Vector-Dominance Model, Partial Conservation of the Axial-Vector Current, and the Decoupling of $\pi\rho\omega$

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We show that partial conservation of the axial-vector current and vector dominance together imply that $g_{\pi\rho\omega} = 0$. We discuss the relevance of this result to the failure of vector dominance in polarized photoproduction of pions. We also provide a simple interpretation of the ρ -meson mass shift in photoproduction experiments based on unitarity.

I. INTRODUCTION

It was noticed some time ago by Sutherland¹ that partial conservation of the axial-vector current (PCAC) and current algebra together imply the decoupling of $\pi^0 - \gamma \gamma$. We note that Sutherland's proof was not really dependent on the explicit form of the current commutators, but only on their general Lorentz behavior. Also, of course, the process $A_1 - \gamma \gamma$ is strictly forbidden by gauge invariance and Bose statistics. Since PCAC may be used to relate A_1 and π matrix elements between given states, Sutherland's result is not completely surprising. On the other hand, the decay $\pi^0 - \gamma \gamma$ is also well understood in a vector-dominance pole model (VDM) as $\pi^0 - \rho \omega$ followed by photon emission.² Thus in the soft-pion limit we would expect that $g_{\pi\rho\omega} = 0$. In this paper we shall show directly without any reference to current algebra that we can derive this decoupling by a straightforward application of PCAC and VDM to the $\pi\rho\omega$ and $A,\rho\omega$ systems. Such a decoupling has already been noted by various authors such as Brown and West³ and Perrin.⁴ They observed that the $\pi \rho \omega$ vertex vanishes in the gauge-field algebra, and hence the fact that it is also forbidden by VDM is not surprising. We only wish to stress in this paper the almost completely "kinematic" nature of the decoupling, as we only appeal to current conservation. We shall deal throughout only with strongly interacting particles, so our result is unaffected by additional terms of order e^2 in the expression for the divergence of the neutral axial-vector current as proposed by Adler.⁵ PCAC is the axial analog of conservation of the vector current (CVC) and should be just as reliable, so we must interpret our result as a failure of VDM; and hence we must say that even without making a large- k^2 extrapolation as in the nucleon form-factor region, already in the classic 0-to- m_v^2 extrapolation range we are witnessing a breakdown of vector dominance. One may suspect that this is of the same nature as the

failure of VDM in polarized photoproduction of pions, and we shall discuss this point later.

Our use of VDM follows closely the technique we have described in a recent publication.⁶ Essentially the procedure is to enforce current conservation and investigate what this entails for processes involving vector mesons. Let $\epsilon_{\mu}(k)T^{\mu}$ be the T matrix of any process involving a vector meson with polarization vector $\epsilon_{\mu}(k)$, and any number of other particles. We may develop T^{μ} in a set of invariants, viz.,

$$T^{\mu} = \sum B_i X_i^{\mu} , \qquad (1)$$

where the number of the invariants B_i depends on the spins of all the other particles involved. A trivial way to force the vector meson to couple to a conserved current is to replace T^{μ} by W^{μ} , where

$$W^{\mu} = T^{\mu} - \frac{(k \cdot T)}{m_{\gamma}^{2}} k^{\mu} , \qquad (2)$$

with the second term making no on-shell contribution to $\epsilon_{\mu}T^{\mu}$. Thus at this stage we have $k_{\mu}W^{\mu}=0$ at the point $k^2 = m_v^2$. As we showed in Ref. 6, the nontrivial implication of vector dominance is that $k_{\mu}T^{\mu} = 0$ also at $k^2 = 0$, and this provides a relation among the various B_i , once they are assumed to be independent of m_V^2 . Essential to this approach is the requirement that the form factors, $B_i(k^2)$, satisfy unsubtracted dispersion relations in k^2 which are vector dominated. Since it is this assumption which leads us to the $\pi \rho \omega$ decoupling, this is the point which we will relax in Sec. III. Thus all of the mass dependence of the continuation is contained in the factors $k_{\mu}X_{i}^{\mu}$. The physical interpretation of this requirement in terms of electric and magnetic couplings is discussed in Ref. 6. Thus in this paper we shall take as a working definition of vector dominance

$$\lim_{n_{V}^{2} \to 0} k_{\mu} T^{\mu} = 0.$$
 (3)

