PHYSICAL REVIEW D VOLUME 4, NUMBER 9 1 NOVEMBER 1971

Analysis of Reggeon-Particle Spin Couplings in Feynman-Diagram Models of Reggeization

Richard A. Morrow
Bennett Hall, University of Maine, Orono, Maine 04473
(Received 12 July 1971)

An attempt is made to fit certain data for the reactions $K^{\pm}p \to K^*(890)\pi^{\pm}p$ and $\pi^{\pm}p \to \rho^{\pm}p$ using simple Regge models incorporating the Reggeon-particle spin couplings of Blankenbecler and Sugar. The aim is to see if a universal form of such spin couplings exists. It is shown that this is not the case if simple, unadorned Regge models are used, but that if absorption is appealed to virtually any type of spin coupling will yield successful fits to existing data.

I. INTRODUCTION

In spite of the support that Regge asymptotic behavior has acquired in scattering reactions, the structure of vertices involving Reggeons and spinning particles is still incompletely understood. All theoretical models are ambiguous to some degree on this important point, thus allowing a certain freedom in fitting experimental data with Regge models.

The purpose of the present work is to study the form of the spin couplings in a comprehensive model, namely, that of Blankenbecler and Sugar.[1] This model is applied to the reactions[2,3]

$$K^{\pm}p \to K^*(890)\pi^{\pm}p \tag{1}$$

and[4,5]

$$\pi^{\pm}p \to \rho^{\pm}p\ , \tag{2}$$

both of which, in the Regge limit, are described by amplitudes involving vertices composed of a spin-one object (Pomeranchukon, K^*, or ρ), a spin-zero particle (K or π) and a Reggeized pion. The aim is to determine if a universal spin coupling exists. The data fitted are various differential distributions for reaction (1) and a certain ratio, $(\text{Re}\rho_{10})/\rho_{00}$, of spin density-matrix elements for reaction (2).

The result is that a universal coupling for the particular vertex involved does not exist in a simple Regge amplitude. That is, using unadorned Regge amplitudes, different couplings are needed in (1) and (2) in order to fit the data. However, it *is* demonstrated in a rough way that, by including absorption, virtually any type of spin coupling can lead to agreement with the data for (1) and (2) simultaneously. This is an unfortunate circumstance indicating, as it does, that to unscramble the nature of the basic universal coupling – if one exists – from the data, incorporation of absorptive effects in reactions of type (1) and (2) must be well understood.

II. THE MODELS

For reactions (1) and (2), Regge amplitudes retaining only the dominant Reggeons and represented by Figs. 1(a) and 1(b), respectively, are envisaged. The theoretical model to be analyzed is that of Blankenbecler and Sugar[1] which, in leading order, is as comprehensive as any other. Thus, both ease of calculation (Feynman diagrams) and some semblance of generality are present. In this model Regge behavior is generated by summing an infinite set of diagrams containing exchanges of particles of physical spin, as suggested by Van Hove[6] and Durand.[7] Drummond *et al.*[8] have in fact shown what form this model predicts for the middle vertex of reaction (1), while Morrow[9] has already used the model in a study of certain aspects of reaction (2). Thus only brief descriptions of the necessary amplitudes need be presented here.

In much of the following the notation will be more general than necessary. Specialization to reactions (1) and (2) will be made at the end of the model building.

Double-Regge Model

The useful kinematic invariants associated with the reaction depicted in Fig. 1(a) are $s=(p_1+p_2)^2$, $s_1=(q_1+q_3)^2$, $s_2=(q_2+q_3)^2$, $t_1=(p_1-q_1)^2$, and $t_2=(p_2-q_2)^2$, where p_i, q_i are the four-momenta and are related to particle masses by $p_i^2=m_i^2$, $q_i^2=\mu_i^2$. The exchanges in this figure are taken to be Reggeons with trajectories $\alpha_1(t_1)$ and $\alpha_2(t_2)$.

