrt{5}}\omega_1\right)\delta_{ba}\delta_{\beta\alpha}Y_t\right], \tag{5.11}$$

where t_c are the isospin matrices of the target and Y_t is the target hypercharge. N_α is the normalization of the state. SU(3)-symmetric vertices require $\omega_1 \equiv 0$ and the baryon algebra requires $\omega_2 = \pm\sqrt{3}$ [see Eqs. (3.16), (3.18), and (4.26)]. To establish our normalization (whenever possible, the same as Bjorken and Drell)[7] we note the S-matrix element for the process α + nucleon → β + nucleon is

$$_{\text{out}}\langle\beta(-q')N_a(-k)|(S-1)|N_b(p)\alpha(q)\rangle_{\text{in}}$$

$$= -\lim_{k^2=M_N{}^2=p^2}\frac{1}{g_N{}^2M_N{}^2}\left(\frac{M_N{}^2}{(2\pi)^6E_pE_k}\right)^{1/2}(2\pi)^4\delta^4(p+k+q+q')\,\overline{u}(-k)(-k\cdot\gamma - M_N)iR_{ab}^{--}(p\cdot\gamma - M_N)\,u(p)$$

$$= -\lim_{k^2=M_N{}^2=p^2}M_N{}^2g_N{}^2(2\pi)^4\delta^4(p+k+q+q')\left(\frac{M_N{}^2}{(2\pi)^6E_pE_k}\right)^{1/2}\overline{u}(-k)L_{ab}^{--}\,u(p), \tag{5.12}$$

where

$$\langle 0|\partial_\mu N_a^\mu|N_d(q)\rangle = \delta_{ad}g_NM_Nu(q) \tag{5.13}$$

and g_N is as yet undetermined. We may now read off the s-wave scattering lengths of physical nucleons. The value of L_{ab}^{--}, Eq. (5.11), is extrapolated to threshold and we recall that the s-wave scattering length a_T^N of total isospin T is defined as $-2\pi i$ times the reduced mass times the coefficient of the δ function in S at threshold. Thus

$$a_T^N = \frac{1}{4\pi}\frac{M_N M_t}{M_N + M_t}\frac{M_N}{g_N{}^2}\frac{(\pm 1)}{2}\left[T(T+1) - t(t+1) - \tfrac{3}{4} + \tfrac{3}{2}Y_t\right], \tag{5.14}$$

where we have used nucleon pole dominance for the $\partial \cdot N$ propagator, i.e., $\rho_3^N(\mu^2) = g_N{}^2 M_N \delta(\mu^2 - M_N{}^2)$, inserted $\pm\sqrt{3}$ in place of ω_2, and the total isospin is given by $\vec{T} = \frac{1}{2}\vec{\tau} + \vec{t}$. Before comparing our result with experiment let us recall Weinberg's expression for pion s-wave scattering lengths,[18]

$$a_T^\pi = -\frac{1}{4\pi}\frac{m_\pi M_t}{m_\pi + M_t}\frac{1}{F_\pi{}^2}\frac{1}{2}\left[T(T+1) - t(t+1) - 2\right] \tag{5.15}$$

and the result of the current-algebra calculation of s-wave soft-kaon scattering lengths,

$$a_T^K = -\frac{1}{4\pi}\frac{M_K M_t}{M_K + M_t}\frac{1}{F_K{}^2}\frac{1}{2}\left[T(T+1) - t(t+1) - \tfrac{3}{4} + \tfrac{3}{2}Y_t\right]. \tag{5.16}$$

The pion decay amplitude F_π is a measurable quantity and is related to the pion-nucleon coupling constant by the Goldberger-Treiman relation. Unfortunately, $g_N{}^2$ is *not* directly measurable and the analog of the Goldberger-Treiman relation would relate $g_N{}^2$ to the pion-nucleon coupling constant and the off-shell $N(1520)$ decay amplitude. Detailed analysis of applications of the baryon algebra to the vertex functions will be presented elsewhere. For the present we shall require consistency between the various expressions for scattering lengths. First of all comparing π-K (Eq. 5.15) with K-π (Eq. 5.16) implies that $F_\pi{}^2 = F_K{}^2$, the SU(3)-symmetric result. If we use $F_\pi \sim 93$ MeV in Eq. (5.15) for π-N, we obtain good agreement with experiment, i.e.,

$$a_{1/2}^{\pi N} = -2a_{3/2}^{\pi N} = 0.20 m_\pi{}^{-1} = 0.28\ \mathrm{F},$$

compared with the experimental value of about $0.17 m_\pi{}^{-1}$. From Eq. (5.16) we find $a_0^{KN} = 0$ (consistent with experiment) and if we use $F_K = F_\pi$, we obtain $a_1^{KN} = 0.75$ F as compared with the experimental value of about 0.3 F. However, experimentally F_K appears to be somewhat larger than F_π and Eq. (5.16) may have to be modified.

