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Theory of a Fixed-Pole Pomeranchukon Bootstrap*†

Richard C. Arnold
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

and

John Steinhoff‡
Department of Physics and Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois 60637 and High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
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A self-consistent model of asymptotic high-energy hadron-hadron scattering determining the t dependence at small angles is formulated and compared with available experimental results. This model takes the Pomeranchuk singularity as a fixed pole, self-generating through unitarity in the s channel. The production mechanisms assumed to be most important in the elastic-scattering unitarity sum are those involving essentially the Pomeranchukon; they are "diffraction dissociation," or "strong bremsstrahlung" with the pion (as the lightest hadron) mass determining the scale for t dependences. In addition to pion propagators, a form factor which is required in the production model may be determined self-consistently by postulating a certain universality and the Chou-Yang hypothesis. Specific predictions for diffraction-peak widths at asymptotic energies are given and are in qualitative agreement with presently available data.

I. INTRODUCTION AND PHYSICAL FORMULATION OF MODEL

There are many competing theoretical schemes for describing or predicting the small-angle elastic scattering amplitudes of hadrons at asymptotically high energies. Such asymptotic behavior is conventionally ascribed to the Pomeranchuk singularity in l, the complex angular momentum variable appropriate to the t channel. The simplest interpretation of total cross-section data, i.e., constant behavior as $s\to\infty$, would specify that this singularity be a simple pole located at $l=1$ for $t=0$. Similarly, the most naive interpretation of πN elastic scattering data between 15 and 25 GeV, where the forward peak seems to have a t dependence unchanging with s, would specify that this pole be fixed at $l=1$, at least for $-1<t<0$ GeV^2.

There are well-known theoretical complications[1] with a simple picture wherein the scattering amplitudes are dominated by such a pole if continuation to physical thresholds in t, such as $4\mu_\pi^2$, is performed. It is necessary that other singularities (moving cuts) be present to avoid contradictions with t-channel unitarity. However, it is completely consistent from a theoretical viewpoint to neglect such cuts compared to the Pomeranchukon pole for $t\leq 0$. Thus, if one has a dynamical scheme involving only $t\leq 0$ for $s\to\infty$, it is presumably allowable to consider only the single fixed-pole singularity. This is our viewpoint in what is to follow.

Calculation of the residue of the Pomeranchukon, $\beta(t)$, is achieved through the s-channel unitarity condition. We assume the unitarity sum is saturated by many-particle intermediate states, and adopt a model for the production matrix elements which contains the off-shell high-energy elastic scattering amplitude A. Equating the sum to ImA, where the form $A = is\,\beta(t)$, $\beta(t)$ real, is assumed as $s \to \infty$, and assuming that A is slowly varying when continued off the mass shell,[2] we obtain the bootstrap (nonlinear self-consistency) equation.

Specifically, for simplicity we consider $\pi\pi$ scattering. Our production model is described as "diffraction dissociation"[3] or "strong bremsstrahlung,"[4] and is pictorially represented in Fig. 1. In the model, each of the produced particles is emitted with a small transverse momentum, such as in the multiperipheral chains of Fig. 1, where each of the squares of invariant momentum transfer $(\mathrm{IMT})^2$, t_i $(i \geq 2)$, on the chain is strongly damped, and the propagators decrease at high subenergy compared to A. These last assumptions make it plausible that the predominant contribution to the unitarity sum comes from those regions of phase space where the squares of invariant mass $(\mathrm{IM})^2$, s_2, $\bar{s}_2$, of the groups of outgoing particles in each direction (in the center-of-mass system) is much smaller than s_1, the $(\mathrm{IM})^2$ of the two scattered pions or resonances in the intermediate state (this will be justified *a posteriori*). We also assume that the dominant contributing intermediate states contain diffractive excitations of these two pions with mass $\mu^*, \bar{\mu}^* \gg \mu$, the mass of the incoming pions; and that $(\mu^*)^2, (\bar{\mu}^*)^2 \gg |t_i|$. (Gundzik,[5] in a different model, has made similar assumptions about the importance of heavy resonant states in the unitarity sum.) If we assume damping of A in $(\mathrm{IMT})^2$, the above assumptions will lead to the kinematic relation [see Eq. (14) below]:

$$s_1 s_2 \bar{s}_2 \approx (\mu^* \bar{\mu}^*)^2 s. \qquad (1)$$

Hence, as $s \to \infty$, s_1 will be large, and A can be consistently given by the Pomeranchukon.[1] This choice of assumptions was not directly motivated by study of production data, but rather by a desire to obtain a closed mathematical model for the Pomeranchukon with somewhat reasonable features.