Neglecting the spins of the external particles for the moment, the invariant amplitude for large s_1 and s_2 was shown by Drummond *et al.*[8] to have the form

$$M \sim \xi_{\alpha_1}\xi_{\alpha_2} f_1(t_1) f_2(t_2) f_3(t_1,t_2,W) \times (z_1/s_{01})^{\alpha_1}(z_2/s_{02})^{\alpha_2}/(\sin\pi\alpha_1 \sin\pi\alpha_2)\ , \tag{3}$$

where

$$z_1 = s_1 - t_2 - m_1^2 - \tfrac{1}{2}(\mu_3^2 - t_1 - t_2) \sim s_1\,,$$
$$z_2 = s_2 - t_1 - m_2^2 - \tfrac{1}{2}(\mu_3^2 - t_1 - t_2) \sim s_2\,,$$
$$W = \tfrac{1}{2}\mu_3^2(4s - 2s_1 - 2s_2 + t_1 + t_2 - 2m_1^2 - 2m_2^2 + \mu_3^2)/(z_1 z_2) \sim 2\mu_3^2 s/(s_1 s_2)\,.$$

Signature τ_i is included as $\xi_{\alpha_i} = 1 + \tau_i e^{-i\pi\alpha_i}$ and the s_{0i} are scaling parameters. In addition, $f_1(t_1)$ and $f_2(t_2)$ are unknown form factors, while $f_3(t_1, t_2, W)$ takes the form

$$f_3(t_1, t_2, W) \sim \int_0^{2\pi} dy\, \psi(\alpha_1, \alpha_2, W, y)\,,$$

with

$$\psi(\alpha_1, \alpha_2, W, y) = G''(\alpha_2, \alpha_1, y) W^{\alpha_1} \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(-\alpha_2)}\, {}_1F_1(-\alpha_1, \alpha_2 - \alpha_1 + 1, W^{-1} e^{iy}) + (\alpha_1 \longleftrightarrow \alpha_2)\,,$$

where $G''(\alpha_2, \alpha_1, y)$ is derived from the (unknown) field coupling used in the interaction Lagrangian.

For reaction (1) in the double-Regge region (s_1 and s_2 large), the Reggeons deemed important are the Pomeranchukon at the proton vertex and the pion at the kaon vertex. Then, with $\alpha_1 \equiv 1.0$, the prediction of the model for the middle vertex simplifies enormously[10]:

$$f_3(t_1, t_2, W) \sim \int_0^{2\pi} dy\, G''(\alpha_2, 1, y) e^{iy}(1 - e^{-iy}\alpha_2 W)\,,$$

which may reasonably be parametrized as

$$f_3(t_1, t_2, W) \sim 1 + \gamma_3 \alpha_2 s/(\alpha' s_1 s_2)\,, \tag{4}$$

with α' the slope of the pion trajectory and γ_3 a real, unknown, function of t_2. In fact, because $\alpha_1 \equiv 1.0$ there are only two field couplings for the middle vertex, and γ_3 is proportional to the ratio of the minimal to the maximal derivative "coupling constants" entering the interaction Lagrangian. An over-all factor omitted in the last step in reaching (4) may be incorporated in $f_2(t_2)$ in (3).

Essentially the same amplitude parametrization results from general arguments based on analyticity and from the Bardakci-Ruegg model[11] of Veneziano couplings, as also shown by Drummond *et al.*[8] The notable exception is that in the latter model $\gamma_3 \equiv +1$ results – an important fact which will be returned to below.

Now in considering reaction (1) it is found that most of the data occur in the domain where s_2 is small. Appealing to duality,[12] the above model will therefore be extended into this region. In so doing it may be argued that the factor $s/(s_1 s_2)$ in (4) should be replaced by something else. To this end, note that this factor derives from $W/(2\mu_3^2)$, which in the small-s_2 region may be approximated by $(s - \frac{1}{2}s_1)/[s_1(s_2 - m_2^2)]$. Also, it is possible to show that the Bardakci-Ruegg model[11] suggests using $(s - s_1)/[s_1(s_2 - m_R^2)]$, where m_R is an effective mass of a resonance in the s_2 channel. Thus a reasonable parametrization for all s_2 is

$$f_3(t_1, t_2, W) \sim 1 + \gamma_3(\alpha_2/\alpha')(s - s_1)/[s_1(s_2 - m_2^2)]\,, \tag{5}$$

which form was used in the calculations described below.

