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Electroproduction from Nucleons in a Relativistic Quark Model*

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A relativistic formulation of the quark model recently proposed by Feynman, Kislinger, and Ravndal is applied to electroproduction of nucleonic resonances. Formulas for the ratio of longitudinal to transverse photoabsorption cross sections that are relevant to both the resonance and deep-inelastic regions are derived. This formulation is found to be much more successful than the nonrelativistic symmetric quark model and is consistent with the existing data.

A relativistic formulation[1] of the symmetric, harmonic-oscillator quark model has recently been proposed by Feynman, Kislinger, and Ravndal (FKR).[2] Such a formulation has several advantages not enjoyed by earlier, nonrelativistic versions[3] of the quark model by virtue of the very fact that it is a relativistic, and therefore covariant theory.

In Ref. 2 Feynman *et al.* use their new formulation to compute matrix elements of vector and axial-vector currents and, in particular, matrix elements appropriate to the description of single-pion photoproduction. Their results are very similar to those of previous nonrelativistic calculations[4] which in turn agree rather well with the available experimental data[5] on photoproduction in the resonance region. However, in the FKR formulation this good agreement is obtained with one less parameter than is involved in the earlier calculations.

Although the photoproduction predictions of the nonrelativistic quark model have been rather impressively confirmed[6] by experiment, the same cannot be said for its predictions for electroproduction. Indeed, the extensive calculations of amplitudes for the electroproduction of nucleonic resonances from protons performed by Thornber[7] are only in rough agreement with the then existing data. Also, as we now know,[8] her prediction for the ratio of the longitudinal to transverse cross sections is too large by about two orders of magnitude. Thus, an extension of the calculations of Feynman *et al.* to the case of electroproduction will serve two ends; namely, to make use of the relativistic covariance of the FKR model's electromagnetic current for the first time, and to attempt to provide a unified and successful picture of both these electromagnetic excitation processes.

Both the nonrelativistic and the FKR versions of the quark model are unable to predict the standard dipole-like dependence of the nucleonic form factors. Thus, we shall concentrate on a feature of electroproduction which does not depend on one's ability to make such predictions. Such a feature is the ratio of the longitudinal to transverse photoabsorption cross sections, $R = \sigma_l/\sigma_t$.

A recent fit[8] to the electroproduction data indicates that this ratio is rather small, $R \leq 0.3$ for photons with momenta such that $-3.0 \leq q^2 \leq -0.5$ GeV^2. It turns out that, unlike the nonrelativistic model, the FKR model is substantially in agreement with this number.

While this work was in progress we received a report by Ravndal[9] on the application of the FKR model to electroproduction. Since we agree with his results, we will emphasize here only those of our own which he does not cover. Moreover, in the interest of clarity, we shall attempt to use the same notation as Ravndal.

Following Bjorken and Walecka,[10] the differential cross section for the process $eN \to eN^*$ can be written as

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2 \cos^2(\frac{1}{2}\theta)}{4E^2 \sin^4(\frac{1}{2}\theta)[1+(2E/m)\sin^2(\frac{1}{2}\theta)]}\left[\frac{q^4}{Q^4}\left|f_c\right|^2 + \left(\frac{M^2}{m^2}\tan^2(\tfrac{1}{2}\theta) - \frac{q^2}{2Q^2}\right)(|f_+|^2 + |f_-|^2)\right]. \tag{1}$$

Here E (E') is the energy of the incident (final) electron in the lab frame, θ is the lab-frame scattering angle, M and m are the resonance and nucleon masses, respectively, $q^2=-4EE'\sin^2(\frac{1}{2}\theta)$ is the invariant four-momentum transfer, and Q^2 is the magnitude of the three-momentum transfer from the electron in the isobar rest frame:

$$Q^2=-q^2+(1/4M^2)(M^2-m^2+q^2)^2\,. \tag{2}$$

The form factors f_c, f_+, and f_- are the helicity amplitudes for the excitation of the resonance via virtual photon absorption by the nucleon. They are given by matrix elements of the corresponding components of the electromagnetic current between appropriate nucleon and resonance states. To be explicit, for nucleon and resonance helicities λ and λ^*, respectively, we have

$$f_\rho=(1/2M)\langle N,\lambda=\tfrac{1}{2}|J_\rho(0)|N^*,\lambda^*\rangle;\quad \rho=+,-,c\,, \tag{3}$$

where the N^* is at rest, the photon momentum is in the z direction, $J_\pm=\mp(J_x\pm iJ_y)/\sqrt{2}$ and $J_c=J_t$.

