Physical and Formal Presentations of Dual Theory of Electromagnetic Currents

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We propose a dynamical Lagrangian theory for the hadron with strong and electromagnetic interactions. We give a simple physical interpretation of our Lagrangian in the parton picture and show explicitly its equivalence with the formal string formulation of the hadron for both the free and interacting Lagrangians. The theory is gauge-invariant with a well-defined conserved current. (The low-energy theorem for currents holds.) The elastic and transition form factors are non-Gaussian and analytic with an infinite series of vector-meson poles. In the two-current channel the electromagnetic amplitude has fixed poles with the imaginary part satisfying rigorously the Dashen–Gell-Mann–Fubini sum rule. In the Bjorken limit our νW_2 scales and is saturated by narrow resonances. A dispersion relation for T_2 continues to hold in the scaling limit and the Gottfried sum rule for $\nu W_2(\omega)$ is exactly satisfied with the fixed-pole contribution. The hadronic amplitude obtained from the off-shell Compton amplitude is equal to a beta function. In the crossed channel our theory generates Regge poles with quantized intercepts which are degenerate with the vector-meson poles in the off-shell photon line which phenomenologically implies the ρ-f-A_2 degeneracy. More generally, we project explicitly hadronic vertices out of our electromagnetic vertex through vector-meson poles and find that these vertices are dual and factorizable.

I. INTRODUCTION

With the recent work on hadron electrodynamics, different approaches to the problem illuminated various aspects of photohadronic processes. On the one hand, Feynman's partons[1] presented an attractive pictorial representation of the hadron as a collection of pointlike objects. On the other hand, formal or phenomenological theories presented schemes for calculations with varying degrees of success. Among these the quark model[2] is at present the best available systematic way of classifying the rich spectrum of observed resonances, the vector-dominance model[3] gives the connection between spin-one resonances and the electromagnetic nucleon form factors, and a great number of approaches[4] are based on the resonance model with the proclaimed goal of presenting the strong, electromagnetic, and weak interactions in a unified representation.

In previous papers[5] the present authors succeeded in constructing a gauge-invariant electromagnetic interaction for the hadron with a well-defined electromagnetic current. We obtained a number of interesting results that the interested reader would find in Ref. 5. On the other hand, some important questions were not satisfactorily treated: (a) The elastic form factor was Gaussian, (b) duality was broken, (c) we failed to connect the strong and electromagnetic amplitudes through vector-meson poles, (d) there were internal symmetry and ghost problems in the theory, and (e) the connection between the formal presentation of our model and a pictorial representation was not accomplished.

In this paper we present a dynamical generalization of our model where we believe we have answered all the above questions except (d) in a satisfactory manner. The summary of the applications of the generalized model was given in a recent letter.[6]

In Sec. II we present the physical model for the theory. The physical picture underlying the present model is the representation of the hadron as a collection of pointlike constituents (following Feynman[1] we call these objects "partons") with the following dynamics: The nth parton is coupled with a frequency $(n-1)\omega$ to the center of mass of the first $n-1$ partons where the potential is approximated by that of a harmonic oscillator and where ω is the frequency of the first- and second-parton potential. The Hamiltonian of this system can be trivially diagonalized with a normal-mode representation corresponding to the Veneziano model.[7] The charge quantum number is assumed to be carried by the first parton. Then we introduce a gauge-invariant electromagnetic interaction by the minimal substitution. The general features of the model are that (i) there is a cluster point of partons around the center of mass of the hadron, (ii) the charge has a peripheral distribution, (iii) in the normal-mode representation of the interaction Lagrangian, the minimal substitution over the parton coordinate

$$m_0\dot{y}_1 \rightarrow m_0\dot{y}_1 - eA(y_1) \tag{1.1}$$

is expressed as

$$m_0\frac{n}{n+1}\dot{z}_n - m_0\frac{n}{n+1}\dot{z}_n - 2\left(m_0\frac{n}{n+1}\right)^{1/2}\sigma_n A(y_1), \tag{1.2}$$

where $\sigma_n \sim 1/n$ is the charge carried by the nth mode and y_1 is a linear function of the z_n's.

In Sec. III we proceed to write the Lagrangian of the free hadron in the formal language of the string model.[7] We assume in addition that the charge distribution over the string is governed by a dynamical variable ζ such that the total free Hamiltonian is given by

$$H = p^2 + \sum_{n=1}^{\infty} n a_n^\dagger a_n + \tfrac{1}{2}\lambda\sum_{i=1}^{L}(\eta_i^2 + \omega_0^2\zeta_i^2) - \tfrac{1}{2}\lambda\omega_0 L. \tag{1.3}$$

In Sec. IV we construct the electromagnetic interaction Lagrangian allowing the quantum-mechanical fluctuation of the normal-mode couplings. We prove that the interaction is gauge-invariant and that in the normal coordinates, the expectation value of the charge distribution reduces to the parton results of Sec. II. We prove current conservation for our electromagnetic current given by

$$j_\mu(k) = \frac{1}{2}\int\left\{\frac{\partial\bar{x}_\mu}{\partial\tau}, e^{ik\bar{x}}\right\}_+ d\tau. \tag{1.4}$$

In Sec. V we show that the elastic form factor and the transition matrices are *not* Gaussian, but rather decrease as a power of the momentum transfer. Typically, the elastic form factor for the ground-state hadron is given by

$$F_L(-k^2) = \frac{2\omega_0^{L/2}}{\Gamma(\frac{1}{2}L)}\int_0^\infty d\zeta\,\zeta^{L-1}e^{-\omega_0\zeta^2}(1-e^{-\zeta^2})^{k^2}. \tag{1.5}$$

Defining a Regge trajectory with a quantized intercept $\alpha_L(-k^2)$ such that

$$\alpha_L(-k^2) = -k^2 + 1 - \tfrac{1}{2}L, \tag{1.6}$$

then $F_L(-k^2)$ has an infinite series of vector-meson poles at $\alpha_L(-k^2) = 1, 2, 3, \ldots, \infty$ and is analytic with an asymptotic decrease

$$F_L(-k^2) \underset{k^2\to\infty}{\sim} (k^2)^{-\omega_0}(\ln k^2)^{L/2-1}. \tag{1.7}$$

For $L = 2$ we have

$$F_2(-k^2) = \frac{\Gamma(\omega_0+1)\Gamma(k^2+1)}{\Gamma(k^2+\omega_0+1)}\,. \tag{1.8}$$

In Sec. VI we investigate the two-current amplitudes, with the following results:

A. The hadronic amplitude $H_2^{(L)}(s,t)$, obtained from the helicity-flip off-shell Compton amplitude $T_2^{(L)}(\nu, t, k_1^2, k_2^2)$ at the lowest poles $-k_1^2 = -k_2^2 = \frac{1}{2}L$, is dual and is given by

$$H_2^{(L)}(s,t) = \frac{2m\omega_0^L}{[\Gamma(\frac{1}{2}L)]^2}\int_0^\infty dy\, e^{y\alpha(s)}(1-e^{-y})^{-\alpha_L(t)+1+L/2} \times (1-e^{-2\lambda\omega_0 y})^{-L/2}, \tag{1.9}$$

where $\alpha(s) = s + \alpha(0)$ and $\alpha_L(t) = t + 1 - \frac{1}{2}L$. The s-channel poles are at $\alpha(s) = N + 2\lambda\omega_0 M$ with $N, M = 0, 1, 2, \ldots, \infty$, whereas the t-channel poles start at $\alpha_L(t) = 2, 3, 4, \ldots, \infty$ due to the double helicity flip of the vector mesons. The particle spectrum has no external mass dependence in either channel, and in the t channel we have the same spectrum as that of the off-shell photon line. The asymptotic behavior of each channel is dictated by the Regge poles in the crossed channel.

When $2\lambda\omega_0 = 1$, the hadronic amplitude is a simple beta function,[8]

$$H_2^{(L)}(s,t) = \frac{2m\omega_0^L}{[\Gamma(\frac{1}{2}L)]^2}B(-\alpha(s), 2-\alpha_L(t)). \tag{1.10}$$

B. We investigate the singularity structure in the t channel of the electromagnetic amplitude and show the presence of fixed poles at $J = 1, 0, -1, \ldots, -\infty$. The leading contribution at $J = 1$ is given by[9]

$$T_2^{(L)}(\nu, t, k_1^2, k_2^2)|_{\text{fixed pole}} = \frac{2m}{-\alpha(s)}F_L(t). \tag{1.11}$$

The imaginary part $W_2^{(L)}(\nu, t, k_1^2, k_2^2)$ of the amplitude satisfies rigorously the Dashen–Gell-Mann–Fubini[10]-type sum rule for the electromagnetic current:

$$\int_0^\infty d\nu\, W_2^{(L)}(\nu, t, k_1^2, k_2^2) = F_L(t). \tag{1.12}$$

C. In the Bjorken scaling limit[11] $\nu W_2^{(L)}$ scales and has a simple analytic expression

$$\nu W_2^{(L)}(\omega) = \frac{2\omega_0^{L/2}}{\Gamma(\frac{1}{2}L)}\frac{\omega(\omega-1)^{\omega_0-1}}{(\omega+1)^{\omega_0+1}}\left[\ln\left(\frac{\omega+1}{\omega-1}\right)\right]^{L/2-1}, \tag{1.13}$$

where $\omega = 2m\nu/k^2$. In this model scaling is saturated by the nondiffractive narrow-resonance contribution. The large-ω behavior is dictated by the ordinary Regge-pole exchange such as $\nu W_2^{(L)}(\omega) \sim \omega^{\alpha_L(0)-1}$, whereas at threshold ($\omega = 1$) $\nu W_2^{(L)}(\omega)$ vanishes like $\sim(\omega-1)^{\omega_0-1}[-\ln(\omega-1)]^{L/2-1}$.

Our $\nu W_2^{(L)}(\omega)$ satisfies exactly the Gottfried sum rule[12]

$$\int_1^\infty \frac{d\omega}{\omega}\nu W_2^{(L)}(\omega) = 1. \tag{1.14}$$

D. The low-energy[10] theorem for currents holds. In the soft-photon limit of $k_1 \to 0$ we have[9]

$$T_2^{(L)}(\nu, t = -k_2^2, k_1^2 = 0, k_2^2)|_{k_1\cdot k_2 = 0} = -\frac{2m}{s-m^2}F_L(-k_2^2). \tag{1.15}$$

E. In Sec. VI E we present the summary of the calculation for $W_1^{(L)}$. The leading contribution to $W_1^{(L)}$ in the Bjorken limit is obtained as

$$W_1^{(L)} = \frac{1}{k^2} f^{(L)}(\omega), \tag{1.16}$$

where $f^{(L)}(\omega)$ is a scaling function.

In Sec. VII we investigate duality and factorization of the theory. We find that duality holds for on-mass-shell vector-meson-hadron vertices. On the other hand, it is minimally broken in the electromagnetic vertex because of the presence of fixed poles in the two-current channel of hadron-current amplitudes. Our electromagnetic vertex specifies the strong vertices in the following manner: The scalar vertex is

$$V_S(k_S) = g_S \frac{\omega_0^{L/2}}{\Gamma(\frac{1}{2}L)} : e^{ik_S \bar{X}(0)}: V_\lambda, \tag{1.17}$$

and the vector vertex is

$$V_\mu(k) = \frac{\omega_0^{L/2}}{\Gamma(\frac{1}{2}L)} \left[\left(p_{\mu,0} + \sum_{n=1}^{\infty} p_{\mu,n} \right) V_\lambda - \frac{k_\mu}{2m_V^2} [H_\lambda, V_\lambda], : e^{ik\bar{X}(0)}: \right]_+ , \tag{1.18}$$

where $\alpha_L(-k_s^2) = 0$ and $\alpha_L(-k^2) = 1$.

In the conclusion, we summarize our approach and present the theoretical curves for the elastic form factor and the structure function $\nu W_2^{(L)}$ and the corresponding data.

II. PHYSICAL MODEL FOR THE THEORY

We will first exhibit the underlying physical model for our theory of the hadron and its electromagnetic interaction.

Let us assume a hadron to be a bound system of $N+1$ pointlike particles ("partons") with mass m_i and four-dimensional coordinates $y_{i,\mu}$, where $i = 1, 2, \ldots, N+1$. Let us further assume all partons to have the same mass $m_0 = m_1 = m_2 = \cdots = m_{N+1}$, that parton 1 interacts with parton 2 through an attractive potential which is here approximated by a harmonic-oscillator potential with a frequency ω, that parton 3 interacts with the center of mass of partons 1 and 2 with frequency 2ω, and more generally that parton n interacts with the center of mass of partons $1, 2, \ldots, n-2$ and $n-1$ with frequency $(n-1)\omega$.

