New Dual Models of Pions with No Tachyon*

M. B. Halpern
Department of Physics, University of California, Berkeley, California 94720
(Received 16 August 1971)

We observe that the σ meson in the recently proposed dual model of Halpern and Thorn can (should) be reinterpreted as a pion in its own right. The resulting model has the π and ρ degenerate at zero mass, no fifth quantum numbers, and a generally reduced spectrum. A related second model has nonmultiplicative internal symmetry, a Pomeranchukon (and exotic states), a ρ at 1 GeV, and a zero-mass pion. Both models can be extended to include fermions.

The recent dual model of Halpern and Thorn[1] pays a certain price for the elimination of the Neveu-Schwarz tachyon.[2] In the first place, it contains a zero-mass σ meson. Further, when extended to include fermions[3] it turns out that (a) the fermions are doubled due to the presence of fifth quantum numbers, and (b) the fermions tell us that what we thought were pions are in fact of mixed parity. Thus, as presented, the model can only be used for a purely meson world. In this paper, we reinterpret the meson model to eliminate all these difficulties. The corresponding fermion world will be reported later.[4]

As the new models will be formulated on the same Hilbert space, we begin by reviewing a few properties of the old model.[1] The Hamiltonian $\hat{J}_0$ is the usual quadratic function of the operators π_m^μ, b_m^μ, π_m^5, and b_m^5. The pion state $\hat{k}\cdot\hat{b}_{-1/2}|0\rangle \sim \hat{G}_{-1/2}|0\rangle$ carries a fifth quantum number ($\pi_0^5\neq 0$), while the σ meson

$$|\sigma\rangle = (\pi_{-1}^5 + \sqrt{2}\, k\cdot b_{-1/2} b_{-1/2}^5)|0\rangle \tag{1}$$

does not ($\pi_0^5 = 0$). It will be useful to give the σ vertex a name[5]

$$\begin{aligned} V \equiv V(k,1) &\equiv V^0[\pi^5(1) + \sqrt{2}\, k\cdot H(1) H^5(1)] \\ &\sim [\hat{G}_n, V^0 H^5(1)]_+ , \end{aligned} \tag{2}$$

where V^0 is the usual orbital vertex (with no fifth operators).

Consider now the n-point functions with all σ's outside:

$$B_n = \langle\sigma| V \frac{1}{\hat{J}_0 - 1} V \cdots V \frac{1}{\hat{J}_0 - 1} V |\sigma\rangle . \tag{3}$$

We see immediately that $B_n = 0$ unless n is even. That is, the σ's carry a hidden quantum number which is not conserved in the original model. We now *choose* to identify these old σ's as the pions of a *new* model, the hidden quantum number becoming the physical G parity.

There are several important observations to make about this new model: (1) Because the old σ (= new pion) was in a zero-fifth-quantum-number sector, the new model contains no fifth quantum numbers. It does of course contain π_m^5 ($m\neq 0$), which are now taken as pseudoscalars.[6] (2) Because the old pion had $\pi_0^5 \neq 0$, it is completely decoupled. All that remains (of the lowest states) is the new pion at zero mass, degenerate with ρ (same state as before). The ω is still at 1 GeV, and, in general, being a subsector of the old model, the spectrum is reduced. (3) Again because the new model is a subsector of the old, we are guaranteed that the full set of spin gauges[1,2,7] are operative.

It will be instructive, however, to see how the gauges work directly in the new model. From now on, we will forget the old name σ and refer simply to "the pion." Observe that the pion state can be rewritten

$$\begin{aligned} |\text{pion}\rangle &= (\pi_{-1}^5 + \sqrt{2}\, k\cdot b_{-1/2} b_{-1/2}^5)|0\rangle \\ &= -\hat{G}_{-1/2} b_{-1/2}^5 |0\rangle , \end{aligned} \tag{4}$$

and one easily establishes that

$$[V, \hat{G}_n] \sim [V^0 H^5(1), \hat{J}_{2n}] , \tag{5}$$

so that all the gauges work in the usual fashion in the shifted form

$$\begin{aligned} B_n &= \langle\text{pion}| V \frac{1}{\hat{J}_0 - 1} V \cdots V \frac{1}{\hat{J}_0 - 1} V |\text{pion}\rangle \\ &\sim \langle 0| b_{+1/2}^5 V \frac{1}{\hat{J}_0 - 1/2} V \cdots V \frac{1}{\hat{J}_0 - 1/2} V b_{-1/2}^5 |0\rangle . \end{aligned} \tag{6}$$

In this model, as in the models of Refs. 1 and 2, we have been tacitly assuming the use of multiplicative[8] isospin factors. Unfortunately, such an introduction of isospin always interprets the "photon"[9] as a zero-mass ρ (which decouples from two pions). We know from the quark models of Bardakci and Halpern[10] that probably the only way of avoiding this is through nonmultiplicative ("internal") isospin. Thus, we now turn our attention to an alternate model of this type.

