Cosmic-Ray Study of Properties of Nuclear Interactions in the 10–300-GeV Energy Range*

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A balloon-borne apparatus consisting of an emulsion target, spark chambers, and an ionization spectrometer has been used to study properties of nuclear interactions of cosmic rays in the energy range 10–300 GeV. The ionization spectrometer, used to measure the energies E_{sp} of the primaries passing through the apparatus, was calibrated at accelerator energies. This calibration was extrapolated to higher energies using Monte Carlo calculations. A comparison of energy estimates obtained using emulsion methods with E_{sp} indicates that for proton interactions the method based on constancy of transverse momentum and inelasticity gives an energy $1.5E_{ch}$ that is too large by a factor of only 1.1 ± 0.2, whereas the method of Castagnoli gives an energy E_{Cast} that is too large by a factor of 5 ± 2. For interactions of $Z > 1$ primaries, estimates (E_p and E_α) based on the opening angle of fragments and the $1.5E_{ch}$ estimates are in good agreement with E_{sp}. It is found that if the depth of the first interaction of each primary is known to within $\pm 1/4$ interaction length, the spectrometer estimate is not sensitive to some other characteristics of the first interaction, such as number and angular distribution of charged secondary particles. For proton interactions with fewer than six black and gray tracks, $E^{1/4}$ and $\ln E$ functions give better fits to the data on the mean number of minimum-ionizing secondaries vs $\langle E_{sp}\rangle$ than the $E^{1/2}$ function does.

I. INTRODUCTION

Apparatus consisting of an emulsion target, spark chambers, and an ionization spectrometer has been built and flown in a series of balloon flights.[1,2] The ionization spectrometer was used to determine the total energy of a particle incident on the apparatus. The spark chambers were used to locate tracks in the emulsions corresponding to cosmic-ray primaries which satisfied certain trigger requirements.

This experiment was the result of a collaboration between the Max Planck Institute for Extraterrestrial Physics at Garching, near Munich, and Louisiana State University in Baton Rouge. This was the first experiment in which emulsions were used with an ionization spectrometer and, except for the Proton I and II satellite experiments, it was the first one in which an ionization spectrometer was used to study primary cosmic rays. The experimental arrangement was superior to previous emulsion experiments for studying the properties of nuclear interactions, because the energy of each primary could be determined independently of emulsion measurements.

The results of using the apparatus to determine the flux of cosmic rays have already been reported.[3,4] The apparatus was also used to determine various properties of ionization spectrometers,[5-11] some properties of nuclear interactions at accelerator energies,[12] and the nucleon-nucleon elastic cross section at 83 GeV.[13,14] Reports[2,15] on the evaluation of the accuracy and reliability of locating primary cosmic rays in an emulsion stack through the use of spark chambers also have been published.

This paper describes some properties of nuclear interactions in the 10–300-GeV energy range which were obtained using the inelastic interactions of primaries in the emulsions. Also presented are comparisons of emulsion methods of energy estimation with the spectrometer energy measurements.[2,16]

II. EXPERIMENTAL PROCEDURE

A. Apparatus

A schematic drawing of the apparatus used in this experiment is shown in Fig. 1. It consists of two main parts: (1) the spark chambers above and below the target, and (2) the ionization spectrometer. The emulsion target was a 6-liter stack of Ilford G-5 emulsion, oriented so that the planes of the emulsions were vertical during the balloon flight.

Each time a high-energy particle satisfied the trigger requirements, the spark chambers were pulsed and photographed using a mirror system and two 16-mm cameras. Two mutually perpendicular views of each event were obtained. In addition, the pictures of each event contained information about the charge, direction, interactions, and energy of the incoming particle as well as the time of occurrence.

In the spectrometer, photomultipliers MI, MII, and MIII (viewing the pairs of scintillators A, B, and C, respectively) were used to measure the light output from the three pairs of scintillators.