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Our use of PCAC is standard.⁷ We take it to mean that $q_{\mu}A^{\mu}(x)$ connects the vacuum to a single and unique pion state with a strength f_{π} . (It is perhaps worth remarking that this is reflected in the apparent experimental absence so far of any other particles with the quantum numbers of the pion.) $A^{\mu}(x)$ may connect the vacuum to this pion state and to any number of resonant or nonresonant 1⁺ continua (which we shall refer to collectively as A) with a strength f_A . For the purposes of this work we shall use the axial-vector meson dominance approximation, and since the number of A_1 's is immaterial we will restrict them to one. Let $|XY\rangle$ be any two-particle state such that $\langle 0 | A^{\mu}(x) | XY \rangle$ has a momentum transfer q_{μ} . Then PCAC as a condition of no subtractions in q^2 requires

$$\begin{split} \lim_{q^{2} \to \infty} \langle 0 | q_{\mu} A^{\mu}(x) | XY \rangle &= \lim_{q^{2} \to \infty} q_{\mu} [\langle 0 | A^{\mu}(x) | A \rangle \langle A | XY \rangle + \langle 0 | A^{\mu}(x) | \pi \rangle \langle \pi | XY \rangle] \\ &= \lim_{q^{2} \to \infty} q_{\mu} \bigg[f_{A} \bigg(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{m_{A}^{2}} \bigg) T_{AXY}^{\nu} \frac{1}{m_{A}^{2} - q^{2}} + f_{\pi}T_{\pi XY}q_{\mu} \frac{1}{m_{\pi}^{2} - q^{2}} \bigg] \\ &= 0 \,, \end{split}$$

where we have put in the A and π propagators and introduced the on-shell vertices

$$\langle A | XY \rangle = \epsilon^{A}_{\mu}(q) T^{\mu}_{AXY},$$

$$\langle \pi | XY \rangle = T_{\pi XY}.$$
(5)

Thus we obtain

$$\frac{f_A}{m_A^2} q_\mu T^\mu_{AXY} - f_\pi T_{\pi XY} = 0$$
 (6)

so that we recover the standard relation

$$\langle 0 | q_{\mu} A^{\mu}(x) | XY \rangle = f_{\pi} T_{\pi XY} \frac{m_{\pi}^{2}}{m_{\pi}^{2} - q^{2}}.$$
 (7)

In the following we shall refer only to Eq. (6) for our use of PCAC. In fact taking $|XY\rangle$ to be the state $|\gamma\gamma\rangle$ we immediately obtain Sutherland's result, to confirm that his derivation was in fact independent of current algebra.

The plan of this paper is to investigate the simultaneous consequences of the application of Eqs. (3) and (6) to the $\pi\rho\omega$ system, in which each of the particles is coupled to a conserved current. We discuss the derivation of our result in Sec. II and its implications in Sec. III, where we also provide a simple explanation of the observed shift to lower mass values of the ρ peak in a photoproduction experiment.

II. THE CALCULATION

We have to study first the kinematics of the $A_{\mu}\rho_{\lambda}\omega_{\nu}$ vertex. We give the three mesons the momenta q, k, and p, respectively, with q=k+p. We may write the most general Lorentz-invariant vertex as

 $T = \sum_{i=1}^{8} B_{i} X^{i} , \qquad (8)$

with

$$\begin{split} X_{1} &= \epsilon_{\lambda}^{0} \epsilon_{\nu}^{\omega} k_{o} p_{\tau} \epsilon^{\lambda \nu \sigma \tau} \epsilon_{\mu}^{\mu} q^{\mu} , \\ X_{2} &= \epsilon_{\lambda}^{0} \epsilon_{\nu}^{\omega} k_{o} p_{\tau} \epsilon^{\lambda \nu \sigma \tau} \epsilon_{\mu}^{4} (k - p)^{\mu} , \\ X_{3} &= \epsilon_{\mu}^{0} \epsilon_{\nu}^{\omega} k_{o} p_{\tau} \epsilon^{\mu \nu \sigma \tau} \epsilon_{\lambda}^{0} (k - p)^{\mu} , \\ X_{4} &= \epsilon_{\mu}^{0} \epsilon_{\nu}^{\omega} k_{o} p_{\tau} \epsilon^{\mu \nu \sigma \tau} \epsilon_{\lambda}^{0} (q + p)^{\lambda} , \\ X_{5} &= \epsilon_{\mu}^{0} \epsilon_{\lambda}^{0} k_{o} p_{\tau} \epsilon^{\mu \lambda \sigma \tau} \epsilon_{\nu}^{\omega} p^{\nu} , \\ X_{6} &= \epsilon_{\mu}^{0} \epsilon_{\lambda}^{0} k_{o} p_{\tau} \epsilon^{\mu \lambda \sigma \tau} \epsilon_{\nu}^{\omega} (q + k)^{\nu} , \\ X_{7} &= \epsilon_{\mu}^{0} \epsilon_{\lambda}^{0} \epsilon_{\nu}^{0} q_{\tau} \epsilon^{\mu \lambda \nu \tau} , \\ X_{8} &= \epsilon_{\mu}^{0} \epsilon_{\lambda}^{0} \epsilon_{\nu}^{\omega} (k - p)_{\tau} \epsilon^{\mu \lambda \nu \tau} . \end{split}$$

