[13]Let $m, n = 1, 0, -1$. Then $D(\psi,\chi)^3{}_m = D(\psi,\chi)^m{}_3 = 0$, $D(\psi,\chi)^3{}_3 = 1$, $D(\psi,\chi)^m{}_n = e^{-im\chi}\bar{d}(\psi)^m{}_n$, and

$$\bar{d}(\psi)^m{}_n = \begin{pmatrix} \frac{1}{2}(\cosh\psi+1) & -\sinh\psi/\sqrt{2} & -\frac{1}{2}(\cosh\psi-1) \\ -\sinh\psi/\sqrt{2} & \cosh\psi & \sinh\psi/\sqrt{2} \\ -\frac{1}{2}(\cosh\psi-1) & \sinh\psi/\sqrt{2} & \frac{1}{2}(\cosh\psi+1) \end{pmatrix}.$$

[14]F. E. Low, Phys. Rev. 120, 582 (1960).

[15]W. A. Bardeen and Wu-Ki Tung, Phys. Rev. 173, 1423 (1968).

[16]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[17]For a recent summary of experimental data, see E. Bloom *et al.*, in paper presented to the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R. (unpublished).

[18]K. M. Watson, Phys. Rev. 88, 1163 (1952).

[19]We note two relevant facts on this point: (i) The next higher partial wave is the D wave, the P wave being forbidden by charge conjugation; (ii) all available estimates of S and D π-π phase shifts support this expectation. On the second point, see for instance D. Morgan and G. Shaw, Phys. Rev. D 2, 520 (1970), and the many contributions to the Proceedings of the Conference on $\pi\pi$ and πK Interactions, Argonne National Laboratories, 1969 (unpublished).

[20]The precise formula for $a^{(B)}$, Eq. (42), has a cut from $-\infty$ to 0 with discontinuity proportional to $(s-4\mu^2)^{1/2}/s^{1/2}$.

[21]Assuming $\delta^I(s) = \pi(s-4\mu^2)/(s+16\mu^2)$ (corresponding to a broad resonance at $s^{1/2} \simeq 5\mu$), we found $-\Delta^I(-c) = 0.2$. In channels where there are no low-energy resonances, it should be much smaller than this.

[22]After the completion of this work, we noticed a report by D. H. Lyth on the process $\gamma\gamma \to \pi\pi$. His general approach was very similar to ours. In particular, he presented some numerical estimates which support the approximations which led to the dispersion relation (46), and also wrote down our Eq. (48). However, he approximated this equation by neglecting the dependence of $\exp[\Delta^I(s)]$ on s and obtained an expression which holds for small values of s where $\delta^I(s)$ remains very small. This may render his subsequent inversion formulas unreliable because they involve integrating over the full range of s. This is true especially if a broad resonance is present near the elastic unitarity region as is believed to be the case. Our results relating $|a^I(s)|$ to $\delta_0^I(s)$, Eqs. (52) and (53), do not suffer from this difficulty.

[23]See the references quoted at the end of footnote 19.

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Weak-Electromagnetic K-Meson Decays

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We discuss the contributions of the combined first-order-weak plus higher-order-electromagnetic effects to several K-meson decays. It is shown that if one uses a chiral SU(3) ×SU(3) pole model to explain both $K_S \to 2\pi$ and $K_L \to 2\gamma$ one needs an effective nonleptonic interaction Lagrangian which contains $\underline{8}$ and $\underline{27}$, $\Delta I = \frac{1}{2}$ pieces. This gives results for $K^+ \to \pi^+ l\bar{l}$ compatible with the experimental data. We also calculate within this model the real part of the $K_L \to l\bar{l}$ amplitude, which results in a very small contribution to the decay rate.

I. INTRODUCTION

In the last few years the rare decays $K_L^0 \to 2\gamma$, $K_L^0 \to l\bar{l}$, and $K^+ \to \pi^+ l\bar{l}$ have become more accessible to experiment. Calling $R(K\to A)$ the fraction for the decay $K \to A$, recent data give[1]

$$R(K_L^0 \to 2\gamma) = 5.2\times10^{-4}, \quad (1.1a)$$

$$R(K^+ \to \pi^+\mu^+\mu^-) < 2.4\times10^{-6}, \quad (1.1b)$$

$$R(K^+ \to \pi^+e^+e^-) < 0.4\times10^{-6}, \quad (1.1c)$$

and for $R(K_L^0 \to l\bar{l})$ we quote the results:

$$R(K_L^0 \to \mu^+\mu^-) < 2.6\times10^{-8}, \quad (1.2)$$

obtained by Darriulat *et al.*,[2] and

$$R(K_L^0 \to \mu^+\mu^-) < 1.82\times10^{-9}, \quad (1.3)$$

measured by Clark *et al.*[3] The last authors also obtained

$$R(K_L^0 \to e^+e^-) < 1.57\times10^{-9}. \quad (1.4)$$

Theoretically the decays $K_L^0 \to l\bar{l}$ and $K^+ \to \pi^+ l\bar{l}$ could proceed through at least three mechanisms:

(a) second-order semileptonic weak interaction;

(b) possible weakly coupled neutral lepton currents;

(c) combined first-order-nonleptonic-weak and higher-order-electromagnetic interactions (WE).

On the other hand, the decay $K_L^0 \to \gamma\gamma$ would proceed predominantly through (c). Since the data are already in the neighborhood of the predictions obtained considering only WE effects, it would be useful to have at least that part of the amplitude for the processes mentioned above. This would help to determine the possible interference of non-electromagnetic contributions.[4] In this paper we will restrict our attention to an estimate of the WE contribution. The basic Feynman diagrams for the processes we consider are shown in Fig. 1. It is clear that to compute diagram (b) we need a model for the behavior of the $K_L^0 \to \gamma\gamma$ form factor for off-shell γ's. Therefore we shall first present our approach to that decay. We shall use a pole model which includes several ingredients:

First, to describe the strong-interaction effects we shall use the effective-Lagrangian approach to broken chiral SU(3)×SU(3). This we discuss in Sec. II. Secondly, we assume the electromagnetic current is dominated by vector mesons and that the decays of pseudoscalar mesons into two photons proceed through PVV couplings, as explained in Sec. III. Finally, as discussed in Sec. IV, we need a model for the weak nonleptonic strangeness-changing couplings. For this purpose we generalize a phenomenological octet Lagrangian used by Sakurai[5] by adding a 27, $\Delta I = \frac12$ piece. We then apply the weak interaction and the prescriptions of Secs. II and III to the calculation of the rate $K_L^0 \to \gamma\gamma$ for the purpose of determining the possible admixture of the 27 representation in the weak Lagrangian, and we make some remarks about the $K_L^0 \to \gamma\gamma$ calculations of other authors.

