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Triple-Regge Coupling and Diffractive Spin Dependence

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From qualitative arguments based on Regge behavior of three-to-three scattering amplitudes, taken together with the assumption that the vacuum trajectory function passes through unity and has a finite slope there, we argue that (1) the vertex function coupling three vacuum trajectories vanishes linearly at zero momentum transfer, and (2) a certain constraint equation equivalent to the vanishing of a single crossed-channel helicity amplitude connects the $j+1$ independent helicity amplitudes for forward diffractive scattering of a boson of spin $j \geq 1$. The appearance of certain spurious poles in the $3\to 3$ amplitude plays an important role in the discussion. We check our arguments in a simple Feynman-diagram model.

I. INTRODUCTION

Much interest in the Regge properties of three-to-three scattering amplitudes has been evident in the recent literature. This arises from Mueller's[1] discovery of a relation between a certain discontinuity of the forward amplitude and the cross section for a corresponding inclusive cross section.

Using a generalized form of unitarity, Mueller showed that

$$\mathrm{disc}_{M^2} F(s, M^2, t)/s \sim E_2 d\sigma/d\vec{p}_2, \tag{1.1}$$

where F is the connected amplitude for the forward three-body process shown in Fig. 1. The squared center-of-mass energy is

$$s = -(p_1 + p)^2, \tag{1.2}$$

the invariant momentum transfer squared to the measured particle is

$$t = -(p - p_2)^2, \tag{1.3}$$

and M is the "missing mass" defined by

$$M^2 = -(p + p_1 - p_2)^2. \tag{1.4}$$

The symbol disc_{M^2} denotes discontinuity with respect to M^2, and $E_2 d\sigma/d\vec{p}_2$ is the differential cross section for the inclusive reaction

$$p_1 + p \to p_2 + X. \tag{1.5}$$

Regge arguments[2] suggest that, in the limit $M^2 \to \infty$, $s/M^2 \to \infty$, $\mathrm{disc}_{M^2} F$ behaves like

$$A \equiv \mathrm{disc}_{M^2} F \sim \beta(t) \frac{|1 \pm e^{-i\pi\alpha(t)}|}{\sin^2 \pi\alpha(t)} (M^2)^{\alpha_v} \left(\frac{s}{M^2}\right)^{2\alpha(t)}, \tag{1.6}$$

where α_v is the vacuum trajectory function evaluated at zero momentum transfer and $\alpha(t)$ is the leading Regge trajectory function in the t channel. The coefficient which couples the trajectories is $\beta(t)$.

Poles in the t variable at integer values of $\alpha(t)$ are explicitly displayed via the factor $[\sin\pi\alpha(t)]^{-2}$. In the remaining coefficients, zeros may occur in the signature factor or in the coupling coefficient $\beta(t)$. One particular zero, it is well known, must occur[3]: If total cross sections approach constant values at large energies ($\alpha_v = 1$), if α_v has a finite slope at zero momentum transfer, and if the t-channel trajectory is itself the vacuum trajectory, so that at $t=0$, $\alpha(t) = \alpha_v = 1$, then $\beta \to 0$ as $t \to 0$.

How this zero comes about dynamically is not fully understood. One knows only that it must be present if $\alpha_v = 1$ is to be a consistent solution of the dynamical equations, whatever they may be.

In contemplating this and other related questions, it is useful to interrogate the Veneziano model for the six-point function.[4] One can explicitly compute there not only the discontinuity function $A(s, M^2, t)$ but the full six-point amplitude $F(s, M^2, t)$ itself. Several interesting features emerge from analysis of the Veneziano model as we shall now discuss.

Spurious t-variable singularities, for one thing, seem to occur in the full amplitude F: From Eq. (1.6) we see that F must have the form

$$F = \beta(t) \frac{|1 \pm e^{-i\pi\alpha(t)}|^2}{\sin\pi[2\alpha(t) - \alpha_v]} \frac{s^{2\alpha(t)}(-M^2)^{\alpha_v - 2\alpha(t)}}{\sin^2\pi\alpha(t)} + F' \tag{1.7}$$

where F' has no asymptotic discontinuity in M^2. Zeros of the sine functions in the denominator appear to produce unexpected poles in F at t values for which $2\alpha(t) - \alpha_v$ is an integer. The Veneziano model in fact produces an amplitude which conforms with Eq. (1.7), with its apparent poles. These poles must, of course, be spurious since there are no particle states with mass $\sqrt{t}$ corresponding to $2\alpha(t) - \alpha_v = \text{integer}$. We can imagine, in advance, that the spurious poles are canceled in either of two ways. Namely, either $\beta(t)$ has zeros at these points or else F' cancels the poles

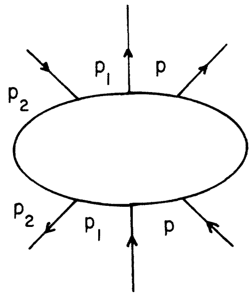

FIG. 1. Forward three-particle scattering process.

in the first term of Eq. (1.7).