The Lagrangian for the system is generally given by

$$L = \tfrac{1}{2} m_1 \dot{y}_1^2 + \tfrac{1}{2} m_2 \dot{y}_2^2 + \cdots + \tfrac{1}{2} m_{N+1} \dot{y}_{N+1}^2 - V(y_1, y_2, \ldots, y_{N+1}), \tag{2.1}$$

where the time derivative $\dot{y}_i$ is taken with respect to the proper time[13] τ of the hadron

$$\dot{y}_i = \frac{\partial y_i}{\partial \tau}. \tag{2.2}$$

Define the center-of-mass coordinate Z and the relative coordinates z_n as

$$\begin{aligned} Z &= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots + m_{N+1} y_{N+1}}{m_1 + m_2 + m_3 + \cdots + m_{N+1}}, \\ z_1 &= y_1 - y_2, \\ z_2 &= \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} - y_3, \\ &\vdots \\ z_N &= \frac{m_1 y_1 + m_2 y_2 + \cdots + m_N y_N}{m_1 + m_2 + \cdots + m_N} - y_{N+1}. \end{aligned} \tag{2.3}$$

Since all masses are equal, the above coordinates are simply expressed as

$$\begin{aligned} Z &= \frac{1}{N+1}(y_1 + y_2 + \cdots + y_{N+1}), \\ z_n &= \frac{1}{n}(y_1 + y_2 + \cdots + y_n) - y_{n+1}, \quad n = 1, 2, \ldots, N. \end{aligned} \tag{2.4}$$

Inversely, $(y_1, y_2, y_3, \ldots, y_{N+1})$ can be expressed as linear functions of $(Z, z_1, z_2, \ldots, z_N)$:

$$\begin{aligned} y_1 &= Z + \tfrac{1}{2} z_1 + \tfrac{1}{3} z_2 + \cdots + \frac{1}{n+1} z_n + \cdots + \frac{1}{N+1} z_N, \\ y_2 &= Z - \tfrac{1}{2} z_1 + \tfrac{1}{3} z_2 + \cdots + \frac{1}{n+1} z_n + \cdots + \frac{1}{N+1} z_N, \\ y_3 &= Z - \tfrac{2}{3} z_2 + \tfrac{1}{4} z_3 + \cdots + \frac{1}{n+1} z_n + \cdots + \frac{1}{N+1} z_N, \\ &\vdots \\ y_i &= Z - \frac{i-1}{i} z_{i-1} + \frac{1}{i+1} z_i + \cdots + \frac{1}{N+1} z_N, \\ &\vdots \\ y_{N+1} &= Z - \frac{N}{N+1} z_N. \end{aligned} \tag{2.5}$$

The Lagrangian of the assumed system written in the normal coordinates $(Z, z_1, z_2, \ldots, z_n, \ldots)$ is then given by

$$\begin{aligned} L = {} & \tfrac{1}{2} m_0 (N+1) \dot{Z}^2 + \tfrac{1}{2} m_0 \tfrac{1}{2} (\dot{z}_1^2 - \omega_1^2 z_1^2) \\ & + \tfrac{1}{2} m_0 \tfrac{2}{3} (\dot{z}_2^2 - \omega_2^2 z_2^2) + \cdots \\ & + \tfrac{1}{2} m_0 \frac{n}{n+1} (\dot{z}_n^2 - \omega_n^2 z_n^2) + \cdots \\ & + \tfrac{1}{2} m_0 \frac{N}{N+1} (\dot{z}_N^2 - \omega_N^2 z_N^2), \end{aligned} \tag{2.6}$$

where

$$\omega_n = n\omega \tag{2.7}$$

and $m_0 n/(n+1)$ is the reduced mass of the $(n+1)$th parton with respect to the center of mass of the first n partons.

The center of mass and relative momenta are given by

$$P=\frac{\partial L}{\partial \dot{Z}}=m_0(N+1)\dot{Z},\qquad \pi_n=\frac{\partial L}{\partial \dot{z}_n}=m_0\frac{n}{n+1}\dot{z}_n\,. \tag{2.8}$$

Then the total Hamiltonian takes the form

$$H=\frac{P^2}{2m_0(N+1)}+\sum_{n=1}^{N}\frac{1}{2}\left(\frac{\pi_n^{\,2}}{m_0 n/(n+1)}+m_0\frac{n}{n+1}\omega_n^{\,2}z_n^{\,2}\right). \tag{2.9}$$

Here we assume the canonical commutation relations

$$[Z_\mu, P_\nu]=i\delta_{\mu\nu}\,,\qquad [z_{m,\mu},\pi_{n,\nu}]=i\delta_{mn}\delta_{\mu\nu}\,. \tag{2.10}$$

Introducing the creation and annihilation operators a_n and $a_n^\dagger$ such that

$$z_n=\left(\frac{1}{2m_0[n/(n+1)]\omega_n}\right)^{1/2}(a_n+a_n^\dagger),\qquad \pi_n=-i\left(\frac{1}{2}m_0\frac{n}{n+1}\omega_n\right)^{1/2}(a_n-a_n^\dagger), \tag{2.11}$$

and

$$[a_{m,\mu}, a^\dagger_{n,\nu}]=\delta_{mn}\delta_{\mu\nu}\,, \tag{2.12}$$

the Hamiltonian can be expressed as

$$H=\frac{P^2}{2M_N}+\omega\sum_{n=1}^{N} n a_n^\dagger a_n\,, \tag{2.13}$$

where M_N is the total mass, and the constant contribution due to zero-point fluctuation is subtracted.

As this point we relax the relation between the total mass M_N and parton mass m_0 by assuming a certain mass renormalization, thus allowing the two quantities to be independent. We shall choose

$$\frac{1}{2M_N}=\omega=1. \tag{2.14}$$

This normalization is such that the fundamental unit of energy is the inverse Regge slope. Then we finally obtain

$$H=P^2+\sum_{n=1}^{N} n a_n^\dagger a_n\,, \tag{2.15}$$

with

$$z_n=\left(\frac{1}{m_0}\frac{n+1}{n}\right)^{1/2}\left(\frac{1}{2n}\right)^{1/2}(a_n+a_n^\dagger),\qquad \pi_n=-i\left(m_0\frac{n}{n+1}\right)^{1/2}\left(\frac{n}{2}\right)^{1/2}(a_n-a_n^\dagger). \tag{2.16}$$

Note the relations

$$\dot{Z}=i[H,Z]=2P,\qquad \dot{z}_n=i[H,z_n]=\frac{1}{m_0}\frac{n+1}{n}\pi_n\,. \tag{2.17}$$

Letting N go to infinity in Eq. (2.15), we notice that this Hamiltonian has the same form as that of the string model (see Sec. III).

Now the hadronic wave function for the system in the ground state is given by

$$\psi(Z, z_1, z_2, \ldots, z_n, \ldots)=\psi_0(Z)\phi_1(z_1)\phi_2(z_2)\cdots\phi_n(z_n)\cdots, \tag{2.18}$$

where $\psi_0(Z)$ is the free-particle wave function corresponding to the center-of-mass motion, and $\phi_n(z_n)$ is the four-dimensional harmonic-oscillator wave function with frequency n:

$$\phi_n(z_n)=\left(m_0\frac{n}{n+1}\,\frac{n}{\pi}\right)\exp\left(-\frac{m_0}{2}\frac{n}{n+1}nz_n^{\,2}\right). \tag{2.19}$$

To understand the model better, let us investigate the clustering properties of partons. The mean-square distance of parton $n+1$ from the center-of-mass point Z is given by

$$\langle (y_{n+1}-Z)^2\rangle=\left(\frac{n}{n+1}\right)^2\langle z_n^{\,2}\rangle+\sum_{i=n+1}^{\infty}\left(\frac{1}{i+1}\right)^2\langle z_i^{\,2}\rangle. \tag{2.20}$$

Using the above wave function, we have

$$\langle z_n^{\,2}\rangle=\frac{1}{m_0}\frac{n+1}{n}\frac{2}{n}\,. \tag{2.21}$$

Thus, we obtain

$$\langle (y_{n+1}-Z)^2\rangle=\frac{1}{m_0}\frac{n}{n+1}\frac{2}{n}+\sum_{i=n+1}^{\infty}\frac{1}{m_0 i(i+1)}\frac{2}{i}\,. \tag{2.22}$$

For $n\to\infty$ the above quantity tends to

$$\langle (y_{n+1}-Z)^2\rangle \underset{n\to\infty}{\simeq} \frac{1}{m_0}\frac{2}{n}+O(1/n^2), \tag{2.23}$$

and most of the partons are very near to Z (clustering point).

Having studied the Hamiltonian for the free system, we now want to introduce the electromagnetic interaction in the model. In the parton coordinates $(y_1, y_2, \ldots, y_i, \ldots)$ we make the minimal substitution

$$m_0\dot{y}_i\to m_0\dot{y}_i-e_iA(y_i), \tag{2.24}$$

where e_i is the charge carried by the ith parton. Gauge invariance is here automatically satisfied. On the other hand, in the normal coordinates

$(Z, z_1, z_2, \ldots, z_n, \ldots)$ the problem is far from trivial and can be local only in special cases since the A field associated with one of the normal coordinates is a superposition of fields at different points and diagonalization is not always possible.

Here let us assume that the charge quantum e is carried by one of the partons, say parton 1; then the interaction Lagrangian is given by

$$\int d\tau\, L_I = e\int d\tau\left(\frac{\partial y_1}{\partial \tau}\right)_\mu A_\mu(y_1). \tag{2.25}$$

Since the transformation $\{y\}\to\{z\}$ is a linear transformation and

$$y_1 = Z + \tfrac{1}{2}z_1 + \tfrac{1}{3}z_2 + \cdots + \frac{1}{n+1}z_n + \cdots, \tag{2.26}$$

the above interaction is also given in the normal coordinates by the substitution

$$\pi_n = m_0\frac{n}{n+1}\dot{z}_n \to \pi_n - 2\left(m_0\frac{n}{n+1}\right)^{1/2}\sigma_n A(y_1), \tag{2.27}$$

where

$$\sigma_n = \frac{1}{2}\frac{1}{n+1}\left(\frac{1}{m_0}\frac{n+1}{n}\right)^{1/2}. \tag{2.28}$$

Thus, we see that *the different normal modes couple with different strength to the electromagnetic field.*

So far, the attempt made in this section has been classical. In the real world, due to parton-parton collisions, the charge distribution may be quantized. We will require that the expectation value of the quantum-mechanical charge-distribution operator ρ_n corresponds to the classical distribution σ_n: Consider an L-dimensional space (ζ_i) $(i=1, 2, \ldots, L)$ corresponding to the charge fluctuation with a harmonic wave function $(\omega_0/\pi)^{L/4} \times \exp(-\frac{1}{2}\omega_0\zeta^2)$. The charge-distribution operator $\rho_n(\zeta) = e^{-n\zeta^2/2}$ with $L=2$ gives for $\sigma_n = \langle\rho_n(\zeta)\rangle_\zeta$ the $1/n$ behavior. This additional degree of freedom gives an extra contribution to the total free Hamiltonian of the system.

In Secs. III and IV we shall present this generalized model in the string language, and then compare the model with the classical picture shown above.

III. DYNAMICAL RESONANCE MODEL FOR FREE HADRON

In the previous formal model[5] considered by the present authors, the hadron was considered to be represented by a collection of world lines that at any proper time τ obey the two-dimensional continuous-string equation with free ends. We will show that the Lagrangian and normal modes of this system correspond to the physical model proposed in Sec. II. In the present generalized dynamical resonance model[6] where we allow *quantum-mechanical fluctuation* in the hadron internal degrees of freedom, we should add to the Lagrangian of the previous model the Lagrangian of a set of scalar harmonic oscillators. We shall prove in Sec. V that these scalar oscillators induce an infinite series of vector-meson poles with *quantized* Regge intercepts in the off-shell photon line of electromagnetic currents when the hadron interacts with the external electromagnetic field.