The longitudinal and transverse photoabsorption cross sections are defined by the Hand[11] formula for the electroproduction from nucleons of a hadronic state of mass W:

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{\text{lab}}=\frac{\alpha}{2\pi^2}\frac{W^2-m^2}{2mq^2}\frac{E'}{E}\frac{1}{1-\epsilon}(\epsilon\sigma_l+\sigma_t)\,, \tag{4}$$

where

$$\epsilon^{-1}=1-(2Q^2/q^2)\tan^2(\tfrac{1}{2}\theta)\,.$$

Equation (1) is just the integral of (4) over the particular resonance state in question. Thus, describing the explicit W dependence of σ_l and σ_t around $W=M$ by an appropriate Breit-Wigner expression, Eqs. (1) and (4) yield the simple relation

$$R=\frac{\sigma_L}{\sigma_t}=-\frac{2q^2}{Q^2}\frac{|f_c|^2}{|f_+|^2+|f_-|^2}\,. \tag{5}$$

We have calculated the form factors f_ρ, $\rho=+,-,c$ and the ratio R for any arbitrary resonance with given quark orbital angular momentum L and total angular momentum J. Our results are tabulated below, but before considering their implications we shall illustrate the nature of these calculations by considering a specific example.

Suppose that a proton target is excited to a state with a symmetric space wave function $\psi^s_{LM_L}$ (i.e., the state belongs to a $\{56\}$ of SU(6)). If the final resonance belongs to a decuplet, it must have quark spin $S=\frac{3}{2}$. Hence, f_c and, therefore, R vanishes. If the final resonance belongs to an octet, it must have $S=\frac{1}{2}$ (and $J=L\pm\frac{1}{2}$) and R may be calculated as follows:

As in Ref. 9, we have

$$\begin{aligned}f_c&=9G\langle N,\lambda=\tfrac{1}{2}|e_aSe^{-\lambda a_z}|N^*,\lambda^*=\tfrac{1}{2}\rangle,\\ f_+&=9G\langle N,\lambda=\tfrac{1}{2}|e_a(Ta_++R\sigma_{a-})e^{-\lambda a_z}|N^*,\lambda^*=\tfrac{3}{2}\rangle,\\ f_-&=9G\langle N,\lambda=\tfrac{1}{2}|e_a(Ta_-+R\sigma_{a+})e^{-\lambda a_z}|N^*,\lambda^*=-\tfrac{1}{2}\rangle,\end{aligned} \tag{6}$$

where

$$R=\sqrt{2}\,Q\frac{M+m}{(M+m)^2-q^2}\,,$$

$$S=(3Mm+q^2-m^2)/6M^2\,,$$

$$T=\frac{1}{3M}(\Omega/2)^{1/2},\quad \lambda=(2/\Omega)^{1/2}Q\,.$$

G is an arbitrary form factor and Ω is the FKR oscillator constant equal to 1.05 GeV^2. In the notation of Ref. 4, a resonance in a $(8,\frac{1}{2})$ state has the wave function

$$\Psi_{JM}(N^*,56(8,\tfrac{1}{2}))=\sum_{M_L+M_S=M}C(L\tfrac{1}{2}J;M_LM_SM)\times(\tfrac{1}{2})^{1/2}(\phi^\lambda_N\chi^\lambda_{M_S}+\phi^\rho_N\chi^\rho_{M_S})\psi^s_{LM_L}\,,$$

where $C(J_1J_2J;M_1M_2M)$ is a Clebsch-Gordan coefficient. Thus, for such a state (6) becomes

$$\begin{aligned}f_c&=3GS\langle\psi^s_{00}|e^{-\lambda a_z}|\psi^s_{L0}\rangle C(L\tfrac{1}{2}J;0\tfrac{1}{2}\tfrac{1}{2}),\\ f_+&=3GT\langle\psi^s_{00}|a_+e^{-\lambda a_z}|\psi^s_{L1}\rangle C(1\tfrac{1}{2}J;1\tfrac{1}{2}\tfrac{3}{2}),\\ f_-&=3G[T\langle\psi^s_{00}|a_-e^{-\lambda a_z}|\psi^s_{L,-1}\rangle C(1\tfrac{1}{2}J;-1\tfrac{1}{2}-\tfrac{1}{2})\\ &\quad+R\langle\psi^s_{00}|e^{-\lambda a_z}|\psi^s_{L0}\rangle C(L\tfrac{1}{2}J;0-\tfrac{1}{2}-\tfrac{1}{2})]\,.\end{aligned} \tag{7}$$