In Secs. V and VI we present our calculation of $K^+ \to \pi^+ l\bar{l}$ and $K_L^0 \to l\bar{l}$, respectively, and finally in Sec. VII we summarize our results.

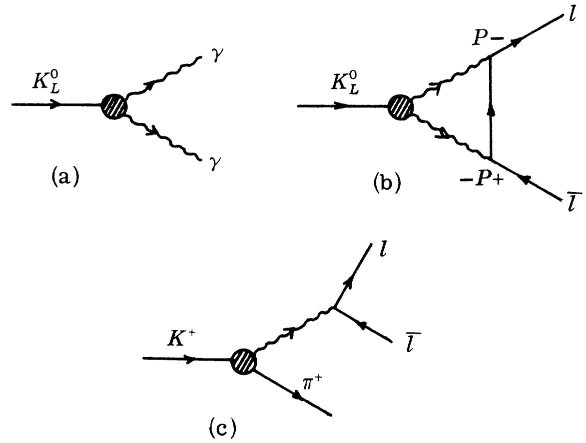

Fig. 1. Feynman diagrams for the weak electromagnetic contribution to $K_L^0 \to \gamma\gamma$, $K_L^0 \to l\bar{l}$, and $K^+ \to \pi^+ l\bar{l}$.

II. CHIRAL SU(3) × SU(3) EFFECTIVE LAGRANGIAN

The charges of the hadronic currents that are usually assumed to enter in the weak and electromagnetic interactions are believed to obey Gell-Mann's[6] chiral SU(3)×SU(3) commutation relations. The matrix elements of these currents are assumed to be dominated[7] by $J^P = 1^+$, 1^-, and 0^- meson poles, and a consistent way to treat these features is provided by the field-algebra hypothesis.[8] As is well known, when one uses the predictions of a field-algebra Lagrangian computed in the "tree approximation" one obtains results equivalent[9] to those of the hard-meson analysis[10] of matrix elements of currents. The effective Lagrangian we shall use is the following one[11]:

$$\mathcal{L} = -\tfrac14 \mathrm{Tr}(K V_{\mu\nu}V^{\mu\nu} + K A_{\mu\nu}A^{\mu\nu}) + \tfrac12 m^2 \mathrm{Tr}(V_\mu V^\mu + A_\mu A^\mu) - (1/2\sqrt{2}) K^{80}\mathrm{Tr}(\lambda_8 V^{\mu\nu})V^0_{\mu\nu} - \tfrac14 K^{00} V^0_{\mu\nu}V^{\mu\nu,0}$$
$$+ \tfrac12 m^2 V^0_\mu V^{\mu,0} + \tfrac12 \mathrm{Tr}(\pi_\mu\pi^\mu + \sigma_\mu\sigma^\mu) + \mathrm{Tr}(f\sigma) + (1/4m^2) g\delta\, \mathrm{Tr}[-iV_{\mu\nu}(\sigma^\mu\sigma^\nu+\pi^\mu\pi^\nu) + A_{\mu\nu}(\sigma^\mu\pi^\nu+\pi^\nu\sigma^\mu)]\,. \qquad (2.1)$$

Except for the SU(3)-singlet vector-meson field V_μ^0, all other fields in $\mathcal{L}$ are SU(3) matrices. V_μ and A_μ are traceless matrices for the $J^P = 1^-$ and 1^+ octet gauge fields, respectively, and they form together a $(1,8)+(8,1)$ representation of chiral SU(3)×SU(3). π and σ denote the $J^P = 0^-$ and 0^+ nonet fields that form the $(3^*,3)+(3,3^*)$ representation. We shall adopt a nonlinear model in which the 0^+ nonet and the pseudoscalar singlet are functions of the independent pseudoscalar octet fields.[12] This functional dependence is realized through the constraints

$$\sigma^2 + \pi^2 = F^2 \qquad (2.2a)$$

and

$$\det(\sigma + i\pi) = F^3\,. \qquad (2.2b)$$

Moreover, in Eq. (2.1) the matrices $V_{\mu\nu}$, $A_{\mu\nu}$, π_μ, and σ_μ are defined as

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - (ig/\sqrt{2})([V_\mu, V_\nu] + [A_\mu, A_\nu])\,, \qquad (2.3a)$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - (ig/\sqrt{2})([V_\mu, A_\nu] - [V_\nu, A_\mu])\,, \qquad (2.3b)$$

$$\pi_\mu = \partial_\mu \pi - (ig/\sqrt{2})[V_\mu, \pi] - (g/\sqrt{2})\{A_\mu, \sigma\}\,, \qquad (2.3c)$$

$$\sigma_\mu = \partial_\mu \sigma - (ig/\sqrt{2})[V_\mu, \sigma] + (g/\sqrt{2})\{A_\mu, \pi\}\,. \qquad (2.3d)$$

Finally, we define the symmetry-breaking matrices

$$K = \underline{1} + \sqrt{3}\,\epsilon_0 \lambda_8 \tag{2.4a}$$

and

$$f = (1/\sqrt{2})(f_0 \lambda_0 + f_8 \lambda_8) . \tag{2.4b}$$

The matrix K produces the usual kinetic-energy breaking for the masses of the vector and axial-vector mesons, while the coefficients K^{80} and K^{00} are related to the ω-φ mixing.[13-17] When relations (2.2) are used, the term containing the matrix f gives rise to pseudoscalar masses obeying the Gell-Mann–Okubo mass formula. As is well known, the term with δ in the Lagrangian is needed to fit the ρ width.