The Lagrangian density of the model is then given by

$$\begin{aligned}\mathcal{L}(\xi) &= \mathcal{L}_0(\xi) + \mathcal{L}_\lambda(\xi)\\ &= \frac{1}{4\pi}\left[\left(\frac{\partial x_\mu}{\partial\tau}\right)^2 - \left(\frac{\partial x_\mu}{\partial\xi}\right)^2\right]\\ &\quad + \frac{1}{2\pi\lambda}\sum_{i=1}^{L}\left[\left(\frac{\partial\zeta_i}{\partial\tau}\right)^2 - \lambda^2\omega_0^2\zeta_i^2\right],\end{aligned} \tag{3.1}$$

where L is the number of independent scalar oscillators, $\lambda\omega_0$ their frequency, and λ an arbitrary constant. The summation over the Lorentz indices is understood to be taken. Applying the principle of least action to the above Lagrangian, we obtain the following equations of motion:

$$\frac{\partial^2 x_\mu}{\partial\tau^2} = \frac{\partial^2 x_\mu}{\partial\xi^2}, \tag{3.2}$$

$$\frac{\partial^2\zeta_i}{\partial\tau^2} = -\lambda^2\omega_0^2\zeta_i, \quad i = 1, 2, \ldots, L. \tag{3.3}$$

We assume the boundary conditions for free ends:

$$\frac{\partial x_\mu}{\partial\xi} = 0 \quad \text{for} \quad \xi = 0, \pi. \tag{3.4}$$

These allow the coordinates $x_\mu(\xi)$ to have the following Fourier decomposition corresponding to the normal modes of the system:

$$x_\mu(\xi) = x_{\mu,0} + 2\sum_{n=1}^{\infty} x_{\mu,n}\cos(n\xi), \tag{3.5}$$

which corresponds to the separation of motion in center-of-mass and relative coordinates. For each mode $x_{\mu,n}(\tau)$ the equation of motion is simply expressed as

$$\frac{\partial^2 x_{\mu,n}}{\partial\tau^2} = -n^2 x_{\mu,n}, \quad n = 0, 1, 2, \ldots, \infty. \tag{3.6}$$

The canonically conjugate momenta associated with these degrees of freedom are given by

$$\begin{aligned}p_\mu(\xi) &= \frac{\partial\mathcal{L}}{\partial(\partial x_\mu/\partial\tau)} = \frac{1}{2\pi}\frac{\partial x_\mu}{\partial\tau}\\ &= \frac{1}{\pi}p_{\mu,0} + \frac{1}{\pi}\sum_{n=1}^{\infty} p_{\mu,n}\cos(n\xi),\end{aligned} \tag{3.7}$$

with the following appropriate commutation relations:

$$[x_{\mu,m}, p_{\nu,n}] = i\delta_{\mu\nu}\delta_{mn}\,. \tag{3.8}$$

Now the total Lagrangian is obtained by integrating $\mathcal{L}(\xi)$ over ξ:

$$L = L_0 + L_\lambda = \int_0^\pi \mathcal{L}(\xi)d\xi$$

$$= \frac{1}{4}\left(\frac{\partial x_{\mu,0}}{\partial\tau}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left[\left(\frac{\partial x_{\mu,n}}{\partial\tau}\right)^2 - n^2 x_{\mu,n}{}^2\right] + \frac{1}{2\lambda}\sum_{i=1}^{L}\left[\left(\frac{\partial\zeta_i}{\partial\tau}\right)^2 - \lambda^2\omega_0{}^2\zeta_i{}^2\right]. \tag{3.9}$$

The Hamiltonian density can be calculated from the Lagrangian density as

$$\mathcal{H}(\xi) = \mathcal{H}_0(\xi) + \mathcal{H}_\lambda(\xi)$$
$$= \pi[p_\mu(\xi)]^2 + \frac{1}{4\pi}\left(\frac{\partial x_\mu}{\partial\xi}\right)^2$$
$$+ \frac{1}{2\pi\lambda}\sum_{i=1}^{L}\left[\left(\frac{\partial\zeta_i}{\partial\tau}\right)^2 + \lambda^2\omega_0{}^2\zeta_i{}^2\right], \tag{3.10}$$

and the total free Hamiltonian is

$$H = H_0 + H_\lambda = \int_0^\pi \mathcal{H}(\xi)d\xi$$
$$= p_{\mu,0}{}^2 + \tfrac{1}{2}\sum_{n=1}^{\infty}(p_{\mu,n}{}^2 + n^2 x_{\mu,n}{}^2) + \tfrac{1}{2}\lambda\sum_{i=1}^{L}(\eta_i{}^2 + \omega_0{}^2\zeta_i{}^2), \tag{3.11}$$

where η_i is defined by

$$\eta_i = \frac{\partial L}{\partial(\partial\zeta_i/\partial\tau)}$$
$$= \frac{1}{\lambda}\frac{\partial\zeta_i}{\partial\tau}. \tag{3.12}$$

ζ_i and η_i obey the canonical commutation relations

$$[\zeta_i, \eta_j] = i\delta_{ij}\,. \tag{3.13}$$

Notice that in the Hamiltonian (3.11) the first term corresponds to the center-of-mass motion and the second and third terms to the internal excitation in terms of an infinite set of four-dimensional harmonic oscillators and a finite set of scalar oscillators.

Let us introduce creation and annihilation operators for the normal modes:

$$x_{\mu,n} = \frac{1}{\sqrt{2n}}(a_{\mu,n} + a_{\mu,n}^\dagger),$$
$$p_{\mu,n} = -i(\tfrac{1}{2}n)^{1/2}(a_{\mu,n} - a_{\mu,n}^\dagger), \tag{3.14}$$

and a similar decomposition for the scalar oscillators:

$$\zeta_i = \frac{1}{\sqrt{2\omega_0}}(b_i + b_i^\dagger),$$
$$\eta_i = -i(\tfrac{1}{2}\omega_0)^{1/2}(b_i - b_i^\dagger). \tag{3.15}$$

These operators satisfy the commutation relations

$$[a_{\mu,m}, a_{\nu,n}^\dagger] = \delta_{\mu\nu}\delta_{mn}\,, \tag{2.12$'$}$$

$$[b_i, b_j^\dagger] = \delta_{ij}. \tag{3.16}$$

Then the Hamiltonian H takes the form

$$H = H_0 + H_\lambda$$
$$= p_{\mu,0}{}^2 + \sum_{n=1}^{\infty} n a_{\mu,n}^\dagger a_{\mu,n} + \lambda\omega_0\sum_{i=1}^{L} b_i^\dagger b_i\,, \tag{3.17}$$

where we dropped the constant term corresponding to the zero-point fluctuation. In particular, in the following H_λ is always defined as

$$H_\lambda = \tfrac{1}{2}\lambda\sum_{i=1}^{L}(\eta_i{}^2 + \omega_0{}^2\zeta_i{}^2) - L\tfrac{1}{2}\lambda\omega_0\,. \tag{3.18}$$

Now it is quite easy to establish the relations between (x_0, p_0), (x_n, p_n) and (Z, P), (z_n, π_n) by comparing Eq. (3.14) with Eq. (2.16). They are given by

$$Z = x_0, \quad P = p_0,$$
$$z_n = \left(\frac{1}{m_0}\frac{n+1}{n}\right)^{1/2} x_n, \quad \pi_n = \left(m_0\frac{n}{n+1}\right)^{1/2} p_n\,. \tag{3.19}$$

The particle spectrum is specified by picking up a class of solutions among those of the proper-time equation that correspond to an eigenvalue $\alpha(0)$, which physically is the intercept of a leading Regge trajectory. Then all resonances lie on the linearly rising trajectories

$$\alpha^M(s) = \alpha(s) - \lambda\omega_0 M, \quad M = 0, 1, 2, \ldots, \infty$$
$$\alpha(s) = \alpha^0(s) = s + \alpha(0), \tag{3.20}$$

and on their daughters. The mass spectrum is given by

$$(m_{M,N})^2 = \sum_{n,\mu} nN_{\mu,n} + \lambda\omega_0 M - \alpha(0). \tag{3.21}$$

We have defined the problem such that the fundamental unit of energy is the inverse Regge slope.

The degeneracy of this model is higher than that[14] of the usual Veneziano model in an unimportant manner. The multiplicity is exponentially increasing and is of the same form as that proposed by Hagedorn[15] in his thermodynamical model.

In the following, for convenience, when we refer to daughters we mean all the nonleading Regge trajectories.

Since we specified the particle spectrum, we can write down the propagator for the hadron. Let us recall that in quantum mechanics the propagator for an intermediate state n is simply given by $1/(E_i - E_n)$. Similarly in our model the propagator for a state with momentum P is

$$\frac{1}{\alpha(0)-H}=\frac{1}{\alpha(0)-P^2-\sum_n n a_n^\dagger a_n - H_\lambda}$$
$$=-\int_0^\infty dy \exp\{-y[\sum_n n a_n^\dagger a_n + H_\lambda + P^2 - \alpha(0)]\}. \tag{3.22}$$

In the conventional Veneziano model without scalar oscillators, the strong coupling with the external scalar field is given by[7]

$$g:e^{ikx(\xi=0)}:, \tag{3.23}$$

where the normal-ordered form is to be taken with respect to the a_n and $a_n^\dagger$ operators. In the present model this coupling should be modified so as to incorporate hadron-scalar interactions. This is achieved through the complete factorization of the hadronic amplitudes which are obtained out of hadron-current amplitudes on top of vector-meson poles in the off-shell photon lines. We shall study this problem in Sec. VII.

IV. ELECTROMAGNETIC CURRENTS AND GAUGE INVARIANCE

In Sec. II we showed explicitly that introducing the electromagnetic interaction through the minimal substitution in the parton picture resulted in an unequal normal-mode coupling (UNMC) with the electromagnetic field evaluated at the point y_1. In this section we shall exhibit the formalism in the string model.

Because of UNMC, the electric charge is distributed over the string. This charge distribution $\rho(\xi)$ is assumed to fluctuate around the point $\rho(\xi) = 2\delta(\xi)$, and its fluctuation to be governed by the Hamiltonian H_λ of the scalar oscillators. In the off-shell photon line, this fluctuation induces vector-meson poles when and only when the charge distribution is localized at $\xi = 0$, thus preserving *duality* in the hadronic amplitude. On the other hand, we shall find that the hadron-current amplitudes are almost, but not quite, dual due to the presence of fixed poles which are characteristic of these physical amplitudes.

Let us denote the charge distribution by $\rho(\xi, \zeta)$ where ζ (ζ_i $i = 1, \ldots, L$) is the L-dimensional dynamical variable introduced already in Sec. III. Then our interaction Lagrangian density is

$$\mathcal{L}_I(\xi) = e\left(\rho(\xi,\zeta)\frac{\partial x_\mu}{\partial\tau} + \frac{\partial\rho(\xi,\zeta)}{\partial\tau}x_\mu\right)A_\mu(\bar{x}), \tag{4.1}$$

where A_μ is the electromagnetic field and $\bar{x}$ is the center of charge defined by

$$\bar{x}_\mu = \int_0^\pi \rho(\xi,\zeta)x_\mu(\xi)d\xi$$
$$= x_{\mu,0} + 2\sum_{n=1}^\infty \rho_n(\zeta)x_{\mu,n}$$
$$\equiv x_{\mu,0} + \bar{X}_\mu(\zeta). \tag{4.2}$$

The charge distribution $\rho(\xi, \zeta)$ is here specified by

$$\rho_n(\zeta) = \int_0^\pi \rho(\xi,\zeta)\cos(n\xi)d\xi$$
$$= e^{-n\zeta^2/2}, \quad n = 0, 1, 2, \ldots, \infty \tag{4.3}$$
$$\rho_0(\zeta) = 1,$$

where the last condition is obtained from the unit total charge

$$\int_0^\pi \rho(\xi,\zeta)d\xi = 1. \tag{4.4}$$

Integrating $\mathcal{L}_I(\xi)$ over ξ, we obtain the interaction Lagrangian

$$L_I = \int_0^\pi \mathcal{L}_I(\xi)d\xi = e\frac{\partial\bar{x}_\mu}{\partial\tau}A_\mu(\bar{x}), \tag{4.5}$$

where

$$\frac{\partial\bar{x}_\mu}{\partial\tau} = \frac{\partial x_{\mu,0}}{\partial\tau} + 2\sum_{n=1}^\infty \rho_n(\zeta)\frac{\partial x_{\mu,n}}{\partial\tau} + 2\sum_{n=1}^\infty \frac{\partial\rho_n(\zeta)}{\partial\tau}x_{\mu,n}. \tag{4.6}$$

The action integral is generally given by

$$S_I = \int L_I d\tau = \int j_\mu(y)A_\mu(y)d^4y. \tag{4.7}$$

Now writing

$$A_\mu(\bar{x}) = \int \delta^4(y-\bar{x})A_\mu(y)d^4y \tag{4.8}$$

in the interaction Lagrangian and taking care of the canonical commutation relations, we obtain the electromagnetic current in the following symmetrized form:

$$j_\mu(y) = \frac{1}{2}\int\left\{\frac{\partial\bar{x}_\mu}{\partial\tau}, \delta^4(y-\bar{x})\right\}_+ d\tau, \tag{4.9}$$

or expressed in momentum space as

$$j_\mu(k) = \int j_\mu(y)e^{iky}d^4y$$
$$= \frac{1}{2}\int\left\{\frac{\partial\bar{x}_\mu}{\partial\tau}, e^{ik\bar{x}}\right\}_+ d\tau. \tag{4.10}$$

To make a correspondence between the present formalism and the classical picture of Sec. II, we first evaluate

$$\langle\bar{x}\rangle_\zeta = x_0 + 2\sum_{n=1}^\infty \langle\rho_n(\zeta)\rangle_\zeta x_n, \tag{4.11}$$

where

$$\langle \rho_n(\zeta)\rangle_\zeta = \int d^L\zeta \left(\frac{\omega_0}{\pi}\right)^{L/2} e^{-\omega_0\zeta^2} e^{-n\zeta^2/2} = \left(\frac{2\omega_0}{n+2\omega_0}\right)^{L/2}. \tag{4.12}$$

For $L=2$ we have

$$\langle \bar{x}\rangle_\zeta = x_0 + 2\sum_{n=1}^{\infty} \frac{2\omega_0}{n+2\omega_0} x_n . \tag{4.13}$$

Comparing this with the relation

$$y_1 = Z + \sum_{n=1}^{\infty} \frac{1}{n+1} z_n = x_0 + \sum_{n=1}^{\infty}\left(\frac{1}{m_0}\frac{n+1}{n}\right)^{1/2} \frac{1}{n+1} x_n \tag{4.14}$$

obtained from Eqs. (2.26) and (3.19), we see the correspondence

$$y_1 \leftarrow \langle \bar{x}\rangle_\zeta \tag{4.15}$$

by choosing

$$\left(\frac{1}{m_0}\right)^{1/2} = 4\omega_0. \tag{4.16}$$

On the same condition we also observe that

$$\frac{\partial y_1}{\partial \tau} \leftarrow \left\langle \frac{\partial \bar{x}}{\partial \tau}\right\rangle_\zeta . \tag{4.17}$$

Now taking the expectation value of the interaction Lagrangian Eq. (4.5) we have

$$\langle L_I\rangle_\zeta = e\left\langle \frac{\partial \bar{x}_\mu}{\partial \tau} A_\mu(\bar{x})\right\rangle_\zeta = e\left\langle \frac{\partial \bar{x}_\mu}{\partial \tau}\right\rangle_\zeta A_\mu(\langle \bar{x}\rangle_\zeta) + \text{(higher correlation terms)} = e\left(\frac{\partial y_1}{\partial \tau}\right)_\mu A_\mu(y_1) + \text{(higher correlation terms)}. \tag{4.18}$$

Thus we have established the correspondence between the generalized model and the classical picture of the parton model.