The sole remaining parameter in Eq. (5.14) is determined by requiring consistency with the soft-pion-nucleon scattering length Eq. (5.15). This implies

$$\omega_N = -\sqrt{3}, \tag{5.17a}$$

$$\frac{g_N{}^2}{M_N} = F_\pi{}^2. \tag{5.17b}$$

With this choice, the "soft-nucleon"–meson scattering lengths are entirely consistent with the "soft-meson"–nucleon scattering lengths. Thus we have an example where soft-nucleon extrapolations are not entirely inappropriate. What happens to the "soft-nucleon" – nucleon s-wave scattering lengths?

$$a_0^{NN} = 0$$

and

$$a_1^{NN} = -\frac{M_N}{4\pi F_\pi{}^2} \simeq -1.1\ \mathrm{F}, \tag{5.18}$$

which is *not* in startling agreement with the experimental values of $a_0^{NN} = 5.5$ F and $a_1^{NN} = -17.5$ F. As expected, the presence of a bound and pseudobound state disallows any "smooth" extrapolation from a "soft" to physical nucleon in this channel.

The value of $g_N{}^2 = M_N F_\pi{}^2$, Eq. (5.17b), is consistent with considerations of Sec. V E.

E. Weinberg-Like Sum Rules

The single-particle-to-vacuum matrix elements of currents and the divergence of currents are in principle measurable via weak or electromagnetic processes. For example, F_π (the vacuum-to-single-pion matrix element of $\partial_\mu A^\mu$) is given by the pion lifetime, whereas g_ρ (the vacuum-to-ρ-meson matrix element of the vector current) is measured in e^+e^- colliding-beam experiments. However, g_{A_1} (the vacuum-to-A_1-meson matrix element of the axial-vector current) will have to await the measurement of a process such as $e^- + \bar{\nu}_e \to A_1^-$ before it is experimentally known. Other single-particle-to-vacuum matrix elements have not been experimentally determined. Nonetheless, one has an important device (within the meson-dominance approximation) to eliminate this arbitrariness in current-algebra calculations, viz., first and second Weinberg sum rules.[5] Weinberg's first sum rule suggests that

$$\frac{g_\rho{}^2}{m_\rho{}^2} = \frac{g_{A_1}{}^2}{M_{A_1}{}^2} + F_\pi{}^2, \tag{5.19}$$

whereas the second sum rule suggests that

$$g_\rho{}^2 = g_{A_1}{}^2. \tag{5.20}$$

One derivation of the first sum rule for a general Lie algebra involves use of the vacuum expectation value of the Jacobi identity between (two time-component and one space-component) currents and the assumption of c-number Schwinger terms.[27] The second sum rule (on less sure footing) relies on an assumption of asymptotic chiral invariance[28] or a specific Lagrangian model.[19] We shall now show that similar sum rules follow (with some additional assumptions) from the baryon algebra.

Consider the Jacobi identity Eq. (3.6a) between $V_a^0(x,t)$, $N_b^0(y,t)$, and $\bar{N}_c^j(z,t)$:

$$\omega_i\begin{pmatrix}8&8&8_i\\b&d&c\end{pmatrix}[V_a^0(\vec{x},t),\,V_d^j(\vec{y},t)]\delta^4(y-z)=\sqrt{3}\begin{pmatrix}8&8&8_2\\c&a&d\end{pmatrix}\{N_b^0(\vec{y},t),\,\bar{N}_d^j(\vec{z},t)\}\delta^4(x-z)$$
$$+\sqrt{3}\begin{pmatrix}8&8&8_2\\a&d&b\end{pmatrix}\{N_d^0(\vec{x},t),\,\bar{N}_c^j(\vec{z},t)\}\delta^4(x-y)+\text{S.T.} \tag{5.21}$$