Since production multiplicity is known to become large as $s \to \infty$, and we assume for simplicity that the values of μ^*, $\bar{\mu}^*$ have some upper limit independent of s, we have the $(\mathrm{IM})^2$ of all particles produced in one direction (in the center-of-mass system), $s_n \gg (\mu^*)^2$. The kinematic inequality $|t_2| \geq (\mu^*)^2 s_n / s_2$ can now be derived [see Eq. (11) below], where t_2 is the $(\mathrm{IMT})^2$ between an incoming pion and the cluster of produced particles moving in the same direction. Since $(\mu^*)^2 \gg |t_2|$, we have $s_n \ll s_2$.

The unitarity sum that we must evaluate is represented by Fig. 2(a), where the circles represent possible final-state interactions and exchanges among the particles in a cluster, and the dashed line denotes the absorptive part. The scattered-pion excitations $(\mu^*, \bar{\mu}^*)$ are assumed not to interact with the cluster since $s_2 \gg s_n$. We now make the simplifying assumption that for small momentum transfers the momenta distributions internal to each cluster can be ignored in the unitarity sum. This assumption results in the diagram of Fig. 2(b) representing the sum, with the clusters replaced by effective hadrons of mass $\sqrt{s_n}$ and $\sqrt{\bar{s}_n}$, respectively. The diagram is integrated over s_n and $\bar{s}_n$.

In order that Fig. 2(b) be a suitable representation for the unitarity sum, it is necessary to show the following:

(a) The important values of s_n are small, so that it is plausible that final-state interactions are

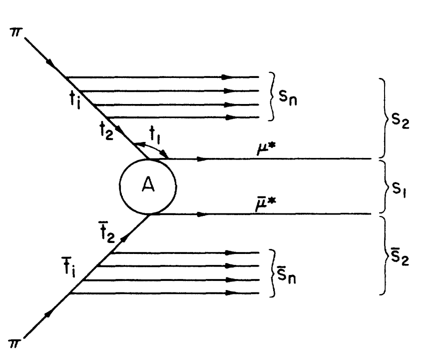

FIG. 1. Production model.

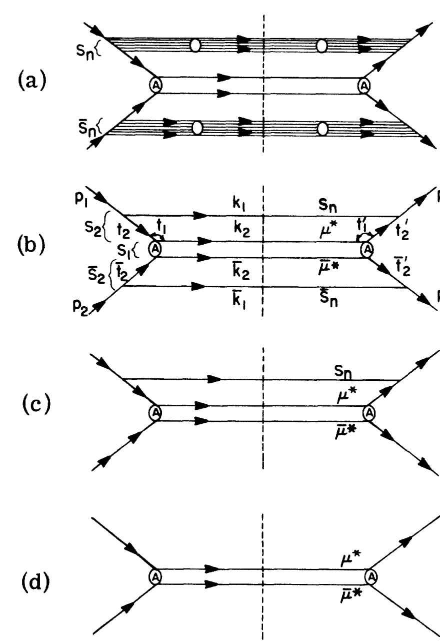

FIG. 2. Unitarity sums: (a) Production in both directions in intermediate states. (b) Strong interaction internal to clusters. (c) Production in one direction, strong interaction. (d) Elastic and quasielastic intermediate states only.

important and the momenta internal to each cluster do not depend on the motion of the cluster's center of mass at small scattering angles. [Another formulation could be expressed as follows: A_n, the amplitude represented by Fig. 3(a), depends very weakly on t for small t, and the dependence on t_2 and t_2' factors. This is the property of the amplitude of Fig. 3(b).]