Turning next to the inclusion of K^* spin, it is readily apparent that the same basic couplings of Reggeons and particles used above for the middle vertex are also appropriate to the $\pi K K^*$ vertex. Letting f^m denote the contribution to $f_2(t_2)$ due to these couplings, for K^* spin projection m along the incident K momentum in the K^* rest frame, it is possible to show[13]

$$\begin{aligned} f^0 &= \lambda(t_2, m_2^2, \mu_2^2)^{1/2}\left[1 - \frac{\gamma_2(\alpha_2/\alpha')(t_2 + \mu_2^2 - m_2^2)}{\lambda(t_2, m_2^2, \mu_2^2)}\right], \\ f^1 &= -f^{-1} = \gamma_2(\alpha_2/\alpha')(-2\mu_2^2 t_2)^{1/2}\lambda(t_2, m_2^2, \mu_2^2)^{-1/2}, \end{aligned} \tag{6}$$

where

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx\,,$$

and where γ_2 replaces γ_3.

Finally, putting (3), (5), and (6) together and assuming that the effect of the protons' spins can be handled by $f_1'(t_1)$ in the large-s_1 limit, the spin-averaged absolute square of the invariant amplitude for reaction (1) has the form

$$\sum |M|^2 = N_0 f_1'(t_1) f_2'(t_2)(z_1/s_{01})^2 \times (z_2/s_{02})^{2\alpha_2} V_2 V_3 (1 - \cos\pi\alpha_2)^{-1}\,, \tag{7}$$

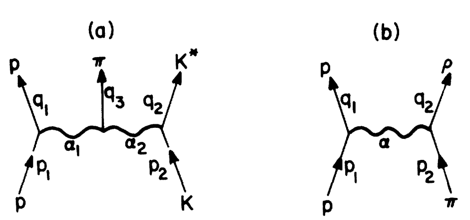

FIG. 1. (a) The double-Regge diagram for the reaction $K^{\pm}p \to p\pi^{\pm}K^*$ with Pomeranchukon (α_1) and pion (α_2) exchanges. (b) The diagram for $\pi^{\pm}p \to \rho^{\pm}p$ with pion (α) exchange.

with

$$V_2=\lambda(m_\pi^2, m_2^2, \mu_2^2)^{-1}[|f^0|^2+|f^1|^2+|f^{-1}|^2]$$
$$=\left[1+\gamma_2(\alpha_2/\alpha')\frac{\gamma_2(\alpha_2/\alpha')-2(t_2+\mu_2^2-m_2^2)}{\lambda(t_2, m_2^2, \mu_2^2)}\right]$$
$$\times\frac{\lambda(t_2, m_2^2, \mu_2^2)}{\lambda(m_\pi^2, m_2^2, \mu_2^2)},$$
$$V_3=\left[1+\gamma_3(\alpha_2/\alpha')\frac{s-s_1}{s_1(s_2-m_2^2)}\right]^2.$$

Specializing further to reaction (1) and following the conventional treatment,[2,3] the amplitude (7) may be normalized at the pion pole ($t_2=m_\pi^2$) to the πKK^* coupling constant and the pion-nucleon scattering cross section. This is accomplished by the following choice of functions, with all remaining factors lumped into the constant N_0:

$$f_1'(t_1)=\exp(8t_1),\quad f_2'(m_\pi^2)=1.0,$$
$$\alpha_2=\alpha'(t_2-m_\pi^2),\quad \alpha'=1.0\ (\mathrm{GeV}/c)^{-2}.$$

In addition, cuts are imposed on certain variables to delimit the domain of applicability of the model: $s_1\geq 3.0\ (\mathrm{GeV}/c)^2$, t_1 and $t_2\geq -1.0\ (\mathrm{GeV}/c)^2$.

The distributions resulting from this model were investigated at a K laboratory momentum of 10 GeV/c; the results are discussed in Sec. III.

Resonance Production Model

The amplitudes for Fig. 1(b) with Reggeized pion exchange are readily available from Eq. (6). In terms of the new kinematic invariants $s=(p_1+p_2)^2$ and $t=(p_1-q_1)^2$ with $p_i^2=m_i^2$ and $q_i^2=\mu_i^2$, the t-channel helicity amplitudes g_t^m, with neglect of the protons' spins, are (up to unnecessary factors)

$$g_t^0=\lambda(t, m_2^2, \mu_2^2)-\gamma(\alpha/\alpha')(t+\mu_2^2-m_2^2),$$
$$g_t^1=-g_t^{-1}=\gamma(\alpha/\alpha')(-2\mu_2^2 t)^{1/2}. \tag{8}$$