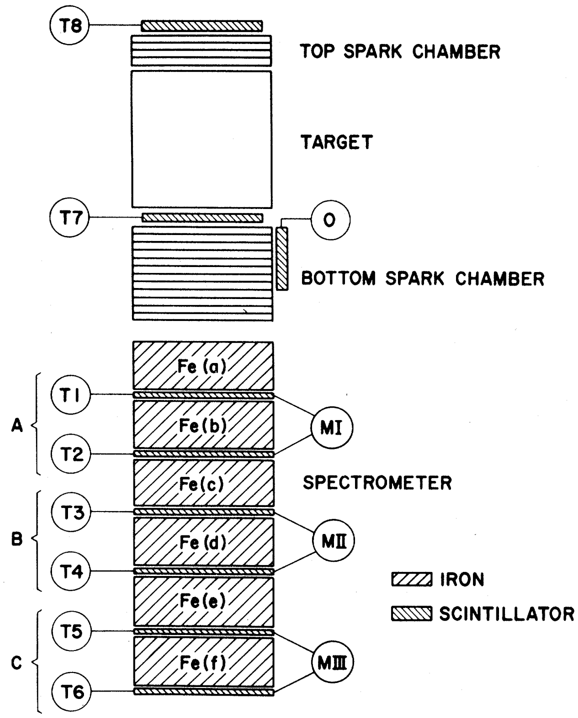

FIG. 1. Schematic diagram of the apparatus. Photomultipliers T1–T8 are for triggering and photomultipliers MI, MII, and MIII are for measuring. Counter 0, located on two sides of the apparatus, was used in anticoincidence.

These signals were used to obtain the energy of each primary. Photomultipliers T1-T6 (each viewing one of the six scintillators) and scintillation counter T7 just below the target were used in the triggering system.

The trigger conditions used for the emulsion flight required a coincidence between

(1) a pulse originating from scintillation counter T7 corresponding to at least one particle,

(2) a pulse originating from scintillation counter T6 corresponding to at least two particles,

(3) a pulse from either of the pairs of scintillators A, B, or C corresponding to the passage of at least 13 particles, and

(4) no pulse from guard counter 0, which was located near two sides of the bottom spark chamber.

Requirement (3) resulted in an energy threshold of approximately 30 GeV.

B. Emulsion Measurements

The apparatus was exposed with the emulsion target for 18 h at an altitude of 35.7 km (5 g/cm^2). A total of 43 proton primaries and 35 $Z>1$ primaries which triggered the apparatus are believed to have interacted in the emulsion target. Twenty-nine of the protons and all of the $Z>1$ primaries were found in the emulsion. The numbers and angular distributions of the tracks appearing in the bottom spark-chamber photographs and the energy distribution for the 14 protons which were not found were similar to those for the 29 protons which were found. Therefore, the results given in Sec. III should not be affected by the fact that some of the primary proton events were not used.

Scintillation counter T8 was used to indicate the charge of the incident primary in four channels corresponding to the charges $Z=0$ or 1, $Z=2$, $Z=3$, and $Z\geq 4$. However, for the primaries in which T8 indicated $Z\geq 3$, the charge was determined by measuring the density of δ rays along the tracks of the primaries in the emulsion.[17-21]

The angular distribution of the charged particles produced in the first interaction of each primary was measured with a Koristka R4 microscope using published methods.[22] The emission angles θ of these secondaries were used to calculate estimates E_{est} of the energy of each primary. Tracks having $\theta \geq 90°$ were not used in the calculations.

Several emulsion methods of estimating the energies were used. The Castagnoli method[22-24] (E_{Cast}) is based on the assumptions that the charged secondaries are emitted with forward-backward symmetry in the center-of-mass system, and that the target is a nucleon. The equations used were

$$\log\gamma_{\text{Cast}} = -\langle \log\tan\theta \rangle$$

and

$$E_{\text{Cast}} = m_p(2\gamma_{\text{Cast}}{}^2 - 1),$$

where m_p is the proton mass.

The E_{ch} energy estimate[22,24] is based on the assumption of constancy of transverse momentum of charged secondaries[24-26] and inelasticity. The equation used was

$$E_{\text{ch}} = \langle p_t \rangle \sum \csc\theta \text{ GeV},$$

where the mean transverse momentum $\langle p_t \rangle$ was taken to be 0.4 GeV/c. The value of E_{ch} is usually multiplied by 1.5 in order to include the energy going to uncharged secondaries (pions). The E_{ch} estimate should be independent of the target mass and secondary interactions inside the target nucleus.[22,24] The E_{Cast} estimate is not independent of either of these.

Black and gray tracks (tracks with grain densities greater than 1.4 times the grain density of the tracks of minimum ionizing particles) were not used in these energy estimations. In the case of the $Z>1$ primaries, only the tracks of secondaries which were not fragments were used in determining E_{Cast} and E_{ch}.

