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Strong Isospin Breaking and Nonleptonic K Decays*

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Assuming (a) an exactly $|\Delta I|=\frac{1}{2}$ weak Lagrangian and (b) the existence of an isospin-breaking term, $\epsilon_3 u_3$, in the strong Hamiltonian, the ratios of reduced decay rates for $K\to 3\pi$ and $K\to 2\pi$ decays, observed experimentally to vary from the $\Delta I=\frac{1}{2}$ predictions by ~10%, are well explained if $\epsilon_3\sim 0.05$. Rare K-decay modes involving two photons are estimated.

I. INTRODUCTION

A number of authors have recently found it necessary to introduce an isospin-breaking term, H_3, into the strong interaction. It is usually added to the strong Hamiltonian H_s,

$$
\begin{aligned}
&H_s = H_0 + H',\\
&H_0 = \mathrm{SU}_3\times\mathrm{SU}_3 \text{ symmetric},\\
&H' = u_0 + \epsilon_8 u_8, \qquad (1)\\
&u_i \in (3,\bar{3}) + (\bar{3},3) \text{ representation of } \mathrm{SU}_3\times\mathrm{SU}_3,\\
&\epsilon_8 \simeq -1.25,
\end{aligned}
$$

so that it belongs to the same representation of $\mathrm{SU}_3\times\mathrm{SU}_3$ as does H':

$$H_3 = \epsilon_3 u_3 . \qquad (2)$$

The motivation for the introduction of H_3 varies considerably. Coleman and Glashow[1] introduced it in their tadpole model to account for the isospin-breaking mass differences of mesons and baryons. Harari,[2] Okubo,[3] and Wilson[4] have shown that it appears as the effective behavior of the electromagnetic interaction at high energy. In the work of Cabibbo and Maiani,[5] and of Gatto *et al.*,[6] it is a consequence of assumed cancellations between leading divergences of higher-order weak interactions; Oakes[7] obtains it from the assumption that $\mathrm{SU}_2\times\mathrm{SU}_2$ symmetry breaking and a nonzero Cabibbo angle are due to a common source. Finally, it has frequently been treated[8] as a phenomenological term demanded by the $\eta\to 3\pi$ decay problem. Independently of the philosophy behind the H_3 term, it is assumed to be present in addition to the conventional electromagnetic Hamiltonian.

The spectrum of proposed values for ϵ_3 is equally wide. Gatto *et al.*[6] find $\epsilon_3\sim 1/137$; the value of Cabibbo and Maiani[5] is $\epsilon_3\sim 1/40$, and that of Oakes is $\epsilon_3\sim 1/18$. Using Dashen's result[9] for the purely electromagnetic part of the kaon mass difference, Osborne and Wallace[10] have performed a more detailed determination of ϵ_3 from the kaon and baryon mass differences. They find a value in agreement with that of Cabibbo and Maiani.

In a chiral-Lagrangian calculation we consider below how H_3 influences the nonleptonic K decays. Specifically, we assume that the nonleptonic weak interaction obeys the $\Delta I=\frac{1}{2}$ rule exactly, and calculate all possible tree graphs involving the pseudoscalar mesons allowed by the weak and strong interactions. The isospin-breaking term, H_3, in the strong Hamiltonian allows $\eta\to\pi^0$ and $\eta\to 3\pi$ virtual transitions which lead to $\Delta I=\frac{3}{2}$ pieces in K-decay amplitudes. The ratios of $K\to 3\pi$ decay rates calculated in this way agree with experiment[11] if Oakes's ϵ_3 is used. However, strong final-state interactions which are considered important in the case of $K\to 3\pi$ are not taken into account. Recent work by Mathews[12] indicates that the strong final-state interactions could provide at least a part of the $\Delta I=\frac{3}{2}$ amplitudes observed. In the $K\to 2\pi$ problem, final-state interactions are not important, but, unlike the $K\to 3\pi$ case, pure electromagnetism contributes. Combining our result for Oakes's ϵ_3 with an estimate by Wallace[13] of the purely electromagnetic contribution, we obtain agreement with experiment.

Our notation is defined in Sec. II; $K\to 3\pi$ decays are treated in III; $K\to 2\pi$ in IV; and rare decay modes of kaons involving two photons are discussed in Sec. V.