We learn from the Veneziano model that both methods of cancellation occur there. The spurious poles at $2\alpha(t)-\alpha_v=1, 2, \ldots$ are canceled by zeros of $\beta(t)$, i.e., the absorptive amplitude A has zeros at these points. On the other hand, the poles at $2\alpha(t)-\alpha_v=0, -1, -2, \ldots$ are explicitly canceled by the "subtraction" function F', i.e., F' has the form

$$F' \sim \frac{s^{\alpha_v}}{2\alpha(t)-\alpha_v} f_1(t) + \frac{M^2}{s} \frac{s^{\alpha_v}}{2\alpha(t)-\alpha_v+1} f_2(t) + \cdots, \tag{1.8}$$

where the coefficient functions $f_1, f_2, \ldots$ are such that the spurious poles at $2\alpha(t)-\alpha_v=0, -1, -2, \ldots$ are exactly canceled. If this latter mechanism were also operative with respect to the spurious poles at $2\alpha(t)-\alpha_v=1, 2, \ldots$ we would need terms F' which behave like s^{α_v+1}/M^2, $s^{\alpha_v+2}/M^4, \ldots$. This would represent not only a highly non-Regge structure for the full three-particle amplitude, but a rather defective asymptotic expansion.[5] We do not expect canceling terms like $s^{2\alpha(t)}/M^2$, $s^{2\alpha(t)}/M^4, \ldots$ because the general Regge behavior for fixed s/M^2 is supposed to be $(M^2)^{\alpha_v} G(s/M^2, t)$. That is, as has been called to our attention by Professor M. L. Goldberger, we are *assuming* this form for the *full* 3-3 amplitude as well as for its absorptive part; and moreover, we are assuming that $G(s/M^2, t)$ is free of spurious poles as $s/M^2 \to \infty$. We are therefore inclined to believe that the occurrence of zeros in $\beta(t)$ at $2\alpha(t)-\alpha_v=1, 2, \ldots$ is a very general phenomenon that goes beyond the Veneziano model.

Related directly to this phenomenon of the zeros of $\beta(t)$ is the vanishing of the leading power of the inclusive cross section at $t=0$ for the case where all three trajectories joined by the triple-Regge vertex pass through $\alpha=1$ at zero momentum transfer.

On the other hand, the elimination of the spurious poles at $2\alpha(t)-\alpha_v=0, -1, -2, \ldots$ must occur, in our view, by the subtraction mechanism, since otherwise $\beta(t)$ would vanish in the physical region $(t<0)$ and we would be confronted by negative cross sections (barring the appearance of double zeros). It appears, then, that the Veneziano model deals with the spurious poles in the most reasonable of ways.

Nothing in the above discussion, nor in the details of the Veneziano model, gives any hint of the mechanism whereby a more general model (e.g., the real world) will produce the expected vanishing of the triple-Regge vertex function $\beta(t)$ at $2\alpha(t)-\alpha_v=1, 2, \ldots$ and the pole-canceling subtraction effect at $2\alpha(t)-\alpha_v=0, -1, -2, \ldots$. Since the $3\to 3$ amplitude is compounded of $2\to 2$ amplitudes as well as the 3-3 parts, it could be that the success of the subtraction mechanism implies precise relations between multi-Regge coefficients and $2\to 2$ amplitude parameters (e.g., coupling constants). In order to study this question, we have performed a simple model calculation based on a single type of multiladder Feynman diagram, shown in Fig. 2.[6] The discontinuity function $A(s, M^2, t)$ is computed in Sec. II. The results are consistent with our hypothesis: $\beta(t)=0$ for $2\alpha(t)-\alpha_v=1, 2, \ldots$. On the other hand, the spurious singularities at $2\alpha(t)-\alpha_v=0, -1, -2, \ldots$ are canceled by naturally occurring subtraction terms in F', without any need to call upon other diagrams and hence without any implication of coupling constant or other parametric identities.

Note added in proof. It has been pointed out to us by A. H. Mueller and T. L. Trueman, and also by M. A. Virasoro, that our general conclusion on the vanishing of the triple Pomeranchukon vertex at $t=0$ ceases to be valid for nonplanar diagrams (analogous to third double spectral-function effects in two-body processes). In this case, spurious singularities at wrong (triple Regge) signature points *can* be present in right- or left-hand M^2 cuts separately, and are canceled when the two cuts are added. This has been verified in a model by A. H. Mueller and T. L. Trueman, and independently by one of us (D.G.).