For the $Z>1$ primaries the emission angles θ_p

of the $Z=1$ fragments and the emission angles θ_α of the $Z=2$ fragments were used to calculate two more energy estimates E_p and E_α. The equations[20,24,27-29] used in these two methods were

$$E_p = 0.12(\langle\theta_p^2\rangle)^{-1/2} \text{ GeV/nucleon}$$

and

$$E_\alpha = 0.06(\langle\theta_\alpha^2\rangle)^{-1/2} \text{ GeV/nucleon.}$$

C. Spectrometer Energy Determination

A calibration of the ionization spectrometer with 10-, 20.5-, and 28-GeV/c protons was carried out using the Alternating Gradient Synchrontron (AGS) at the Brookhaven National Laboratory.[5] As a result of this calibration it was determined that the best parameter for obtaining the energy of the primary is $\sum N \equiv N_1+N_2+N_3$, where N_1, N_2, and N_3 are the normalized pulse heights from measuring photomultipliers MI, MII, and MIII, respectively. Since the scintillators were equally spaced within the iron absorber, $\sum N$ is a measure of the total ionization produced by a cascade within the spectrometer.

The calibration at accelerator energies was extrapolated to higher energies with the help of three-dimensional Monte Carlo calculations,[7-11] which were fitted with the AGS measurements. Basically, the cascade model incorporated existing information about mean values and fluctuations of the nuclear interaction parameters for high-energy hadron interactions as well as the conversion of energies of π^0 mesons into electromagnetic cascades.

The interaction parameters considered in the cascade model included multiplicity, inelasticity, nuclear evaporation energy, and the interaction length of the interacting particle. Lateral spreading was determined by taking into account (1) the angles of emission of the secondary particles from individual interactions, (2) the angular separation of gamma rays from π^0 decay, (3) the angular separation of the electrons created in pair production, and (4) multiple scattering of the strongly interacting particles passing through the absorber.

The particles produced in each interaction were assumed to be pions only. The calculations were performed for single particles incident on the absorber by following each incident particle and all the created charged pions through successive interactions until either they stopped or passed out of the absorber.

Neutral pions were considered to decay instantaneously into two γ rays, each of which underwent electron pair production. Each of these electrons in turn was taken to form an independent electromagnetic cascade with properties calculated using Rossi's approximation B.[30] No attempt was made to follow individual particles in the electromagnetic cascade.

Figure 2 shows a comparison of the calculations with the AGS measurements. This comparison involves distributions of $\sum N$ for 28-GeV protons which interacted in a carbon target ($\frac{1}{2}$ interaction length thick). The relative standard deviations $\sigma(\sum N)/\langle\sum N\rangle$ of the distributions are about 43%.

The determination of the energy E_{sp} was made for each cosmic-ray proton as follows: A preliminary estimate of the energy was obtained based on a linear extrapolation of the energy dependence of $\langle\sum N\rangle$ determined from the AGS measurements. Then, Monte Carlo calculations were made for two or three trial energies which bracketed the preliminary estimate. These calculations incorporated the known interaction point, the known angle of incidence, and the known number of charged secondary particles, which in turn had known directions. For each trial energy E, 500 Monte Carlo events were used. A log-log plot of E vs $\langle\sum N\rangle$ was used to obtain the energy E_{sp} corresponding to the measured value of $\sum N$. The relative standard deviation $\sigma_{E_{sp}}$ of E_{sp} was obtained from plots of E vs $\langle\sum N\rangle \pm\sigma(\sum N)$.

In addition, the energies of these protons were determined from Monte Carlo calculations using mean values of the incident angles and randomly

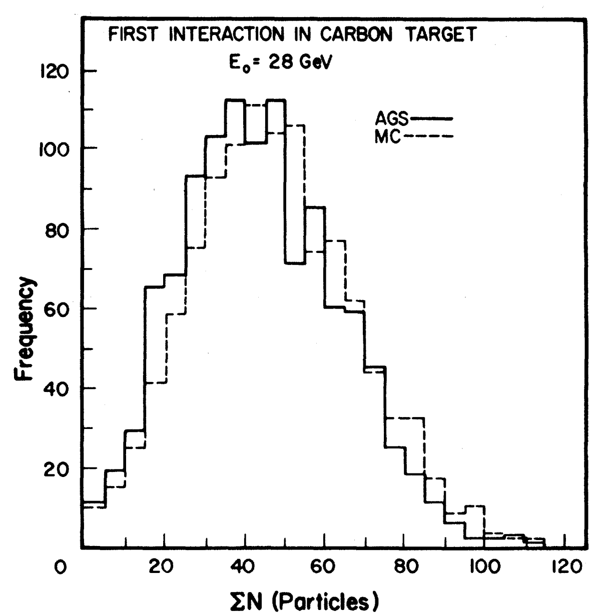

FIG. 2. Comparison of Monte Carlo calculations with AGS spectrometer measurements for 28-GeV primary protons which interacted in a carbon target. Plotted are the distributions of $\sum N$ recorded by the spectrometer (solid curve) and predicted by the calculations (dashed curve).