If we now make use of the operator identities

$$[L_\pm,e^{-\lambda a_z}]=\sqrt{2}\,\lambda a_\mp e^{-\lambda a_z}\,, \tag{8}$$

where $L_\pm$ are the raising and lowering operators for the orbital angular momentum, we can relate the spatial matrix elements appearing in (7) as follows:

$$\langle\psi^s_{00}|a_\pm e^{-\lambda a_z}|\psi^s_{L,\pm1}\rangle=-\frac{[L(L+1)]^{1/2}}{\lambda\sqrt{2}}\langle\psi^s_{00}|e^{-\lambda a_z}|\psi^s_{L0}\rangle\,. \tag{9}$$

We therefore see that for any given resonance we need only calculate one spatial matrix element to determine all three of its electromagnetic form factors. Further, if we are only interested in the cross-section ratio R, we need not evaluate *any* spatial matrix elements. Indeed, for a $56(8,\frac{1}{2})$ state excited from protons, Eqs. (5), (7), and (9) yield

$$R_p[56(8,\tfrac{1}{2})]=\frac{-2(q^2/Q^2)S^2[C(L\tfrac{1}{2}J;\,0\tfrac{1}{2}\tfrac{1}{2})]^2}{[L(L+1)/2\lambda^2]\,T^2[C(L\tfrac{1}{2}J;\,-1\,\tfrac{1}{2}\,-\tfrac{1}{2})]^2+[RC(L\tfrac{1}{2}J;\,0\,-\tfrac{1}{2}\,-\tfrac{1}{2})-\{T[L(L+1)]^{1/2}/\sqrt{2}\,\lambda\}C(L\tfrac{1}{2}J;\,-1\,\tfrac{1}{2}\,-\tfrac{1}{2})]^2}\,. \tag{10}$$

In an exactly analogous manner one may calculate the form factors appropriate to the excitation of any arbitrary resonance belonging to a {56} or {70} of SU(6). The results of performing such a calculation are shown in Table I for both proton and neutron targets and can be immediately used to calculate the corresponding ratio of cross sections, R.

One can apply the formulas of Table I to each of the well-known nucleon resonances in the 1–2-GeV mass region.[12] However, all of the recently analyzed experiments involving the resonance[8] or deep-inelastic regions[13] observe only the scattered electron and, therefore, sum over *all* contributing resonances (as well as the nonresonant background). Thus, in order to provide a meaningful comparison with experiment, we shall perform a similar sum over resonances (but with the background necessarily omitted).

Before doing so, we note that for any given resonance R has zeros at $q^2=0$ and $q^2=-(3M-m)m$ (as noticed in Ref. 9). The latter zero has the effect of suppressing the magnitude of R so that, for any given resonance, R is never greater than 0.5 for $0\le -q^2 \le 4.0$ GeV^2. In view of the analysis of Brasse *et al.*,[8] this is a very important achievement of the model. It is interesting to note that this zero also occurs in the electric form factor of the proton $G_E^p(q^2)$ which is predicted to vanish at $q^2=-2m^2$. This is a rather curious result since historically one of the supporting arguments[14] for the symmetric quark model was that it did *not* predict a zero in the elastic form factors.[15] At large negative q^2, the R for each resonance is proportional to $(-q^2)$ and, therefore, increases without limit.

To illustrate how the various resonant contributions to R may be added together for fixed values of M, we shall now sum over all {56} states which arise from a given quark state of definite L. In the denominator of Eq. (5), we must add incoherently the contributions to f_+ and f_- arising from $I=\frac{3}{2}$ states with all values of J (i.e., $J=L\pm\frac{3}{2}$, $L\pm\frac{1}{2}$) and from $I=\frac{1}{2}$ states with $J=L\pm\frac{1}{2}$. In the numerator, f_c receives contributions only from the $I=\frac{1}{2}$ states. Therefore, making use of Table I and the unitarity of Clebsch-Gordan coefficients, we obtain

$$R_p[56,L]=\frac{-2q^2}{Q^2}\,\frac{9S^2}{17R^2+9(T^2/\lambda^2)L(L+1)} \tag{11}$$

and $R_n[56,L]=0$ for proton and neutron targets,

TABLE I. Amplitudes for resonance electroproduction.