From the Lagrangian (2.1) one obtains straightforwardly the field-current identities

$$J_\mu^V = -(m^2/g) V_\mu , \tag{2.5a}$$

$$J_\mu^A = -(m^2/g) A_\mu . \tag{2.5b}$$

Only the nonstrange components of J_μ^V are conserved. For the axial-vector current J_μ^A we have the PCAC (partial conservation of axial-vector current) relation

$$\langle 0 | \partial^\mu J_\mu^A(0) | P^a \rangle = m_a{}^2 c_a / (2p_0)^{1/2} , \tag{2.6}$$

with $c_\pi = c_K = c_\eta$. From Eqs. (2.2a) and (2.3c) one obtains the SU(3)-symmetric axial-vector–pseudoscalar mixing which is removed by the substitution

$$A_\mu^a = (1/\sqrt{K_A}) \mathcal{A}_\mu^a + (g c_\pi / m^2)(\partial_\mu P^a + g f^{abc} V_\mu^b P^c) , \tag{2.7}$$

where $P^a = \pi^a [1 - (c_\pi{}^2 g^2 / m^2)]^{1/2}$ and $\mathcal{A}_\mu^a$ are the renormalized interpolating fields for the 0^- and 1^+ meson octets.

The masses of the axial-vector mesons are given by

$$m_a{}^2 = m_A{}^2 / K_a , \tag{2.8a}$$

$$m_A{}^2 = \frac{m^2}{1 - (c_\pi{}^2 g^2 / m^2)} , \tag{2.8b}$$

where the K_a are the elements of the diagonal matrix K of Eq. (2.4a). Relation (2.8b) is the Lagrangian analog of the one obtained by Weinberg.[18] We identify the axial-vector mesons with the $A_1(1070)$, $D(1285)$, and $K_A(1240)$.

Taking into account the ω-φ mixing, we get for the interpolating 1^- meson fields

$$V_\mu^{1,2,3} = \frac{1}{\sqrt{K_\rho}} \rho_\mu^{1,2,3} , \quad V_\mu^{4,5,6,7} = \frac{1}{\sqrt{K_{K^*}}} K^{*4,5,6,7} , \tag{2.9a}$$

$$V_\mu^8 = -\frac{\sin\theta}{\sqrt{K_\omega}} \omega_\mu + \frac{\cos\theta}{\sqrt{K_\varphi}} \varphi_\mu , \quad V_\mu^0 = \frac{\cos\theta}{\sqrt{K_\omega}} \omega_\mu + \frac{\sin\theta}{\sqrt{K_\varphi}} \varphi_\mu ,$$

with

$$m_i{}^2 = m^2 / K_i \quad (i = \rho, K^*, \omega, \varphi)$$

and (2.9b)

$$m = 847 \text{ MeV} , \quad \theta = 27.5^\circ , \quad \epsilon_0 = 0.18 .$$

We have in this model $K_\rho = K_{A_1}$ and $K_{K^*} = K_{K_A}$.

III. PVV INTERACTION

An effective interaction vertex (PVV) involving one pseudoscalar and two vector mesons plays an important role in the vector-dominance picture[19] of production and decay processes of mesons. The PVV interaction cannot be written as part of a chiral-invariant Lagrangian but can be introduced in a $(3^*, 3) + (3, 3^*)$-type breaking term.[20] This term leads to a consistent parametrization of the known strong radiative meson decays as discussed in Ref. 17. We shall adopt this form of the PVV interaction which is given by

$$\mathcal{L}_{PVV} = \tfrac{1}{4} \epsilon^{\alpha\beta\mu\nu} (h D^{abc} V_{\alpha\beta}^a V_{\mu\nu}^b P^c + \lambda D^{ab} V_{\alpha\beta}^0 V_{\mu\nu}^a P^b) , \tag{3.1}$$

with

$$D^{abc} = d^{abc} + \sqrt{3}\,\epsilon_1 d^{abd} d^{d8c} + \tfrac{1}{2}\sqrt{3}\,\epsilon_2 (d^{acd} d^{d8b} + d^{bcd} d^{d8a}) + (1/\sqrt{3})\epsilon_3 \delta^{ab} \delta^{c8} ,$$

and

$$D^{ab} = \delta^{ab} + \sqrt{3}\,\epsilon_4 d^{ab8} .$$

In Ref. 17 it was found that

$$g^2/4\pi = 3.35 , \quad (h^2 m_\pi{}^2 / 4\pi)(1 + \epsilon_1)^2 = 0.10 ,$$
$$\lambda = -(2h/\sqrt{3}) \cot\theta \left(\frac{1 + \epsilon_1}{1 + \epsilon_4} \right) , \tag{3.2}$$

and that there are two sets of values for the ϵ's:

$$\text{(a)} \quad \epsilon_1 = 0.77 , \quad \epsilon' \equiv \epsilon_2 + \epsilon_3 = 2.1 , \quad -0.3 < \epsilon_4 < 6 , \tag{3.3a}$$

and

$$\text{(b)} \quad \epsilon_1 = 1.3 , \quad \epsilon' \equiv \epsilon_2 + \epsilon_3 = -2.7 , \quad \epsilon_4 > 0.2 \text{ or } \epsilon_4 < -3.3 . \tag{3.3b}$$

It is interesting that with m and g given by Eqs. (2.9b) and (3.2), respectively, and $c_\pi \cong 94$ MeV, we obtain $c_\pi{}^2 g^2 / m^2 = 0.51$, which is analogous to the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation.[21] From Eq. (2.8b) it follows that $m_A{}^2 \cong 2m^2$.