Now take the vacuum expectation value and Fourier transform of the above equation:

$$\omega_i\begin{pmatrix}8&8&8_i\\b&d&c\end{pmatrix}\delta_{a,-d}(-1)^{Q_a}q_1^j(C_J^1-C_J^0)+\text{S.T.}=\sqrt{3}\begin{pmatrix}8&8&8_2\\c&a&b\end{pmatrix}C^{0j}(q_2)+\sqrt{3}\begin{pmatrix}8&8&8\\a&c&b\end{pmatrix}C^{0j}(q_1+q_2), \tag{5.22}$$

where $C_J^{1(0)}=\int\sigma_J^{1(0)}(\mu^2)/\mu^2$ and $\sigma_J^{1(0)}$ is the spin-1 (-0) spectral function. In Eq. (5.21), Schwinger terms (S.T.) may arise in $\{N,\bar{N}\}$ or in $[V,\bar{N}]$. In general such terms may be q-number terms whose equal-time (anticommutators) commutators with (N) V may indeed have nonvanishing vacuum expectation values and contribute to Eq. (5.22). We will invoke (revoke) such terms whenever their presence (absence) is required. The vacuum expectation value of equal-time (anticommutators) commutators of (BIF) currents are given in Eqs. (A3) and (A10). The first observation appropriate to Eq. (5.22) is that the SU(3) tensor structure requires

$$\omega_1\equiv 0. \tag{5.23}$$

As emphasized in Sec. III [see Eqs. (3.16) and (3.18)] this implies that $\omega_2=\pm\sqrt{3}$. Thus Eq. (5.22) implies ($\pm$ corresponding to the sign of ω_2)

$$\pm q_1^j(C_J^1-C_J^0)=C_N^{0j}(q_2)-C_N^{0j}(q_1+q_2)+\text{S.T.}, \tag{5.24}$$

where [Eq. (A3)]

$$C_N^{0j}(q_2)=(C_1^N-\tfrac{4}{3}C_3^N)[q^j+\tfrac{1}{3}\vec{q}\cdot\vec{\gamma}\gamma^j]$$
$$+\tfrac{2}{3}(C_2^N-\tfrac{8}{3}C_4^N)q^j\vec{q}\cdot\vec{\gamma}-\tfrac{1}{3}C_4'^N\gamma^j. \tag{5.25}$$

If we equate terms proportional to q_1^j in Eq. (5.24) and allow the remainder to be canceled by the S.T., we obtain

$$\pm(C_J^1-C_J^0)=-(C_1^N-\tfrac{4}{3}C_3^N). \tag{5.26}$$

Equation (5.26) holds for all members of the octet of currents and BIF. In particular, using pole dominance[29] [ρ meson on the left-hand side and $N^*(1520)$ and the nucleon on the right-hand side], we obtain

$$\pm\frac{-g_\rho^2}{m_\rho^2}=\frac{g_N^2(1520)}{M(1520)}+\frac{4}{3}\frac{g_N^2}{M_N}, \tag{5.27}$$

where

$$[(2\pi)^2 2E_\rho]^{1/2}\langle 0|V^\mu|\rho\rangle=g_\rho\epsilon^\mu$$

and the single-particle contributions to the BIF spectral functions follow from Eqs. (2.9) and (2.10). From Eq. (5.27) one concludes that

$$\omega_2=-\sqrt{3}. \tag{5.28}$$

The sign is the same as required by our study of $N\pi$ and NK *scattering lengths* in Sec. V D. Furthermore, the left-hand side of Eq. (5.27) is approximately[24] equal to $2F_\pi^2$, whereas in Sec. V D we determined the second term on the right-hand side to be equal to $\frac{4}{3}F_\pi^2$ and hence of the *same* order of magnitude. There is no *a priori* reason to expect the sign and the magnitudes to be consistent. In precisely the same manner one can eliminate many of the unknown single-particle-to-vacuum matrix elements of BIF. We record the result in the following manner (using pole dominance)[29]:

$$\frac{g_\rho^2}{M_\rho^2}=\frac{g_{N(1520)}^2}{M(1520)}+\frac{4}{3}\frac{g_N^2}{M_N}$$
$$=\frac{g_{N(1860)}^2}{M(1860)}+\frac{4}{3}\frac{g_{N(1535)}^2}{M(1535)}$$
$$=\zeta\left[\frac{g_{\Delta(1235)}^2}{M(1235)}+\frac{4}{3}\frac{g_{\Delta(1650)}^2}{M(1650)}\right]$$
$$=\zeta\left[\frac{g_{\Delta(1670)}^2}{M(1670)}+\frac{4}{3}\frac{g_{\Delta(1910)}^2}{M(1910)}\right], \tag{5.29}$$