TABLE I. Properties of the primary interactions found in the emulsion target. All energies are in GeV.

Charge	N_e [a]	E_{sp} range	$\langle E_{sp}\rangle$	$\langle\sigma_{E_{sp}}\rangle$ (%)	$\langle n_s\rangle$	$\langle N_h\rangle$
1	29	14–297	64	32	5.0	4.0
2	8	6–32	16	25	7.4	3.9
2 Frag [b]	9	6–29	13	29	2.6	1.9
>2 Frag [b]	9	2–11	6	19	5.0	2.6

[a] Number of events used.
[b] Primaries which fragmented.

selected depths of the first interactions in the target. These calculations were not made separately for each primary. In Sec. III A these latter energies E_{sp0} are compared with the energies E_{sp} obtained using the known characteristics of the first interactions.

The calculation for each $Z>1$ primary nucleus was made using only the incident direction and the location of the first interaction. The number of nucleons participating in the first interaction was determined randomly from a uniform distribution. After the first interaction, the nucleons were assumed to behave independently. This is a reasonable assumption for α particles, but it is probably too crude for heavier nuclei.

III. RESULTS

A. Energy Comparisons

Properties of the primary interactions found in the emulsion target are given in Table I. The first column indicates the primary charge. It also indicates whether or not the $Z>1$ primaries underwent fragmentation. The mean values of the number n_s of minimum-ionizing secondaries and of the number N_h of black and gray tracks are given in columns six and seven, respectively.

The results of the comparison[31] of energy estimates obtained using emulsion methods with spectrometer measurements E_{sp} are given in Table II. The events used in the "E_p" and "E_α and E_p" groups are events listed as "Frag" in Table I. Eight of the $Z=2$ and one of the $Z>2$ primaries were not used in the comparison because of the small number (2) of secondaries. For the $Z=1$ and $Z=2$ primaries the methods of E_{Cast} and $1.5E_{ch}$ are used for the same group of events. When the contribution of $\sigma_{E_{sp}}$ has been removed from the values of σ and 10^σ for $\log_{10}(E_{est}/E_{sp})$, the values in the last two columns of Table II are obtained. These values can be considered to represent σ and 10^σ for $\log_{10}(E_{est}/E_0)$, with E_0 being the true energy of the primary.

Although not given here, additional comparisons were made for the proton events by grouping them according to the values of E_{sp}, n_s, and N_h. These comparisons indicate that all of the variations in 10^{mean} and 10^σ among various groups (using the same energy estimation method) are small compared to the statistical errors involved. However, the variations are much smaller for $1.5E_{ch}$ than for E_{Cast}.

Comparisons of $1.5E_{ch}$, E_{sp}, and E_{sp0} for protons are given in Table III. The values of 10^{mean} are within one standard deviation of 1.0. Both E_{sp} and E_{sp0} give similar values of 10^σ when compared with $1.5E_{ch}$. The value of $\langle\sigma_{E_{sp0}}\rangle$ was not statistically different from the value of $\langle\sigma_{E_{sp}}\rangle$. The good agreement of E_{sp0} with E_{sp} and $1.5E_{ch}$ implies that if the depth of the first interaction is known to within $\pm\frac{1}{4}$ interaction length, little improvement in the accuracy of each measurement is obtained by including the exact depth of the interaction, the angle of incidence of the primary, and the number and directions of the charged secondaries.

B. Multiplicity

In the 29 proton interactions there were a total of 145 charged, minimum-ionizing particles. This

TABLE II. Comparison of energy estimates obtained using emulsion methods with the spectrometer measurements E_{sp}. The errors given are statistical.

Z [a]	E_{est} [b]	N_e [c]	$\log_{10}(E_{est}/E_{sp})$ Mean	σ	10^{mean}	10^σ	$\log_{10}(E_{est}/E_0)$ [d] σ	10^σ
1	E_{Cast}	29	0.7 ± 0.2	0.9 ± 0.1	5 ± 2	7 ± 2	0.9 ± 0.1	7 ± 2
1	$1.5E_{ch}$	29	0.03 ± 0.07	0.39 ± 0.05	1.1 ± 0.2	2.4 ± 0.3	0.36 ± 0.05	2.3 ± 0.2
2	E_p	9	−0.25 ± 0.08	0.23 ± 0.05	0.6 ± 0.1	1.7 ± 0.2	0.19 ± 0.05	1.6 ± 0.2
2	E_{Cast}	8	0.2 ± 0.3	0.7 ± 0.2	1.6 ± 0.9	5 ± 2	0.7 ± 0.2	5 ± 2
2	$1.5E_{ch}$	8	0.2 ± 0.1	0.4 ± 0.1	1.4 ± 0.5	2.6 ± 0.6	0.4 ± 0.1	2.5 ± 0.6
>2	E_α and E_p	9	−0.1 ± 0.1	0.33 ± 0.08	0.8 ± 0.2	2.1 ± 0.4	0.32 ± 0.08	2.1 ± 0.4

[a] Primary charge.
[b] Emulsion method of energy estimation.
[c] Number of events used.
[d] The values of the mean and 10^{mean} are the same as the values for $\log_{10}(E_{est}/E_{sp})$.