(b) The important values of s_2 are not too large,[6] so that the propagators corresponding to t_2 and t_2' can be given by elementary pion propagators.[7] Both (a) and (b) can be justified *a posteriori* by checking the important regions of integration in evaluating our diagrams.

If we evaluate the diagram in Fig. 2(b) as $s\to\infty$ for fixed values of s_n and $\bar{s}_n$, the (IM)^2 of the clusters of produced particles, and μ^* and $\bar{\mu}^*$ (the masses of the excited pions in the intermediate state), we find the function

$$\Sigma_n = sG^2(s_n,\mu^2)G^2(\bar{s}_n,\mu^2)\frac{1}{s_n^{\,2}}\frac{1}{\bar{s}_n^{\,2}}F(t)C^2(\mu^*)C^2(\bar{\mu}^*), \qquad (2)$$

where the amplitude of Fig. 3(b) is assumed to factorize into $G^2(s_n,\mu^2)g(t_2)g(t_2')$ with $g(0)=1$ and $F(t)$ is independent of $s, s_n, \bar{s}_n$, and the masses μ, μ^*, and $\bar{\mu}^*$. Thus, if A_n, the amplitude represented by Fig. 3(a), has an s_n dependence such that $A_n/s_n^{\,2}$ becomes large only for small values of $|t|$ and s_n, and if A_n becomes roughly independent of t in that region, then the dominant contribution to the unitarity sum

$$\Sigma = \int ds_n \int d\bar{s}_n\, \Sigma_n$$

comes from regions of phase space where A_n can be represented by Fig. 3(b). We find that values of s_2 such that $s_2 \sim s_n(\mu^*)^2/|t_2|$ dominate the sum, so that these effective s_2 values do not become large as $s\to\infty$. Freund,[8] in his formulation of the multi-Regge Pomeranchukon bootstrap, replaces the sum over exchanges internal to the clusters by sums over resonant states in s_n, $\bar{s}_n$ and obtains a similar result, although he prefers to consider vector and tensor Regge exchanges rather than pions.

Our bootstrap condition equates Σ, summed over intermediate states with excited pions of mass μ^* and $\bar{\mu}^*$, with ImA. A consistent assumption for A is that the dependence on the external masses is small for small values of these masses and factors for large values. This assumption was used in deriving Eq. (2). Thus,

$$A(\mu_i^j,\mu_f^j,s,t)=\prod_{j=1}^{2} C(\mu_i^j)C(\mu_f^j)A(s,t),$$

where $j=1,2$ and μ_i^j (μ_f^j) are the masses of the incoming (outgoing) particles and $C(\mu)\approx C(t_1)\approx 1$. Since $A(s,t)=is\beta(t)$, we have

$$\beta(t)=D^2(\mu^2)F(t), \qquad (3)$$

where

$$D(\mu^2)=\left(\int ds_n G^2(s_n,\mu^2)/s_n^{\,2}\right)\left(\int d(\mu^*)^2\, C^2(\mu^*)\right). \qquad (4)$$

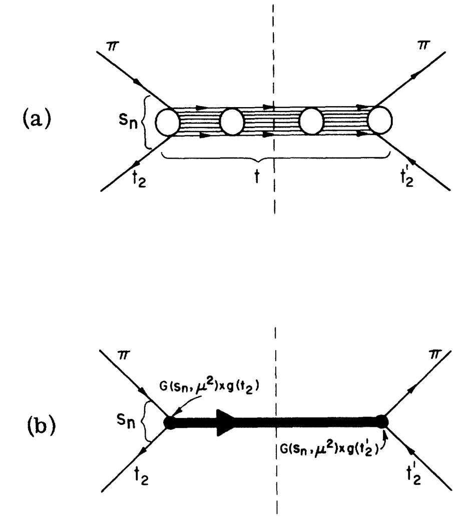

FIG. 3. Contribution of produced particles to unitarity sum: (a) General. (b) Strong interaction internal to cluster.

We find that to justify neglect of diagrams such as Figs. 2(c) and 2(d) in our sum, and intermediate states with unexcited pions ($\mu^*=\mu$), we must have each factor in $D(\mu^2)\gg 1$, respectively.