where $\zeta=1$ or $\frac{25}{4}$ corresponding to solution 1, Eq. (3.16), or solution 2, Eq. (3.18), respectively. The last two equalities are a consequence of the Jacobi identity between V^0, ${}_+\Delta^0$, ${}_+\bar{\Delta}^j$, and V^0, ${}_-\Delta^0$, ${}_-\bar{\Delta}^j$, respectively. In Eq. (5.29) we have exhibited only nonstrange baryons; however, identical relations are suggested for the low-lying strange baryons. Further analysis of consequences of these sum rules and the analog of Weinberg's second sum rule will be given in a separate communication. We have thus demonstrated how one can determine many of the vacuum-to-single-particle matrix elements of BIF that arise in the context of baryon dominance.

VI. DISCUSSION

Our primary objective has been to propose an algebraic framework within which one can ultimately describe low-energy baryonic processes in a manner similar to the successful description of many mesonic processes. Wherever possible we have proceeded by analogy with current algebra.

Thus we have focused our attention on spin-$\frac{3}{2}$ fields so that their divergences could be used as interpolators for spin-$\frac{1}{2}$ particles just as the spin-0 components of some of the vector and axial-vector currents serve as interpolating fields for spin-0 particles. We have imposed an ansatz of linearity for the various commutators and anticommutators of the BIF and have required that the baryon algebra also be closed. As an exercise we have constructed a Lagrangian model whose anticommutation relations produce terms *linear* in spin-1 fields. However, it would be interesting to find a Lagrangian model that reproduces the baryon algebra without the need to consider various limiting processes.

Several questions remain unanswered. The properties of linear algebras involving fermions unfortunately have not been studied in the literature (to the author's knowledge), and it would be instructive to examine in detail their consequences; in particular, the representation theory of such algebras may be of physical interest.

We have pointed out that the baryon algebra implies exact theorems for "soft" spin-$\frac{1}{2}$ objects such as zeros in nucleon amplitudes and an expression for "soft-nucleon" scattering lengths. Can this limit be understood in terms of some new kind of symmetry? In the limit where the axial-vector current is conserved one can think in terms of Goldstone particles. Is there an analog in the limit $\partial \cdot N$ equal to zero?

In order to construct a nontrivial linear algebra we found it necessary to introduce BIF of opposite parity. We have taken advantage of this circumstance by assuming that the low-lying baryons couple to BIF with the same parity. Is there a fundamental reason why opposite-parity fields appear?

Weinberg sum rules are not low-energy theorems and we have shown that under certain assumptions concerning Schwinger terms similar sum rules follow from the baryon algebra. If one is hesitant to enter a world of "soft nucleons" and other "soft baryons," the appropriate way to incorporate the algebra into a physical theory may be indeed via a study of "hard nucleons or baryons," the analog of the study of "hard pions."[30] The procedure to be followed is formally very similar to the Ward-identity techniques used in current algebra. However, just as current algebra does not uniquely specify an amplitude,[31] so too the baryon algebra will leave undetermined parameters. However, as has recently been shown, undetermined parameters of the hard-pion approach *can* be fixed by a simple smoothness assumption for pion vertices.[32] Similar smoothness assumptions can be posited for baryon vertices.

Some of these questions will be discussed elsewhere.

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APPENDIX A: VACUUM EXPECTATION VALUES AND PROPAGATORS

In addition to the results given in Sec. II, the following relations are useful. Single baryon pole dominance of the covariant spin-$\frac{3}{2}$ propagator yields [see Eq. (2.12)]

$$P^{\mu\nu}(q) = -\frac{g_\alpha{}^2}{M_\alpha{}^2 - q^2}\left[g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{3} - \frac{1}{3M_\alpha{}^2}(q^\mu\gamma^\nu q\cdot\gamma + q\cdot\gamma\gamma^\mu q^\nu)\right](M_\alpha - q\cdot\gamma) - \frac{g_\alpha{}^2}{3M_\alpha{}^2}(q^\mu\gamma^\nu - q^\nu\gamma^\mu). \tag{A1}$$