We choose to attach an internal-symmetry label α to the *fifth* modes of the previous model – i.e., instead of π_m^5, b_m^5, we write π_m^α, b_m^α; α runs from 1 to 3 for SU(2), from 1 to 8 for SU(3), etc.[11] We sum over all α in constructing $\hat{G}_m, \hat{J}_m$, and choose as the pion vertex

$$V^\alpha \equiv V^0[\pi^\alpha(1)+\sqrt{2}\,k\cdot H(1)H^\alpha(1)] \sim [\hat{G}_n, V^0 H^\alpha(1)]_+ , \quad [V^\alpha, \hat{G}_n] \sim [V^0 H^\alpha(1), \hat{J}_{2n}] . \tag{7}$$

The pion state can be rewritten as above,

$$(\pi_{-1}^\alpha + \sqrt{2}\,k\cdot b_{-1/2} b_{-1/2}^\alpha)|0\rangle = -\hat{G}_{-1/2} b_{-1/2}^\alpha |0\rangle . \tag{8}$$

Thus, on the shifted form, we have

$$\overline{B}_n \equiv \langle 0 | b_{+1/2}^{\alpha_1} V^{\alpha_2} \frac{1}{\hat{J}_0 - 1/2} V^{\alpha_3} \cdots V^{\alpha_{n-2}} \frac{1}{\hat{J}_0 - 1/2} V^{\alpha_{n-1}} b_{-1/2}^{\alpha_n} |0\rangle . \tag{9}$$

In this model, the old ρ meson of Refs. 1 and 2 is the spin-one state of the exchange-degenerate partner of the Pomeranchukon. Of course, this whole trajectory decouples after Bose symmetrization. The real (charged) ρ appears at 1 GeV. Unfortunately, the spectrum of this model is not good. In the first place, the ω is at 2 GeV. To make matters worse, the lowest exotic state appears at 1 GeV (spin zero).[12] Thus, if spectrum is the prime consideration, we take the *first* model rather more seriously. Still, the second model is unique in that it has the Pomeranchukon with no tachyon, and all gauges working.

We would now like to make some concluding comments. In the first place, we remark that the spectrum of our *first* model (multiplicative isospin) so far looks like the dual quark model of Bardakci and Halpern, after using the spin gauges discovered by Bardakci.[13] The two models are constructed with different operators, but I would hazard a guess that they may be the same. As a final comment, I want to note the by now obvious fact that all the extant dual pion models (including the models of this paper) have an intrinsic G parity which limits them to SU(2); in every case $B_n = 0$ for n odd. This problem is worth future focus.

It is a pleasure to thank Charles Thorn for helpful conversations and for reading the manuscript.

*Research supported by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under Contract No. F44620-70-C-0028.

[1]M. B. Halpern and C. B. Thorn, Phys. Letters (to be published).

[2]A. Neveu and J. H. Schwarz, Nucl. Phys. B31, 86 (1971).

[3]M. B. Halpern and C. B. Thorn (unpublished).

[4]M. B. Halpern and C. B. Thorn (unpublished).

[5]The elegant anticommutator form of the vertex, and the resultant (see later in text) elegant proof of the gauges was pointed out to me by C. B. Thorn.

[6]In the meson world this is an assumption, but the fermion couplings of Ref. 4 dictate this. Moreover, b_m^μ continues to be an axial vector, but b_m^5 becomes scalar. This being the proper assignment, we see why the "old pion" $(k\cdot b_{-1/2} - k_5 b_{-1/2}^5)|0\rangle$ comes out with mixed parity.

[7]A. Neveu, J. H. Schwarz, and C. B. Thorn (unpublished).

[8]J. E. Paton and Chan Hong-Mo, Nucl. Phys. B10, 516 (1969).

[9]M. B. Halpern, Phys. Rev. D 3, 3068 (1971).

[10]K. Bardakci and M. B. Halpern, Phys. Rev. D 3, 2493 (1971).

[11]A ninth α can be added, but it decouples from octet scattering. See also our concluding remarks. We mean $[\pi_l^\alpha, \pi_m^\beta] = l\delta^{\alpha\beta}\delta_{l,-m}$, etc.

[12]This compares unfavorably with exotic states at 2 GeV expected in the quark model of Ref. 10.

[13]K. Bardakci (unpublished).