We shall use for our calculations the formalism of vector-meson dominance of the hadronic electromagnetic current developed by Kroll, Lee, and Zumino.[14,15] This leads, in the absence of weak interactions, to the effective coupling

$$\mathcal{L}_{\rm em}^{(S)} = e[J_\mu^{V(3)} + (1/\sqrt{3})J_\mu^{V(8)}]\phi^\mu$$
$$= -e(m^2/g)[(1/\sqrt{K_\rho})\rho_\mu^0 - (3K\omega)^{-1/2}\sin\theta\,\omega_\mu + (3K_\varphi)^{-1/2}\cos\theta\,\varphi_\mu]\Phi^\mu\,, \qquad (3.4)$$

where Φ^μ is the electromagnetic field.

IV. NONLEPTONIC WEAK INTERACTION

As a model for the strangeness-changing nonleptonic (NL) interaction we shall adopt a modified version of an octet Lagrangian used by Sakurai[5] in a pole approximation and by Hara and Nambu[22] in the current-algebra approach. Their interaction Lagrangian has the form

$$\mathcal{L}_{\rm NL} = 2\frac{G_{\rm NL}}{\sqrt{2}} d^{6ab} J_\mu^a J^{\mu(b)}\,, \qquad (4.1)$$

where

$$J_\mu^a = J_\mu^{V(a)} + J_\mu^{A(a)}\,. \qquad (4.2)$$

By using vector-meson dominance and PCAC, Sakurai was able to reproduce Hara and Nambu's relations among the different K-meson decay amplitudes and to fit the $K_1^0 \to 2\pi$ and the S-wave nonleptonic baryon decays, obtaining for $G_{\rm NL}$ the values

$$G_{\rm NL} = 1.1\times 10^{-5}/m_p^{\,2} \qquad (4.3a)$$

and

$$G_{\rm NL} = 1.4\times 10^{-5}/m_p^{\,2}\,, \qquad (4.3b)$$

respectively.

The Lagrangian (4.1) has been used in several calculations of the rate for $K_L^0 \to 2\gamma$. In the works of Greenberg[23] and of Savoy and Zimerman[24] there are contributions from axial-vector meson poles obtained from $A \to \gamma\gamma$ amplitudes which, in fact, violate gauge invariance. In Rockmore's[25] calculation the axial-vector contributions are absent. He finds that $G_{\rm NL}$, given by Eq. (4.3), is about five times too large to fit $K_L^0 \to \gamma\gamma$. It is important to point out that in all of the mentioned calculations there are large contributions from a K^* pole diagram which contains a direct K^*-γ transition. We found, instead, as shown below, that such a transition vanishes for a real photon due to gauge invariance and that the only contributions come from π^0 and η poles. Even with only these two contributions the rate turns out to be too large when the weak interaction defined by Eqs. (4.1) and (4.3) is used. However, it should be noted that the results obtained in the pole approximation with La-

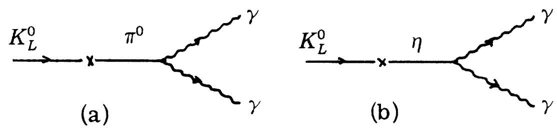

FIG. 2. Pole diagrams contributing to $K_L^0 \to 2\gamma$.

grangian (4.1) for $K_1^0 \to 2\pi$, $K \to 3\pi$, and S-wave baryonic decays,[5,26] except for electromagnetic corrections, are not affected if we employ instead a Lagrangian which includes a term belonging to the $\underline{27}$ representation of SU(3) with the restriction $\Delta I = \frac{1}{2}$. This Lagrangian has the form

$$\mathcal{L}_{\rm NL} = 2(G_{\rm NL}/\sqrt{2})[\tfrac{1}{4}(3+\beta)d^{6ab} + (1-\beta)d^{6al}d^{8lb}]J_\mu^a J^{\mu(b)}\,, \qquad (4.4a)$$

or equivalently

$$\mathcal{L}_{\rm NL} = (G_{\rm NL}/\sqrt{2})\{J_\mu^{(4-i5)}J^{\mu(1+i2)} + {\rm H.c.} - 2J_\mu^6[J^{\mu(3)} + (\beta/\sqrt{3})J^{\mu(8)}]\}\,, \qquad (4.4b)$$

where the hadronic currents are given in this model by

$$J_\mu^a = -\frac{m^2}{g}\left[V_\mu^a + \frac{\mathcal{A}_\mu^a}{\sqrt{K_a}} + \frac{gc_\pi}{m^2}(\partial_\mu P^a + gf^{abc}V_\mu^b P^c)\right]. \qquad (4.4c)$$

From Eq. (4.4b) it is clear that the $\underline{27}$, $\Delta I = \frac{1}{2}$ piece (i.e., $\beta \neq 1$) only affects the coupling $J_\mu^6 J^{\mu(8)}$, which does not enter in the above-mentioned decays unless one considers corrections due to electromagnetic effects, e.g., an η-π^0 transition.[27,28]

The coupling $J_\mu^6 J^{\mu(8)}$ plays, instead, an important role in the $K_2^0 \to \gamma\gamma$ decay.[29] Figure 2 shows the diagrams that contribute to this decay in the tree approximation. We do not have any contribution from a pole diagram with the sequence $K_2^0 \to \gamma K^*$ and $K^* \to \gamma$. This is a consequence of the extension of the gauge-invariant formalism of Refs. 14 and 15 to the total Lagrangian that now includes the weak interaction (4.4). In this case the effective electromagnetic interaction (3.4) is modified to the form

$$\mathcal{L}_{\rm em} = e\left[J_\mu^{V(3)} + \frac{1}{\sqrt{3}}J_\mu^{V(8)} - \frac{2G_{\rm NL}}{\sqrt{2}}\frac{m^2}{g^2}\left(1+\frac{\beta}{3}\right)J_\mu^{(6)}\right]\Phi^\mu\,. \qquad (4.5)$$

The additional last term produces a K^*-γ transition, shown in Fig. 3, of the form

$$\langle K^*|\gamma\rangle = 2\frac{G_{\rm NL}}{\sqrt{2}}\frac{em^4}{g^3}e_\mu(K^*)e^\mu(\gamma)\left[\left(1+\frac{\beta}{3}\right) + \frac{m_\rho^{\,2}}{k^2 - m_\rho^{\,2}} + \frac{\beta}{3}\left(\frac{m_\omega^{\,2}\sin^2\theta}{k^2 - m_\omega^{\,2}} + \frac{m_\varphi^{\,2}\cos^2\theta}{k^2 - m_\varphi^{\,2}}\right)\right], \qquad (4.6)$$

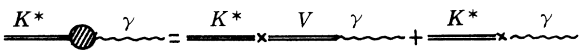

FIG. 3. Off-shell K^*-γ transition.

which clearly vanishes for a real photon.