TABLE III. Comparison of E_{ch}, E_{sp}, and E_{sp0} for protons.

Ratio	$\log_{10}$(ratio) 10^{mean}	10^{σ}
$1.5E_{ch}/E_{sp}$	1.1 ± 0.2	2.4 ± 0.3
$1.5E_{ch}/E_{sp0}$	1.1 ± 0.2	2.3 ± 0.2
E_{sp0}/E_{sp}	0.95 ± 0.05	1.34 ± 0.05

number included five particles (from three interactions) in which θ was greater than 90°.

In Table IV the observed values of mean multiplicity $\langle n_s \rangle$ are given for various groupings of E_{sp} and N_h. The use of $N_h = 5$ as a criterion for separating collisions involving light nuclei from collisions involving heavy nuclei has been suggested in several papers.[32,33] As might be expected, the observed value of n_s is somewhat dependent upon N_h. (A high N_h value indicates that the interaction involved a heavy nucleus of the emulsion with secondary interactions having occurred within that nucleus.)

Monte Carlo calculations were used to correct these values of $\langle n_s \rangle$ for effects of the apparatus triggering efficiency. The dependence of the triggering efficiency on primary energy is shown by the lower curve in Fig. 3. The extrapolation above 100 GeV is based on the result that, for all primary energies, about 86% of the events satisfied the guard counter condition. The upper curve in Fig. 3 shows the energy dependence of the ratio $R = (\langle n_s \rangle$ for Monte Carlo events which satisfied the trigger requirements)/($\langle n_s \rangle$ for all Monte Carlo events). This curve indicates that $\langle n_s \rangle$ was not significantly dependent on the triggering efficiency. The correction factors R and the corrected $\langle n_s \rangle$ values $\langle n_s \rangle_{corr}$ are given in Table IV also. Plotted in Fig. 4 are the values of $\langle n_s \rangle_{corr}$ and $\langle E_{sp} \rangle$ for the group of events containing all interactions (solid circles) and for the group of events containing only interactions with $N_h \leq 5$ (open circles). Weighted least-squares fits were made to these values of $\langle n_s \rangle_{corr}$ and $\langle E_{sp} \rangle$. Three functions $AE^{1/2}$, $BE^{1/4}$, and $C \ln E$ were used.[34-36] The fits were made separately for all interactions and for interactions with $N_h \leq 5$. However, for the same type of energy dependence the two values of the constants (A, B, and C) were not statistically different. The resulting curves are plotted in Fig. 4. For all interactions the best fit is obtained using the $E^{1/2}$ function, whereas for interactions with $N_h \leq 5$ better fits are obtained using the $E^{1/4}$ and $\ln E$ functions, both of which fit the data equally well.

The result that $E^{1/4}$ and $\ln E$ functions give better fits than an $E^{1/2}$ function for interactions with $N_h \leq 5$ can be compared with the results of other experiments. Hayakawa[34] states that the ICEF results[24] at high energies indicate a $\ln E$ dependence

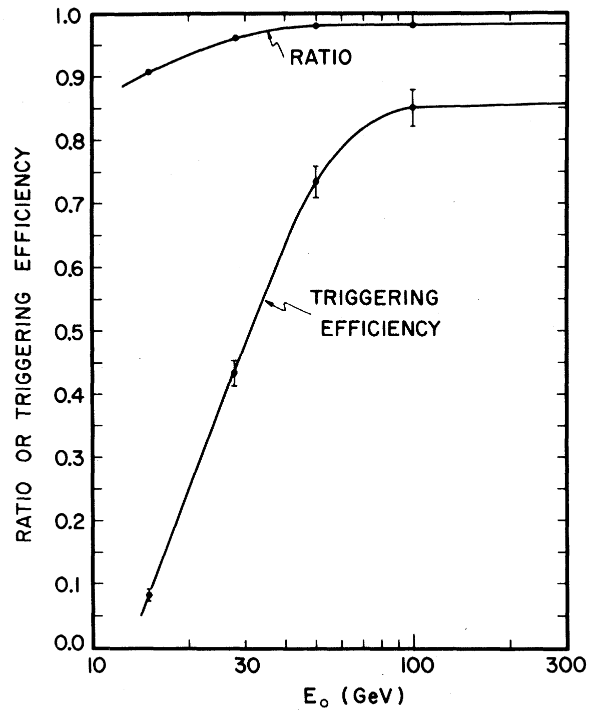

FIG. 3. Apparatus triggering efficiency. The lower curve shows the dependence of the triggering efficiency on primary energy. The upper curve shows the energy dependence of the ratio ($\langle n_s \rangle$ for Monte Carlo events which satisfied the trigger requirements)/($\langle n_s \rangle$ for all Monte Carlo events).