The total Lagrangian in the presence of the electromagnetic interaction is simply given by

$$L_{\text{tot}} = L + L_I . \tag{4.19}$$

Substituting the above Lagrangian in the relation

$$H_{\text{tot}} = \frac{\partial x_{\mu,0}}{\partial \tau}\frac{\partial L_{\text{tot}}}{\partial(\partial x_{\mu,0}/\partial\tau)} + \sum_{n=1}^{\infty}\frac{\partial x_{\mu,n}}{\partial \tau}\frac{\partial L_{\text{tot}}}{\partial(\partial x_{\mu,n}/\partial\tau)} + \sum_{i=1}^{L}\frac{\partial \zeta_i}{\partial \tau}\frac{\partial L_{\text{tot}}}{\partial(\partial \zeta_i/\partial\tau)} - L_{\text{tot}}, \tag{4.20}$$

we obtain the total Hamiltonian

$$H_{\text{tot}} = H + H_I = [p_{\mu,0} - eA_\mu(\bar{x})]^2 + \tfrac{1}{2}\sum_{n=1}^{\infty}\{[p_{\mu,n} - 2e\rho_n(\zeta)A_\mu(\bar{x})]^2 + n^2 x_{\mu,n}{}^2\} + \tfrac{1}{2}\lambda\sum_{i=1}^{L}\left[\left(\eta_i - 2e\sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\zeta_i} x_{\mu,n}A_\mu(\bar{x})\right)^2 + \omega_0{}^2\zeta_i{}^2\right], \tag{4.21}$$

where the interaction Hamiltonian H_I is defined by

$$\begin{aligned} H_I = &- e\{p_{\mu,0}, A_\mu(\bar{x})\}_+ + e^2 A_\mu(\bar{x})A_\mu(\bar{x}) - e\sum_{n=1}^{\infty}\{\rho_n(\zeta)p_{\mu,n}, A_\mu(\bar{x})\}_+ + 2e^2\sum_{n=1}^{\infty}[\rho_n(\zeta)]^2 A_\mu(\bar{x})A_\mu(\bar{x}) \\ &- e\sum_{i=1}^{L}\left\{\lambda\eta_i, \sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\zeta_i} x_{\mu,n}A_\mu(\bar{x})\right\}_+ + 2e^2\lambda\sum_{i=1}^{L}\left(\sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\zeta_i} x_{\mu,n}A_\mu(\bar{x})\right)^2 \\ = &- L_I - e^2\left[\delta_{\mu\nu} + 2\sum_{n=1}^{\infty}[\rho_n(\zeta)]^2\delta_{\mu\nu} + 2\lambda\sum_{i=1}^{L}\left(\sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\zeta_i}x_{\mu,n}\right)\left(\sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\zeta_i}x_{\nu,n}\right)\right]A_\mu(\bar{x})A_\nu(\bar{x}). \end{aligned} \tag{4.22}$$

It is worth noting that the interaction Hamiltonian, in addition to the $j\cdot A$ term, has the *contact terms*. Having split the total Hamiltonian H_{tot} into two parts, H, the Hamiltonian in the absence of the electromagnetic interaction, and H_I, the interaction Hamiltonian, we shall always consider the problem in the interaction representation.

In the following, let us prove current conservation. The matrix element of the divergence of the electromagnetic current between the two eigenstates $|\alpha\rangle$ and $|\alpha'\rangle$ of the Hamiltonian H is

$$\langle\alpha'|ik_\mu j_\mu(k)|\alpha\rangle = \left\langle \alpha'\left|\int d\tau \frac{\partial}{\partial\tau} e^{ik\bar{x}}\right|\alpha\right\rangle. \tag{4.23}$$

Using the equation of motion, the above gives

$$\left\langle\alpha'\left|\int d\tau\, i[H, e^{ik\bar{x}}]\right|\alpha\right\rangle = i(\alpha'-\alpha)\int d\tau\,\langle\alpha'|e^{ik\bar{x}}|\alpha\rangle = 2\pi i(\alpha'-\alpha)\delta(\alpha'-\alpha)\langle\alpha'|e^{ik\bar{x}(\tau=0)}|\alpha\rangle \equiv 0, \tag{4.24}$$

where we have explicitly used the translational invariance of the current in the τ space or the conservation of the eigenvalue α of H.

Similarly, we can prove the gauge invariance of the theory beginning with the action integral S_I. Gauge invariance requires that the substitution

$$A_\mu(\bar{x}) \to A_\mu(\bar{x}) + \frac{\partial \Lambda(\bar{x})}{\partial \bar{x}_\mu} \tag{4.25}$$

should at most change the action by a total derivative. This condition is here trivially satisfied since

$$e\int d\tau \frac{\partial \bar{x}_\mu}{\partial \tau}\frac{\partial \Lambda(\bar{x})}{\partial \bar{x}_\mu} = e\int d\bar{x}_\mu \frac{\partial \Lambda(\bar{x})}{\partial \bar{x}_\mu}. \tag{4.26}$$

For computational purposes, we will write down the matrix element of the current between the ground state and the excited state. First we split the current into three parts:

$$j_\mu(k) = j_\mu^{(0)}(k) + j_\mu^{(1)}(k) + j_\mu^{(2)}(k), \tag{4.27}$$

where

$$j_\mu^{(0)}(k) = \int \{p_{\mu,0}, e^{ik\bar{x}}\}_+ d\tau, \tag{4.28}$$

$$j_\mu^{(1)}(k) = \int \sum_{n=1}^{\infty} \{\rho_n(\zeta) p_{\mu,n}, e^{ik\bar{x}}\}_+ d\tau, \tag{4.29}$$

$$j_\mu^{(2)}(k) = \int \sum_{i=1}^{L} \left\{\lambda\eta_i, \sum_{n=1}^{\infty} \frac{\partial \rho_n(\zeta)}{\partial \zeta_i} x_{\mu,n} e^{ik\bar{x}}\right\}_+ d\tau. \tag{4.30}$$

Then

$$\begin{aligned}\Gamma_\mu &= \langle m, n_\lambda, p' | j_\mu(k) | 0, 0, p\rangle \\ &= \Gamma_\mu^{(0)} + \Gamma_\mu^{(1)} + \Gamma_\mu^{(2)} \\ &= (p+p')_\mu \langle m, n_\lambda, p' | e^{ik\bar{X}(\zeta)} | 0, 0, p\rangle + \sum_{n=1}^{\infty} \left\langle m, n_\lambda, p' \middle| \rho_n(\zeta) \left\{-i\left(\frac{n}{2}\right)^{1/2} (a_n - a_n^\dagger)_\mu, e^{ik\bar{X}(\zeta)}\right\}_+ \middle| 0, 0, p\right\rangle \\ &\quad + \sum_{i=1}^{L}\sum_{n=1}^{\infty} \left\langle m, n_\lambda, p' \middle| \left\{\lambda\eta_i, \frac{\partial \rho_n(\zeta)}{\partial \zeta_i} e^{ik\bar{X}(\zeta)}\right\}_+ \frac{1}{\sqrt{2n}} (a_n + a_n^\dagger)_\mu \middle| 0, 0, p\right\rangle,\end{aligned} \tag{4.31}$$

with $p' = p + k$. The first term corresponds to the center of mass and is equivalent to the pointlike electrodynamical vertex. The second term was investigated in Ref. 5. We will outline here its important properties. Noting that

$$\rho_n(\zeta)\{-i(\tfrac{1}{2}n)^{1/2}(a_n - a_n^\dagger)_\mu, e^{ik\bar{X}(\zeta)}\}_+ = [-2\rho_n(\zeta)^2 k_\mu - 2i\rho_n(\zeta)(\tfrac{1}{2}n)^{1/2}(a_n - a_n^\dagger)_\mu] e^{ik\bar{X}(\zeta)} \tag{4.32}$$

and by the property of the coherent states, $\Gamma_\mu^{(1)}$ reduces to

$$\Gamma_\mu^{(1)} = \sum_{n=1}^{\infty} \langle m, n_\lambda, p' | 2i\rho_n(\zeta)(\tfrac{1}{2}n)^{1/2}(a_n^\dagger)_\mu e^{ik\bar{X}(\zeta)} | 0, 0, p\rangle. \tag{4.33}$$

The vertex contribution from the scalar oscillators is distinctively different from the above. Recall that

$$\zeta_i = \frac{1}{\sqrt{2\omega_0}}(b_i + b_i^\dagger), \quad \eta_i = -i\left(\frac{\omega_0}{2}\right)^{1/2}(b_i - b_i^\dagger); \tag{3.15}$$

then an elementary calculation yields

$$\Gamma_\mu^{(2)} = \sum_{i=1}^{L}\sum_{n=1}^{\infty} \left\langle m, n_\lambda, p' \middle| i\sqrt{2\omega_0}\, \lambda b_i^\dagger \frac{\partial \rho_n(\zeta)}{\partial \zeta_i} e^{ik\bar{X}(\zeta)} \frac{1}{\sqrt{2n}} (a_n^\dagger)_\mu \middle| 0, 0, p\right\rangle. \tag{4.34}$$

Let us introduce here a function $K_L(\zeta_1, \zeta_2; y)$ which is characteristic of our model. The propagator in the ζ space is given by

$$\left\langle \zeta_2 \middle| \frac{1}{\alpha(0) - H} \middle| \zeta_1 \right\rangle = -\int_0^\infty dy\, e^{-y[H_0 - \alpha(0)]} K_L(\zeta_1, \zeta_2; y), \tag{4.35}$$

where the correlation function $K_L(\zeta_1, \zeta_2; y)$ is defined and given by

$$K_L(\zeta_1,\zeta_2;y)\equiv\langle\zeta_2|e^{-yH_\lambda}|\zeta_1\rangle$$
$$=\left[\frac{\omega_0}{\pi(1-e^{-2\lambda\omega_0 y})}\right]^{L/2}\exp[-\tfrac14\omega_0(\zeta_1+\zeta_2)^2\tanh(\tfrac12\lambda\omega_0 y)-\tfrac14\omega_0(\zeta_1-\zeta_2)^2\coth(\tfrac12\lambda\omega_0 y)]$$
$$=\left[\frac{\omega_0}{\pi(1-e^{-2\lambda\omega_0 y})}\right]^{L/2}\exp\left[-\tfrac12\omega_0(\zeta_1{}^2+\zeta_2{}^2)\coth(\lambda\omega_0 y)+\omega_0\frac{\zeta_1\zeta_2}{\sinh(\lambda\omega_0 y)}\right]. \quad (4.36)$$

As is apparent from the above, $K_L(\zeta_1,\zeta_2;y)$ is the propagation function of an L-dimensional harmonic oscillator in coordinate space.

We shall write down useful properties of the function $K_L(\zeta_1,\zeta_2;y)$ below.

$$-\frac{\partial}{\partial y}K_L=H_\lambda K_L, \quad (4.37)$$

$$\left(\omega_0\zeta_1+\frac{\partial}{\partial\zeta_1}\right)_i K_L=e^{-\lambda\omega_0 y}\left(\omega_0\zeta_2-\frac{\partial}{\partial\zeta_2}\right)_i K_L, \quad (4.38)$$

$$-\frac{\partial}{\partial y}(e^{-\lambda\omega_0 y}K_L)=\tfrac12\lambda\sum_{i=1}^{L}\left(\omega_0\zeta_1+\frac{\partial}{\partial\zeta_1}\right)_i\left(\omega_0\zeta_2+\frac{\partial}{\partial\zeta_2}\right)_i K_L, \quad (4.39)$$

$$-\psi_0(\zeta_1)\psi_0(\zeta_2)\frac{\partial}{\partial y}K_L=\tfrac12\lambda e^{-\lambda\omega_0 y}\sum_{i=1}^{L}\left(\frac{\partial}{\partial\zeta_1}\right)_i\left(\frac{\partial}{\partial\zeta_2}\right)_i[\psi_0(\zeta_1)\psi_0(\zeta_2)K_L], \quad (4.40)$$

where $\psi_0(\zeta)$ is the ground-state wave function of the scalar oscillators:

$$\psi_0(\zeta)=\left(\frac{\omega_0}{\pi}\right)^{L/4}e^{-\omega_0\zeta^2/2}. \quad (4.41)$$

In Sec. V we will study specific applications.

V. ELECTROMAGNETIC FORM FACTOR

In Sec. IV we defined the electromagnetic current and the vertex function. The simplest application will be to evaluate the elastic form factor for the ground-state hadron.