Since $F(t)$ in Eq. (3) is bilinear in β, we can define $\beta'(t)=C_a^{-1}D^2\beta(t)$, and write

$$\beta'(t)=C_aF'(t), \qquad (5)$$

where $F'(t)$ contains β', and C_a is a constant related to the normalization of $\beta'(t)$. In (5), the only unknown quantity is $g(t)$, the form factor. To obtain a closed system, we propose a "universal" form factor g and, in addition, adopt the Chou-Yang hypothesis[9]: $\beta(t)=\text{const}\times(\text{form factor})^2$. This form of the equation is consistent only if we assume that the amplitude A is small, which we have already done in neglecting diagrams such as Figs. 2(c) and 2(d) in the unitarity sum. Equation (5) can now, in principle, be solved for the residue of the Pomeranchukon, $\beta(t)$, within an unknown multiplicative constant.

II. THE INTEGRAL EQUATION

We introduce m_f, the form-factor scale, such that $g(t) \to 0$ for $-t \gg m_f^2$, and we assume $\mu < m_f \ll \mu^*$. We will see later that these are consistent assumptions. The inequalities contained in our model can now be summarized:

$$\mu^2 < m_f^2 \ll (\mu^*)^2 \ll s_n \ll s_2 \ll s_1, \tag{6}$$

for the important regions of integration.

To obtain our integral equation, we compute Σ_n of Eq. (2), represented by Fig. 2(b). A variable with an overbar in the following will pertain to the lower part of the diagram in Fig. 2(b), and a primed variable to the right-hand side. We have

$$\Sigma_n = D' \int d^4k_1 d^4k_2 d^4\bar{k}_1 d^4\bar{k}_2 \delta^4(k_1+k_2+\bar{k}_1+\bar{k}_2-p_1-p_2) \times \delta(k_1^2-s_n)\delta(\bar{k}_1^2-\bar{s}_n)\delta(k_2^2-\mu^{*2})\delta(\bar{k}_2^2-\bar{\mu}^{*2}) \times s_1^2\beta(t_1)\beta(t_1')f(t_2)f(t_2')f(\bar{t}_2)f(\bar{t}_2'), \tag{7}$$

where $(p_1+p_2)^2 = s$, k_1 is the momentum of a cluster, and k_2 that of a scattered pion in the intermediate state. Also,

$$D' = G^2(s_n, \mu^2)G^2(\bar{s}_n, \mu^2)C^2(\mu^*)C^2(\bar{\mu}^*)$$

and

$$f(t) = g(t)\frac{1}{\mu^2 - t}.$$

Since $\mu^2, |t_1|, |t_2| \ll (\mu^*)^2, s_n$ we can factor Eq. (7).

$$\Sigma_n = D' \int_0^\infty ds_2 \int_0^\infty d\bar{s}_2\, s_1^2 B_0 B\bar{B}, \tag{8}$$

where

$$B_0 = \int d^4k\, \delta(k^2 - s_2) \times \int d^4\bar{k}\, \delta(\bar{k}^2 - \bar{s}_2)\delta^4(k+\bar{k}-p_1-p_2)\beta(t_1)\beta(t_1'), \tag{9}$$

$$B = \int d^4k_1\, \delta(k_1^2 - s_n) \times \int d^4k_2\, \delta(k_2^2-\mu^{*2})\delta^4(k_1+k_2-k^*)f(t_2)f(t_2'), \tag{10}$$

and similarly for $\bar{B}$. In the above expressions $(k^*)^2 = s_2$ and $(\bar{k}^*)^2 = \bar{s}_2$.