Resonance multiplet	$(f_c/G)\langle 0\vert e^{-\lambda a_z}\vert L\,0\rangle$	$(f_+/G)\langle 0\vert e^{-\lambda a_z}\vert L\,0\rangle$	$(f_-/G)\langle 0\vert e^{-\lambda a_z}\vert L\,0\rangle$
		A. Proton target	
56(8, $\frac{1}{2}$)	$3SC(L\frac{1}{2}J;\,0\frac{1}{2}\frac{1}{2})$	$-\frac{3}{\sqrt{2}}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,1\frac{1}{2}\frac{3}{2})$	$3\left(-\frac{T[L(L+1)]^{1/2}}{\sqrt{2}\lambda}C(L\frac{1}{2}J;\,\bar{1}\frac{1}{2}\bar{\frac{1}{2}})+RC(L\frac{1}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})\right)$
56(10, $\frac{3}{2}$)	0	$-\sqrt{6}\,RC(L\frac{3}{2}J;\,0\frac{3}{2}\frac{3}{2})$	$\sqrt{2}\,RC(L\frac{3}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$
70(8, $\frac{3}{2}$)	0	0	0
70(10, $\frac{1}{2}$)	$\frac{3}{\sqrt{2}}SC(L\frac{1}{2}J;\,0\frac{1}{2}\frac{1}{2})$	$\frac{3}{2}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,1\frac{1}{2}\frac{3}{2})$	$-\frac{3}{2}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,1\bar{\frac{1}{2}}\frac{1}{2})-\frac{1}{\sqrt{2}}RC(L\frac{1}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$
70(8, $\frac{1}{2}$)	$\frac{3}{\sqrt{2}}SC(L\frac{1}{2}J;\,0\frac{1}{2}\frac{1}{2})$	$-\frac{3}{2}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,1\frac{1}{2}\frac{3}{2})$	$-\frac{3}{2}\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,1\bar{\frac{1}{2}}\frac{1}{2})+\frac{3}{\sqrt{2}}RC(L\frac{1}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$
		B. Neutron target	
56(8, $\frac{1}{2}$)	0	0	$-2RC(L\frac{1}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$
56(10, $\frac{3}{2}$)	0	$\sqrt{6}RC(L\frac{3}{2}J;0\frac{3}{2}\frac{3}{2})$	$-\sqrt{2}RC(L\frac{3}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$
70(, $\frac{3}{2}$)	0	$-\frac{1}{2}\sqrt{3}RC(L\frac{3}{2}J;\,0\frac{3}{2}\frac{3}{2})$	$(1/\sqrt{2})RC(L\frac{3}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$
70(10, $\frac{1}{2}$)	$-\frac{3}{\sqrt{2}}SC(L\frac{1}{2}J;\,0\frac{1}{2}\frac{1}{2})$	$-\frac{3}{2}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,1\frac{1}{2}\frac{3}{2})$	$\frac{3}{2}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,\bar{1}\frac{1}{2}\bar{\frac{1}{2}})+\frac{1}{\sqrt{2}}RC(L\frac{1}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$
70(8, $\frac{1}{2}$)	$-\frac{3}{\sqrt{2}}SC(L\frac{1}{2}J;\,0\frac{1}{2}\frac{1}{2})$	$+\frac{3}{2}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;1\frac{1}{2}\frac{3}{2})$	$\frac{3}{2}T\frac{[L(L+1)]^{1/2}}{\lambda}C(L\frac{1}{2}J;\,\bar{1}\frac{1}{2}\bar{\frac{1}{2}})-\frac{1}{\sqrt{2}}RC(L\frac{1}{2}J;\,0\bar{\frac{1}{2}}\bar{\frac{1}{2}})$

respectively.

The same calculation may be repeated for resonances belonging to a $\{70\}$, in which case we find

$$R_p[70, L]=\frac{-2q^2}{Q^2}\frac{9S^2}{5R^2+9(T^2/\lambda^2)L(L+1)} \tag{12}$$

and

$$R_n[70, L]=\frac{-2q^2}{Q^2}\frac{9S^2}{3R^2+9(T^2/\lambda^2)L(L+1)}\,. \tag{13}$$

In the calculation of these formulas we have implicitly assumed the existence of an SU(6)×O(3) mass degeneracy (i.e., we ignore the possibility of spin-orbit coupling). This is known to be a reasonable approximation even in the 1–2-GeV mass region.

The denominators of (11), (12), and (13) contain two separate terms: one which is due to the orbital motion of the quarks and proportional to $L(L+1)$, and the other due to the quark spins. It turns out that, even for the highest L value allowed for a fixed M by the FKR mass formula, the orbital term is always less than 10% of the spin term at values of $q^2 \leq -1.0$ GeV^2. Thus, we can conveniently neglect this term and thereby render our ratios independent of L.

