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[12]For example, in R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970).

[13]Our example of an $h_{\mu\nu}(x,p)$ which has $\delta(x_0)\,h_{00}(x,p) \neq 0$ leads to a $T^*_{\mu\nu}$ which cannot be Wick rotated. However, if $h_{\mu\nu}(x,p)$ is modified to

$$\tilde{h}_{\mu\nu}(x,p) = i\,(g_{\mu\nu}\Box - \partial_\mu\,\partial_\nu)[(p\cdot\partial)^2 - m^2\Box]\Delta(x,m),$$

then the expression for $h_{00}(x,p)\delta(x_0)$ remains unmodified but the $T^*_{\mu\nu}$ obtained from it *can be Wick rotated.* This example clearly indicates that the condition that $h_{00}(x,p) \times \delta(x_0)$ vanish in all frames is a sufficient but not a necessary condition for Wick rotation of the contour to be permitted.

[14]Doing the q_0 integration first is the "natural" thing to do as it leads to Feynman rules in a perturbative framework. We would like to thank J. Sucher for pointing this out to us.

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Self-Consistent Pomeranchon Coupling Ratios in the Multiperipheral Model*

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Given the two leading eigenvalues and eigenfunctions of the resonance (low-subenergy) component of a multiperipheral kernel and assuming lower eigenvalues to be unimportant, it is shown how the mixture corresponding to the Pomeranchon eigenfunction may be calculated from considerations of self-consistency. The method is illustrated in a multiperipheral model with pseudoscalar-meson links by associating the two leading unperturbed eigenstates with the 2^+ particles $f(1260)$ and $f'(1514)$.

I. INTRODUCTION

A recently developed multiperipheral model describes the leading Regge singularity with vacuum quantum numbers, the so-called "Pomeranchon," as arising from a self-generating perturbation that splits an idealized "primitive" Regge-pole spectrum into fine structure.[1,2] The primitive spectrum belongs to a simplified multiperipheral integral equation whose kernel is truncated so as to lack the Pomeranchon-generated long-range correlations in longitudinal rapidity. When only short-range (resonance) correlations are included in the kernel, the generated Regge singularities consist entirely of well-spaced poles. Inclusion of the weak but long-range Pomeranchuk component of the kernel introduces a weak branch point in crossed-channel angular momentum, and in the associated multisheeted J Riemann surface, poles occur at slightly different positions on the different sheets. This combination of branch point with closely spaced poles, first identified by Frazer and Mehta,[3] is what we have called "fine structure."

It has been conjectured that the Pomeranchon (P) should be identified with a real pole on the first (physical) sheet of the J surface while the nearest pole on the second sheet, whose position is slightly complex, should be identified with the P'. The lower-lying P'' would similarly be located on the second sheet. If the separation between P and P' is small compared with the distance to other poles, it can be shown that the ratios of P and P' couplings should be almost equal – that is, the P and P' "vectors" should be almost "parallel." A rough parallelism between P and P' couplings is an established experimental fact, but the observed P and P' vectors appear not to be exactly parallel.[4] For example, the P' is often associated with the 2^+ $f(1260)$ and described (e.g., in the quark model) by a vector whose $K\bar{K}$ component has half the amplitude of the $\pi\bar{\pi}$ component, while the observed P seems closer to being an SU_3 singlet, with equal $\pi\bar{\pi}$ and $K\bar{K}$ components. Recently it has been suggested by Carlitz, Green, and Zee[5] (CGZ) that the P vector should have a significant component parallel to the $f'(1514)$ as well as a component parallel to $f(1270)$. The special model employed by CGZ, however, is inconsistent in its treatment of the Pomeranchon vector in that these authors begin by assuming the Pomeranchon to be a pure SU_3 singlet while their finally calculated vector has a substantial octet component. The present paper explores the general CGZ suggestion within a multiperipheral framework and shows how self-consistency can be achieved for the Pomeranchon coupling ratios.

II. MIXING OF THE UNPERTURBED EIGENSTATES

We employ the notation of Sec. V of Ref. 2 where

the partially diagonalized multiperipheral equation is written schematically as

$$A = V + VSA\,, \tag{2.1}$$

the "potential"

$$V = V^R + V^P \tag{2.2}$$

being the sum of a large "resonance" component V^R and a small "Pomeranchuk" component V^P. The diagonal operator S may be described as a "propagator." The compact notation of Eq. (2.1) has suppressed a variety of conserved crossed-channel quantum numbers, including angular momentum J, squared momentum transfer t, charge conjugation, and natural parity, as well as internal quantum numbers like charge, hypercharge, and baryon number. Also suppressed are nondiagonal indices labeling the particle types and masses that characterize a particular link in the crossed-channel chain. It is the dependence on the discrete particle-type indices that is the special concern of this paper. Dependence on the continuous link mass (momentum transfer) has been dealt with elsewhere.