Going beyond the specifics of our model, we consider in Sec. III the implications of the presumed vanishing of the triple-Regge function $\beta(t)$ in the

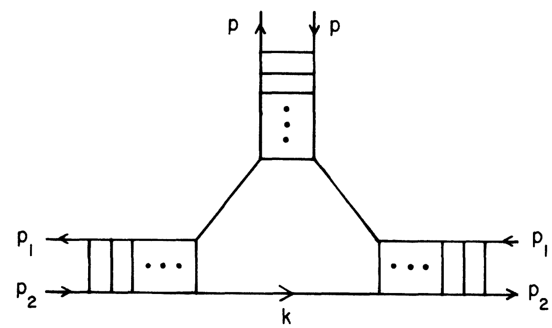

FIG. 2. Model Feynman diagram.

timelike region ($t>0$) at the points $2\alpha(t)-\alpha_v=1, 2, \ldots,$ in particular, with $\alpha_v=1$, at the boson-particle points $\alpha(t)=1, 2, \ldots .$ At such points, we are in effect considering the forward elastic scattering on a spinless target of a particle of mass $\sqrt{t}$ and spin $j=\alpha(t)=1, 2, \ldots .$ Let f_m^j be the forward amplitude for s-channel helicity m (which is of course conserved). We learn for the vacuum-trajectory contribution to the f_m^j that

$$\sum_{m=-j}^{j}\frac{(-1)^m}{(j-m)!(j+m)!}f_m^j=0\,. \tag{1.9}$$

That is, at zero momentum transfer, the combination of s-channel helicity amplitudes indicated above decouples from the vacuum trajectory. For the special case of vector particles ($j=1$) this result implies that the amplitudes are independent of helicity in the high-energy limit where the vacuum trajectory dominates. For particles of higher spin ($j>1$), helicity independence is not necessarily required by Eq. (1.9) but is *consistent* with it,[5] as we see from the identity

$$\sum_{m=-j}^{j}\frac{(-1)^m}{(j-m)!(j+m)!}=0\,.$$

It may also be remarked that Eq. (1.9) is equivalent, via well-known crossing relations,[7] to vanishing in the crossed ($\bar{p}p$) channel of the amplitude corresponding to *maximum* helicity difference ($\lambda-\bar{\lambda}=\pm 2j$).[8] This result in fact also follows directly from the interpretation of Eq. (1.6) in terms of helicity poles.[9]

II. THE MODEL

Our model is based on the diagram of Fig. 2, where the boxes describe off-mass-shell $2\to 2$ amplitudes for which we adopt the Regge properties corresponding to ladder graphs. The Regge trajectory function associated with momentum transfer t is $\alpha(t)$ and the unsignatured amplitude associated with the corresponding two boxes is

$$R_t=\frac{-[(k-p_2)^2]^{\alpha(t)}}{\sin\pi\alpha(t)}\beta_t(k^2, (k+p_1-p_2)^2, t)\,. \tag{2.1}$$

The trajectory function associated with momentum transfer zero is α_v and the amplitude associated with the corresponding box is

$$R_v=\frac{-[(k+p+p_1-p_2)^2]^{\alpha_v}}{\sin\pi\alpha_v}\beta_v((k+p_1-p_2)^2, (k+p_1-p_2)^2)\,. \tag{2.2}$$

For the moment we are regarding α_v and $\alpha(t)$ as adjustable, so, e.g., α_v has not yet been set equal to unity. But we are already anticipating, for $M^2\to\infty$, $s/M^2\to\infty$, that R_t and R_v can be approximated by their high-energy forms. In Eqs. (2.1) and (2.2) we have suppressed the factorized dependence on external masses.

The forward $3\to 3$ amplitude is given by

$$F=\int\frac{d^4k}{(2\pi)^4 i}R_t^2R_v\frac{1}{[(k+p_1-p_2)^2+\mu_0^2]^2}\frac{1}{k^2+\mu_0^2}\,. \tag{2.3}$$

In order to carry out the k integration, it is convenient to use a spectral representation for the integrand based on the identity

$$\frac{-(-s)^\alpha}{\sin\pi\alpha}=\frac{1}{\pi}\int_0^\infty ds'\frac{s'^\alpha}{s'-s-i\epsilon}, \quad -1<\alpha<0\,. \tag{2.4}$$