We sandwich the current Eq. (4.27) between the ground-state bra and ket:

$$|0,0,p\rangle=|0,p\rangle_a\int_{-\infty}^{\infty}d^L\zeta\,|\zeta\rangle\langle\zeta|0\rangle_\lambda. \quad (5.1)$$

The current is diagonal in the ζ space making one of the ζ integrals trivial. Using the coordinate-space representation for the wave function

$$\langle\zeta|0\rangle_\lambda=\left(\frac{\omega_0}{\pi}\right)^{L/4}e^{-\omega_0\zeta^2/2} \quad (5.2)$$

and the polar-coordinate expression for the L-dimensional volume element

$$\int_{-\infty}^{\infty}d^L\zeta=\int_0^\infty d\zeta\,\zeta^{L-1}\frac{2\pi^{L/2}}{\Gamma(\frac12 L)}, \quad (5.3)$$

the elastic form factor is then given by

$$F_L(-k^2)=\frac{2(\omega_0)^{L/2}}{\Gamma(\frac12 L)}\int_0^\infty d\zeta\,\zeta^{L-1}e^{-\omega_0\zeta^2}(1-e^{-\zeta^2})^{k^2}. \quad (5.4)$$

Here we define a Regge trajectory $\alpha_L(-k^2)$ such that

$$\alpha_L(-k^2)=-k^2+1-\tfrac12 L,$$

with daughters (5.5)

$$\alpha_L(-k^2)-n,\quad n=1,2,3,\ldots,\infty;$$

then $F_L(-k^2)$ has an infinite series of vector-meson poles at $\alpha_L(-k^2)=1,2,3,\ldots,\infty$. (For $L=1$ the lowest pole starts at the ρ-meson mass $-k^2=\frac12$.) Furthermore, it is analytic with an asymptotic decrease

$$F_L(-k^2)\underset{k^2\to\infty}{\sim}(k^2)^{-\omega_0}(\ln k^2)^{L/2-1}. \quad (5.6)$$

Therefore our form factor satisfies an unsubtracted dispersion relation

$$F_L(t)=\frac1\pi\int_0^\infty dt'\,\frac{\operatorname{Im}F_L(t')}{t'-t},\quad t=-k^2 \quad (5.7)$$

where the discontinuity is given by an infinite sum of δ functions.

Notice that in the present model the elastic form factor and the transition matrices are not Gaussian; they are analytic and their rate of decrease is mostly determined by ω_0.

For the special case of $L=2$, $F_L(-k^2)$ takes the simple form given by

$$F_2(-k^2)=\frac{\Gamma(\omega_0+1)\Gamma(k^2+1)}{\Gamma(k^2+\omega_0+1)}. \quad (5.8)$$

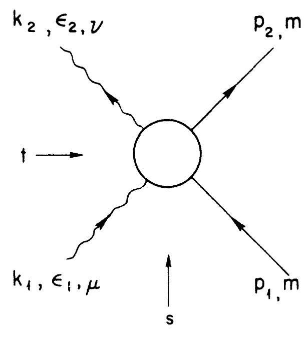

FIG. 1. Off-shell Compton scattering.

VI. TWO-CURRENT AMPLITUDE

We next examine the two-current amplitude in our model. We calculate the Compton scattering amplitudes for off-shell photons on a target particle in the ground state with mass m (see Fig. 1).

We define

$$P=\tfrac{1}{2}(p_1+p_2)\,,\quad s=-(p_1+k_1)^2\,,\quad u=-(p_1-k_2)^2\,,$$
$$2m\nu=-2P\cdot k_1=-2P\cdot k_2=(s-m^2+k_1\cdot k_2)\,, \tag{6.1}$$

with momentum conservation

$$p_1+k_1=p_2+k_2 \tag{6.2}$$

and mass-shell conditions

$$p_1^{\,2}=-m^2\,,\quad p_2^{\,2}=-m^2\,,\quad \alpha(m^2)=0\,,$$
$$k_1=(\vec{k}_1,[\vec{k}_1^{\,2}+(-k_1^{\,2})]^{1/2})\,, \tag{6.3}$$
$$k_2=(\vec{k}_2,[\vec{k}_2^{\,2}+(-k_2^{\,2})]^{1/2})\,.$$

Then, using the gauge invariance of the theory, the off-shell Compton scattering amplitudes T_1 and T_2 are defined such that

$$T_{\mu\nu}=T_1\left(\delta_{\mu\nu}-\frac{k_{2,\mu}k_{1,\nu}}{k_1\cdot k_2}\right)+\frac{T_2}{m^2}\left(P_\mu-\frac{k_1\cdot P}{k_1\cdot k_2}k_{2,\mu}\right)\left(P_\nu-\frac{k_2\cdot P}{k_1\cdot k_2}k_{1,\nu}\right). \tag{6.4}$$

The propagator here is given by

$$\left\langle\zeta_2\left|\frac{1}{\alpha(0)-H}\right|\zeta_1\right\rangle=-\int_0^\infty dy\,e^{-y[H_0-\alpha(0)]}K_L(\zeta_1,\zeta_2;y)\,, \tag{6.5}$$

where the correlation function $K_L(\zeta_1,\zeta_2;y)$ was defined in Eq. (4.36). Following the steps in Ref. 5, the helicity-flip amplitude $T_2^{(L)}$ can be calculated as

$$T_2^{(L)}(\nu,t,k_1^{\,2},k_2^{\,2})=2m\left(\frac{\omega_0}{\pi}\right)^{L/2}\int_{-\infty}^{\infty}d^L\zeta_1\int_{-\infty}^{\infty}d^L\zeta_2\int_0^\infty dy\,e^{-\omega_0\zeta_1^{\,2}/2}e^{-\omega_0\zeta_2^{\,2}/2}(1-e^{-\zeta_1^{\,2}})^{k_1^{\,2}}(1-e^{-\zeta_2^{\,2}})^{k_2^{\,2}}$$
$$\times(1-e^{-\zeta_1^{\,2}/2}e^{-\zeta_2^{\,2}/2}e^{-y})^{-2k_1\cdot k_2}(e^{y\alpha(s)}+e^{y\alpha(u)})K_L(\zeta_1,\zeta_2;y)\,. \tag{6.6}$$

The helicity-nonflip amplitude $T_1^{(L)}$ will be discussed later. In the following we will consider the s-channel contribution since the expression is manifestly symmetric in s and u.

A. Hadronic Amplitude

Let us first evaluate the hadronic amplitude corresponding to the lowest vector-meson poles at $-k_1^{\,2}=-k_2^{\,2}=\tfrac{1}{2}L$ in the off-shell photon lines. In Eq. (6.6), perform the angular integration in the ζ space and then expand the integrand around $\zeta_1=\zeta_2=0$. Upon integration over ζ_1 and ζ_2 we get the pole contribution corresponding to the factor $1/(2k_1^{\,2}+L)(2k_2^{\,2}+L)$ whose residue gives the hadronic amplitude

$$H_2^{(L)}(s,t)=\frac{2m\omega_0^{\,L}}{[\Gamma(\tfrac{1}{2}L)]^2}\int_0^\infty dy\,e^{y\alpha(s)}(1-e^{-y})^{-\alpha_L(t)+1+L/2}(1-e^{-2\lambda\omega_0 y})^{-L/2}\,, \tag{6.7}$$

where

$$\alpha(s)=s+\alpha(0)\,, \tag{6.8}$$

and

$$\alpha_L(t)=t+1-\tfrac{1}{2}L\,. \tag{6.9}$$

(i) *s-channel singularities*. In the s channel $H_2^{(L)}$ has an infinite series of poles at

$$\alpha(s)=N+2\lambda\omega_0 M\,,\quad N,M=0,1,2,\ldots,\infty\,. \tag{6.10}$$

This can simply be seen by taking $y\to\infty$; denoting $x=e^{-y}$,

$$H_2^{(L)}(s,t)=\frac{2m\omega_0^{\,L}}{[\Gamma(\tfrac{1}{2}L)]^2}\int_0^1 dx\,x^{-\alpha(s)-1}(1-x)^{-\alpha_L(t)-1+L/2}(1-x^{2\lambda\omega_0})^{-L/2}\,. \tag{6.11}$$

From this expression we obtain the above result by expanding the integrand around $x=0$. Notice that the scalar oscillators are excited in pairs since our electromagnetic current conserves a parity $(-)^{b^\dagger b}$.

(ii) *t-channel singularities*. In the t channel $H_2^{(L)}$ has an infinite series of poles which starts at

$$\alpha_L(t)=2,3,4,\ldots,\infty\,. \tag{6.12}$$

Note that we have no poles at the nonsense points $\alpha_L(t)=0$ and 1 due to the double helicity flip of the vector mesons. To prove this assertion, make a change of variable $1-x=z$ in Eq. (6.11). Then

$$H_2^{(L)}(s,t)=\frac{2m\omega_0^{\,L}}{[\Gamma(\frac{1}{2}L)]^2}\int_0^1 dz(1-z)^{-\alpha(s)-1}z^{-\alpha_L(t)-1+L/2}[1-(1-z)^{2\lambda\omega_0}]^{-L/2}, \tag{6.13}$$

and writing the last term of the integrand as

$$[1-(1-z)^{2\lambda\omega_0}]^{-L/2}=(2\lambda\omega_0)^{-L/2}z^{-L/2}[1+O(z)], \tag{6.14}$$

the location of the poles is seen immediately.

(iii) *Regge asymptotics*. The asymptotic behavior of $H_2^{(L)}$ is controlled by the Regge poles $\alpha(s)$, $\alpha_L(t)$, and their daughters. Specifically, the asymptotic behavior in the s channel for fixed t is obtained by letting $\alpha(s)\to-\infty$; then the $y\simeq 0$ region contributes and gives

$$H_2^{(L)}(s,t)\underset{s\to\infty;\ \text{fixed } t}{\sim}\frac{2m\omega_0^{\,L}}{[\Gamma(\frac{1}{2}L)]^2(2\lambda\omega_0)^{L/2}}\,\Gamma(2-\alpha_L(t))[-\alpha(s)]^{\alpha_L(t)-2}. \tag{6.15}$$

On the other hand, in the limit of $t\to\infty$ with fixed s, the nonvanishing contribution comes from the region $x\simeq 0$ $(y\to\infty)$ and the amplitude behaves like

$$H_2^{(L)}(s,t)\underset{t\to\infty;\ \text{fixed } s}{\sim}\frac{2m\omega_0^{\,L}}{[\Gamma(\frac{1}{2}L)]^2}\,\Gamma(-\alpha(s))[-\alpha_L(t)]^{\alpha(s)}. \tag{6.16}$$

(iv) *Duality*. From the above investigation it is obvious that $H_2^{(L)}$ is dual.

(v) *ρ-f-A_2 degeneracy*. It is important to note that the particle spectrum in the t channel is degenerate with that of the off-shell photon line. Phenomenologically this implies the ρ-f-A_2 degeneracy. This is one of the severe restrictions to be satisfied by dual off-shell Compton scattering amplitudes.

When $2\lambda\omega_0=1$, the hadronic amplitude $H_2^{(L)}$ is given by a simple Veneziano representation[8]:

$$H_2^{(L)}(s,t)=\frac{2m\omega_0^{\,L}}{[\Gamma(\frac{1}{2}L)]^2}\,B(-\alpha(s),2-\alpha_L(t)). \tag{6.17}$$

B. Fixed Poles

Now we study the singularity structure of the two-current amplitude. Most of the properties listed above for the hadronic amplitude are preserved for the off-shell Compton scattering amplitude. However, duality here is minimally broken due to the presence of fixed poles.

The pole structure in the s channel is obvious by construction. In the following, let us look at the singularity structure in the t channel.