The amplitudes B_0, B, and $\bar{B}$ have the form of the absorptive part of the box diagram of Fig. 2(d), with the appropriate values for the masses and variables. Using the inequalities (6), we have[10]

$$-t_2 = \vec{q}_2^{\,2} + s_n\mu^{*2}/s_2,$$
$$-t_2' = (\vec{q}_2 - \vec{Q})^2 + s_n\mu^{*2}/s_2, \tag{11}$$

and

$$-t = \vec{Q}^2,$$

where

$$\vec{q}_2 = (\tfrac{1}{4}s_2)^{1/2}(\hat{p}_1 - \hat{k}_1)$$

and

$$\vec{Q} = (\tfrac{1}{4}s_2)^{1/2}(\hat{p}_1 - \hat{p}_1').$$

In the above, $\hat{p}_1$, $\hat{p}_1'$ and $\hat{k}_1$, $\hat{k}_2$ are unit momentum vectors of the external pions and of the particles in the intermediate state of Fig. 2(d), evaluated in the center-of-mass frame where $\vec{k}_1+\vec{k}_2 = 0$. Damping in $|t_2|$ implies, by Eq. (11), $s_2 \gg (\sqrt{s_n}+\mu^*)^2$ and Eq. (10) becomes

$$B = \frac{1}{2s_2}\int d^2q_2\, f\left(\vec{q}_2^{\,2} + \frac{s_n\mu^{*2}}{s_2}\right) f\left((\vec{q}_2 - \vec{Q})^2 + \frac{s_n\mu^{*2}}{s_2}\right). \tag{12}$$

Inserting corresponding values for B_0, and noting that s is large enough so that $s_2\bar{s}_2/s \ll \mu^2$, we have

$$B_0 = \frac{1}{2s}\int d^2q\, \beta(\vec{q}^{\,2})\beta((\vec{q} - \vec{Q})^2). \tag{13}$$

If we assume inequalities (6) hold, s_1 assumes a simple form. Defining $s_T = (k_1+k_2+\bar{k}_2)^2$, we can derive the expressions[11]

$$s_T\bar{s}_2 = (\bar{\mu}^*)^2 s$$

and

$$s_1 s_2 = (\mu^*)^2 s_T.$$

Hence, we have

$$s_1 = \frac{(\mu^*)^2(\bar{\mu}^*)^2}{s_2\bar{s}_2}\, s\,. \tag{14}$$

We then have

$$\Sigma_n = \frac{D'}{s_n^2\bar{s}_n^2}\, sU^2(\vec{Q}^2)B_\beta(\vec{Q}^2),$$

where

$$U(\vec{Q}^2) = \tfrac{1}{2}\int_0^\infty x\,dx \int d^2q_2 f(\vec{q}_2^{\,2}+x)f((\vec{q}_2 - \vec{Q})^2 + x) \tag{15}$$

and

$$B_\beta(\vec{Q}^2) = \tfrac{1}{2}\int d^2q\, \beta(\vec{q}^{\,2})\beta((\vec{q} - \vec{Q})^2)\,. \tag{16}$$

Because of damping in $f(\vec{q}_2^{\,2}+x)$, we see that values of $x = s_n\mu^{*2}/s_2$ such that $x \sim \mu^2$ are important, or $s_n\mu^{*2} \sim s_2\mu^2$, justifying our earlier assumption. We can now identify $U^2(\vec{Q}^2)B_\beta(\vec{Q}^2)$ with $F(t)$ in Eqs. (2) and (3).

Our bootstrap equation is

$$\beta(\vec{Q}^2) = \lambda' U^2(\vec{Q}^2)\tfrac{1}{2}\int d^2q\, \beta(\vec{q}^{\,2})\beta((\vec{q} - \vec{Q})^2)\,, \tag{17}$$

where λ' is a constant, and $U(\vec{Q}^2)$ is given by Eq. (15). Adopting the "universal Chou-Yang hypothesis," $\beta(\vec{q}^{\,2}) = c[g(\vec{q}^{\,2})]^2$, c = constant, we can, in principle, solve Eq. (17) for $\beta(\vec{q}^{\,2})$ within a multiplicative constant.