For α_v and $2\alpha(t)$ in the interval -1 to 0 we therefore have

$$R_t^2R_v\frac{1}{[(k+p_1-p_2)^2+\mu_0^2]^2}\frac{1}{k^2+\mu_0^2}=-\frac{\sin 2\pi\alpha(t)}{\sin^2\pi\alpha(t)}\frac{1}{\pi^2}\int d\mu_1^2d\mu^2dm_1^2dm_2^2\rho(\mu_1^2, \mu^2, t)$$
$$\times\frac{(m_1^2)^{2\alpha(t)}(m_2^2)^{\alpha_v}}{[(k+p_1-p_2)^2+\mu^2]^2(k^2+\mu_1^2)[m_1^2+(p_2-k)^2][m_2^2+(p+p_1-p_2+k)^2]}\,. \tag{2.5}$$

The double spectral function $\rho(\mu_1^2, \mu^2, t)$ absorbs the free propagator functions and provides for off-mass-shell behavior of the residue functions β_t and β_v. We may note here that any falloff of these functions for large values of the off-shell mass variables will reflect itself in superconvergence for the spectral function ρ. This is central to our main result, and we shall return to it at the appropriate time. Concerning the representation of Eq. (2.4), we emphasize that it holds only for $-1<\alpha<0$. Later on, we will want to consider the situation for $\alpha>0$, which we reach by analytic continuation after the k-space integration has been carried out. On the other hand continuation to α_v or $2\alpha(t)<-1$ is not allowed, since in this regime the amplitude is no longer controlled by the high-energy properties of the Regge boxes.

With integrand given by Eq. (2.5), the k-space integration of Eq. (2.3) is standard. It is equivalent to the integration associated with the square-box diagram of Fig. 3. It will be convenient to work in the totally spacelike region where p_1^2, p_2^2, p^2 and

$$\bar{M}^2 \equiv -M^2, \quad \bar{s} \equiv -s, \quad \bar{t} \equiv -t \tag{2.6}$$

are all positive. We later continue to the appropriate physical region. The amplitude $F(s, M^2, t)$ is given by

$$F = -\frac{\cot\pi\alpha(t)}{(2\pi)^4}\int d\mu_1^2 d\mu^2 dm_1^2 dm_2^2 \rho(\mu_1^2, \mu^2, t)(m_1^2)^{2\alpha(t)}(m_2^2)^{\alpha_v}\int \prod_{i=1}^{4} dx_i \frac{2x_4}{\bar{D}^3}\delta(1-\sum x_i), \tag{2.7}$$

where

$$\bar{D} = \bar{M}^2 x_2 x_3 + \bar{s} x_1 x_2 + \bar{t} x_3 x_4 + p_1^2 x_1 x_4 + p_2^2 x_1 x_3 + p^2 x_2 x_4 + m_1^2 x_1 + m_2^2 x_2 + \mu_1^2 x_3 + \mu^2 x_4 . \tag{2.8}$$

Carrying out the integrations over m_1^2 and m_2^2 we find

$$F = \frac{-2\cot\pi\alpha(t)}{(2\pi)^4}\Gamma(\alpha_v + 1)\Gamma(2\alpha(t)+1)\Gamma(1-\alpha_v - 2\alpha(t))$$
$$\times \int d\mu_1^2 d\mu^2 \rho(\mu_1^2, \mu^2, t)\int \prod_i dx_i \delta(1-\sum x_i)\frac{x_4 x_1^{-1-2\alpha(t)} x_2^{-1-\alpha_v}}{D^{1-2\alpha(t)-\alpha_v}}, \tag{2.9}$$

where

$$D = \bar{M}^2 x_2 x_3 + \bar{s} x_1 x_2 + \bar{t} x_3 x_4 + p_1^2 x_1 x_4 + p_2^2 x_1 x_3 + p^2 x_2 x_4 + \mu_1^2 x_3 + \mu^2 x_4 . \tag{2.10}$$

Let us now compute the M^2 discontinuity:

$$A = \frac{1}{2i}[F(M^2 + i\epsilon) - F(M^2 - i\epsilon)]. \tag{2.11}$$

The discontinuity of $D^{-\gamma}$ is given by

$$\mathrm{disc}_{M^2}\int \frac{1}{D^\gamma} = \int \frac{\sin\pi\gamma\Theta(-D)}{(-D)^\gamma}, \tag{2.12}$$

where the integral symbol here includes all the numerator complications of Eq. (2.9) and $\gamma = 1 - 2\alpha(t) - \alpha_v$. When $\gamma > 1$ Eq. (2.12) implies an integral to within ϵ of the point where D vanishes, plus a semicircular contour of radius ϵ which cancels the singularity as $\epsilon \to 0$. The analytic continuation to $\gamma < 1$ omits the semicircular contribution. We now suppose that $2\alpha(t) + \alpha_v$ has been continued to positive values, so that Eq. (2.12) has no contribution from the tiny semicircle. Let us next study the denominator function of Eq. (2.10) with $\bar{M}^2$ replaced by $-M^2$. Introduce the new variables u and z, defined by