These are sufficient for present purposes for, as shown elsewhere,[9,14] the ratio $(\mathrm{Re}\rho_{10})/\rho_{00}$ of ρ-meson spin density-matrix elements for reaction (2) are, at large s, dependent only on pion exchange and on the $\pi\pi\rho$ vertex. Thus the quantity to be compared with data at 8-GeV/c π laboratory momentum is

$$-(\mathrm{Re}\rho_{10})/\rho_{00}=-g_t^1/g_t^0$$
$$=\gamma(\alpha/\alpha')(-2\mu_2^2 t)^{1/2}[\lambda(t, m_2^2, \mu_2^2)-\gamma(\alpha/\alpha')(t+\mu_2^2-m_2^2)]^{-1}, \tag{9}$$

with $\alpha=\alpha'(t-m_\pi^2)$ and $\alpha'=1.0\ (\mathrm{GeV}/c)^{-2}$.

III. CALCULATIONS

It is well known that good fits to the data[2,3] of reaction (1) result from use of (7) if maximum derivative coupling ($\gamma_2=\gamma_3=0$) alone is used and if the form factor

$$f_2'(t_2)=\lambda(m_\pi^2, m_2^2, \mu_2^2)/\lambda(t_2, m_2^2, \mu_2^2)$$

is adopted. Typical distributions for this model are shown in Fig. 2 as full lines. Since these curves fit the data reasonably well[2,3] they will be used as a standard against which curves for other values of the parameters are compared.

The difficulty referred to in the Introduction concerning a universal coupling is immediately evident when maximum derivative coupling ($\gamma=0$) is also adopted in (9) for reaction (2). In this case $(\mathrm{Re}\rho_{10})/\rho_{00}=0$ is predicted in noticeable disagreement with the data[4,5] as seen in Fig. 3.

At this stage a parameter search might be attempted to see if a universal form for the γ exists. Instead two additional specific cases will be presented to show the scope of the dilemma. A particularly suggestive choice is $\gamma=\gamma_2=\gamma_3=+1$, which corresponds to using Veneziano-type residues in the spin couplings, as remarked on earlier. As shown elsewhere[14] and reproduced in Fig. 3, the agreement of model and data for reaction (2) is fair. On the other hand, trying this choice in (7) for reaction (1) – and using an *ad hoc* $f_2'(t_2)$ to fit the t_2 distribution – leads to obvious disagreement with the data, as shown by the long-dashed curves in Fig. 2. Note in particular the ϕ distribution.

The third parameter choice explored was $\gamma=\gamma_2=\gamma_3=-1$. Here reasonable fits to the data of reaction (1) result – again if an *ad hoc* $f_2'(t_2)$ is used to fit the t_2 distribution – as shown by the short-dashed curves in Fig. 2. As for reaction (2) it is straightforward to show that (9) predicts $-(\mathrm{Re}\rho_{10})/\rho_{00}<0$ in disagreement with the data.

It is therefore tentatively concluded that a universal field coupling in the Blankenbecler-Sugar model does not exist when the resulting vertices are used in a simple Regge amplitude. In Sec. IV this distressing point is resolved by appealing to absorption.

IV. ABSORPTION

Since there is no unique prescription available for incorporating absorptive effects into scattering reactions, let alone production reactions, absorption will be treated only roughly here. A simple