TABLE IV. Observed values of mean multiplicity $\langle n_s \rangle$ in proton interactions for various ranges of primary energy. The results are shown for the group of events containing all interactions and for the group of events containing only interactions with $N_h \leq 5$ (in parentheses). In the last column $\langle n_s \rangle$ has been corrected for triggering efficiency. The errors given are statistical.

Energy range (GeV)	N_e	$\langle E_{sp} \rangle$	$\langle n_s \rangle$	R [a]	$\langle n_s \rangle_{corr}$ [b]
11–300	29	64 ± 12	5.0 ± 0.7	0.980	5.1 ± 0.7
	(20)	(46 ± 7)	(3.8 ± 0.3)	(0.978)	(3.9 ± 0.3)
11–50	15	27 ± 3	3.3 ± 0.3	0.959	3.5 ± 0.3
	(13)	(28 ± 3)	(3.5 ± 0.3)	(0.961)	(3.6 ± 0.3)
50–300	14	104 ± 20	7 ± 1	0.982	7 ± 1
	(7)	80 ± 9	(4 ± 1)	(0.981)	(5 ± 1)

[a] Ratio of $\langle n_s \rangle$ for events satisfying trigger requirements to $\langle n_s \rangle$ for all events at the value of $\langle E_{sp} \rangle$ given in column 3.

[b] $\langle n_s \rangle_{corr} = \langle n_s \rangle / R$.

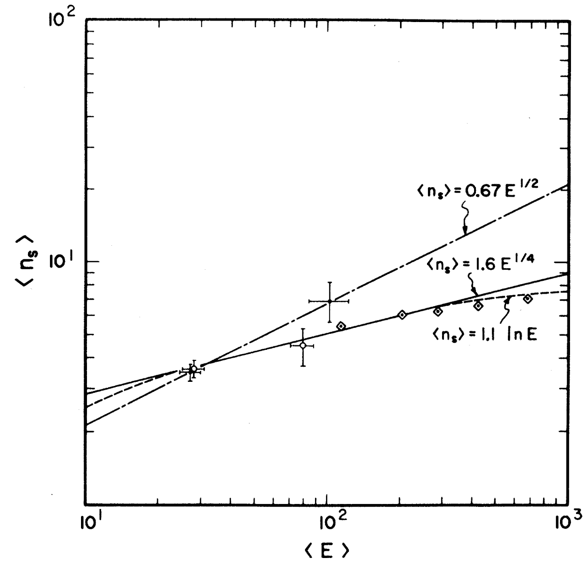

FIG. 4. Plot of $\langle n_s \rangle_{corr}$ vs $\langle E_{sp} \rangle$ for the primary protons. The solid circles are for all interactions and the open circles are for interactions with $N_h \leq 5$. The lines plotted are the results of weighted least-squares fits. The diamonds are the values obtained in the Echo Lake experiment.[39,40]

while some experiments at energies near and below 100 GeV have indicated an $E^{1/2}$ dependence. This $E^{1/2}$ dependence has been found by Kaneko and Okazaki[37] and by Guseva *et al.*[38] (in an ionization spectrometer experiment).

Perhaps the most significant results on multiplicity at ultrahigh energies come from the Echo Lake experiment.[39,40] This experiment involved a hydrogen target, spark chambers, and an ionization spectrometer. The energy range was 90–800 GeV. It was found that a $\ln E$ function gives the best fit to the $\langle n_s \rangle$ data (see Fig. 4). The values of $\langle n_s \rangle$ seem to be consistent with the values of $\langle n_s \rangle_{corr}$ obtained in this experiment for interactions with $N_h \leq 5$. A $\ln E$ function gives a very good fit to the combined data.

In the other experiments mentioned above, either the energy was estimated by some emulsion method or the actual interaction was not visible. If the actual interaction is not visible the values of n_s (and N_h) cannot be observed directly, but must be estimated from spark- (or cloud-) chamber photographs showing the secondaries which emerged from a separate target. In addition, corrections must be made for secondaries which miss the chambers and for secondaries from interactions other than the first interaction.