$$x_1 = uz, \quad x_3 = u(1-z), \tag{2.13}$$

and observe that

$$\int dx_1 dx_3 \to \int u\,du \int dz . \tag{2.14}$$

Then

$$-D = C - Bz + u^2 z^2 p_2^2, \tag{2.15}$$

where

$$C = M^2 x_2 u - (\bar{t} x_4 u + p^2 x_2 x_4 + \mu_1^2 u + \mu^2 x_4), \tag{2.16}$$

$$B = M^2 x_2 u + \bar{s} x_2 u - \bar{t} x_4 u + p_1^2 u x_4 - \mu_1^2 u + p_2^2 u^2 . \tag{2.17}$$

With respect to the variable z, $-D$ has roots z_+ and z_- given by

$$z_\pm = \frac{B \pm (B^2 - 4p_2^2 u^2 C)^{1/2}}{2p_2^2 u^2}, \tag{2.18}$$

so we write

$$-D = p_2^2 u^2 (z - z_+)(z - z_-). \tag{2.19}$$

We take p_2^2 to be positive and not too large ($p_2^2 < \frac{1}{3}\mu_1^2$), later continuing in p_2^2 to its (negative) physical value. It can then be established that $z_+ > 1$ and $0 < z_- < 1$, so that the z integration runs over the interval $0 < z < z_-$. For the discontinuity function A we now find

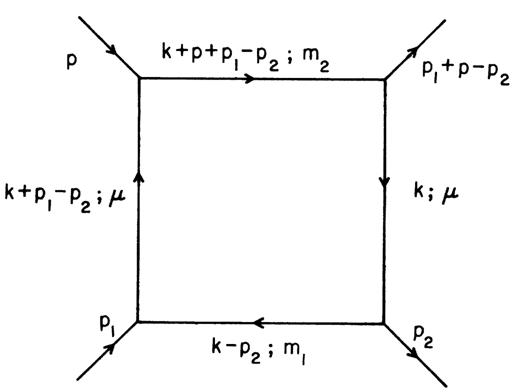

FIG. 3. Equivalent square-box diagram; see text.

$$A = G(t)\int d\mu_1^2 d\mu^2 \rho(\mu_1^2, \mu^2, t)\int u\,du\, x_4 dx_4 dx_2\, x_2^{-1-\alpha_v} u^{-1-2\alpha(t)}\delta(u + x_2 + x_4 - 1) \times \Theta(M^2 x_2 u - \bar{t}x_4 u - p^2 x_2 x_4 - \mu^2 x_4 - \mu_1^2 u) I\,, \tag{2.20}$$

where

$$G(t) = \frac{-2}{(2\pi)^4}\cot\pi\alpha(t)\sin\pi[2\alpha(t) + \alpha_v]\Gamma(\alpha_v + 1)\Gamma(2\alpha(t) + 1)\Gamma(1 - \alpha_v - 2\alpha(t)), \tag{2.21}$$

and

$$I = \int_0^{z_-} dz \frac{z^{-1-2\alpha(t)}}{[p_2^2 u^2 (z_+ - z)(z_- - z)]^{1-2\alpha(t)-\alpha_v}}. \tag{2.22}$$

Let us now pass to the first of our limits, $s/M^2 \to \infty$. The Θ function in Eq. (2.20) ensures that $p_2^2 u^2 (z_+ - z)$ is well approximated by $\bar{s}x_2 u$ and z_- by $C/x_2 u\bar{s}$. With these approximations the z integration is easily carried out, and we find

$$I \to \frac{(x_2 u\bar{s})^{2\alpha(t)}}{C^{1-\alpha_v}}\,\frac{\Gamma(-2\alpha(t))\Gamma(2\alpha(t) + \alpha_v)}{\Gamma(\alpha_v)}. \tag{2.23}$$

Finally, using the identity

$$\Gamma(a)\Gamma(1 - a) = \frac{\pi}{\sin\pi a},$$

we find

$$A \xrightarrow[s/M^2 \to \infty]{} \frac{\alpha_v(\bar{s})^{2\alpha(t)}}{16\pi^2 \sin^2\pi\alpha(t)}\int d\mu_1^2 d\mu^2 \rho \int dx_2 dx_4\, x_2^{-\alpha_v + 2\alpha(t) - 1} x_4 \int du\, \delta(u + x_2 + x_4 - 1)\Theta(C) C^{-1+\alpha_v}, \tag{2.24}$$

where, as before,

$$C = M^2 x_2 u - \bar{t}x_4 u - p^2 x_2 x_4 - \mu^2 x_4 - \mu_1^2 u > 0\,. \tag{2.25}$$