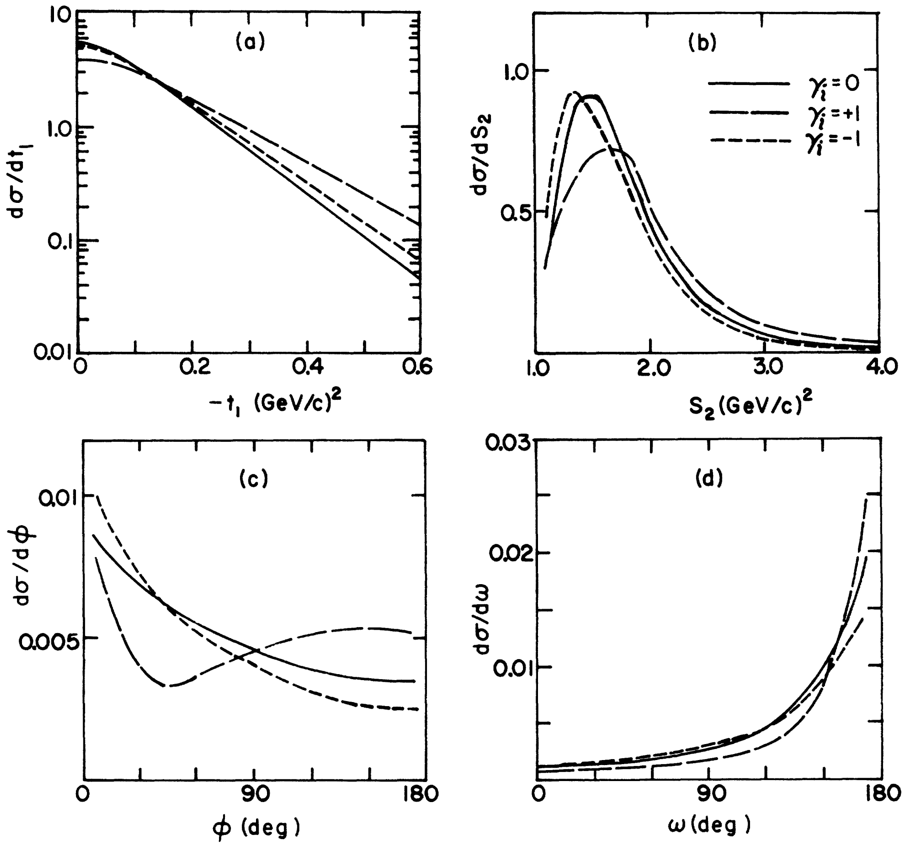

FIG. 2. Typical distributions resulting from the diagram of Fig. 1(a) for three values of the spin coupling functions: full curves for $\gamma_2=\gamma_3=0$, long-dashed curves for $\gamma_2=\gamma_3=+1$, short-dashed curves for $\gamma_2=\gamma_3=-1$. The conventional model, with $\gamma_2=\gamma_3=0$, is in reasonable agreement with the data (see Refs. 2 and 3). With absorption (OPE $-\delta$) all values of the spin coupling functions yield curves very close to those for $\gamma_2=\gamma_3=-1$. The Treiman-Yang angle ϕ in (c) is defined by $\cos\phi=(\vec{p}_1\times\vec{q}_1)\cdot(\vec{p}_2\times\vec{q}_2)/[|\vec{p}_1\times\vec{q}_1||\vec{p}_2\times\vec{q}_2|]$ in the frame $\vec{q}_1+\vec{q}_3=0$, while the Toller angle ω in (d) is defined by the same expression but evaluated in the frame $\vec{q}_3=0$. Over-all normalization, depending on N_0 in (7), is arbitrary in this figure.

prescription (OPE $-\delta$) for finding at least the suggestive effects of absorption for OPE (one-pion exchange) scattering amplitudes has been given by Williams[15] in which he demands the absence of all J-plane Kronecker-δ terms in the s-channel partial-wave amplitudes. In the present case of Regge amplitudes it might be conjectured that this prescription is applied as described below.

Double-Regge Model

The parts of the production amplitudes most likely to be affected by absorption are those describing the inelastic scattering of the K from the Pomeranchukon (P) resulting in the πK^* final state. The helicity-dependent pieces of these scattering amplitudes are shown in the Appendix to be, for large s_2,

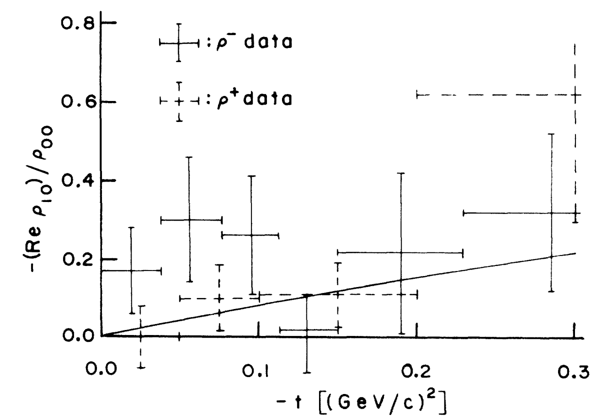

FIG. 3. A comparison of the prediction of the model of Fig. 1(b) with the data of Refs. 4 and 5. The curve is from Eq. (9) for the spin coupling function $\gamma=+1$. It is also the result for all values of γ when absorption (OPE $-\delta$) is imposed.