We choose all the diagonal internal quantum numbers to be those of the vacuum, leaving J and t as the only diagonal parameters to be varied. A Regge pole occurs at values of (J, t) where one of the eigenvalues μ_i of the kernel VS is equal to unity. Near such a point the solution behaves as

$$A \underset{\mu_i \to 1}{\sim} \frac{|\psi_i\rangle\langle\psi_i|}{1-\mu_i}\,, \tag{2.3}$$

where ψ_i is the eigenfunction of VS associated with μ_i, normalized so that

$$\langle\psi_i|S|\psi_j\rangle = \delta_{ij}\,. \tag{2.4}$$

The eigenvalues μ_i are analytic functions of (J, t), apart from isolated singularities, so from Eq. (2.3) it follows that Regge-pole residues are determined by the eigenfunctions ψ_i. That is,

$$A(J, t) \underset{J \to \alpha_i(t)}{\sim} k_i(t)\frac{|\psi_i\rangle\langle\psi_i|}{J-\alpha_i(t)}\,, \tag{2.5}$$

$\alpha_i(t)$ being that value of J for which $\mu_i = 1$, while

$$k_i(t) \equiv -\left(\frac{\partial\mu_i}{\partial J}\right)^{-1}_{\mu_i=1}. \tag{2.6}$$

We are, in particular, interested in the eigenvalue which becomes equal to 1 for the largest real J corresponding to the Regge pole designated as P. The associated eigenfunction contains the coupling ratios of interest to us.

The "unperturbed" (resonance) kernel V^RS also possesses a spectrum of eigenvalues μ_i^R, with associated eigenfunctions ψ_i^R, and our objective may be described as understanding the perturbative effect of V^P on the leading eigenvalues and eigenfunctions of V^RS. We shall assume, generalizing slightly the reasoning of Ref. 1, that the two leading eigenvalues of the resonance kernel correspond to Regge trajectories of normal slope (≈ 1 GeV^{-2}) passing, respectively, close to the 2^+ particles designated $f(1260)$ and $f'(1514)$. The basis for such an assumption is the smallness of the perturbation V^P except close to the infinite Amati-Fubini-Stanghellini (AFS) logarithmic branch point.[6] Near $J = 2$, $t = m_f^2$, $m_{f'}{}^2$, one is far from this branch point so the physical and unperturbed amplitudes may be presumed there to be nearly equal.

We interpret the Carlitz, Green, and Zee suggestion,[5] within our context, as the assignment to the two leading unperturbed eigenvectors ψ_1^R and ψ_2^R of the same dependence on particle-type indices near $t = 0$ as at the physical $J = 2$ point. In other words, everywhere along the "primitive" trajectories these authors propose adopting coupling ratios equal to the ratios of f and f' (reduced) partial widths. Given this, or some other specific assignment of the unperturbed couplings, we now set ourselves the task of determining the consequences for Pomeranchon coupling ratios.

Neglecting all but the first two unperturbed eigenvectors, one may easily calculate the mixing induced by the perturbation V^P, as well as the corresponding shift of eigenvalues. Defining a mixing angle θ such that

$$\begin{aligned}\psi_1 &= \cos\theta\,\psi_1^R + \sin\theta\,\psi_2^R\,,\\ \psi_2 &= -\sin\theta\,\psi_1^R + \cos\theta\,\psi_2^R\,,\end{aligned} \tag{2.7}$$

we find

$$\tfrac{1}{2}\tan 2\theta = \frac{\mathfrak{V}_{12}^P}{\mu_1^R + \mathfrak{V}_{11}^P - \mu_2^R - \mathfrak{V}_{22}^P}\,, \tag{2.8}$$

where the symbol $\mathfrak{V}_{ij}^P$ has the meaning

$$\mathfrak{V}_{ij}^P \equiv \langle\psi_i^R|SV^PS|\psi_j^R\rangle\,. \tag{2.9}$$