We next want to pass to the large-M^2 limit. With respect to the x_2 integration, it appears that the contributions from small x_2, namely $x_2 u \sim 1/M^2$, produce an M^2 dependence expressed by $(M^2)^{\alpha_v - 2\alpha(t)}$. The contributions from x_2 of order unity produce a dependence $(uM^2)^{\alpha_v - 1}$ and since we have continued to $\alpha_v > 0$ this term will be convergent for $u \to 0$. Notice however that the coefficient of the $(M^2)^{\alpha_v - 1}$ term is independent of the variables μ_1^2 and μ^2 and, upon integration over μ_1^2 and μ^2 the term would vanish if the following superconvergence relation obtains:

$$\int \rho(\mu_1^2, \mu^2, t) d\mu_1^2 d\mu^2 = 0\,. \tag{2.26}$$

In the absence of this superconvergence, we would be confronted with a crossover phenomenon at $2\alpha(t) = 1$, the asymptotic M^2 dependence being $(M^2)^{\alpha_v - 2\alpha(t)}$ for $2\alpha(t) < 1$, as expected, and $(M^2)^{\alpha_v - 1}$ for $2\alpha(t) > 1$, an unexpected result. Indeed, in continuing to still larger values of $\alpha(t)$ we would encounter further crossover phenomena at $2\alpha(t) = 2, 3, \ldots,$ unless the superconvergence relations keep pace, e.g., for t such that $2\alpha(t) = 2$, unless

$$\int \rho\mu^2 d\mu^2 d\mu_1^2 = 0\,, \quad 2\alpha(t) = 2, \qquad \int \rho\mu_1^2 d\mu^2 d\mu_1^2 = 0\,, \tag{2.27}$$

and so on.

The presumed absence of crossovers suggests that the superconvergence relations do indeed keep pace. One can try to justify this assumption by appealing to dynamical models of Regge behavior. At issue is the behavior of the residue functions β_t and β_v of Eqs. (2.1) and (2.2) in their dependence on off-shell mass variables. For any such residue functions, call it r, let k_1^2 and k_2^2 be the off-shell masses, and let α be the trajectory function. In the ladder approximation to ϕ^3 theory, for example, one finds for large masses that[10]

$$r(k_1^2, k_2^2) \sim \left(\frac{i}{k_{\max}^2}\right)^{\alpha+1}, \tag{2.28}$$

where $k_{\max}^2$ is the larger of the two masses. Since in other respects our model is essentially based on the ϕ^3 theory, we shall adopt this off-mass-shell behavior, although it is in fact stronger than we need. We are then led to the following superconvergence relations.

(1) Let n be an *integer* satisfying

$$n < 2\alpha(t) + \alpha_v + 3\,. \tag{2.29}$$

Then

$$\int d\mu^2 \rho(\mu_1{}^2, \mu^2, t)(\mu^2)^n = 0 . \tag{2.30}$$

(2) Let m be an integer *satisfying*

$$m < 2\alpha(t) + 2 . \tag{2.31}$$

Then

$$\int d\mu_1{}^2 \rho(\mu_1{}^2, \mu^2, t)(\mu_1{}^2)^m = 0 . \tag{2.32}$$

Returning to Eq. (2.24) we now see that the large-M^2 behavior is determined, thanks to the superconvergence, by the behavior of the integrand for small x_2. Hence in the δ function we can set $x_2 = 0$ and similarly in Eq. (2.25) we set $x_2 = 0$ except in the term $M^2 x_2 u$. The upper limit on the x_2 integration can now be extended to infinity and the x_2 integration becomes elementary. We find

$$A \underset{s/M^2\to\infty;\ M^2\to\infty}{\sim} \frac{(M^2)^{\alpha_v}(\bar{s}/M^2)^{2\alpha(t)}}{16\pi^2 \sin^2 \pi\alpha_t} \frac{\Gamma(\alpha_v+1)\Gamma(1-2\alpha(t))}{\Gamma(1+\alpha_v-2\alpha(t))}$$
$$\times \int d\mu_1{}^2 d\mu^2 \rho(\mu_1{}^2, \mu^2, t) \int du\, x_4 dx_4 \delta(u + x_4 - 1) u^{\alpha_v - 2\alpha(t)} [\bar{t} x_4 u + \mu^2 x_4 + \mu_1{}^2 u]^{2\alpha(t)-1} . \tag{2.33}$$

We see a remaining hint of the crossover danger in the factor $\Gamma(1-2\alpha(t))$. However, at $2\alpha(t) = 1$ the integrand is independent of μ^2 and $\mu_1{}^2$ and superconvergence eliminates the pole; similarly, the poles in $\Gamma(1-2\alpha(t))$ at $2\alpha(t) = 2, 3, \ldots$ are eliminated by superconvergence.