To determine a final formula for the cross-section ratio that will be appropriate even to deep-inelastic scattering, we must also eliminate the dependence on the SU(6)-multiplet assignments of the resonances. This requires that we account for the relative size of contributions arising from $\{56\}$'s and $\{70\}$'s. It is known[16] that the spectrum of a three-particle harmonic-oscillator system contains twice as many $\{70\}$'s as $\{56\}$'s at high excitation levels. This suggests that the contributions of a $\{56\}$ and a $\{70\}$ should be averaged with relative weights of 1 : 2 in both the numerator and denominator of R. If we now make the assumption that the spatial matrix element for a resonance in a $\{56\}$ does not appreciably differ from that for a $\{70\}$, we can obtain our final formula by separately averaging the numerators and denominators of our previous formulas. Thus, we find that

$$R_p=\frac{-2q^2}{Q^2}\frac{S^2}{R^2} \tag{14}$$

and

$$R_n=\frac{-2q^2}{Q^2}\frac{S^2}{R^2}\,. \tag{15}$$

These expressions for R_p and R_n are only accurate to the same extent as our assumption about the spatial matrix elements. However, the values taken by $R_N[56]$ and $R_N[70]$ $(N=p, n)$ provide not too distant lower and upper bounds on the true ratios. In addition, it is quite unlikely that the true R_p and R_n will lie close to these bounds.

Equations (14) and (15) obviously predict that the ratio of longitudinal to transverse photoabsorption cross sections be the same for both proton and neutron targets. This prediction is, of course, not testable at present.

In Fig. 1 we have plotted R_p as a function of q^2 for several different values of the resonance mass, M. For $M=1.7$ GeV (i.e., at the mass of the "third resonance"), R_p achieves a maximum of 0.32 at $q^2=-0.3$ GeV^2. It then remains less than 0.2 over the region $1.0 \leq -q^2 \leq 9$ GeV^2 following which it rises linearly with $(-q^2)$. This behavior is in good agreement with the analysis of Ref. 8.

For higher values of M (e.g., $M=2.0$ or 3.0 GeV), R_p is rather small, ≤ 0.1, over the interval $1.0 \leq -q^2 \leq 10.0$ GeV^2. In addition, R_p decreases sufficiently rapidly with increasing M that in the (Bjorken) limit as $-q^2 \to \infty$ with $-\nu/q^2$ fixed, R_p tends to zero like $-q^2/\nu^2$. These results are in rough agreement with, but consistently lower than, the data of Bloom *et al.*[13] who quote an average value for R_p of 0.18 ± 0.18. The errors on the measured values of R_p are so large that only qualitative comparisons can be made. In addition, there is a significant diffractive background present which this model is unable to account for. However, it is perhaps worth noting that, for fixed M and as a function of q^2, the R_p of Ref. 13 appears to have a dip at about the same value of q as the zero predicted by this model. More accurate data are, of course, required before a definite identification of these dips can be made.

In conclusion, we have found that the FKR model agrees reasonably well with the existing data and indeed, much better than the nonrelativistic formulation of the quark model. The qualitative

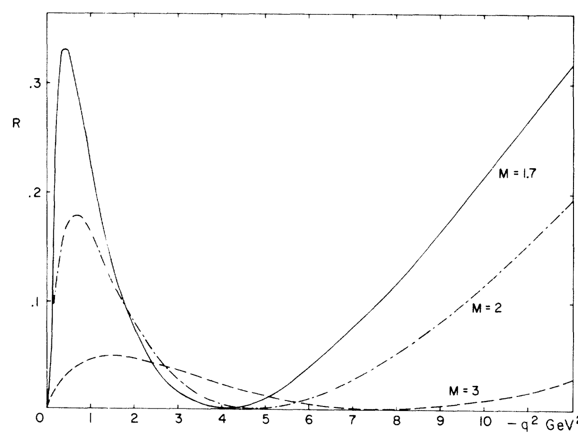

FIG. 1. The ratio $R=\sigma_l/\sigma_t$ plotted as a function of four-momentum transfer squared, $-q^2$, for several values of the mass of the final state.

and rough quantitative agreement at high M and $-q^2$ seems to indicate that resonances play a very significant role in deep-inelastic electroproduction. This is a possibility that has already been emphasized by a number of authors.[17]

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