(i) *t-channel poles*. In Eq. (6.6), perform the angular integration in the ξ space and make the following change of variables:

$$y=zx\,,\quad \zeta_1^{\,2}=z(1-x)w\,,\quad \zeta_2^{\,2}=z(1-x)(2-w)\,. \tag{6.18}$$

Then we obtain

$$\begin{aligned}T_2^{(L)}=2m\left(\frac{\omega_0}{\pi}\right)^{L/2}&\left(\frac{2\pi^{L/2}}{\Gamma(\frac{1}{2}L)}\right)\\ &\times\frac{1}{4}\int_0^\infty dz\,(1-e^{-z})^{-2k_1k_2}\int_0^1 dx\int_0^2 dw\, z^2(1-x)[z(1-x)]^{L-2}[w(2-w)]^{L/2-1}e^{-\omega_0 z(1-x)}\\ &\quad\times(1-e^{-z(1-x)w})^{k_1^2}(1-e^{-z(1-x)(2-w)})^{k_2^2}e^{zx\alpha(s)}\left(\frac{\omega_0}{\pi(1-e^{-2\lambda\omega_0 zx})}\right)^{L/2}\exp[-\omega_0 z(1-x)\coth(\lambda\omega_0 zx)]\\ &\quad\times\frac{2\pi^{(L-1)/2}}{\Gamma((L-1)/2)}\int_0^\pi d\theta(\sin\theta)^{L-2}\exp\left(\frac{\omega_0 z(1-x)[w(2-w)]^{1/2}}{\sinh(\lambda\omega_0 zx)}\sin\theta\right).\end{aligned} \tag{6.19}$$

Expanding the integrand around $z=0$ and using the relation

$$-2k_1k_2 = -\alpha_L(t) + 1 - \tfrac{1}{2}L - k_1{}^2 - k_2{}^2, \quad t = -(k_1 - k_2)^2, \tag{6.20}$$

we can see the pole structure in $\alpha_L(t)$ which starts at

$$\alpha_L(t) = 2, 3, 4, \ldots, \infty \tag{6.21}$$

due to the double helicity flip of the off-shell photons. The amplitude does not have any unphysical singularity in t in contrast to some other attempts done before.[16]

(ii) *Fixed poles*. The amplitude has fixed poles at $J = 1, 0, -1, \ldots, -\infty$.[17] The leading contribution at $J = 1$ is given by

$$T_2^{(L)}(\nu, t, k_1{}^2, k_2{}^2)|_{\text{fixed pole}} = 2m\left(\frac{\omega_0}{\pi}\right)^{L/2}\int_{-\infty}^{\infty} d^L\zeta_1 \int_{-\infty}^{\infty} d^L\zeta_2 \int_0^\infty dy\, e^{-\omega_0\zeta_1{}^2}(1 - e^{-\zeta_1{}^2})^{(k_1-k_2)^2} e^{y\,\alpha(s)}\delta^L(\zeta_1 - \zeta_2)$$
$$= \frac{2m}{-\alpha(s)} F_L(t). \tag{6.22}$$

The fixed-pole residue has no structure in $k_1{}^2$ and $k_2{}^2$.[18]

(iii) *Current-algebra sum rule*. Let us denote the imaginary part of $T_2^{(L)}$ by

$$W_2^{(L)}(\nu, t, k_1{}^2, k_2{}^2) \equiv \frac{1}{\pi}\,\mathrm{Im}\,T_2^{(L)}(\nu, t, k_1{}^2, k_2{}^2). \tag{6.23}$$

Recalling that $T_2^{(L)}(\nu, t, k_1{}^2, k_2{}^2)$ satisfies a dispersion relation in ν and using the simple superconvergence technique, we obtain the Dashen–Gell-Mann–Fubini[10]-type sum rule for the electromagnetic current:

$$\int_0^\infty d\nu\, W_2^{(L)}(\nu, t, k_1{}^2, k_2{}^2) = F_L(t). \tag{6.24}$$

Another approach to prove the sum rule is to use the *no-correlation* property of the product of two currents:

$$j_0(\vec{y}, y_0)j_0(\vec{y}\,', y_0) = \delta(\vec{y} - \vec{y}\,')j_0(\vec{y}, y_0). \tag{6.25}$$

The above property is very important in that it enables us to construct a local current algebra.[5]

C. Bjorken Limit and Deep-Inelastic e-p Scattering

Of special interest to deep-inelastic e-p scattering is the forward off-shell Compton scattering amplitude. An interesting limit to high-energy physics here is the Bjorken limit[11]; both $\alpha(s)$ and k^2 tend to infinity with their ratio kept fixed. Experimentally, this is the region where the SLAC data proves scaling for the structure functions of the process.

In Eq. (6.6), let us put $k_1 = k_2 = k$ corresponding to the forward case. Replace $\alpha(s)$ by $-k^2(1-\omega)$, where

$$\omega \equiv 2m\nu/k^2, \tag{6.26}$$

and let k^2 go to infinity keeping ω in the region $\omega < 1$. (By analytic continuation we can go to $\omega > 1$.) The factors with k^2 dependence are

$$\left[\frac{(1 - e^{-\zeta_1{}^2})(1 - e^{-\zeta_2{}^2})}{(1 - e^{-\zeta_1{}^2/2}e^{-\zeta_2{}^2/2}e^{-y})^2}e^{-y(1-\omega)}\right]^{k^2}; \tag{6.27}$$

the only surviving contribution in the limit of $k^2 \to \infty$ comes from $y \simeq 0$ and $\zeta_1{}^2 = \zeta_2{}^2$. Introducing the variable $z = k^2 y$, then,

$$\lim_{k^2\to\infty} K_L(\zeta_1, \zeta_2; z/k^2) = \delta^L(\zeta_1 - \zeta_2). \tag{6.28}$$

Using

$$\lim_{k^2\to\infty}\left(\frac{1 - e^{-\zeta_1{}^2}e^{-z/k^2}}{1 - e^{-\zeta_1{}^2}}\right)^{-2k_1{}^2} = \lim_{k^2\to\infty}\left(1 + \frac{e^{-\zeta_1{}^2}}{1 - e^{-\zeta_1{}^2}}\frac{z}{k^2}\right)^{-2k^2} = \exp\left(-\frac{2e^{-\zeta_1{}^2}}{1 - e^{-\zeta_1{}^2}}z\right), \tag{6.29}$$

we obtain for $T_2^{(L)}$

$$\lim_{k^2\to\infty} k^2 T_2^{(L)}(\nu, k^2) = \frac{4m\omega_0{}^{L/2}}{\Gamma(\frac{1}{2}L)}\int_0^\infty d\zeta_1 \frac{\zeta_1{}^{L-1}e^{-\omega_0\zeta_1{}^2}}{2e^{-\zeta_1{}^2}/(1 - e^{-\zeta_1{}^2}) + 1 - \omega}. \tag{6.30}$$

Introducing the new variable

$$\omega' \equiv 1 + \frac{2e^{-\zeta_1{}^2}}{1 - e^{-\zeta_1{}^2}}, \tag{6.31}$$

we have

$$\lim_{k^2\to\infty} k^2 T_2^{(L)}(\nu, k^2) = 2m\int_1^\infty \frac{d\omega'}{\omega'}\frac{\nu W_2^{(L)}(\omega')}{\omega' - \omega}, \tag{6.32}$$

where the scaling function $\nu W_2^{(L)}(\omega)$ is defined by

$$\nu W_2^{(L)}(\omega) = \lim_{k^2\to\infty} \nu W_2^{(L)}(\nu, k^2) = \lim_{k^2\to\infty}\frac{1}{\pi}\nu\,\mathrm{Im}\,T_2^{(L)}(\nu, k^2) \tag{6.33}$$

and is given by the expression

$$\nu W_2^{(L)}(\omega) = \frac{2\omega_0^{L/2}}{\Gamma(\frac{1}{2}L)} \frac{\omega(\omega-1)^{\omega_0-1}}{(\omega+1)^{\omega_0+1}}\left[\ln\left(\frac{\omega+1}{\omega-1}\right)\right]^{L/2-1}. \tag{6.34}$$

Thus, $\nu W_2^{(L)}(\nu, k^2)$, the deep-inelastic e-p scattering structure function, scales and has the simple analytic expression in the Bjorken limit. Notice that in the present model the scaling curve is saturated by the nondiffractive narrow-resonance contribution.

In the following let us look at some specific properties of our structure function $\nu W_2^{(L)}(\omega)$.

(i) *Asymptotic behavior.* $\nu W_2^{(L)}(\omega)$ has the asymptotic form dictated by the ordinary Regge-pole exchange such that

$$\nu W_2^{(L)}(\omega) \underset{\omega\to\infty}{\sim} \frac{(2\omega_0)^{L/2}}{\Gamma(\frac{1}{2}L)} \omega^{\alpha_L(0)-1}. \tag{6.35}$$

(ii) *Threshold behavior.* For $\omega = 1$, $\nu W_2^{(L)}(\omega)$ vanishes in agreement with the usual conjecture. The rate of rise of this function is given by

$$\nu W_2^{(L)}(\omega) \underset{\omega\to 1}{\sim} \frac{\omega_0^{L/2}}{\Gamma(\frac{1}{2}L)2^{\omega_0}}(\omega-1)^{\omega_0-1}[-\ln(\omega-1)]^{L/2-1}. \tag{6.36}$$

The relationship between this rate of increase and the asymptotic decrease of the elastic form factor does not agree with the field-theoretical model of Drell, Levy, and Yan.[19] In the Bloom-Gilman[20] derivation of this relation the effect of the infinite clustering of resonances at $\omega = 1$ when $k^2 \to \infty$ was neglected, giving the discrepancy of an extra power of $(\omega-1)^p$, where the power p is a model-dependent parameter which is closely related to the density of higher-resonance states.

In our model, since the propagation function $K_L(\zeta_1, \zeta_2; y)$ reduces to the uncorrelated form $\delta^L(\zeta_1-\zeta_2)$ in the Bjorken limit and furthermore our electromagnetic current satisfies the no-correlation property [Eq. (6.25)], the threshold behavior is determined by the asmyptotic decrease of $F_L(t)$ rather than by the asymptotic decrease of its square $F_L(t)^2$. This noncorrelation may be understood in the parton picture of Feynman. In the high-energy Bjorken limit of the off-shell Compton amplitude, the only surviving contributions are those where a single parton absorbs and emits the off-shell currents almost *instantaneously without disturbing the configuration of other partons.* Mathematically, this condition is realized in our model by putting the time variable in the propagator as $y = z/k^2 \to 0$, $k^2 \to \infty$ [see Eqs. (6.28) and (6.29)]. Then we have the limiting form, $\delta^L(\zeta_1-\zeta_2)$, of noncorrelation for $K_L(\zeta_1, \zeta_2; y)$. This means that the hadron excited into a certain configuration specified by ζ_1 by absorbing the current through a parton is de-excited from the same configuration specified by $\zeta_2 = \zeta_1$ by emitting the current through the same parton.

Experimentally the rate of increase of $\nu W_2^{(L)}(\omega)$ at threshold is anywhere between 2 and 5.

(iii) *Gottfried sum rule.* Our $\nu W_2^{(L)}(\omega)$ satisfies exactly the sum rule[12] given by

$$\int_1^\infty \frac{d\omega}{\omega}\nu W_2^{(L)}(\omega) = 1. \tag{6.37}$$

This can be proved by making the change of variable

$$x = \ln\left(\frac{\omega+1}{\omega-1}\right) \tag{6.38}$$

in the left-hand side of Eq. (6.37) and by using the formula

$$\int_0^\infty x^{L/2-1}e^{-\omega_0 x}dx = \omega_0^{-L/2}\Gamma(\tfrac{1}{2}L). \tag{6.39}$$

The above sum rule is a special case of the Dashen-Gell-Mann-Fubini-type sum rule for the electromagnetic currents Eq. (6.24).

D. Low-Energy Theorem for Currents

Since our theory of electromagnetic currents is gauge-invariant and the obtained amplitude is analytic, the low-energy theorem for currents is naturally expected to hold.[10]

Using gauge invariance in the soft-photon limit of $k_1 \to 0$, the low-energy theorem states that the two-current amplitude reduces to the external-line insertion of one current-matrix element.

From Eq. (6.6) one can immediately write down the expression for $T_2^{(L)}(\nu, t, k_1^2, k_2^2)$ at $k_1^2 = 0 = k_1\cdot k_2$ and $t = -k_2^2$:

$$T_2^{(L)}(\nu, t=-k_2^2, k_1^2=0, k_2^2)|_{k_1\cdot k_2=0} = 2m\left(\frac{\omega_0}{\pi}\right)^{L/2}\int_{-\infty}^{\infty} d^L\zeta_1\int_{-\infty}^{\infty} d^L\zeta_2\int_0^\infty dy\, e^{-\omega_0\zeta_1^2/2} \times e^{-\omega_0\zeta_2^2/2}(1-e^{-\zeta_2^2})^{k_2^2}e^{y\alpha(s)}K_L(\zeta_1, \zeta_2; y). \tag{6.40}$$

Now using the energy representation of $K_L(\zeta_1, \zeta_2; y)$ given by

$$K_L(\zeta_1, \zeta_2; y) = \sum_{n_\lambda}\langle\zeta_2|n_\lambda\rangle\langle n_\lambda|e^{-yH_\lambda}|n_\lambda\rangle\langle n_\lambda|\zeta_1\rangle \tag{6.41}$$

and the orthogonality of the harmonic-oscillator wave functions over the ζ_1 space, we finally obtain

$$T_2^{(L)}(\nu, t=-k_2^2, k_1^2=0, k_2^2)|_{k_1\cdot k_2=0} = 2m\left(\frac{\omega_0}{\pi}\right)^{L/2}\int_{-\infty}^{\infty} d^L\zeta_2 \int_0^\infty dy\, e^{-\omega_0\zeta_2^2}(1-e^{-\zeta_2^2})^{k_2^2} e^{y\alpha(s)}$$
$$= -\frac{2m}{s-m^2}F_L(-k_2^2), \tag{6.42}$$

which is a verification of the low-energy theorem in our model.

E. Evaluation of $W_1^{(L)}$

All considerations in this chapter so far pertained to the helicity-flip amplitude $T_2^{(L)}$. We will next study the contribution to the helicity-nonflip amplitude $T_1^{(L)}$. The calculations are technically very tedious and lengthy. We will only highlight the important steps for calculating the forward amplitude.