Equation (2.33) shows the promised zeros at $2\alpha(t) - \alpha_v = 1, 2, \ldots$; these arise from the denominator factor $\Gamma(1+\alpha_v - 2\alpha(t))$. To be sure, at $2\alpha(t) - \alpha_v = 1, 2, \ldots$ the u integration diverges as $u \to 0$ and this would appear to cancel the above zeros. But at $u \to 0$ superconvergence with respect to $\mu_1{}^2$ causes the integral to vanish and the zeros are reinstated.

We conclude this section with a few comments.

The triple-Regge vertex function $\beta(t)$ has zeros, as we have seen, at the points $2\alpha(t) - \alpha_v = 1, 2, \ldots$. In particular, if $\alpha_v = 1$ there is a zero at $\alpha(t) = 1$, i.e., the triple vacuum-trajectory coupling vanishes at zero momentum transfer. It is of course well known, for $\alpha_v = 1$, that such a vanishing is required for consistency with the constancy of asymptotic total cross sections. However, our model has not in any obvious way taken this consistency requirement into account. Moreover, the consistency condition can be satisfied by a zero of any order, whereas in the model we have found that the zero is a simple one in $\alpha(t)$. The same feature arises in the Veneziano model, where the results closely resemble what has been found here. On the other hand, the vanishing of the triple vacuum-trajectory coupling (with $\alpha_v = 1$) does *not* seem to emerge from Amati-Fubini-Stanghellini (AFS) type of model if one computes the discontinuity function $A(s, M^2, t)$ directly, in the manner indicated in Fig. 4, where the cut is along the wavy line.[11] We believe that this situation resembles the one that arises in connection with the AFS[12] vs Mandelstam[13] Regge-cut controversy. The diagram in Fig. 4, if it is treated as a Feynman diagram, has many M^2-discontinuity contributions other than the one symbolized by the wavy line. Consistency seems to require that these be taken into account since, with $\alpha_v = 1$, the zero must occur. We leave it to the reader to deal with this confusion as best he can.

III. COUPLING OF PARTICLES TO THE VACUUM TRAJECTORY

The vanishing of the triple-Regge vertex function at $2\alpha(t) - \alpha_v = 1, 2, \ldots$ has interesting implications for the coupling of the vacuum trajectory to physical particles with integer spin greater than zero, i.e., for the particle-particle-Pomeranchukon vertex at zero momentum transfer.[5] We are assuming, of course, that the vacuum trajectory passes through unity at zero momentum transfer. To see how this goes, let us continue our discontinuity function $A(s, M^2, t)$ to positive values of t, a con-

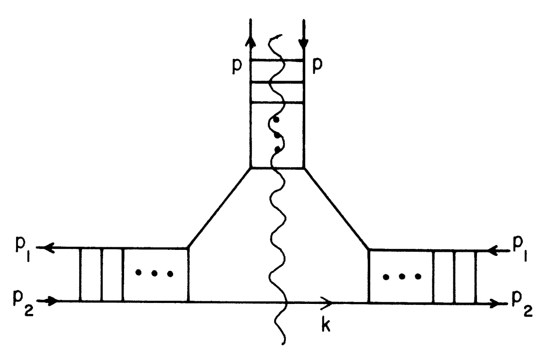

FIG. 4. The wavy line symbolizes one of the contributions to the M^2 discontinuity.

tinuation which we have already anticipated and discussed in Sec. II. In this region, we are describing forward scattering on a spinless target of momentum p of a diparticle (p_1, p_2) whose invariant mass is $\sqrt{t}$. The s-channel energy variable is now M^2, and we are in the region where M^2 is large. Let θ and ϕ be the polar and azimuthal angles which describe the orientation of the vector $\vec{p}_1 - \vec{p}_2$ in the rest frame of the diparticle, with z axis along the vector $\vec{p}$ in that frame. The amplitude can be written in the form

$$\mathcal{A} = \sum_{j', j'', m} Y_{j'm}(\theta, \phi) Y^*_{j''m}(\theta, \phi) f^{j', j''}_{m\to m}(\alpha(t), M^2). \tag{3.1}$$

We now continue in t to the *particle* pole corresponding to $\alpha(t) = j$. We observe that the asymptotic variable s/M^2 is linear in $z = \cos\theta$. Thus, at the pole, and for $z \to \infty$, we find

$$\mathcal{A} \sim C^j \sum_m \frac{(-1)^m}{(j-m)!(j+m)!} f^j_{m\to m}(M^2) z^{2j}. \tag{3.2}$$