The corresponding shifted eigenvalues are

$$\begin{aligned}\mu_1 &= \cos^2\theta\,(\mu_1^R + \mathfrak{V}_{11}^P) + \sin^2\theta\,(\mu_2^R + \mathfrak{V}_{22}^P)\\ &\quad + 2\sin\theta\cos\theta\,\mathfrak{V}_{12}^P\,,\end{aligned} \tag{2.10}$$

$$\begin{aligned}\mu_2 &= \sin^2\theta\,(\mu_1^R + \mathfrak{V}_{11}^P) + \cos^2\theta\,(\mu_2^R + \mathfrak{V}_{22}^P)\\ &\quad - 2\sin\theta\cos\theta\,\mathfrak{V}_{12}^P\,.\end{aligned} \tag{2.11}$$

Also useful is the following unsymmetrical formula for the mixing angle:

$$\tan\theta = \frac{\mathfrak{V}_{12}^P}{\mu_1 - \mu_2^R - \mathfrak{V}_{22}^P}\,, \tag{2.12}$$

which may be deduced from formulas (2.8) and (2.10).

The reader may be puzzled as to where the advertised fine structure will appear since the per-

turbation of the eigenvalue spectrum is following the adiabatic rule in preserving the number of eigenstates. The splitting occurs in the mapping between eigenvalues and J poles, which is not one-to-one. In formula (2.10) the matrix elements of V^P contain a logarithmic singularity in J dependence at $J=\alpha_c$, where α_c is the AFS branch point. There is then more than one point in the J Riemann surface where μ_1, for example, assumes the value 1. Presumably only a single such value occurs on the physical sheet, at a point $J=\alpha_P(t)$ real and slightly to the right of α_c, but according to the hypothesis of Ref. 1 the eigenvalue μ_1 will again be equal to 1 at a second sheet point $J=\alpha_{P'}(t)$ that has a small displacement from the real axis.

In order to justify the statements in the Introduction, it is necessary that the mixing angle θ be negligible for J near $\alpha_{P'}$ even though significant for J near α_P. Such a circumstance is to be expected if the "distance" $|\alpha_P-\alpha_c|$ is much smaller than the distance $|\alpha_{P'}-\alpha_c|$ because all matrix elements of the perturbation V^P are tiny except when J is very close to the logarithmic singularity at α_c. Assuming α_P and α_c both to be near 1 while $\alpha_{P'}$ is near 0.6, we may thus identify

$$\psi_P=\psi_1 \quad (J=\alpha_P), \tag{2.13}$$

$$\psi_{P'}=\psi_1 \quad (J=\alpha_{P'}) \approx \psi_1^R . \tag{2.14}$$

The answer to the originally posed question evidently resides in the value of the mixing angle θ_P for $J=\alpha_P$.

III. STRUCTURE OF THE PERTURBATION

The perturbation V^P has the structure shown in Fig. 1, where m and n are the heretofore-suppressed link indices, while k and l represent physical particles in the unitarity sum underlying the multiperipheral equation. To simplify that problem let us assume that the Pomeranchon couples only to particle-antiparticle pairs, so the link index can be taken to be a single-particle index and Fig. 1 replaced by Fig. 2. To the extent, furthermore, that the logarithmic singularity dominates V^P, the dependence on m and n approximately factorizes and we have

$$V^P_{mn} \propto |\psi_P(m)|^2 |\psi_P(n)|^2 , \tag{3.1}$$

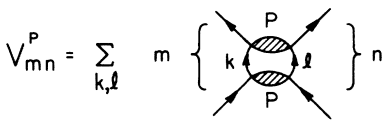

FIG. 1. Structure of the perturbation V^P.

since each vertex in the diagram is proportional to the vector ψ_P. It follows from formula (2.9) that

$$v^P_{ij}=g_i g_j , \quad i,j=1,2 \tag{3.2}$$

with

$$g_i \propto \int \sum_m \psi_i^R(m)[\psi_P(m)]^2 S(m) , \tag{3.3}$$

the integral running over link momentum transfers, as described in detail in Ref. 2. Because

$$\psi_P=\cos\theta_P \psi_1^R + \sin\theta_P \psi_2^R , \tag{3.4}$$

it follows that g_i is proportional to a linear combination of quantities C_{ijk}, defined by the formula

$$C_{ijk} \equiv \int \sum_m \psi_i^R(m)\psi_j^R(m)\psi_k^R(m)S(m) , \quad i,j,k=1,2 . \tag{3.5}$$

(Note the lack of dependence on the permutation of the three indices.) Specifically,

$$g_i \propto C_{i11}\cos^2\theta_P + C_{i22}\sin^2\theta_P + 2C_{i12}\sin\theta_P\cos\theta_P . \tag{3.6}$$

We shall see in Sec. IV that such a proportionality formula for g_i is sufficient for our purposes.