Expressing the invariant amplitude for the decay of a $J^{PC}=0^{-+}$ meson P into two photons as

$$F(P)\epsilon_{\alpha\beta\mu\nu}e^{\alpha}(\gamma_1)k_1^{\beta}e^{\mu}(\gamma_2)k_2^{\nu}\,, \tag{4.7a}$$

with the decay rate given by

$$\Gamma(P\to\gamma\gamma)=\frac{m_p^{\,3}}{64\pi}\,|F(P)|^2\,, \tag{4.7b}$$

we get for $K_2^0\to\gamma\gamma$

$$F(K_L^0)=\sqrt{2}\,G_{\rm NL}\,c_\pi^{\,2}\left(\frac{F(\pi^0)}{1-\dfrac{m_\pi^{\,2}}{m_K^{\,2}}}+\frac{\beta}{\sqrt{3}}\,\frac{F(\eta)}{1-\dfrac{m_\eta^{\,2}}{m_K^{\,2}}}\right). \tag{4.8}$$

From the experimental rates[1] we can obtain for the $F(P)$'s:

$$|F(\pi^0)|=(2.6\pm0.4)\times10^{-5}\ ({\rm MeV})^{-1}\,, \tag{4.9a}$$

$$|F(\eta)|=(3.5\pm0.5)\times10^{-5}\ ({\rm MeV})^{-1}\,, \tag{4.9b}$$

$$|F(K_L^0)|=(3.1\pm0.2)\times10^{-12}\ ({\rm MeV})^{-1}\,. \tag{4.9c}$$

Using $G_{\rm NL}=1.15\times10^{-5}/m_p^{\,2}$, obtained from fitting $K_1^0\to2\pi$, and $c_\pi=94$ MeV, we get from Eq. (4.8) the following values of β:

$$\beta=\pm\begin{cases}0.1\ \ \pm0.06\\ 0.49\pm0.12\,,\end{cases} \tag{4.10}$$

where the positive and negative values correspond, respectively, to assuming the same or the opposite sign for $F(\pi^0)$ and $F(\eta)$. It can be seen from (4.10) that the case $\beta=1$, which would correspond to a pure octet $\mathcal{L}_{\rm NL}$, is clearly excluded by the experimental data. On the other hand, one could assume $\beta=1$ and fix $G_{\rm NL}$ from $K_L^0\to\gamma\gamma$ alone, obtaining

$$G_{\rm NL}=\begin{cases}(0.32\pm0.1)\times10^{-5}/m_p^{\,2}\\ (0.18\pm0.04)\times10^{-5}/m_p^{\,2}\,,\end{cases} \tag{4.11}$$

where the upper and lower values correspond, respectively, to equal or opposite signs for $F(\pi^0)$ and $F(\eta)$. In this case the pole model could not account for the $K_1^0\to2\pi$ rate. It is amusing that the values of $G_{\rm NL}$, Eq. (4.11), are roughly equal to $G_F\sin\theta\cos\theta$, where G_F is the Fermi coupling constant and θ is the Cabibbo angle.

V. $K^+\to\pi^+l\bar{l}$ DECAYS

The decays $K^+\to\pi^+l\bar{l}$ receive contributions from weak electromagnetic processes[30] in which both the parameter β and the off-shell K^*-γ transition,

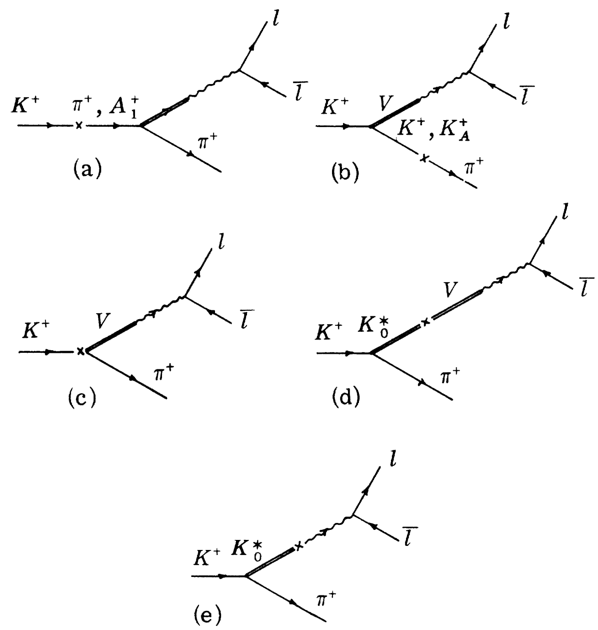

FIG. 4. Pole diagrams for $K^+\to\pi^+l\bar{l}$.

Eq. (4.6), play an important role. The corresponding pole-model diagrams are displayed in Fig. 4. Diagrams (a) and (b) contain the pseudoscalar and axial-vector pole terms arising from the bilinear weak-interaction Lagrangian (4.4), while the contact term in diagram (c) is required by gauge invariance and in our model appears directly because of the substitution of Eq. (4.4c) in (4.4a). Diagrams (d) and (e) represent the K^* contribution. In Appendix A we give the strong and weak couplings necessary for the calculation of the decay matrix element corresponding to the diagrams of Fig. 4. We found that the contribution from the axial-vector poles is small compared with that of the pseudoscalar poles throughout the final-state phase space, while the K^* contribution, which is the only one that depends on β, is of the same order. Table I shows the calculated branching ratios $R(K^+\to\pi^+l\bar{l})$ as functions of the parameter β. We can see from the table that for $\beta=1$, i.e., no 27, and with $G_{\rm NL}$ fixed by $K_1^0\to2\pi$, we obtain for the $\pi^+e^+e^-$ decay mode a value much larger than the experimental upper bound. Instead, any one of the negative β values of Eq. (4.10) yields, for both the π^+e^+e and $\pi^+\mu^+\mu^-$ modes, branching ratios below the experimental upper bounds. On the other hand, by using the $G_{\rm NL}$ of Eq. (4.11), fixed by $K_L^0\to2\gamma$ only, we also obtain values compatible with experiment.