$$\end{split}$$

For the moment, $\epsilon^{A}_{\mu}(q)$, $\epsilon^{\rho}_{\lambda}(k)$, and $\epsilon^{\omega}_{\nu}(p)$ are introduced solely as a device to represent the Lorentz properties of the particles, and we shall go to the mass shell later on. This set of eight invariants is not linearly independent. There exist two kinematic relations among them, viz.,

$$2X_{1} - X_{3} - X_{4} + X_{5} + X_{6} + (p^{2} - k^{2})X_{7} + q^{2}X_{8} = 0,$$

$$2X_{2} - 3X_{3} + X_{4} - 3X_{5} + X_{6}$$
(10)

$$+ (q^{2} - 2k^{2} - 2p^{2})X_{7} + (k^{2} - p^{2})X_{8} = 0,$$

where we have used the general relation⁸

$$(a^{\mu}\epsilon^{\nu\lambda\sigma\kappa} - a^{\nu}\epsilon^{\mu\lambda\sigma\kappa})a_{\lambda}b_{\sigma}c_{\kappa}$$
$$= \epsilon^{\mu\nu\lambda\sigma}[-a^{2}b_{\lambda}c_{\sigma} - (a\cdot c)a_{\lambda}b_{\sigma} + (a\cdot b)a_{\lambda}c_{\sigma}].$$
(11)

We have checked that our final result does not depend on when we use these relations, so we shall

(4)

apply them immediately by eliminating B_4 and B_6 . To check that we have the correct number of invariants we note that in on-shell $A \rightarrow \rho \omega$ there are indeed three independent amplitudes, which we may choose as

$$\epsilon^{A}_{\mu}(q)T^{\mu}_{A\rho\omega} = X_{2}g^{A\rho\omega}_{2} + X_{7}g^{A\rho\omega}_{7} + X_{8}g^{A\rho\omega}_{8}.$$
 (12)

Finally we also introduce the $\pi \rho \omega$ vertex

$$T_{\pi\rho\omega} = \epsilon^{\rho}_{\lambda} \epsilon^{\omega}_{\nu} k_{\sigma} p_{\tau} \epsilon^{\lambda\nu\sigma\tau} g_{\pi\rho\omega}$$
(13)

so that we are now ready to perform the calculation. From Eq. (6) we obtain

$$f_{\pi}g_{\pi\rho\omega} - \frac{f_{A}}{m_{A}^{2}} [(m_{\rho}^{2} - m_{\omega}^{2})g_{2}^{A\rho\omega} - 2g_{8}^{A\rho\omega}] = 0 \quad (14)$$

and from Eq. (3) applied first for ρ and then for ω , we have

$$g_7^{A\rho\omega} - g_8^{A\rho\omega} = 0,$$

$$g_7^{A\rho\omega} + g_8^{A\rho\omega} = 0.$$
(15)

Thus in the limit $m_{\rho}^{2} = m_{\omega}^{2}$ we conclude that $g_{\pi\rho\omega}$ =0. It is easy to see that this result is not altered if we replace A by a sum on 1^+ states. We note that only in the limit of equal masses for the ρ and ω (which is the case in the soft-pion limit⁹) and in the limit of both ρ and ω being coupled to conserved currents do we obtain our result. This is a reflection of the fact that any $A \rightarrow \gamma \gamma$ decay is forbidden by gauge invariance and Bose statistics. Note however that unless we appeal directly to Bose statistics for the ρ and ω themselves, the on-shell coupling $g_2^{A\rho\omega}$ is not forbidden (X₂ being antisymmetric under the exchange of k and p). We should stress immediately that our derivation is open to one possible query, which is whether we can treat the demands of current conservation for each of A^3_{μ} , V^3_{λ} , and V^8_{ν} separately; i.e., whether we may disperse in each of q^2 , k^2 , and p^2 in turn while holding the other two fixed.