$$f^{mm'} \sim \left[\tfrac{1}{2}\delta_{m0}(\mu_3{}^2 - t_1 - t_2 + \gamma_3\alpha_2/\alpha')/\sqrt{t_1} + \delta_{m1}(-\tfrac{1}{2}t_2)^{1/2} - \delta_{m-1}(-\tfrac{1}{2}t_2)^{1/2}\right]$$
$$\times\left[\tfrac{1}{2}\delta_{m'0}(m_2{}^2 - \mu_2{}^2 - t_2 + \gamma_2\alpha_2/\alpha')/\mu_2 - \delta_{m'1}(-\tfrac{1}{2}t_2)^{1/2} + \delta_{m'-1}(-\tfrac{1}{2}t_2)^{1/2}\right], \qquad (10)$$

where m (m') is the helicity of the Pomeranchukon (K^*). Then the OPE-δ prescription requires (10) to have the form

$$f_A^{mm'} \sim (-t_2)^{|m+m'|/2} \times (\text{term in which } t_2 \text{ is replaced by } m_\pi{}^2).$$

The absorbed scattering amplitudes are thus, recalling $\alpha_2 = \alpha'(t_2 - m_\pi{}^2)$,

$$f_A^{mm'} \sim \left[\tfrac{1}{2}\delta_{m0}(\mu_3{}^2 - t_1 - m_\pi{}^2)/\sqrt{t_1} + \delta_{m1}(-\tfrac{1}{2}t_2)^{1/2} - \delta_{m-1}(-\tfrac{1}{2}t_2)^{1/2}\right]$$
$$\times\left[\tfrac{1}{2}\delta_{m'0}(m_2{}^2 - \mu_2{}^2 - m_\pi{}^2)/\mu_2 - \delta_{m'1}(-\tfrac{1}{2}t_2)^{1/2} + \delta_{m'-1}(-\tfrac{1}{2}t_2)^{1/2}\right] + \tfrac{1}{2}(\delta_{m1}\delta_{m'-1} + \delta_{m-1}\delta_{m'1})(t_2 - m_\pi{}^2), \qquad (11)$$

where it may be noted that there is no dependence on γ_2 or γ_3.

It then follows, as indicated in the Appendix, that the production amplitudes contain a term of the form (A2) with $f^{mm'}$ replaced by $f_A^{mm'}$. The net result is to have $\sum|M|^2$ of the form (7) with the replacement

$$V_2V_3 \to V_A = (1+Z)^2 + 4\mu_2{}^2 t_1 Z/\lambda(m_\pi{}^2, m_2{}^2, \mu_2{}^2) - 4\mu_2{}^2(\mu_3{}^2 - m_\pi{}^2)Z(1+Z)/\lambda(m_2{}^2, \mu_2{}^2, m_\pi{}^2), \qquad (12)$$

where[16]

$$Z = (t_2 - m_\pi{}^2)(s - s_1)/[s_1(s_2 - m_2{}^2)].$$

This is the result of applying the helicity-dependent part of the OPE − δ absorption prescription. There is still an unknown helicity-independent part which may be taken to be given by $f_2'(t_2)$. A Dürr-Pilkuhn form[17] for this latter function was adopted:

$$f_2'(t_2) = \frac{1 + R_c{}^2\lambda(m_\pi{}^2, m_2{}^2, \mu_2{}^2)}{1 + R_c{}^2\lambda(t_2, m_2{}^2, \mu_2{}^2)},$$

with "vertex radius" $R_c = \sqrt{0.2}$ GeV/c chosen to fit the t_2 distribution. One further modification in the amplitude was found to be necessary in fitting the data. Recall from Sec. II that the functions describing the spin couplings at the vertices depended on the momentum-transfer variables only via the Regge trajectories. This is decidedly not true for (12) because of the factor t_1 in the second term. To make (12) independent of t_1 (since $\alpha_1 \equiv 1.0$ is used) this variable was replaced by a parameter τ which if desired may be viewed as an absorptive parameter. Curves (not shown) for $\tau = -0.13$ (GeV/c)^2 were then found to come very close to the short-dashed curves of Fig. 2, which are acceptable themselves. Thus all spin couplings (all values of γ_2 and γ_3) in the model lead to reasonable agreement with the data.