II. BASIC FORMULAE

In a notation close to that of Cronin,[14] we write the strong-interaction Lagrangian as

$$\mathcal{L}_s = \frac{1}{8f^2}\left[-\mathrm{Tr}\,\partial_\alpha M^\dagger \partial^\alpha M + \mathrm{Tr}(a\lambda_0 + b\lambda_3 + c\lambda_8)(M+M^\dagger)\right], \qquad (3)$$

where, up to fourth order,

$$M(\phi) = 1 + 2if\phi + 2(if\phi)^2 + \tfrac{4}{3}(if\phi)^3 + \tfrac{2}{3}(if\phi)^4$$

and ϕ is the traceless pseudoscalar-meson matrix

$$\phi = \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i\varphi_i .$$

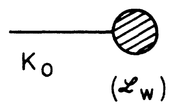

FIG. 1. Tadpole graph allowed by a general weak Lagrangian, $\mathcal{L}_w$.

The constants a and c are determined by the pseudoscalar masses:

$$a=(1/\sqrt{6})(2m_K^{\,2}+m_\pi^{\,2}),$$

$$c=(2/\sqrt{3})(m_\pi^{\,2}-m_K^{\,2}),$$

while $b=\epsilon_3 a$. The constant f is found by Cronin[14] to be $f=m_\pi^{-1}$.

A weak Lagrangian that transforms as the sixth component of an octet is

$$\mathcal{L}_w=\frac{g}{\sqrt{2}}\,\frac{1}{4f^4}\mathrm{Tr}(\lambda_6\partial_\alpha M\partial^\alpha M^\dagger). \tag{4}$$

$\mathcal{L}_w$ is normalized so that g is equal to the Fermi constant. An interesting point is the absence[15] of the $\sin^2\theta_{\mathrm{Cab}}$ factor expected from the Cabibbo[16] theory of weak interactions. However, the derivatives in Eq. (4) are motivated by the current $\times$ current form of leptonic and semileptonic weak interactions. We note that this model for $\mathcal{L}_w$ is not of the most general form: Tadpole graphs such as $K\to$ vacuum, Fig. 1, are not allowed by $\mathcal{L}_w$ of Eq. (4).

We comment here on the reason for working with only eight mesons: Since we are dealing with K decays whose mass is close to that of the η, the propagators of η intermediate states will involve $(m_K^{\,2}-m_\eta^{\,2})^{-1}$, which is to be compared to $m_K^{\,2}$. This ratio has the value 4.5, whereas an η' virtual state would contribute $m_K^{\,2}(m_K^{\,2}-m_{\eta'}^{\,2})^{-1}\simeq 0.36$. For similar reasons one is able to ignore the spin-one mesons as intermediate states.[17]

III. $K\to 3\pi$ DECAYS

As early as 1963 Bouchiat, Nuyts, and Prentki[18] pointed out that the η propagator is important in the $K\to 3\pi$ problem. Given the assumptions of broken chiral symmetry for the strong interaction, a $\Delta I=\frac{1}{2}$ weak Lagrangian, and the isospin-breaking piece of the strong Hamiltonian, H_3, as discussed above, we are now able to perform their calculation without any additional phenomenological information.

We begin by noting that the purely electromagnetic contribution to $\eta\to 3\pi$ decay can be neglected by the Sutherland-Veltman theorem.[19] We thus consider only tree graphs allowed by $\mathcal{L}_w$ and $\mathcal{L}_s$. The $\eta\to\pi^0$ and $\eta\to 3\pi$ vertices are allowed by the H_3 term. The nine graphs contributing to $K\to 3\pi$ decay are shown in Fig. 2. The amplitudes of Figs. 2(a1)–2(a3) have been calculated by Cronin[14]; they are larger than the transitions of Figs. 2(b) and 2(c) by a factor of ϵ_3^{-1} and lead, of course, to an exactly valid $\Delta I=\frac{1}{2}$ rule. The remaining graphs act as a correction to these results.