III. SOLUTION OF THE INTEGRAL EQUATION

For mathematical simplicity, we choose a one-parameter pole form for the form factor:

$$g(\vec{Q}^2) = (1+\vec{Q}^2/m_f^2)^{-1}. \tag{18}$$

Then, we have

$$f(\vec{Q}^2) = (1+\vec{Q}^2/m_f^2)^{-1}(\mu^2+\vec{Q}^2)^{-1} \tag{19}$$

and

$$\beta(\vec{Q}^2) = c(1+\vec{Q}^2/m_f^2)^{-2}. \tag{20}$$

We expect the above parametrization to be valid only for small $\vec{Q}^2$, and we may use Eq. (17) to solve for m_f^2 by looking at the logarithmic derivative at $\vec{Q}^2=0$. We define $y=\vec{Q}^2/m_f^2$, $\gamma=m_f/\mu$,

$$\beta'(y) = (1+y)^{-2}, \tag{21}$$

and

$$B_{\beta'}(y) = \tfrac{1}{2}\int d^2q\, \beta'(\vec{q}^2)\beta'((\vec{q}-\vec{Q})^2). \tag{22}$$

Rewriting Eq. (17), we have

$$\frac{\beta'(y)}{B_{\beta'}(y)/B_{\beta'}(0)} = \frac{U_\gamma^2(y)}{U_\gamma^2(0)}, \tag{23}$$

where the dependence on γ of $U^2(y)$ is denoted, and the Pomeranchukon residue $\beta=\text{const}\times\beta'$.

Defining $\delta_1=\gamma^2$ and $\delta_2=1$, we calculate from Eqs. (15) and (19),

$$U_\gamma(y) = \frac{\pi}{24}(I_0+I_{11}-2I_{12}+I_{22})\frac{\mu^2}{y^2}, \tag{24}$$

where

$$I_0 = 6\gamma^4(\gamma^2-1)\ln\gamma^2 y^2 + \gamma^2(\gamma^2-1)[6(\gamma^2+1)\ln\gamma^2+4(\gamma^2-1)]y + 2(\gamma^2-1)^3\ln y^2,$$

$$I_{ij} = a_{ij}^3\ln\Delta_{ij}, \quad i,j=1,2,$$

$$a_{ij} = [(y^2+\delta_i+\delta_j)^2-4\delta_i\delta_j]^{1/2},$$

$$\Delta_{ij} = \frac{y^2+2y(\delta_i+\delta_j)+a_{ij}(y+\delta_i+\delta_j)+\delta_i^2+\delta_j^2}{2\delta_i\delta_j}.$$

Also, from Eqs. (21) and (22),

$$B_{\beta'}(y) = \frac{4\pi\gamma^2\mu^2}{y(y+4)^2} \times\left\{\frac{4(y+1)}{[y(y+4)]^{1/2}}\ln\left[\frac{(y+4)^{1/2}}{2}+\frac{y^{1/2}}{2}\right]+\frac{y}{2}-1\right\}. \tag{25}$$

We see that the left-hand side of Eq. (23) is independent of γ; this function is plotted in Fig. 4,

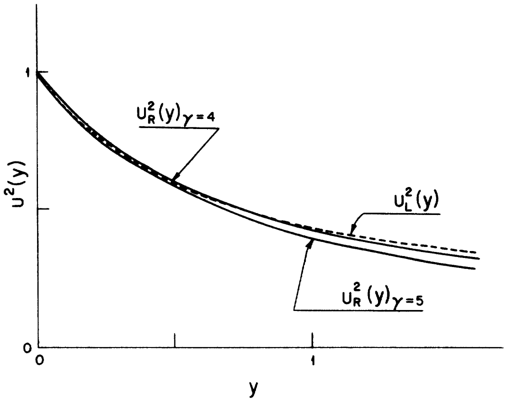

FIG. 4. Left- and right-hand sides of integral equation (23).

where it is denoted $U_L^2(y)$. The right-hand side, $U_\gamma^2(y)/U_\gamma^2(0)\equiv U_R^2(y)_\gamma$ is also plotted in Fig. 4 for $\gamma=4$ and $\gamma=5$. We see that $\gamma\approx4.7$ satisfies Eq. (23) very well for small y, and reasonably well up to $y\approx0.8$, which corresponds to $|t|\approx0.35$ GeV^2.

Note that we have a stable solution to Eq. (22) only at small $|t|$ and for our one-parameter model, and we cannot claim that an exact solution exists.