VI. THE DECAY $K_L^0\to l\bar{l}$

The decay $K_L^0\to\mu^+\mu^-$ has recently attracted considerable attention because the latest reported

TABLE I. $K^{\pm}\to\pi^{\pm}l\,\bar{l}$ branching ratios and parameters G_{NL} and β of the weak, nonleptonic Lagrangian.

$10^5 G_{NL} m_p^2$	β	$10^6 R(K^{\pm}\to\pi^{\pm}\mu^+\mu^-)$	$10^6 R(K^{\pm}\to\pi^{\pm}e^+e^-)$
1.15	1	2.01	1.41
	0.49	1.24	0.80
	0.1	0.75	0.44
	−0.1	0.58	0.30
	−0.49	0.25	0.10
0.32	1	0.56	0.39
0.18	1	0.31	0.22

experimental upper bound[3] is significantly smaller than the lower bound obtained from unitarity.[31] The estimates of the imaginary part of the amplitude (neglecting CP nonconservation[32]) indicate that at least 80% of the unitarity bound is contributed by the two-photon intermediate state.[33] Therefore if we retain only this contribution both the real and imaginary parts of $K_2^0\to l\bar{l}$ will be given by the Feynman diagram of Fig. 1(b). Assuming that the $K_2^0\to\gamma\gamma$ amplitude is purely real,[34] one can relate the imaginary part of the diagram 1(b) to the physical $K_2^0\to\gamma\gamma$ amplitude as

$$\mathrm{Im}\,T(K_2^0\to\mu^+\mu^-)=\frac{\alpha}{8}\frac{m_K}{A}\ln\frac{1+A}{1-A}F(K_2^0)\,, \qquad (5.1)$$

where $A=(1-4m_\mu^2/m_K^2)^{1/2}$, and the rate is given by

$$\Gamma(K_2^0\to\mu^+\mu^-)=\frac{m_\mu^2}{2\pi m_K}A\,|T|^2\,. \qquad (5.2)$$

Using only the imaginary part of T given by (5.1) and $F(K_2^0)$ from Eq. (4.9), one obtains from (5.2) a lower bound for the branching ratio

$$R(K_2^0\to\mu^+\mu^-)>6.21\times10^{-9}\,. \qquad (5.3)$$

To date there has been no definite explanation of the discrepancy between the result (5.3) and experiment. In any case it is important to know the additional contribution from the real part of the amplitude. This contribution comes from the Feynman integral over the virtual-photon states in Fig. 1(b) which gives a logarithmic divergence when the $K_2^0\gamma\gamma$ vertex-momentum dependence is given by (4.7a) with $F(K_2^0)$ constant. The integral has been estimated by Sehgal[31] and by Quigg and Jackson,[35] the former using a cutoff and the latter incorporating vector-meson dominance but without detailed reference to the phenomenological features of the PVV vertex or the weak interaction. Both of these calculations found that the contribution from the real part is an increasing function of the cutoff mass and that when this is of the order of the vector-meson masses, the branching ratio is about 20%,[35] or more,[31] larger than the unitarity lower bound.

In general, the Feynman integral for diagram (b) of Fig. 1 can be written as

$$T=\frac{e^2}{(2\pi)^4}\int F(p,k)\epsilon^{\mu\nu\alpha\beta}p_\alpha k_\beta[\bar{u}(p_-)\gamma_\mu(\not{k}-\not{p}-m_\rho)^{-1}\gamma_\nu v(p_+)]\frac{d^4k}{(k^2+i\epsilon)[(p-k)^2+i\epsilon]}\,, \qquad (5.4)$$

where $F(p,k)=F(K_2^0)$ for $k^2=(p-k)^2=0$. Our model provides a functional form for $F(p,k)$ off the mass shell which is described graphically in Fig. 5. The dominance of the form factor by the vector mesons makes the integral convergent, as found by Quigg and Jackson.[35] The numerical contributions of the different diagrams are taken in our calculation from the PVV effective interaction, Eq. (3.1), and from the weak-interaction model of Eq. (4.4), which provide the additional K^* contribution shown in diagrams (b) and (c) of Fig. 5. In Appendix B we give details of the calculation. The various couplings that appear in the K_2^0 form factor $F(p,k)$ depend on β and on the PVV breaking parameters ϵ_i; ϵ_1 and ϵ' are given in Eqs. (3.3), while values $\epsilon_2=12.4$ and $\epsilon_2=16.4$ corresponding, respectively, to solutions (a) and (b) for ϵ_1 have been obtained from the K^+-K^0 mass difference[36] and the $K^*\to K\pi\pi$ decay.[37] Recently, the value $\epsilon_4=2.8\pm0.6$, which falls in the middle of the allowed range, has been found by Brown, Costanzi, and Deshpande[28] to fit the $\eta\to3\pi$ decay.

In Tables II and III are shown the results of our calculations of the real part of the amplitude K_2^0

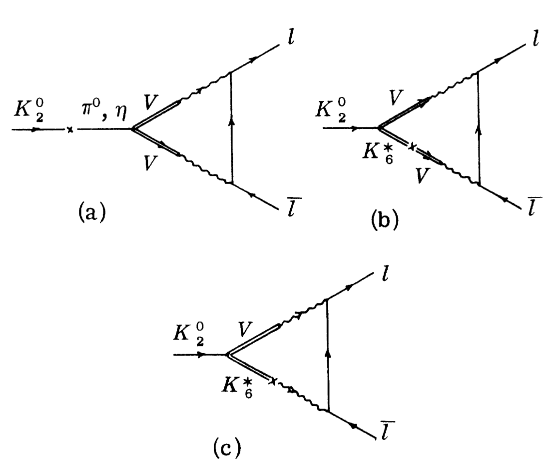

FIG. 5. Pole-model Feynman diagrams for $K_2^0\to l\bar{l}$.