IV. CONCLUSIONS

This experiment suffers from limited statistics, yet it is felt that some significant conclusions can be made.

The fact that the $1.5E_{ch}$ estimates are more accurate than the E_{Cast} estimates implies that the assumption of constancy of transverse momentum and inelasticity is more consistently valid than the assumption of forward-backward symmetry in the center-of-mass system. Also, the fact that the results for $1.5E_{ch}$ are independent of n_s and N_h implies that the $1.5E_{ch}$ estimate is independent of the target mass and secondary collisions within the nucleus. Since the values of 10^{mean} for $\log_{10}(1.5E_{ch}/E_{sp})$ are very close to 1.0, the surviving primary, the produced charged pions, and all other produced charged particles must have about the same mean transverse momentum (approximately 0.4 GeV/c). In the case of fragmentations of $Z \geq 2$ nuclei, the E_p and E_α estimates are in good agreement with E_{sp}. The result that the $1.5E_{ch}$ estimates are more accurate than the E_{Cast} estimates is in agreement with a 20-GeV pion experiment[22] and with Monte Carlo calculations involving higher energies.[32]

The good agreement of E_{sp0} with E_{sp} and $1.5E_{ch}$ implies that if the depth of the first interaction of each primary is known to within $\pm\frac{1}{4}$ interaction length, the spectrometer estimate is not significantly improved by incorporating knowledge of additional characteristics of the first interaction such as the multiplicity and the angular distribution of the charged secondaries.

The study of multiplicities has indicated that the functional dependence of $\langle n_s \rangle$ on E is related to the values of N_h. For the group of events containing all interactions, the function $\langle n_s \rangle = AE^{1/2}$ gives the best fit to the data. For the group of events containing interactions with $N_h \leq 5$, the functions $\langle n_s \rangle = BE^{1/4}$ and $\langle n_s \rangle = C \ln E$ give better fits to the data. The values of $\langle n_s \rangle$ for interactions with $N_h \leq 5$ are consistent with the values obtained in the Echo Lake experiment[39,40] for a slightly higher energy range. The best fit to the combined data is obtained with a $\ln E$ function.

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[1]K. Pinkau, U. Pollvogt, W. Schmidt, and R. W. Huggett, in *Proceedings of the Ninth International Conference on Cosmic Rays, 1965*, edited by A. C. Stickland (The Institute of Physics and The Physical Society, London, England, 1966), Vol. 2, p. 821.

[2]E. R. Goza, R. W. Huggett, S. Krzywdzinski, E. Stafford, V. Jones, K. Pinkau, U. Pollvogt, and W. Schmidt, Bull. Am. Phys. Soc. 14, 90 (1969).

[3]W. K. H. Schmidt, K. Pinkau, U. Pollvogt, and R. W. Huggett, Phys. Rev. 184, 1279 (1969).

[4]K. Pinkau, U. Pollvogt, W. K. H. Schmidt, and R. W. Huggett, Acta Phys. Acad. Sci. Hung. 29, Suppl. 1, 291 (1970).

[5]W. V. Jones, K. Pinkau, U. Pollvogt, W. K. H. Schmidt, and R. W. Huggett, Nucl. Instr. Methods 72, 173 (1969).

[6]W. V. Jones, K. Pinkau, U. Pollvogt, W. K. H. Schmidt, and R. W. Huggett, Acta Phys. Acad. Sci. Hung. (to be published); Paper No. TE 27/3, presented at the Eleventh International Conference on Cosmic Rays, Budapest, 1969 (unpublished).

[7]W. V. Jones, Phys. Rev. 187, 1868 (1969).

[8]W. V. Jones, Acta Phys. Acad. Sci. Hung. (to be published); Paper No. TE 27/1, presented at the Eleventh International Conference on Cosmic Rays, Budapest, 1969 (unpublished).

[9]W. V. Jones, Acta Phys. Acad. Sci. Hung. (to be published); Paper No. TE 27/2, presented at the Eleventh International Conference on Cosmic Rays, Budapest, 1969 (unpublished).

[10]C. R. Gillespie, R. W. Huggett, and W. V. Jones, Nucl. Instr. Methods 81, 270 (1970).

[11]W. V. Jones, Phys. Rev. D 1, 2201 (1970).

[12]W. V. Jones, K. Pinkau, U. Pollvogt, W. K. H. Schmidt, and R. W. Huggett (unpublished).

[13]E. R. Goza, R. W. Huggett, and E. G. Stafford, Bull. Am. Phys. Soc. 15, 1331 (1970).