IV. COMPARISON WITH DATA: IMPROVEMENTS

We can obtain the residue of the Pomeranchukon which satisfies available scattering data from any empirical model which relies on a fixed-pole Pomeranchukon. Taking πN fits[12] using the hybrid model[13] as a rough guide to $\pi\pi$ scattering, we have the form[14]

$$\beta_e(\vec{Q}^2) = (1+\vec{Q}^2/m_e^2)^{-4}, \tag{26}$$

where $m_e\approx1.1$ GeV.

To compare our solution with fits to the data, we fix $\gamma=4.7$, which corresponds to $m_f=0.66$ GeV in Eq. (18), and plot the left-hand side $[U_L^2(\vec{Q}^2)]$ and the right-hand side $[U_R^2(\vec{Q}^2)]$ of Eq. (23) in Fig. 5. We now compute $B_{\beta_e}(\vec{Q}^2)$ using the form (26) and plot

$$\frac{\beta_e(\vec{Q}^2)}{B_{\beta_e}(\vec{Q}^2)/B_{\beta_e}(0)} = U_e^2(\vec{Q}^2)$$

also in Fig. 5. We see that agreement between our solution and $U_e(\vec{Q}^2)$, obtained from experiment, is satisfactory for small $\vec{Q}^2$.

Perhaps a more meaningful comparison would be the logarithmic derivative of the forward differential cross section;

$$\left.\frac{d^2\sigma_{el}}{dt^2}\right|_{t=0}\Big/\left.\frac{d\sigma_{el}}{dt}\right|_{t=0}.$$

Our solution for $\beta(t)$ gives a value $4/m_f^2=9.2$ GeV^{-2}, whereas the form (26) gives $8/m_e^2=6.7$ GeV^{-2}.

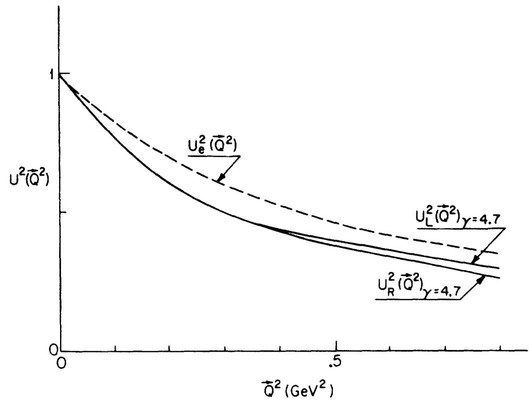

FIG. 5. Comparison of solution to Eq. (23) with hybrid model fit to data.

The agreement between these two values is only qualitative, but not unsatisfactory.

One obvious reason for this discrepancy is our neglect of diagrams such as Figs. 2(c) and 2(d) in our unitarity sum. Had we included them instead of $U(\vec{Q}^2)$ in our bootstrap Eq. (17), we would have had $U'(\vec{Q}^2)=U(\vec{Q}^2)+C_u$, where C_u = constant and $C_u^2/U'^2(0)=\sigma_{\text{elastic}}/\sigma_{\text{total}}$. This addition gives us an undetermined parameter which, if $\sigma_{\text{el}}/\sigma_{\text{tot}}=0.2$, results in $C_u\approx 0.8U(0)$. We see that this correction is hardly small if we require such a ratio for $\sigma_{\text{el}}/\sigma_{\text{tot}}$. This value of C_u results in a slope for $U_R^2(y)$ (Fig. 4) at $y=0$ about half of that plotted. We then find a new solution $\gamma \gtrsim 10$ which is too large, where $\gamma=4.7$ was too small. Thus, we see that our solution depends strongly on this additional parameter, and an estimate of it or, equivalently, of $(\int ds_n G^2(s_n)/s_n^2)/C(\mu^*)$ is necessary before a more detailed model can be developed. Also, poles at masses heavier than μ_π(e.g., μ_ρ) in the propagators would give us additional terms in $U(\vec{Q}^2)$ resulting in a smaller slope of $U_R^2(y)$, and a $\gamma>4.7$. To include these additional terms, their relative strengths would have to be estimated. Additional terms, such as C_u, if important, imply that $\beta(\vec{Q}^2)$ may be large, which would not allow us to use the form (20).