If the field Ψ_a^μ carries an internal-symmetry index a, then the spectral function Eq. (2.6) has the form

$$\rho_{ab}^{\mu\nu}(q) = \delta_{ab}\rho^{\mu\nu}(q). \tag{A2}$$

The vacuum expectation value of the various commutators may be calculated from Eq. (2.6) and Eq. (2.7):

$$\begin{aligned}\delta_{ab}C^{0\nu}(q) \equiv \int dx\, e^{-iq\cdot x}\langle 0|\{\Psi_a^0(x),\ \bar{\Psi}_b^\nu(0)\}\,\delta(x^0)|0\rangle = \delta_{ab}\{&(C_1 - \tfrac{4}{3}C_3)[q^\nu - \tfrac{1}{3}q\cdot\gamma\gamma^\nu - \tfrac{1}{3}(\gamma^0 q\cdot\gamma g^{\nu 0} + \gamma^\nu\gamma^0 q^0)]\\ &- \tfrac{2}{3}(C_2 - \tfrac{8}{3}C_4)[q^0 g^{\nu 0} q^\sigma\gamma^\sigma - q^\nu q^0\gamma^0 - q^2 g^{\nu 0}\gamma^0 + q^\nu q\cdot\gamma]\\ &- \tfrac{1}{3}C_4'(\gamma^\nu - 4g^{\nu 0}\gamma^0)\},\end{aligned} \tag{A3}$$

where C_i is defined in Eq. (2.13),

$$-i\delta_{ab}C^{0-}(q) \equiv \int dx\, e^{-iq\cdot x}\langle 0|\{\Psi_a^0(x),\ \partial\bar{\Psi}_b(0)\}\,\delta(x^0)|0\rangle = +i\delta_{ab}(C_3' + \tfrac{4}{3}C_4'\vec{q}\cdot\vec{\gamma}). \tag{A4}$$

Note that in contradistinction to the spin-1 field commutator one obtains Schwinger terms for all four components of $C^{0\nu}$. In fact positivity requires $C^{00}(q) > 0$.

The propagators and vacuum expectation values of a current J_a^μ are given in terms of the spectral functions (spherical basis)

$$\sigma^{\mu\nu}_{mn}(q) \equiv (2\pi)^3 \sum_{n'} \delta^4(q - p_{n'})\langle 0|J^{\mu}_{m}(0)|n'\rangle\langle n'|J^{\nu}_{n}(0)|0\rangle = (-1)^{Q_m}\delta_{m,-n}\sigma^{\mu\nu}(q), \tag{A5}$$

where

$$\sigma^{\mu\nu}(q) = \sigma^J_1(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \sigma^J_0(q^2)\,\frac{q^\mu q^\nu}{q^2}\,,$$

$$Q_m = I_{Z_m} + \tfrac{1}{2}Y_m \tag{A6}$$

and the adjoint of the current is

$$J^{\mu\dagger}_{m} = (-1)^{Q_m} J^{\mu}_{-m}\,. \tag{A7}$$

Thus,

$$\int dx\, e^{-iq\cdot x}\langle 0|TJ^{\mu}_{m}(x)J^{\nu}_{n}(0)|0\rangle = -i(-1)^{Q_m}\delta_{m,-n}\left[\int \frac{\sigma^J_1(\mu^2)d\mu^2}{\mu^2 - q^2 - i\epsilon}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{\mu^2}\right) + q^\mu q^\nu \int \frac{\sigma^J_0(\mu^2)d\mu^2}{\mu^2(\mu^2 - q^2 - i\epsilon)} - g^{\mu 0}g^{\nu 0}(C^1_J - C^0_J)\right], \tag{A8}$$

where

$$C^{1(0)}_J = \int \sigma^J_{1(0)}(\mu^2)\frac{d\mu^2}{\mu^2}\,. \tag{A9}$$

Finally,

$$\int dx\, e^{-iq\cdot x}\langle 0|[J^0_m(x), J^\nu_n(0)]\delta(x^0)|0\rangle = (-1)^{Q_m}\delta_{m,-n}(q^\nu - q^0 q^{\nu 0})(C^1_J - C^0_J). \tag{A10}$$

APPENDIX B: LINEAR ANTICOMMUTATORS

We record the most general linear anticommutation relations consistent with covariance and C, P, and T invariance.