Let us denote the polarization vectors of the two off-shell photons by ϵ_1 and ϵ_2. Then the forward Compton amplitude $T_1^{(L)}(\nu, k^2)$ is given by the term proportional to $\delta_{\mu\nu}$:

$$\int_{-\infty}^{\infty} d^L\zeta_1 \int_{-\infty}^{\infty} d^L\zeta_2 \sum_m \langle 0, 0, p|\epsilon_{2,\nu}(\Gamma^{(1)}(k)+\Gamma^{(2)}(k))_\nu|m, \zeta_2, p+k\rangle$$
$$\times\int_0^\infty dy\,\langle m,\zeta_2,p+k|e^{-yH_0}e^{-yH_\lambda}|m,\zeta_1,p+k\rangle\langle m,\zeta_1,p+k|\epsilon_{1,\mu}(\Gamma^{(1)}(k)+\Gamma^{(2)}(k))_\mu|0,0,p\rangle$$
$$\equiv \epsilon_{1,\mu}\delta_{\mu\nu}\epsilon_{2,\nu}\int_{-\infty}^{\infty} d^L\zeta_1\int_{-\infty}^{\infty} d^L\zeta_2(\bar{A}+\bar{B})K_L(\zeta_1,\zeta_2;y)(A+B), \tag{6.43}$$

where

$$\bar{A}K_LA = 2\left[\frac{\partial^2}{\partial y^2}g(\zeta_1,\zeta_2;y)\right]\langle\zeta_2|e^{-yH_\lambda}|\zeta_1\rangle F, \tag{6.44}$$

$$\bar{A}K_LB = -2\lambda\sqrt{2\omega_0}\left[\frac{\partial}{\partial\zeta_1{}^2}\frac{\partial}{\partial y}g(\zeta_1,\zeta_2;y)\right]\langle\zeta_2|e^{-yH_\lambda}(b^\dagger\cdot\zeta)|\zeta_1\rangle F, \tag{6.45}$$

$$\bar{B}K_LA = -2\lambda\sqrt{2\omega_0}\left[\frac{\partial}{\partial\zeta_2{}^2}\frac{\partial}{\partial y}g(\zeta_1,\zeta_2;y)\right]\langle\zeta_2|(\zeta\cdot b)e^{-yH_\lambda}|\zeta_1\rangle F, \tag{6.46}$$

$$\bar{B}K_LB = 4\lambda^2\omega_0\left[\frac{\partial}{\partial\zeta_1{}^2}\frac{\partial}{\partial\zeta_2{}^2}g(\zeta_1,\zeta_2;y)\right]\langle\zeta_2|(\zeta\cdot b)e^{-yH_\lambda}(b^\dagger\cdot\zeta)|\zeta_1\rangle F, \tag{6.47}$$

with

$$g(\zeta_1,\zeta_2;y) = \sum_{n=1}^\infty \rho_n(\zeta_1)\rho_n(\zeta_2)\frac{1}{n}e^{-ny}$$
$$= -\ln(1-e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}), \tag{6.48}$$

and

$$F = \psi_0(\zeta_1)\psi_0(\zeta_2)\exp[-k^2g(\zeta_1,\zeta_1;0)]\exp[-k^2g(\zeta_2,\zeta_2;0)]\exp[2k^2g(\zeta_1,\zeta_2;y)](e^{y\alpha(s)}+e^{y\alpha(u)})$$
$$=\psi_0(\zeta_1)\psi_0(\zeta_2)(1-e^{-\zeta_1^2})^{k^2}(1-e^{-\zeta_2^2})^{k^2}(1-e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y})^{-2k^2}(e^{y\alpha(s)}+e^{y\alpha(u)}). \tag{6.49}$$

The summation over n is standard and the matrix element $\langle\zeta_2|e^{-yH_\lambda}|\zeta_1\rangle$ has been considered in Sec. IV. Here we use the operator relation

$$b_i = \left(\frac{\omega_0}{2}\right)^{1/2}\zeta_i + \left(\frac{1}{2\omega_0}\right)^{1/2}\frac{\partial}{\partial\zeta_i},$$
$$b_i^\dagger = \left(\frac{\omega_0}{2}\right)^{1/2}\zeta_i - \left(\frac{1}{2\omega_0}\right)^{1/2}\frac{\partial}{\partial\zeta_i}. \tag{6.50}$$

Then, using the differential equation (4.39) for $K_L(\zeta_1,\zeta_2;y)$ in the $\bar{B}K_LB$ term, we can make partial integration in y. In the terms $\bar{A}K_LB$ and $\bar{B}K_LA$ we make partial integrations in ζ_1 and ζ_2. Thus, we finally obtain the following expression:

$$\bar{A}K_LA=\frac{2e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}{(1-e^{-\zeta_1^2/2}e^{-\zeta_1^2/2}e^{-y})^2}FK_L(\zeta_1,\zeta_2;y)\,, \tag{6.51}$$

$$\bar{B}K_LA=\lambda e^{-\lambda\omega_0 y}\frac{e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}{(1-e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y})^2}(\zeta_1\cdot\zeta_2)FK_L(\zeta_1,\zeta_2;y)$$
$$\times\left[\left(1+\frac{2e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}{(1-e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y})}\right)-2k^2\left(\frac{e^{-\zeta_2^2}}{1-e^{-\zeta_2^2}}-\frac{e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}{1-e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}\right)\right], \tag{6.52}$$

$$\bar{A}K_LB+\bar{B}K_LB=2\lambda\zeta_1^2\frac{e^{-\zeta_1^2}}{(1-e^{-\zeta_1^2})^2}\psi_0(\zeta_1)\psi_0(\zeta_1)\delta^L(\zeta_1-\zeta_2)+\lambda e^{-\lambda\omega_0 y}\frac{e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}{(1-e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y})^2}(\zeta_1\cdot\zeta_2)K_L(\zeta_1,\zeta_2;y)$$
$$\times\left[\frac{\partial F}{\partial y}+2k^2F\left(\frac{e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}{1-e^{-\zeta_1^2/2}e^{-\zeta_2^2/2}e^{-y}}-\frac{e^{-\zeta_2^2}}{1-e^{-\zeta_2^2}}\right)\right]. \tag{6.53}$$

There are two contact-term contributions to be added, to the second-order electromagnetic interaction, and these are given by

$$\text{contact terms}=-2\left(\frac{2e^{-\zeta_1^2}}{1-e^{-\zeta_1^2}}\right)\psi_0(\zeta_1)\psi_0(\zeta_1)\delta^L(\zeta_1-\zeta_2)-2\lambda\zeta_1^2\frac{e^{-\zeta_1^2}}{(1-e^{-\zeta_1^2})^2}\psi_0(\zeta_1)\psi_0(\zeta_1)\delta^L(\zeta_1-\zeta_2)\,. \tag{6.54}$$

These two terms should be added to the two-current amplitude to obtain a gauge-invariant Compton amplitude. These only contribute to the real part of the amplitude and are actually canceled by the surface terms of $\bar{A}K_LA$ and $\bar{B}K_LB$ after the y partial integration.

Now the structure function $W_1^{(L)}(\nu,k^2)$ is defined as

$$W_1^{(L)}(\nu,k^2)=\frac{1}{\pi}\,\text{Im}\,T_1^{(L)}(\nu,k^2)\,. \tag{6.55}$$

In order to obtain the Bjorken scaling limit of $W_1^{(L)}$, we follow the similar steps to those prescribed in the derivation of $\nu W_2^{(L)}(\omega)$. Then we find that the leading term, corresponding to the scaling form, vanishes. The next leading term is equal to

$$2mW_1^{(L)}=\frac{1}{k^2}\frac{\omega_0^{L/2}}{\Gamma(\frac{1}{2}L)}\left(\frac{\omega-1}{\omega+1}\right)^{\omega_0}\left[\ln\left(\frac{\omega+1}{\omega-1}\right)\right]^{L/2-1}$$
$$\times\left[1+\tfrac{1}{4}\lambda L+\tfrac{1}{2}\lambda(\omega+2\lambda\omega_0-\omega_0)\ln\left(\frac{\omega+1}{\omega-1}\right)\right]. \tag{6.56}$$

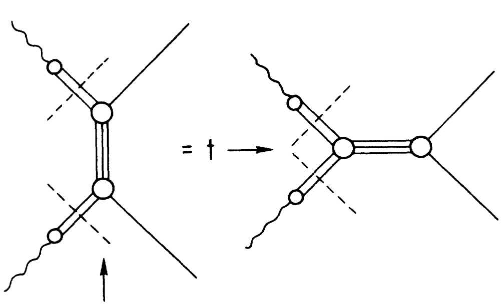

FIG. 2. Duality scheme on top of vector-meson poles in off-shell Compton amplitudes.

Therefore, our result for $W_1^{(L)}$ in the scaling limit is in contradiction with the experimental data from SLAC. The asymptotic behavior of the quantity R defined as

$$R=\sigma_s/\sigma_L$$

is similar to that predicted by the vector-dominance model.

However, we do not regard this nonscaling behavior of $W_1^{(L)}$ as a serious defect of the present model. We rather think that this is due to the fact that in our framework we have not really treated the target particle as a spin-$\frac{1}{2}$ object. A similar situation usually happens for a spin-0 target. For example, in the ladder model of the ϕ^3 theory allowing the vector-photon coupling, the calculation[21] shows that in the Bjorken limit νW_2 scales, whereas W_1 behaves like k^{-2} times a scaling function up

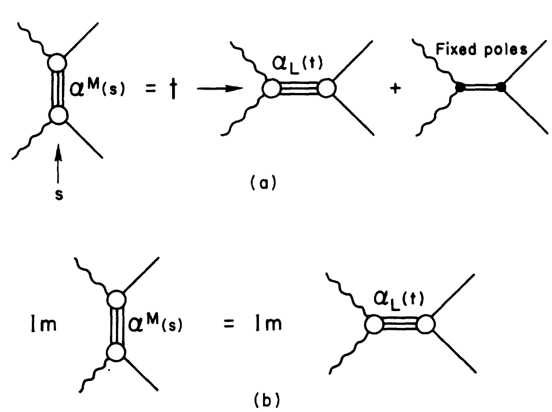

FIG. 3. (a) In the two-current amplitudes of the present model duality is minimally broken due to the presence of fixed poles; (b) duality in the local average sense is preserved for the imaginary parts of the amplitudes.

to logarithmic factors.

VII. DUALITY AND FACTORIZATION

Electromagnetic interaction does not necessarily require duality to hold in off-shell photon-hadron amplitudes. The minimal requirement there is that duality should hold for the on-mass-shell vector-meson–hadron vertices which are obtained on top of vector-meson poles in the off-shell photon line. This is what we have achieved in our model; it is schematically shown in Fig. 2 for the case of off-shell Compton amplitudes.

However, we point out that our electromagnetic vertex is already almost dual in its maximally possible extent as may be seen in the specific applications given in Sec. VI. The exact duality is only broken in the real part of the amplitude by the presence of fixed poles in the two-current channel, which is also the required property characteristic of any physical hadron-current amplitude. Figure 3 illustrates this.

First let us recall some of the formulas that we have obtained. The electromagnetic current is given by

$$j_\mu(k)=\frac{1}{2}\int\left\{\frac{\partial\bar{x}_\mu}{\partial\tau},e^{ik\bar{x}}\right\}_+d\tau, \tag{4.10}$$

where

$$\frac{\partial\bar{x}_\mu}{\partial\tau}=\frac{\partial x_{\mu,0}}{\partial\tau}+2\sum_{n=1}^{\infty}\rho_n(\zeta)\frac{\partial x_{\mu,n}}{\partial\tau}+2\sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\tau}x_{\mu,n} \tag{4.6}$$

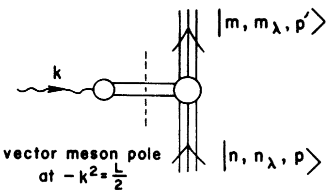

FIG. 4. Projection of the hadronic vertex out of the electromagnetic vertex through the vector-meson pole at $-k^2=\frac{1}{2}L$.

and

$$\rho_n(\zeta)=e^{-n\zeta^2/2}. \tag{4.3}$$

In the interaction representation we have the following expression for the current:

$$j_\mu(k)=j_\mu^{(0)}(k)+j_\mu^{(1)}(k)+j_\mu^{(2)}(k) \tag{4.27}$$

where

$$j_\mu^{(0)}(k)=\int\{p_{\mu,0},e^{ik\bar{x}}\}_+d\tau, \tag{4.28}$$

$$j_\mu^{(1)}(k)=\int\sum_{n=1}^{\infty}\{\rho_n(\zeta)p_{\mu,n},e^{ik\bar{x}}\}_+d\tau, \tag{4.29}$$

$$j_\mu^{(2)}(k)=\int\sum_{i=1}^{L}\left\{\lambda\eta_i,\sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\tau}x_{\mu,n}e^{ik\bar{x}}\right\}_+d\tau. \tag{4.30}$$