[14]E. R. Goza and E. G. Stafford Phys. Rev. D 3, 2577 (1971).

[15]E. R. Goza, S. Krzywdzinski, and E. G. Stafford, Rev. Sci. Instr. 41, 219 (1970).

[16]E. R. Goza, R. W. Huggett, W. V. Jones, and E. G. Stafford, Bull. Am. Phys. Soc. 15, 619 (1970).

[17]N. F. Mott, Proc. Roy. Soc. (London) A124, 425 (1929).

[18]D. A. Tidman, E. P. George, and A. G. Herz, Proc. Phys. Soc. (London). A66, 1019 (1953).

[19]O. B. Young and W. C. Ballowe, Am. J. Phys. 24, 157 (1956).

[20]C. F. Powell, P. H. Fowler, and D. H. Perkins, *The Study of Elementary Particles by the Photographic Method* (Pergamon, New York, 1959).

[21]O. Mathiesen, Arkiv Fysik 17, 441 (1960).

[22]E. R. Goza, S. Krzywdzinski, C. O. Kim, and J. N. Park, Phys. Rev. D 2, 1838 (1970).

[23]C. Castagnoli, G. Cortini, C. Franzinetti, A. Manfredini, and D. Moreno, Nuovo Cimento 10, 1539 (1953).

[24]See the entire issue of Nuovo Cimento Suppl. 1, No. 4 (1963), which is devoted to the results of the International Cooperative Emulsion Flight (ICEF).

[25]B. Edwards, J. Losty, D. H. Perkins, K. Pinkau, and J. Reynolds, Phil. Mag. 3, 237 (1958).

[26]D. H. Perkins, in *Progress in Elementary Particle and Cosmic Ray Physics*, edited by J. G. Wilson and S. A. Wouthuysen (North-Holland, Amsterdam, 1960), Vol. V, Chap. 4, pp. 257–363.

[27]M. F. Kaplon, B. Peters, H. L. Reynolds, and D. M. Ritson, Phys. Rev. 85, 295 (1952).

[28]B. Peters, in *Progress in Cosmic Ray Physics*, edited by J. G. Wilson (North-Holland, Amsterdam, 1952), Vol. I, Chap. 4, pp. 191–242.

[29]K. Rybicki, Nuovo Cimento 49B, 203 (1967).

[30]B. Rossi, *High Energy Particles* (Prentice-Hall, Englewood Cliffs, N. J., 1952).

[31]The logarithmic distribution of ratios is used in the comparisons because it is generally more nearly symmetrical and Gaussian than the linear distribution (Refs. 22 and 32). The antilogarithm (10^{mean}) of the mean of this distribution is the mean factor by which one estimate is greater than the other, and is equal to the geometric mean of the distribution of the ratios of the estimates. The antilogarithm (10^{σ}) of the standard deviation is the factor which defines the approximate 68% confidence interval for statistical fluctuations of individual values of the ratios about the mean factor.

[32]R. D. Settles and R. W. Huggett, Phys. Rev. 133, B1305 (1964).

[33]J. Gierula, M. Miesowicz, and P. Zielinski, Nuovo Cimento 18, 102 (1960).

[34]S. Hayakawa, *Cosmic Ray Physics* (Wiley, New York, 1969).

[35]E. M. Friedländer, invited papers and rapporteur's talks in Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, 1969 (unpublished).

[36]R. E. Gibbs, J. J. Lord, and E. R. Goza, in *Proceedings of the Sixth Interamerican Seminar on Cosmic Rays, La Paz, Bolivia, 1970* (Publication Department, Universidad Mayor de San Andrês Laboratoria de Fisica Cósmica, La Paz, Bolivia, 1970), Vol. III, p. 639.

[37]S. Kaneko and S. Okazaki, Nuovo Cimento 8, 521 (1958).

[38]V. V. Guseva, N. A. Dobrotin, N. G. Zelevinskaya, K. A. Kotelnikov, A. M. Lebedev, and S. A. Slavatinsky, J. Phys. Soc. Japan 17, Suppl. A-III, 373 (1962).

[39]K. N. Erickson, Ph.D. thesis, Colorado State University, 1970 (unpublished); University of Michigan Report No. UM HE 70-4 (unpublished).

[40]L. W. Jones, A. E. Bussian, G. D. DeMeester, B. W. Loo, D. E. Lyon, Jr., P. V. Ramana Murthy, R. F. Roth, J. G. Learned, F. E. Mills, D. D. Reeder, K. N. Erickson, and B. Cork, Phys. Rev. Letters 25, 1679 (1970); 26, 213(E) (1971).