The point of this paper is not in fact to claim that there is a $\pi\rho\omega$ decoupling, but only to point out that a naive application of PCAC and VDM to the (A_{u}^{3}) V_{λ}^{3} , V_{ν}^{8}) vertex runs into trouble. Indeed, as has been stressed by Wilson,¹⁰ the vertex is controlled by the behavior of operator products of local currents at small separations, and is a highly singular object. Wilson has suggested that the gauge-field algebra is not capable of producing a singular enough behavior near the light cone, and we may infer that this is in part a kinematic effect once we treat the vector mesons as local operators. We are now studying the light-cone interpretation of the proposed modification of VDM given in Sec. III. We note that the light-cone considerations of Wilson are only relevant in the presence of two or more local operators, and hence VDM may be applied, at least in the small- k^2 region, to matrix elements of a single vector current. In the presence of two or more local currents (i.e., in $\pi\rho\omega$ or in the interference terms in polarized photoproduction of pions), VDM may not be applied even in the small- k^2 region, and specifically in such cases the simultaneous dependence on all the necessary mass variables is relevant.

III. SOME IMPLICATIONS OF THE RESULT

We feel that we must interpret our result as a failure of vector dominance. When an electromagnetic form factor, $f^{XY}(k^2)$, is not dominated by a single meson there are two obvious possibilities. One is that the form factor simply requires subtractions in k^2 . This is undesirable as we then have no knowledge of how many subtractions to make or how we may calculate the subtraction constants. Alternatively there may be other states (i.e., more mesons or a continuum) also contributing. Following our knowledge of the $k^2 \rightarrow -\infty$ behavior of the nucleon form factor we shall only consider this latter possibility. The simplest solution which gives rapidly falling form factors is to take a set of poles, i.e.,

$$f^{XY}(k^2) = \sum_{i} \frac{m_i^2 f_i g_i^{XY}}{m_i^2 - k^2}$$
(16)

such that

$$f^{XY}(0) = \sum_{i} f_{i} g_{i}^{XY},$$

$$0 = \sum_{i} m_{i}^{2} f_{i} g_{i}^{XY},$$

$$0 = \sum_{i} m_{i}^{4} f_{i} g_{i}^{XY},$$
(17)

and so on for the higher moments. Here the set $\{i\}$ of mesons have couplings f_i to the current and g_i^{XY} to the hadrons. For electric form factors $f^{XY}(0)$ is the electric charge when X = Y and is zero otherwise.⁶ The more mesons we include the more superconvergent we can make $f^{XY}(k^2)$. Taking a clue from duality we might even expect a whole set $\{\rho, \rho', \ldots\}$ of parallel daughter trajectories to couple in. (Of course the author claims no originality for most of the above remarks.) From the theoretical viewpoint there appears to be nothing against this possibility, though as of yet its formulation is not particularly adequate. In simple cases like the pion electric form factor we are able to relate all the $g_i^{\pi\pi}$, but there seems to be no guiding principle for calculating the f_i . Since the pion form factor is Hermitian we may infer that all the f_i are relatively real, at least in the narrow-resonance approximation to Eq. (16), since the $g_i^{\pi\pi}$ are in phase. However, superconvergence can only be guaranteed if there are sign changes, and we have no way

of calculating the signs. From the experimental viewpoint there are also difficulties as the ρ' is notorious in that it has not been observed. However, we feel that this is a consequence of unitarity and shall elaborate on this in more detail.