Resonance Production Model

To apply the OPE − δ absorption prescription in this case the s-channel helicity amplitudes g_s^m are needed. They are found from the t-channel amplitudes of (8) by the Trueman-Wick crossing relations[18]:

$$g_s^m = \sum_n d^1_{nm}(\chi) g_t^n,$$

where, in the large-s limit,

$$\cos\chi \to (t + \mu_2{}^2 - m_2{}^2)/\lambda(t, m_2{}^2, \mu_2{}^2)^{1/2}.$$

The results are

$$g_s^0 = \lambda(t, m_2{}^2, \mu_2{}^2)^{1/2}[t + \mu_2{}^2 - m_2{}^2 - \gamma\alpha/\alpha'],$$
$$g_s^1 = -g_s^{-1} = \lambda(t, m_2{}^2, \mu_2{}^2)^{1/2}(-2\mu_2{}^2 t)^{1/2}.$$

Next the absorbed amplitudes are found by the requirement

$$g_{As}^m \sim (-t)^{|m|/2} \times (\text{term in which } t \text{ is replaced by } m_\pi{}^2)$$

and are crossed[18] back to the t channel by

$$g_{At}^m = \sum_n d^1_{mn}(\chi) g_{As}{}^n$$

to yield the absorbed t-channel helicity amplitudes

$$g_{At}^0 = \lambda(t, m_2{}^2, \mu_2{}^2) - (\alpha/\alpha')(t + \mu_2{}^2 - m_2{}^2),$$
$$g_{At}^1 = -g_{At}^{-1} = (\alpha/\alpha')(-2\mu_2{}^2 t)^{1/2},$$

up to a common factor

$$\lambda(m_\pi{}^2, m_2{}^2, \mu_2{}^2)^{1/2}/\lambda(t, m_2{}^2, \mu_2{}^2)^{1/2}.$$

Thus $-(\mathrm{Re}\rho_{10})/\rho_{00}$ for these absorbed amplitudes is precisely the same as for the unabsorbed result (9) with Veneziano-type couplings ($\gamma = +1$). As already shown (Fig. 3) this result does fit the data to a fair degree. Hence again all spin couplings (all values of γ) in the model work equally well.

IV. CONCLUSIONS

The calculations presented in the last section, while admittedly crude, nevertheless do suggest that absorption strongly masks the nature of the underlying spin couplings of Reggeons and particles from observation. It will therefore be rather difficult to determine whether a universal coupling of the spins exists or not. To this end incorporation of absorption will have to be understood much better.

APPENDIX

The determination of the $P+K\to\pi+K^*$ helicity amplitudes for large s_2 and pion exchange can be carried out very conveniently using the Feynman-diagram Reggeization technique.[1] To do this first define the four-vectors

$$Q=\tfrac{1}{2}(p_1+q_1),\quad Q'=\tfrac{1}{2}(p_2+q_2),$$

$$P=p_1-q_1,\quad P'=p_2-q_2.$$

Then in the πK^* center-of-mass frame select[19] $\vec{p}_2$ to lie in the $+z$ direction and $\vec{q}_2$ to lie in the x-z plane with positive x component and making an angle θ_2 with the $+z$ axis. A set of polarization vectors for the $P(e^m)$ and the $K^*(\epsilon^{m'})$, where m and m' are the respective helicities, are

$$e^0=(|\vec{P}|,0,0,-P_0)/\sqrt{t_1},\quad e^1=-(0,-1,-i,0)/\sqrt{2},\quad e^{-1}=(0,-1,i,0)/\sqrt{2},$$

$$\epsilon^0=(|\vec{q}_2|,q_{20}\sin\theta_2,0,q_{20}\cos\theta_2)/\mu_2,\quad \epsilon^1=-(0,\cos\theta_2,i,-\sin\theta_2)/\sqrt{2},\quad \epsilon^{-1}=(0,\cos\theta_2,-i,-\sin\theta_2)/\sqrt{2}.$$

Since both P and K^* have spin 1, there are two ways each polarization vector can couple to the other two particles at the vertex. Denoting the two field coupling constants by a min and max notation, the required amplitude may be written, in part (taking the exchanged particle to have spin J),

$$f^{mm'}\sim[g_3(\max)e^m\cdot q_3P_{\mu_1}P_{\mu_2}\cdots P_{\mu_J}+g_3(\min)e^m_{\mu_1}P_{\mu_2}P_{\mu_3}\cdots P_{\mu_J}]\Gamma^J_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}$$
$$\times[g_2(\max)\epsilon^{m'*}\cdot p_2Q'_{\nu_1}Q'_{\nu_2}\cdots Q'_{\nu_J}+g_2(\min)\epsilon^{m'*}_{\nu_1}Q'_{\nu_2}Q'_{\nu_3}\cdots Q'_{\nu_J}].$$