$$\{{}_\pm N^0_a(x),\, {}_\pm\bar{N}^\mu_b(y)\}\,\delta(x^0 - y^0) = \begin{pmatrix} 8 & 8 & 8_j \\ a & c & b\end{pmatrix}[\omega_{j\pm}V^\mu_c(x) + i\omega^3_{j\pm}\gamma^5\sigma^\mu{}_\nu\, {}^\nu_c(x)]\delta^4(x-y)$$

$$= \{{}_\pm N^\mu_a(x),\, {}_\pm\bar{N}^0_b(y)\}\,\delta(x^0 - y^0), \tag{B1}$$

$$\{{}_\pm N^0_a(x),\, {}_\mp\bar{N}^\mu_b(y)\}\,\delta(x^0 - y^0) = \begin{pmatrix} 8 & 8 & 8_j \\ a & c & b\end{pmatrix}[(\phi_j g^\mu{}_\nu \pm \phi^1_j\sigma^\mu{}_\nu)A^\nu_c(x) + (\phi^2_j\sigma^\mu{}_\nu \pm \phi^3_j g^\mu{}_\nu)\gamma^5 V^\nu_c(x)]\delta^4(x-y)$$

$$= \{{}_\pm N^\mu_a(x),\, {}_\mp\bar{N}^0_b(y)\}\,\delta(x^0 - y^0), \tag{B2}$$

$$\{{}_\pm \Delta^0_a(x),\, {}_\pm\bar{\Delta}^\mu_b(y)\}\,\delta(x^0 - y^0) = \begin{pmatrix} 10 & 8 & 10 \\ a & c & b\end{pmatrix}[\theta_\pm V^\mu_c(x) + i\theta'_\pm\gamma^5\sigma^\mu{}_\nu A^\nu_c(x)]\delta^4(x-y)$$

$$= \{{}_\pm \Delta^\mu_a(x),\, {}_\pm\bar{\Delta}^0_b(y)\}\,\delta(x^0 - y^0), \tag{B3}$$

$$\{{}_\pm \Delta^0_a(x),\, {}_\mp\bar{\Delta}^\mu_b(y)\}\,\delta(x^0 - y^0) = \begin{pmatrix} 10 & 8 & 10 \\ a & c & b\end{pmatrix}[(kg^\mu{}_\nu \pm k_1\sigma^\mu{}_\nu)A^\nu_c(x) + (k_2\sigma^\mu{}_\nu \pm k_3 g^\mu_\nu)\gamma^5 V^\nu_c(x)]\delta^4(x-y)$$

$$= \{{}_\pm \Delta^\mu_a(x),\, {}_\pm\bar{\Delta}^0_b(y)\}\,\delta(x^0 - y^0), \tag{B4}$$

$$\{{}_\pm N^0_a(x),\, {}_\pm\bar{\Delta}^\mu_b(y)\}\,\delta(x^0 - y^0) = \begin{pmatrix} 8 & 8 & 10 \\ a & c & b\end{pmatrix}[(\sigma_\pm g^\mu_\nu + i\sigma_{1\pm}\sigma^\mu{}_\nu)V^\nu_c(x) + (\sigma_{2\pm}g^\mu{}_\nu + i\sigma_{3\pm}\sigma^\mu{}_\nu)\gamma^5 A^\nu_c(x)]\delta^4(x-y)$$

$$= \{{}_\pm N^\mu_a(x),\, {}_\pm\bar{\Delta}^0_b(y)\}\,\delta(x^0 - y^0), \tag{B5}$$

and

$$\{{}_\pm N_a^0(x), {}_\mp\bar{\Delta}_b^\mu(y)\}\,\delta(x^0-y^0) = \begin{pmatrix}8 & 8 & 10\\ a & c & b\end{pmatrix}[(\tau_{\pm}g^\mu{}_\nu + i\tau_{1\pm}\sigma^\mu{}_\nu)A_c^\nu(x) + (\tau_{2\pm}g^\mu{}_\nu + i\tau_{3\pm}\sigma^\mu{}_\nu)\gamma^5 V_c^\nu(x)]\,\delta^4(x-y)$$

$$= \{{}_\pm N_a^\mu(x), {}_\mp\Delta_b^0(y)\}\,\delta(x^0-y^0). \quad \text{(B6)}$$

All of the parameters $\omega_{j\pm}, \ldots, \tau_{3\pm}$ introduced in the above equations are real. The number of parameters is greatly reduced by the requirement that the Jacobi identity Eq. (3.6a) be satisfied. The results are recorded in Sec. III [Eqs. (3.12)–(3.14) and the discussion following].

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