Unitarity has an influence both in the production process and in the subsequent ρ' decay. We may produce the ρ' in $\gamma p - \pi^+ \pi^- p$ or in the collidingbeam experiment $e^+e^- \rightarrow \pi^+\pi^-$. The photoprocess is a pseudoelastic diffraction dissociation. There is now a reasonable amount of evidence to suggest a falloff in the diffractive amplitude as we go to higher ω^2 (= $m_{\pi\pi}^2$). This is presumably a unitarity effect, in the sense that only in the fully elastic process (with $\omega^2 = 0$) do we feel the full impact of the unitarity shadow. As we go to higher ω^2 the feedback from the inelastic channels in the unitarity sum falls because we are going further away from pure elasticity. So we expect the diffractive cross section to vary as some inverse power of ω^2 . Unfortunately from unitarity alone it is very difficult to predict the specific power. Various models have been constructed¹¹ to calculate the power, and they suggest ω^{-4} . Such a factor has the effect of cutting the higher- ω^2 contributions in the ρ mass region causing the cross section of $\gamma p \rightarrow \pi^+ \pi^- p$ to peak some 30 MeV below the standard ρ mass value, in rough agreement with experiment If we then assume that this factor simulates the effect of unitarity for all ω^2 , we deduce that

$$\frac{\sigma(\gamma p - \rho' p)}{\sigma(\gamma p - \rho p)} = \left(\frac{m_{\rho}}{m_{\rho'}}\right)^4 = \frac{1}{9}$$
(18)

so that the $\rho'(\rho'',...)$ cross sections are very much damped. This loss in intensity is also consistent with the apparent absence of the g meson in photoproduction experiments.

The other way that unitarity has an effect is in the decay $\rho' \rightarrow \pi\pi$, i.e., through the presence of many open decay modes. Not only do we not know the branching ratio, but also another severe difficulty may be the full width. It has been suggested⁹ that this may be of the order of 2 $(\text{GeV}/c)^2$, which would then make it impossible to pick up the ρ' above any nonresonating background. Further in the colliding-beam experiment the rate for ρ' production also depends on $f_{\rho'}^2$, which is still unknown. Most of these considerations also afflict possible ω -like daughters. One possible piece of encouragement is to note that the A_2 is usually seen as split in its charged states, but not in its neutral one.¹² In such cases there is also the possible presence of ω' production which may be filling in between two A_2^0 peaks.

Now once we are granted the presence of these other mesons then our selection rule $g_{\pi\rho\omega}=0$ may be relaxed. In fact, a small violation of ρ and ω

dominance can make $g_{\pi\rho\omega}$ as large as we like as its value becomes very sensitive to $\{\rho_{i}, \omega_{j}\}$ interference terms and to the absolute value of the other $g_{\pi\rho_{i}\omega_{j}}$. Thus our selection rule is a far more sensitive test of VDM than its use even in $\pi^{0} \rightarrow \gamma\gamma$ and elsewhere, where VDM can still be reliable. We suspect that exactly the same effect is present in applications of VDM to polarized photoproduction of pions. As we discussed in Ref. 6, VDM for the Ball amplitudes yields VDM in the helicity frame at large s, giving the standard predictions

$$\sigma_{\perp}(\gamma N \to \pi N) = f_{\rho}^{2}(\rho_{11}^{H} + \rho_{1-1}^{H})\sigma(\pi^{-}p \to \rho^{0}n) ,$$

$$\sigma_{\parallel}(\gamma N \to \pi N) = f_{\rho}^{2}(\rho_{11}^{H} - \rho_{1-1}^{H})\sigma(\pi^{-}p \to \rho^{0}n) , \qquad (19)$$

$$\sigma_{\rm un} = \frac{1}{2}(\sigma_{\perp} + \sigma_{\parallel}) = f_{\rho}^{2}\rho_{11}^{H}\sigma(\pi^{-}p \to \rho^{0}n) .$$

Experimentally the unpolarized prediction is fitted satisfactorily while the polarized predictions fail. It is precisely in the polarized cross sections that we will feel most strongly any $\{\rho_i, \rho_j\}$ interference terms, making this a sensitive test of VDM. The unpolarized cross sections will be effectively diagonal in the ρ_i so that σ_{un} is modified to

$$\sigma_{\rm un} = \sum_{i} f_i^2 \rho_{11}^{Hi} \sigma(\pi^- p - \rho_i^0 n) \,. \tag{20}$$

Now if the ρ_i really are the daughters of the ρ then by exchange degeneracy all the cross sections in Eq. (20) will have exactly the same t behavior, so that this successful feature of the fit is maintained. As for the scale of the two sides of Eq. (20), a lot depends on how much of the measured f_{ρ} in $e^+e^- \rightarrow \pi^+\pi^-$ is really due to the ρ and how much is due to background effects coming from, say, a 2 (GeV/c)²-wide ρ' . Thus the "standard value" of f_{ρ} could already be a smear, and the successes of VDM so far could have resulted because we have only kept the ρ but used this smeared value which then simulates the effects of the other ρ_i in the small- k^2 region.