Now let us calculate the matrix elements of $j_\mu^{(i)}(k)$ ($i=0, 1, 2$) in the λ space:

$$\Gamma_\mu^{(0)}=\langle m_\lambda|j_\mu^{(0)}(k)|n_\lambda\rangle$$
$$=\int_{-\infty}^{\infty}d^L\zeta\{p_{\mu,0},:e^{ik\bar{X}(\zeta)}:\}_+(1-e^{-\zeta^2})^{k^2}\langle m_\lambda|\zeta\rangle\langle\zeta|n_\lambda\rangle, \tag{7.1}$$

$$\Gamma_\mu^{(1)}=\langle m_\lambda|j_\mu^{(1)}(k)|n_\lambda\rangle$$
$$=\int_{-\infty}^{\infty}d^L\zeta\sum_{n=1}^{\infty}\{\rho_n(\zeta)p_{\mu,n},:e^{ik\bar{X}(\zeta)}:\}_+(1-e^{-\zeta^2})^{k^2}\langle m_\lambda|\zeta\rangle\langle\zeta|n_\lambda\rangle, \tag{7.2}$$

$$\Gamma_\mu^{(2)}=\langle m_\lambda|j_\mu^{(2)}(k)|n_\lambda\rangle$$
$$=\int_{-\infty}^{\infty}d^L\zeta\sum_{i=1}^{L}\sum_{n=1}^{\infty}\frac{\partial\rho_n(\zeta)}{\partial\zeta_i}x_{\mu,n}:e^{ik\bar{X}(\zeta)}:(1-e^{-\zeta^2})^{k^2}\frac{\lambda}{i}\left[\langle m_\lambda|\zeta\rangle\left(\frac{\overrightarrow{\partial}}{\partial\zeta_i}-\frac{\overleftarrow{\partial}}{\partial\zeta_i}\right)\langle\zeta|n_\lambda\rangle\right], \tag{7.3}$$

where the normal-ordered form refers to a_n and $a_n^\dagger$, i.e.,

$$:e^{ik\bar{X}(\zeta)}:=\exp\left[ik\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^{1/2}\rho_n(\zeta)a_n^\dagger\right]\exp\left[ik\sum_{n=1}^{\infty}\left(\frac{2}{n}\right)^{1/2}\rho_n(\zeta)a_n\right]. \tag{7.4}$$

As a specific example, here we try to project out of the above expressions the vector-meson–hadron vertex corresponding to the lowest vector-meson pole at $-k^2=\frac{1}{2}L$ in the off-shell photon line (see Fig. 4) and study the duality and factorization properties. $\Gamma_\mu^{(0)}$ and $\Gamma_\mu^{(1)}$ simply give the following contributions at the pole:

$$\Gamma_\mu^{(0)}\underset{-k^2=L/2}{\simeq}\frac{1}{k^2+\frac{1}{2}L}\frac{\pi^{L/2}}{\Gamma(\frac{1}{2}L)}\{p_{\mu,0},:e^{ik\bar{X}(0)}:\}_+\langle m_\lambda|0\rangle\langle 0|n_\lambda\rangle, \tag{7.5}$$

$$\Gamma_\mu^{(1)} \underset{-k^2=L/2}{\simeq} \frac{1}{k^2+\frac{1}{2}L} \frac{\pi^{L/2}}{\Gamma(\frac{1}{2}L)} \sum_{n=1}^{\infty} \{p_{\mu,n}, : e^{ik\bar{X}(0)} : \}_+ \langle m_\lambda|0\rangle\langle 0|n_\lambda\rangle , \tag{7.6}$$

where

$$\bar{X}_\mu(0) = 2\sum_{n=1}^{\infty} x_{\mu,n} = \sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^{1/2} (a_{\mu,n} + a_{\mu,n}^\dagger) . \tag{7.7}$$

$\Gamma_\mu^{(2)}$ should be treated more carefully. Consider the divergence $ik_\mu\Gamma_\mu^{(2)}$ and pick up the vector-meson pole of that quantity. Then we get

$$\begin{aligned} ik_\mu\Gamma_\mu^{(2)} &= \frac{1}{2}\int_{-\infty}^{\infty} d^L\zeta \sum_{i=1}^{L}\sum_{n=1}^{\infty} \frac{\partial}{\partial\zeta_i} e^{ik\bar{X}(\zeta)} \frac{\lambda}{i}\left[\langle m_\lambda|\zeta\rangle\left(\frac{\overrightarrow{\partial}}{\partial\zeta_i} - \frac{\overleftarrow{\partial}}{\partial\zeta_i}\right)\langle\zeta|n_\lambda\rangle\right] \\ &= -\frac{1}{2}\int_{-\infty}^{\infty} d^L\zeta\, e^{ik\bar{X}(\zeta)}\frac{\lambda}{i}\left[\langle m_\lambda|\zeta\rangle\sum_{i=1}^{L}\left(\frac{\overrightarrow{\partial}^2}{\partial\zeta_i^{\,2}} - \frac{\overleftarrow{\partial}^2}{\partial\zeta_i^{\,2}}\right)\langle\zeta|n_\lambda\rangle\right] \\ &= -\int_{-\infty}^{\infty} d^L\zeta : e^{ik\bar{X}(\zeta)} : (1-e^{-\zeta^2})^{k^2}\frac{\lambda}{i}\left[\langle m_\lambda|\zeta\rangle(m_\lambda\omega_0 - n_\lambda\omega_0)\langle\zeta|n_\lambda\rangle\right] \\ &\underset{-k^2=L/2}{\simeq} \frac{1}{k^2+\frac{1}{2}L}\frac{\pi^{L/2}}{\Gamma(\frac{1}{2}L)} : e^{ik\bar{X}(0)} : i\lambda\omega_0(m_\lambda - n_\lambda)\langle m_\lambda|0\rangle\langle 0|n_\lambda\rangle . \end{aligned} \tag{7.8}$$

Recalling the mass-shell condition

$$-k^2 = \tfrac{1}{2}L \equiv m_V^{\,2} , \quad \alpha_L(m_V^{\,2}) = 1 \tag{7.9}$$

in the pole residue, we finally obtain

$$ik_\mu\Gamma_\mu^{(2)} \underset{-k^2=m_V^2}{\simeq} -ik_\mu \frac{1}{k^2+m_V^{\,2}} \frac{\pi^{L/2}}{\Gamma(\frac{1}{2}L)} \frac{k_\mu}{m_V^{\,2}} : e^{ik\bar{X}(0)} : \lambda\omega_0(m_\lambda - n_\lambda)\langle m_\lambda|0\rangle\langle 0|n_\lambda\rangle . \tag{7.10}$$

Therefore, totally we have

$$\Gamma_\mu \underset{-k^2=m_V^2}{\simeq} \frac{(\delta_{\mu\nu} + k_\mu k_\nu/m_V^{\,2})}{k^2+m_V^{\,2}} \langle m_\lambda| V_\nu(k)|n_\lambda\rangle , \tag{7.11}$$

where

$$\langle m_\lambda| V_\mu(k)|n_\lambda\rangle = \frac{\pi^{L/2}}{\Gamma(\frac{1}{2}L)} \left\{ p_{\mu,0} + \sum_{n=1}^{\infty} p_{\mu,n} - k_\mu \frac{\lambda\omega_0}{2m_V^{\,2}}(m_\lambda - n_\lambda), : e^{ik\bar{X}(0)} : \right\}_+ \langle m_\lambda|0\rangle\langle 0|n_\lambda\rangle . \tag{7.12}$$

The spin factor $\delta_{\mu\nu} + k_\mu k_\nu/m_V^{\,2}$ is explicitly shown in the vector-meson propagator where we have used the conservation of the vector current for on-mass-shell hadron states:

$$k_\mu\langle m_\lambda| V_\mu(k)|n_\lambda\rangle \equiv 0 . \tag{7.13}$$

From the expression Eq. (7.12) we can deduce the scalar-meson vertex corresponding to the mass-shell condition $\alpha_L(-k_S^{\,2}) = 0$, which is given by

$$\langle m_\lambda| V_S(k_S)|n_\lambda\rangle = g_S \frac{\pi^{L/2}}{\Gamma(\frac{1}{2}L)} : e^{ik_S\bar{X}(0)} : \langle m_\lambda|0\rangle\langle 0|n_\lambda\rangle , \tag{7.14}$$

with

$$\alpha_L(-k_S^{\,2}) = 0 . \tag{7.15}$$

g_S is an undetermined constant.

Instead of having these matrix representations of the scalar and vector vertices, it is possible to obtain the operator expressions for them. Let us recall the elementary relation of the generating function for the Hermite polynomial

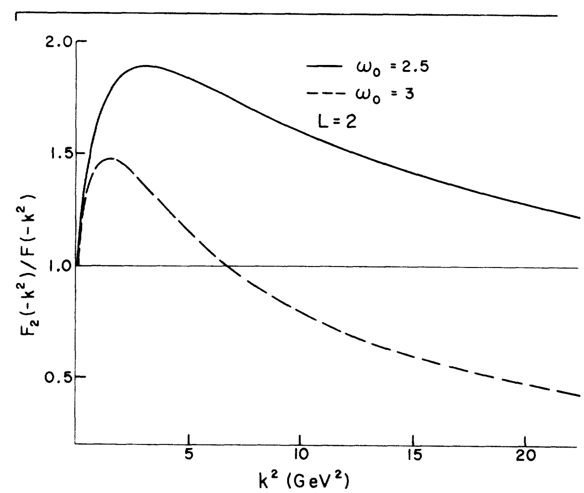

FIG. 5. The variation of our elastic form factor $F_2(-k^2)$ versus the dipole formula $F(-k^2) = (1 + k^2/0.71)^{-2}$.

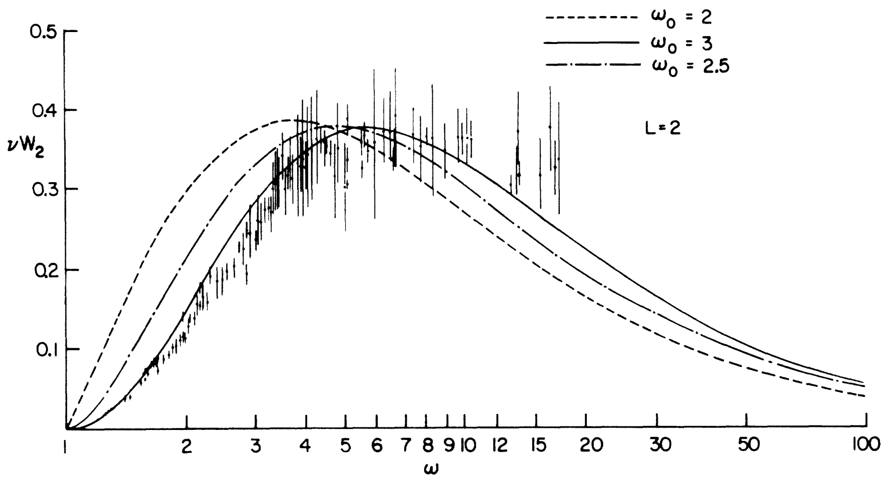

FIG. 6. Theoretical scaling curves of $\nu W_2^{(L)}(\omega)$ for $L=2$ versus SLAC data (Ref. 24) for different values of ω_0. The curves are saturated by the nondiffractive narrow-resonance contribution.

$$e^{2zt-t^2}=\sum_{n=0}^{\infty}\frac{H_n(z)}{n!}t^n. \tag{7.16}$$

Then we can immediately write down the following operator form for these hadronic vertices:

$$V_S(k_S)=g_S\frac{\omega_0{}^{L/2}}{\Gamma(\frac{1}{2}L)}:e^{ik_S\bar{X}(0)}:V_\lambda, \tag{7.17}$$

$$V_\mu(k)=\frac{\omega_0{}^{L/2}}{\Gamma(\frac{1}{2}L)}\left\{p_{\mu,0}+\sum_{n=1}^{\infty}p_{\mu,n}-\frac{k_\mu}{2m_V{}^2}[H_\lambda,V_\lambda],:e^{ik\bar{X}(0)}:\right\}_+, \tag{7.18}$$

where

$$V_\lambda=e^{-\frac{1}{2}b^\dagger\cdot b^\dagger}\Omega e^{-\frac{1}{2}b\cdot b}, \tag{7.19}$$

and

$$\Omega=|\text{vacuum}\rangle\langle\text{vacuum}|, \tag{7.20}$$

with

$$b\cdot b=\sum_{i=1}^{L}b_ib_i, \quad \text{etc.}$$

These vertices are dual and factorizable, and in the crossed channel generate the Regge pole $\alpha_L(t)$ with the quantized intercept and their daughters spaced by integers. The vector current is similar to that studied before in Ref. 22. It can be shown that a full N-point amplitude which is constructed out of these vertices satisfies SU(1, 1) invariance[23] under the specific conditions for the parameters:

$$2\lambda\omega_0=1, \tag{7.21}$$

$$\alpha(0)=\alpha_L(0), \tag{7.22}$$

which imply that all the Regge poles and their daughters are completely degenerate up to spacing by integers.

By considering higher moments in ξ in the integrands of Eqs. (7.1)–(7.3), one can similarly obtain the hadronic vertices for higher vector-meson poles.

VIII. CONCLUSION

In conclusion we would like to stress that *all our results* were rigorously derived from the Lagrangian of the model. We did not have to make any further assumption to arrive at a particular result. We find it interesting that we can exhibit explicitly many of the conjectured properties of hadrons out of our model.

To compare with experiment the internal symmetry should be incorporated in the model. We trust that our physical picture would underline these generalizations. As a guideline, we compare our theoretical curves with the experimental results,[24] in Fig. 5 the elastic form factor and in Fig. 6 the deep-inelastic structure function $\nu W_2^{(L)}(\omega)$ for $L=2$ for different values of ω_0.

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