The factor $(-1)^m$ arises from the fact that $\sin^2\theta \to -z^2$ for large z. In the above expression C^j absorbs m-independent factors. At the pole, we are describing forward elastic scattering of a particle of spin j on a spinless target. In the direct channel (s channel) $f^j_{m\to m}$ is the helicity amplitude for helicity m. We have argued, however, that the amplitude vanishes for $2\alpha_t - \alpha_v = 1, 2, \ldots,$ so with $\alpha_v = 1$, $\alpha(t) = j$, we argue that the amplitude vanishes for spin j integral and larger than zero. Thus, for $j \geq 1$ we have

$$\sum_{m=-j}^{j} \frac{(-1)^m}{(j-m)!(j+m)!} f^j_{m\to m} \to 0, \quad M^2 \to \infty. \tag{3.3}$$

This is the result which was announced in the Introduction and discussed in some detail there.

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[1]A. H. Mueller, Phys. Rev. D 2, 2963 (1970). Also see H. P. Stapp, *ibid.* 3, 3177 (1971), and C.-I Tan, *ibid.* 4, 2412 (1971).

[2]C. E. DeTar, C. E. Jones, F. E. Low, C.-I Tan, J. H. Weis, and J. E. Young, Phys. Rev. Letters 26, 675 (1971); D. Silverman and C.-I Tan, Nuovo Cimento 2A, 489 (1971) and Phys. Rev. D 3, 991 (1971); H. D. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Letters 26, 937 (1971) and Phys. Rev. D 4, 2988 (1971); N. F. Bali, A. Pignotti, and D. Steele, *ibid.* 3, 1167 (1971). Silverman *et al.*, Abaranel *et al.*, and Bali *et al.* discuss this limit from the point of view of the multiperipheral model.

[3]V. N. Gribov and A. A. Migdal, Yad. Fiz. 8, 1002, 1213 (1968) [Sov. J. Nucl. Phys. 8, 583, 783 (1969)]; C. E. DeTar, D. Z. Freedman, and G. Veneziano, Phys. Rev. D 4, 906 (1971); G. Veneziano, invited talk at the International Conference on Duality and Symmetry in Hadron Physics, Tel-Aviv, Israel, 1971 (unpublished); Abarbanel *et al.* (Ref. 2).

[4]L. N. Chang, P. G. O. Freund, and Y. Nambu, Phys. Rev. Letters 24, 628 (1970); D. Gordon and G. Veneziano, Phys. Rev. D 3, 2116 (1971); M. A. Virasoro, *ibid.* 3, 2843 (1971); C. E. DeTar, K. Kang, C.-I Tan, and J. H. Weis, *ibid.* 4, 425 (1971); R. C. Brower and R. E. Waltz, CERN Report No. CERN-TH-1335, 1971 (unpublished).

[5]Many of the points taken up in the present paper have been discussed also by C. E. DeTar and J. H. Weis [Phys. Rev. D 4, 3141 (1971)] who reach similar conclusions, in particular concerning the helicity structure of particle-particle-Pomeranchukon vertices. Their approach places special emphasis on the question of nonsense zeros in these vertices. In their approach the unwanted sequence s^{α_v+1}/M^2, s^{α_v+2}/M^4 would arise from nonsense-eliminating fixed poles at $J = 1, 2, \ldots$. We may also recall the important theorem of Y. Hara, Phys. Letters 23, 696 (1966). He shows that the particle-particle-Pomeranchukon vertices at zero momentum transfer are totally helicity-independent, provided that the vacuum trajectory chooses sense at nonsense points.

[6]D. Dorren [Weizmann Institute of Science report, 1971 (unpublished)] has performed a weak-coupling (all α's ~ -1) calculation of this diagram. Our purposes require a calculation which permits analytic continuation to large values of the trajectories.

[7]T. L. Trueman and G. C. Wick, Ann. Phys. (N.Y.) 26, 332 (1964); I. Muzinich, J. Math. Phys. 5, 1481 (1964).

[8]This vanishing, in the context of the dual-resonance model has been noted by M. A. Virasoro and C. B. Thorn, private communication.

[9]C. E. Jones, F. E. Low, and J. E. Young, Phys. Rev. D 4, 2358 (1971); C. E. DeTar (Ref. 2).

[10]G. Tiktopoulos and S. B. Treiman, Phys. Rev. 135, B711, (1964); 136, B1217 (1964).

[11]H. D. Abarbanel *et al.* (Ref. 2).

[12]D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962).

[13]S. Mandelstam, Nuovo Cimento 30, 1128 (1963).