Our selection rule on the $\pi\rho\omega$ system may have an amusing consequence in pion photoproduction. All the purely strong interactions such as $\pi N \rightarrow VN$ exhibit an energy behavior typical of the exchange of the leading natural-parity trajectories. On the other hand, $\gamma N \rightarrow \pi N$ has $\alpha_{eff}(t) = 0.^{13}$ Our rule may imply a decoupling of the ρ and ω trajectories from the photoprocess in which we make a sum on i and j, with each individual i(j) amplitude not being constrained. Then by exchange degeneracy all the leading trajectories will approximately decouple from photoproduction of pions (consistent with the observed absence of the main nucleon resonances in forward-region photoproduction¹³), while no constraint at all is obtained in any of the related strong interactions $\pi N \rightarrow V_i N$. This is also consistent with the failure of the prediction for σ_1 ,¹⁴ as it

only couples to natural-parity exchanges. Once this possibility is granted then the energy dependence must be dominated by the unnatural-parity trajectories or a fixed pole generated, say, by the gauge-invariant electric Born term to give the observed $\alpha_{eff}(t)$. Of course our selection rule strictly applies only at the pole so we would not expect it to be valid over the whole t region. Since we would like to retain some interpretation of the dip, no-dip structure,¹³ we would expect that our effect would have been lost by t = -0.5 (GeV/c)². However, by then the energy dependence of the leading trajectories has also dropped to $\alpha_{eff}(t) = 0$, and also we note that fits to σ_1 are acceptable in this region.¹⁴

In conclusion we must state that we are well aware that most of the remarks of this section are at best speculative or highly qualitative. We present them only because we feel that they may merit further consideration once we have a more detailed understanding of the nucleon form factor.

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Errata

Theory of $\eta - \pi$ Mixing with Applications to Meson Decays, Laurie M. Brown, Nilendra G. Deshpande, and Frank A. Costanzi [Phys. Rev. D4, 146 (1971)]. The value of ϵ_4 in the Appendix was deduced incorrectly from Eq. (A9). The correct value for the case $M = (-4.3 \pm 1.5) \times 10^3$ MeV² is $\epsilon_4 = 15 \pm 5$; for $M = (9.1 \pm 3) \times 10^3$ MeV² it is $\epsilon_4 = -30 \pm 10$. This gives decay rates [Eqs. (5) and (6)]: $\Gamma(\varphi \rightarrow \eta_{\gamma}) = 4 \pm 2$ keV (instead of 10 keV), $\Gamma(\omega \rightarrow \eta_{\gamma}) = 49 \pm 4$ keV (instead of 40 keV). Other parts of our paper are not affected. We wish to thank Professor Paul Singer for calling this error to our attention. We also take this opportunity to point out that Dr. David Greenberg [Phys. Rev. <u>178</u>, 2190 (1969)] has also considered pole models for κ and η decays. Fixed Pole in the Virtual Compton Amplitude A_2 , R. Rajaraman and G. Rajasekaran [Phys. Rev. D3, 266 (1971)]. Equation (22) should read as follows:

$$\tilde{v}_{1}(q^{2},\nu) + q^{2}\tilde{v}_{2}(q^{2},\nu)$$

$$\equiv v_{1} + q^{2}v_{2} - \sum_{i}(-i\pi)\frac{\beta_{i}(q^{2})}{q^{2}}\nu^{\alpha_{i}}\frac{(e^{i\pi\alpha_{i}}\pm 1)}{\sin\pi\alpha_{i}}.$$
(22)

Field-Theoretic Model for Low-Energy $J = \frac{3}{2}^+ K^+ p$ Scattering, L. B. Rédei [Phys. Rev. D 3, 1650 (1971)]. In the Acknowledgments, Nathor Foundation should read Nathorst Foundation. The author is indebted to Professor L. Hulthén for pointing out to him the error in spelling.