At large s_2 the propagator term $\Gamma^J_{\mu;\nu}$ reduces to a sum of $J!$ terms, each of which is of the form $g_{\mu_i\nu_j}g_{\mu_k\nu_l}\cdots$ (J factors). Thus $\Gamma^J_{\mu;\nu}$ connects the μ indices with the ν indices in all possible ways. The resulting amplitude has the form (setting $J=\alpha_2$ after the usual Reggeization)

$$f^{mm'}=\{e^m\cdot q_3+\delta_{m0}[2g_3(\min)/g_3(\max)]/(2\sqrt{t_1})\}\{\epsilon^{m'*}\cdot p_2+\delta_{m'0}[2g_2(\min)/g_2(\max)]/(2\mu_2)\},\tag{A1}$$

where a factor $(\tfrac{1}{2}s_2)^{\alpha_2}g_3(\max)g_2(\max)$ has been omitted.

Identifying $2g_i(\min)/g_i(\max)=\gamma_i\,\alpha_2/\alpha'$ for $i=2,3$ allows (A1) to be put in the form of Eq. (10) of the text.

The production amplitude follows when the ppP coupling is incorporated. Neglecting the protons' spin there is only one way for the P to couple to the protons, and so the essential part of the production amplitude is

$$M^{m'}\sim\sum_m Q\cdot e^{m*}f^{mm'}.\tag{A2}$$

It is straightforward to show that $\sum_{m'}|M^{m'}|^2$ contains the spin-dependent terms V_2 and V_3 of (7).

---

[1]R. Blankenbecler and R. L. Sugar, Phys. Rev. 168, 1597 (1968).

[2]J. Andrews, J. Lack, T. Ludlam, J. Sandweiss, H. D. Taft, and E. L. Berger, Phys. Rev. Letters 22, 731 (1969); at 12.6-GeV/c K^- laboratory momentum.

[3]C. Y. Chien, E. I. Malamud, D. J. Mellema, F. D. Rudnick, P. E. Schlein, W. E. Slater, D. H. Stork, H. K. Ticho, and T. G. Trippe, Phys. Letters 29B, 433 (1969); at 7.3-GeV/c K^+ laboratory momentum.

[4]S. J. Barish, W. Selove, N. N. Biswas, N. M. Cason, P. B. Johnson, V. P. Kenney, J. A. Poirier, W. D. Shepard, and H. Yuta, Phys. Rev. 184, 1375 (1969); Phys. Rev. D 1, 375(E) (1970); at 8-GeV/c π^- laboratory momentum.

[5]Aachen-Berlin-CERN Collaboration, Phys. Letters 22, 533 (1966); at 8-GeV/c π^+ laboratory momentum.

[6]L. Van Hove, Phys. Letters 24B, 183 (1967).

[7]L. Durand, III, Phys. Rev. 154, 1537 (1967); 161, 1610 (1967).

[8]I. T. Drummond, P. V. Landshoff, and W. J. Zakrzewski, Nucl. Phys. B11, 383 (1969).

[9]R. A. Morrow, Phys. Rev. 185, 1764 (1969).

[10]Use

$${}_1F_1(a,b,x)=1+\frac{a}{b}x+\frac{a(a+1)}{2!\,b(b+1)}x^2+\cdots.$$

[11]K. Bardakci and H. Ruegg, Phys. Letters 28B, 342 (1968).

[12]G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

[13]The f^m may be derived from (5) by appropriate changes of variables or directly from the Blankenbecler-Sugar model or the Bardakci-Ruegg model. These latter derivations were done in Refs. 9 and 14.

[14]R. A. Morrow, Phys. Rev. D 1, 2113 (1970).

[15]P. K. Williams, Phys. Rev. 181, 1963 (1969); L. Chan and P. K. Williams, *ibid.* 188, 2455 (1969). See also J. E. Rush, Jr., *ibid.* 188, 2535 (1969).

[16]Actually $Z=(t_2-m_\pi^2)s/(s_1s_2)$ emerges, but this is modified for the same reason that led from (4) to (5).

[17]H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).

[18]T. L. Trueman and G. C. Wick, Ann. Phys. (N.Y.) 26, 322 (1964).

[19]A four-vector is written as $X=(X_0,\vec{X})$ with $X^2=X_0^2-\vec{X}^2$.