IV. PARTIAL BOOTSTRAP

We are dealing with a problem of self-consistency, attempting to find a mixing angle θ_P that gives matrix elements v^P_{ij} through Eqs. (3.2) and (3.6), which then lead by Eq. (2.8) back to θ_P. Because the unperturbed eigenfunctions and eigenvalues are regarded as given quantities, the bootstrap is only partial.

Let us introduce a quantity ν defined as the ratio,

$$\nu \equiv g_2/g_1 . \tag{4.1}$$

We then rewrite formulas (2.10) and (2.12) for the special case of interest, using (3.2) and (4.1) and remembering that $\mu_1=1$:

$$1=\cos^2\theta_P(\mu_1^R+v^P_{11})+\sin^2\theta_P(\mu_2^R+\nu^2 v^P_{11}) + 2\sin\theta_P\cos\theta_P\,\nu\, v^P_{11} , \tag{4.2}$$

$$\tan\theta_P=\frac{\nu v^P_{11}}{1-\mu_2^R-\nu^2 v^P_{11}} . \tag{4.3}$$

Finally, we have from (4.1) and (3.6) the following equation for ν:

$$\nu=\frac{C_{112}\cos^2\theta_P+C_{222}\sin^2\theta_P+2C_{122}\sin\theta_P\cos\theta_P}{C_{111}\cos^2\theta_P+C_{122}\sin^2\theta_P+2C_{112}\sin\theta_P\cos\theta_P} . \tag{4.4}$$

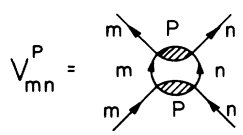

FIG. 2. An approximate relation for V^P based on Pomeranchon coupling only to particle-antiparticle pairs.

Given ratios of the C_{ijk} together with values for μ_1^R and μ_2^R, the foregoing three equations determine the three quantities v_{11}^P, ν, and θ_P. Provided there exists a solution to these nonlinear equations, we have in principle solved the stated problem. To achieve a concrete numerical estimate for θ_P, of course, requires numerical specification of the unperturbed eigenvalues and sufficient specification of the particle-type dependence of the corresponding unperturbed eigenfunctions so that ratios of the C_{ijk} can be calculated.

An approximate explicit solution to the foregoing bootstrap equations is possible if we assume v_{11}^P and θ_P to be small and of a common order of magnitude. Linearizing in θ_P, formula (4.4) becomes

$$\nu \approx \frac{C_{112}}{C_{111}} + 2\theta_P\left(\frac{C_{122}}{C_{111}} - \frac{C_{112}^2}{C_{111}^2}\right). \tag{4.5}$$

To a corresponding order, Eq. (4.2) is

$$1 \approx \mu_1^R + v_{11}^P, \tag{4.6}$$

while Eq. (4.3) becomes

$$\theta_P\left[1 - \mu_2^R - v_{11}^P\frac{C_{112}^2}{C_{111}^2}\right] = v_{11}^P\left[\frac{C_{112}}{C_{111}} + 2\theta_P\left(\frac{C_{122}}{C_{111}} - \frac{C_{112}^2}{C_{111}^2}\right)\right]. \tag{4.7}$$

These equations may be solved to yield

$$\theta_P = \frac{\frac{C_{112}}{C_{111}}R}{1 - R\left(2\frac{C_{122}}{C_{111}} - \frac{C_{112}^2}{C_{111}^2}\right)}, \tag{4.8}$$

where

$$R \equiv \frac{1 - \mu_1^R(J = \alpha_P)}{1 - \mu_2^R(J = \alpha_P)}. \tag{4.9}$$

V. QUARK-MODEL UNPERTURBED COUPLINGS

Following CGZ[5] let us take, for a specific example of unperturbed coupling ratios, the standard f and f' couplings to the pseudoscalar octet. Translated into the notation of the present paper, the quark-model ratios are

$$\psi_1^R(\pi):\psi_1^R(K):\psi_1^R(\eta):\psi_2^R(\pi):\psi_2^R(K):\psi_2^R(\eta) = 1:1/2:1/3:0:1/\sqrt{2}:4/3\sqrt{2}. \tag{5.1}$$

Because each of the three pion types, $\pi^{+,-,0}$, is equivalent, as is each of the four kaon types, we may confine attention to three different components rather than eight.