TABLE II. $K_2^0 \to l\bar{l}$ amplitudes and branching ratios with the corresponding weak-coupling and PVV interaction parameters for $\epsilon_1 = 0.77$ [sgn $F(\pi^0)$ = sgn $F(\eta)$].

			$K_2^0 \to \mu^+\mu^-$			$K_2^0 \to e^+e^-$		
ϵ_2	$10^5 G_{NL} m_p^2$	β	10^{12} ImT	10^{12} ReT	$10^9 R(K_2^0 \to \mu^+\mu^-)$	10^{11} ImT	10^{11} ReT	$10^{12} R(K_2^0 \to e^+e^-)$
12.4	1.15	0.5	−4.85	−0.58	6.30	−2.01	1.42	4.13
12.4	1.15	0.1	4.85	1.11	6.53	2.01	−1.19	3.73
0	1.15	0.5	−4.85	−0.17	6.22	−2.01	1.47	4.24
0	1.15	0.1	4.85	1.50	6.81	2.01	−1.14	3.66
12.4	0.32	1	−4.85	−0.77	6.37	−2.01	1.34	3.98
0	0.32	1	−4.85	−0.65	6.32	−2.01	1.35	4.01

TABLE III. $K_2^0 \to l\bar{l}$ amplitudes and branching ratios with the corresponding weak-coupling and PVV interaction parameters for $\epsilon_1 = 1.3$ [sgn $F(\pi^0)$ = −sgn $F(\eta)$].

			$K_2^0 \to \mu^+\mu^-$			$K_2^0 \to e^+e^-$		
ϵ_2	$10^5 G_{NL} m_p^2$	β	10^{12} ImT	10^{12} ReT	$10^9 R(K_2^0 \to \mu^+\mu^-)$	10^{11} ImT	10^{11} ReT	$10^{12} R(K_2^0 \to e^+e^-)$
16.4	1.15	−0.5	−4.85	−0.95	6.45	−2.01	1.33	3.98
16.4	1.15	−0.1	4.85	0.68	6.33	2.01	−1.29	3.90
0	1.15	−0.5	−4.85	−0.58	6.30	−2.01	1.37	4.05
0	1.15	−0.1	4.85	1.08	6.52	2.01	−1.25	3.82
16.4	0.18	1	4.85	0.80	6.38	2.01	−1.31	3.93
0	0.18	1	4.85	0.87	6.41	2.01	−1.30	3.91

$\to l\bar{l}$ and the corresponding branching ratios. Table II corresponds to solution (a) of Eq. (3.3a), for which $F(\pi^0)$ and $F(\eta)$ have the same sign, while Table III shows the results for solution (b) of Eq. (3.3b), for which the signs of $F(\pi^0)$ and $F(\eta)$ are unequal. We have chosen $\epsilon_4 = 2.8$, but different values within the range $0.2 < \epsilon_4 < 6$ do not change the rate by more than 2%. For ϵ_2, besides the values mentioned above, we show for comparison the results corresponding to $\epsilon_2 = 0$. The results for $\beta = 1$ were obtained by using G_{NL}, Eq. (4.11), for which $K_1^0 \to 2\pi$ is not fitted by the octet weak Lagrangian. It can be seen from the tables that the maximum contribution to the rate from the real part of the amplitude is only of about 10% for $K_L^0 \to \mu^+\mu^-$, while for $K_L^0 \to e^+e^-$ we have much larger contributions.

VII. CONCLUSIONS

Our work indicates that one can successfully relate the experimental $K_1^0 \to 2\pi$, $K_2^0 \to \gamma\gamma$, and $K^\pm \to \pi^\pm l\bar{l}$ decay rates in the tree approximation by using a phenomenological weak nonleptonic Lagrangian with a $\underline{27}$, $\Delta I = \frac{1}{2}$ piece. On the other hand, an octet weak Lagrangian cannot simultaneously fit the 2π mode and the 2γ and $\pi l\bar{l}$ modes. We have also shown that a modification of the field dependence of the electromagnetic current is necessary to preserve gauge invariance when one adds to the strong gauge-field Lagrangian a weak coupling term involving the nonstrange vector gauge fields. This result is a feature of gauge-field Lagrangian models in which the minimal electromagnetic coupling is achieved through a mixing of the nonstrange gauge fields with the photon. In particular, we obtain a vanishing K^*-pole contribution to $K_2^0 \to \gamma\gamma$.

We also estimate the weak-electromagnetic contribution to the real part of the $K_2^0 \to l\bar{l}$ amplitude. A definite prediction cannot be made because the coupling constants of the octet-broken PVV interaction are not yet fully determined. Nevertheless, the values of the real ($K_2^0 \to \mu^+\mu^-$) amplitude calculated for several choices of the PVV parameters are always small, ranging from about 10 to 25% of the imaginary amplitude. The same calculation for the $K_2^0 \to e^+e^-$ decay gives a much larger contribution of the real part of the amplitude.