Another obvious oversimplification that we made was to use the form (18) for $g(\vec{Q}^2)$ instead of the more realistic dipole form. The dipole form factor corresponds to the phenomenological amplitude of (26). Trial functions with more adjustable parameters would be used in a more sophisticated model, as well as the exact Chou-Yang hypothesis.

Aside from specific fits, the model with $U(\vec{Q}^2)$ alone and only the pion propagator can be considered as predicting a lower bound for widths of diffraction peaks at asymptotic energies. Presently available data all show such widths of the same order as that exhibited by our solution: We regard this as an argument for a model such as this one which uses the pion mass as a scale.

More general forms of our model can be developed by dropping the assumption that the outer parts of the diagram in Fig. 2(a) be strictly of box form. One may, for example, consider

$$\int f\left(\vec{q}^2+\frac{s_n\mu^{*2}}{s_2}\right)f\left((\vec{q}-\vec{Q})^2+\frac{s_n\mu^{*2}}{s_2}\right)d^2q$$

to be some pion-hadron scattering amplitude. However, it is not a scattering amplitude for real particles, and there must not be a Pomeranchukon contribution; otherwise, the proposed model becomes inconsistent.

V. DISCUSSION

We have demonstrated that a qualitatively successful bootstrap theory of a fixed-pole Pomeranchuk singularity can be constructed using multiperipheral dynamical ideas.

The scale of the diffraction-peak width is established by the pion mass, although reliable quantitative estimates for the parameters of the Pomeranchukon residue $\beta(t)$ cannot be made. In order to avoid unknown constants in the theory, it was necessary to assume that $\beta(t)$ was small, or equivalently, that our coupling constants were large. We have seen that this assumption does not lead to realistic ratios for the cross sections. Also, the neglect of the pion ground state in the intermediate states may not be justified. In addition, other poles besides that of the pion may be important in the determination of $U(\vec{Q}^2)$.

Refinements of this model and extension to processes other than $\pi\pi$ scattering are clearly called for. The question of residue factorization is nontrivial, and the consistency of the Chou-Yang hypothesis in the context of our model has only been assumed. Also, there may be some connection between $U(\vec{Q}^2)$ and low-energy production cross sections, and there may be some way to determine the unknown quantities avoided in our present model. Even within the framework of this model, the existence of solutions to the integral equation (23) should be studied, or at least a more realistic function, such as a dipole form, should be used for $g(\vec{Q}^2)$.

An apparent prediction of this model is that multiplicity and distribution of the low-subenergy produced particles should be independent of s asymptotically.

Our general ideas are in accordance with the hypothesis that non-Pomeranchukon production mechanisms[7,8,15] are not as important asymptotically as those involving a Pomeranchukon exchange, together with low-subenergy "bremsstrahlung"[4] or "fragmentation"[16] in the non-Pomeranchukon parts of the production diagrams. These features cannot yet be reliably tested since multiparticle production data of high accuracy are required.

During the course of the development of this model some other publications have appeared in which attempts to construct self-consistent theories for a fixed Pomeranchukon are presented. Hwa[17] has proposed that the Pomeranchukon be a fixed cut; by such means, one may achieve self-consistency through unitarity in the t channel. However, in such a model it is necessary that the total cross section decrease indefinitely as s increases. No definite predictions of peak shapes are obtained.

Ball and Zachariasen[18] have constructed a theory based on unitarity in the s channel (such as ours), but they start from a factorized form for the amplitude and *a priori* restrictions on each production cross section, without appealing to a specific physical model. In their results, the scale of the diffraction-peak width is not directly related to the particle masses.

Gundzik[5] has developed a scheme relying on a detailed model of classical emission processes, together with an inhomogeneous term, which yields satisfactory diffraction-peak widths, but contains adjustable parameters. He is able to fit data, but the sensitivity of his scheme to variations of parameters is too great to allow much prediction.

Recently, two groups[19] have suggested fixed-pole Pomeranchukon theories very close in spirit to ours, but not as specific.

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†This supersedes a preprint of a previous version of such a theory by one of us (R.C.A.) dated January 1970.

‡AUA-ANL Fellow.

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