The quark model predicts couplings to all non-exotic multiplets, not only to the pseudoscalar octet, but implicit in the multiperipheral model is the assumption of kernel dominance by the least massive particle links. [A theoretical basis lies in the propagator $S(m)$ which is largest for the lowest-mass particles.] The simplest version of a multiperipheral model, in fact, would keep only pion links. Such an approximation is supported by the experimental observation that most of the produced particles from high-energy collisions are pions, but the consequence here would be $\theta_P = 0$, because the vanishing of $\psi_2^R(\pi)$ causes all the coefficients C_{ijk}, except C_{111}, to vanish.

A natural next approximation is to keep both π's and K's. We then calculate the following ratios from (5.1) and (3.5):

$$\frac{C_{112}}{C_{111}} = \frac{\sqrt{2}}{7}, \tag{5.2}$$

$$\frac{C_{122}}{C_{111}} = \frac{2}{7}, \tag{5.3}$$

$$\frac{C_{222}}{C_{111}} = \frac{2\sqrt{2}}{7} \tag{5.4}$$

if the difference between $S(\pi)$ and $S(K)$ is ignored. Attention to the π-K mass difference presumably would make all the foregoing ratios smaller, but for the sake of simplicity in our illustration here we proceed without regard for such complications. The linear approximation to θ_P, given by formula (4.8), is then

$$\theta_P \approx \frac{\frac{1}{7}\sqrt{2}R}{1 - \frac{26}{49}R}. \tag{5.5}$$

Inclusion of the η according to (5.1), with status equivalent to π and K, does not significantly change this result.

It may be noted that the quark-model ratios (5.1) do not correspond to orthogonal vectors ψ_1^R and ψ_2^R if the vector space is strictly limited to (π, K, η). The two "eight-vectors" given by (5.1) are roughly orthogonal to each other, nevertheless, in the sense that ψ_1^R points predominantly in the pion "direction," while ψ_2^R contains no π component.

Since we expect the unperturbed eigenvalues to lie between 0 and 1 when J is near 1, with $\mu_1^R > \mu_2^R$, it follows that the quantity R, as defined by (4.9), lies between zero and unity. The result (5.5) thus implies a positive value for the mixing angle, with θ_P sufficiently small as to justify *a posteriori* the linear approximation. A positive sign for θ_P means that in the Pomeranchon vector,

$$\psi_P \approx \psi_1^R + \theta_P\psi_2^R, \tag{5.6}$$

the amplitude ratio,

$$\frac{\psi_P(K)}{\psi_P(\pi)} \approx \frac{1}{2} + \frac{1}{\sqrt{2}}\theta_P, \tag{5.7}$$

is larger than the P' value of $\frac{1}{2}$ (assuming $\psi_{P'} \approx \psi_1^R$). Such a shift would be in the desired direction, Regge-pole analyses of high-energy data assigning a value to the $K:\pi$ Pomeranchon coupling ratio that is less than unity but definitely larger than $\frac{1}{2}$.[4]

VI. CONCLUSION

We shall not attempt here a quantitative estimate of R, this parameter being excessively dependent on details of the multiperipheral model. CGZ[5] suggest the formula

$$R \approx \frac{1-\alpha_f}{1-\alpha_{f'}}, \tag{6.1}$$

which is equivalent to our (4.9) if the unperturbed eigenvalues vary linearly with angular momentum, the constant of proportionality being the same, and if $\alpha_1^R \approx \alpha_f$, $\alpha_2^R \approx \alpha_{f'}$. Although the latter identifications are reasonable in view of the large distances from the AFS branch point, even the crudest multiperipheral models do not give a linear relation between eigenvalue and J over the wide J range (from 1 nearly down to 0) that is involved. It nevertheless seems unlikely that formula (6.1) could be in error by an order of magnitude, so the size of R should be somewhere in the neighborhood of CGZ's estimate of 0.6.[5] With such a choice formula (5.5) leads to $\theta_P \approx 0.2$.

In view of the various uncertainties that have been emphasized we do not propose taking seriously such a value for the mixing angle, but we point out that the corresponding CGZ estimate,[5] ignoring self-consistency (with the same value for R), was roughly twice as large. It therefore appears that self-consistency constitutes a significant consideration.

When and if there is developed a detailed multiperipheral model for an unperturbed resonance kernel, whose leading eigenfunctions are realistically associable with the f and f', the methods developed in this paper can be used to predict the corresponding Pomeranchon couplings. Conversely, experimental observations of the P, P', and P'' coupling ratios can be translated by our methods into constraints on the multiperipheral kernel.

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