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APPENDIX A

The strong three-particle effective interactions needed for the calculation of the decays $K^\pm \to \pi^\pm l\bar{l}$ are obtained from the Lagrangian (2.1) through the substitution (2.7) for the axial-vector gauge field. Only

PPV and AVP couplings contribute, namely,

$$\mathcal{L}_{PPV}=gf^{abc}V_\mu^a P^b\partial^\mu P^c-\frac{g}{2m^2}f^{abc}\left(\frac{c_\pi{}^2g^2}{m^2}K^{ad}+\tfrac{1}{2}\delta\delta^{ad}\right)\overline{V}^d_{\mu\nu}\partial^\mu P^b\partial^\nu P^c-\frac{c_\pi{}^2g^3}{2m^4}K^{80}V^0_{\mu\nu}f^{8bc}\partial^\mu P^b\partial^\nu P^c \tag{A1}$$

and

$$\mathcal{L}_{\mathfrak{A}VP}=-\frac{c_\pi g^2}{m^2}\left(f^{abc}(K^{ad}+\delta\delta^{ad})\overline{V}^d_{\mu\nu}\frac{\mathfrak{A}^{\mu(b)}}{\sqrt{K_b}}\partial^\nu P^c+K^{80}V^0_{\mu\nu}f^{8bc}\frac{\mathfrak{A}^{\mu(b)}}{\sqrt{K_b}}\partial^\nu P^c+\tfrac{1}{2}f^{abc}K^{ad}\frac{\overline{\mathfrak{A}}^d_{\mu\nu}}{\sqrt{K_d}}V^{\mu\nu(b)}P^c\right), \tag{A2}$$

where $\varphi_{\mu\nu}\equiv\partial_\mu\varphi_\nu-\partial_\nu\varphi_\mu$. Equation (A1) leads to a $\rho\pi\pi$ coupling

$$f_{\rho\pi\pi}=\frac{g}{\sqrt{K_\rho}}\left[1-\frac{p^2}{4m_\rho{}^2}\left(1+\frac{\delta}{K_\rho}\right)\right], \tag{A3}$$

where p^2 is the four-momentum squares of the ρ. Using the ρ width of 125 MeV, we get from the formula $\Gamma\cong 52f_{\rho\pi\pi}{}^2/4\pi$ MeV that $\delta=-0.92$ or $\delta=8.3$. The last value gives a too-large $A_1\rho\pi$ coupling and therefore we adopt the first one. Notice that relation (A3) is the same as the one obtained in SU(2)×SU(2) when $K_\rho=1$.[10] From the weak Lagrangian (4.4) the relevant couplings can be written as

$$\mathcal{L}_{\rm NL}^{PP}=c_\pi{}^2D^{6ab}\partial_\mu P^a\partial^\mu P^b, \tag{A4a}$$

$$\mathcal{L}_{\rm NL}^{P\mathfrak{A}}=2\frac{m^2}{g}c_\pi D^{6ab}\frac{\mathfrak{A}^a_\mu}{\sqrt{K_a}}\partial^\mu P^b, \tag{A4b}$$

$$\mathcal{L}_{\rm NL}^{VV}=\frac{m^2}{g^2}D^{6ab}V^a_\mu V^{\mu(b)}, \tag{A4c}$$

$$\mathcal{L}_{\rm NL}^{PPV}=2c_\pi{}^2gD^{6ab}f^{am\mu}V^m_\mu P^\mu\partial^\mu P^b, \tag{A4d}$$

with

$$D^{6ab}=\frac{2G_{\rm NL}}{\sqrt{2}}\left(\frac{3+\beta}{4}d^{6ab}+\frac{1-\beta}{2}(d^{6al}d^{18b}+d^{6bl}d^{18a})\right). \tag{A5}$$

APPENDIX B

The $K_2^0\to l^+l^-$ amplitude of Eq. (5.4) can be expressed as

$$T=t_{\pi\rho\omega}I(\rho,\omega)+t_{\eta\rho\rho}I(\rho,\rho)+t_{\eta\omega\omega}I(\omega,\omega)+t_{\eta\varphi\varphi}I(\varphi,\varphi)+t_{\eta\omega\varphi}I(\omega,\varphi)+\sum_{i=\rho,\omega,\varphi}\tau_iI(i,K^*)+\sum_{i,j=\rho,\omega,\varphi}\tau_{ij}I(i,j,K^*), \tag{B1}$$

where

$$I(i,j)=-i4\left(\frac{\alpha}{\pi}\right)^2m_K\int \mathcal{G}(i,j)\,d^4k, \tag{B2}$$

$$I(i,j,K^*)=-i4\left(\frac{\alpha}{\pi}\right)^2m_K\int\frac{\mathcal{G}(i,j)}{k^2-m_{K^*}{}^2}d^4k, \tag{B3}$$

with

$$\mathcal{G}(i,j)=(\vec{k})^2\{k^2(p-k)^2[(k-p_+)^2-m_\rho{}^2][(p-k)^2-m_j{}^2][k^2-m_i{}^2]\}^{-1}. \tag{B4}$$

The numerator of Eq. (5.4) has been simplified by taking the K meson at rest.

The coefficients of the integrals are

$$t_{Pij}=-W_{KP}G_{Pij}E_iE_j[1-(m_P{}^2/m_K{}^2)]^{-1}, \tag{B5}$$

$$\tau_i=-G_{KiK^*}W_{K^*\gamma}E_i, \tag{B6}$$

$$\tau_{ij}=G_{KiK^*}W_{K^*j}E_iE_j. \tag{B7}$$

The G_{Pij} coupling constants come from the PVV interaction in Eq. (3.1); e.g.,

$$G_{\pi\rho\omega}=(-2h\sin\theta D^{383}+\lambda\cos\theta D^{32})(K_\rho K_\omega)^{-1/2}, \tag{B8}$$

with $D^{383}=(1/\sqrt{3})(1+\epsilon_1)$ and $D^{33}=1+\epsilon_4$. We note that the choice of λ, Eq. (3.2), leads to $G_{\pi\rho\varphi}=0$. The W's are weak-coupling parameters which can be obtained from Eq. (A5), for example, $W_{\pi K}=\sqrt{2}G_{\rm NL}c_\pi{}^2$; and $W_{K^*\gamma}=\sqrt{2}G_{\rm NL}m^4(1+\frac{1}{3}\beta)/g^3$ from Eq. (4.6). Finally, the E_i are the vector-meson–photon couplings from

(3.4) (without the factor e).

As shown in Ref. 35, the integrands in (B2) and (B3) can be expanded in partial fractions resulting in integrals over the product of three poles only. Using the standard Feynman techniques these can be reduced to one-variable integrals which can be solved numerically. Our results are discussed in Sec. VI.

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