We now list the matrix elements needed for the 2(b) and 2(c) graphs:

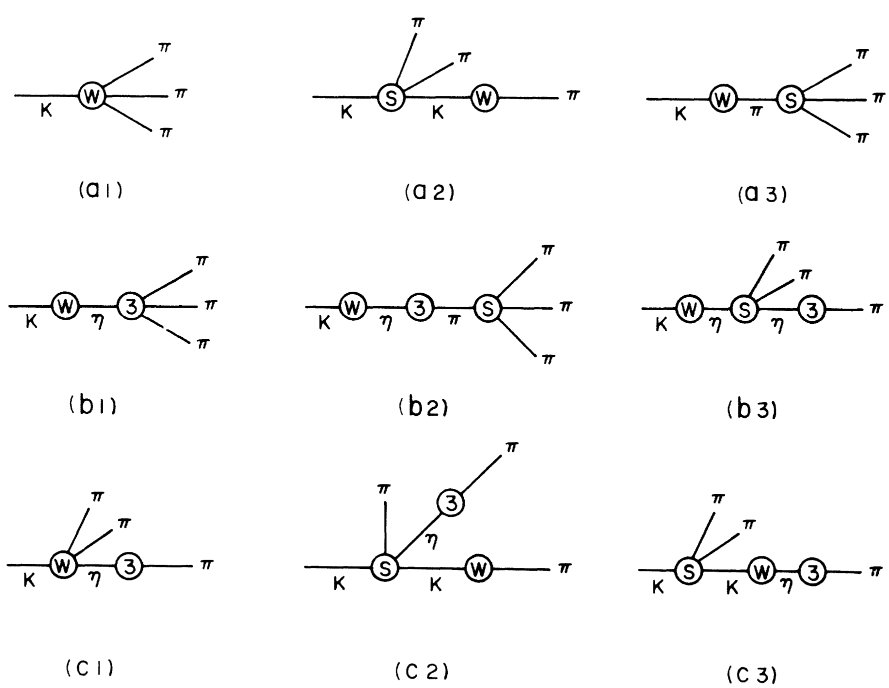

FIG. 2. Feynman graphs contributing to $K\to 3\pi$ decay. S denotes a strong vertex, W a weak vertex, and 3 the isospin-breaking vertex.

$$\langle \pi_a(q_1)\pi_c(q_2)|\mathfrak{L}_s(0)|\pi_b(q_3)\pi_d(q_4)\rangle = 2f^2[\delta_{ab}\delta_{cd}(t-m_\pi^2)+\delta_{ad}\delta_{bc}(u-m_\pi^2)+\delta_{ac}\delta_{bd}(s-m_\pi^2)] + \tfrac{2}{3}f^2(\delta_{ab}\delta_{cd}+\delta_{ad}\delta_{cb}+\delta_{ac}\delta_{bd})\left(4m_\pi^2+\sum_{i=1}^{4}q_i^2\right), \tag{5}$$

$$\langle \pi_a(q_1)K_\alpha(q_2)|\mathfrak{L}_s(0)|\pi_b(q_3)K_\beta(q_4)\rangle = \tfrac{1}{2}f^2\left[(2t+s+u-2m_K^2-2m_\pi^2)\delta_{ab}\delta_{\alpha\beta} + i(s-u)\epsilon_{bal}\sigma^l_{\beta\alpha}+\tfrac{4}{3}\delta_{ab}\delta_{\alpha\beta}\left(2m_K^2+2m_\pi^2+\sum_{i=1}^{4}q_i^2\right)\right], \tag{6}$$

$$\langle \pi_a(q_1)\eta(q_2)|\mathfrak{L}_s(0)|\pi_b(q_3)\eta(q_4)\rangle = 2f^2\delta_{ab}\left(\tfrac{1}{4}\sum_{i=1}^{4}q_i^2+\tfrac{1}{3}t\right), \tag{7}$$

$$\langle K_\alpha(q_1)\pi_a(q_2)|\mathfrak{L}_s(0)|K_\beta(q_3)\eta(q_4)\rangle = 2f^2\delta_{ab}(m_\pi^2-m_K^2)\sigma^b_{\alpha\beta} \tag{8}$$

where $s=-(q_1+q_2)^2$, $t=-(q_1-q_3)^2$, and $u=-(q_1-q_4)^2$. $\sigma^a_{\alpha\beta}$ stands for the $\alpha\beta$ component of the ath Pauli matrix. These results follow from the expansion of Eq. (3) to fourth order in the pseudoscalar fields.

The weak amplitudes are listed next:

$$\langle K_L(q_1)\eta(q_2)|\mathfrak{L}_w(0)|\pi^+(q_3)\pi^-(q_4)\rangle = (\tfrac{2}{3})^{1/2}g(s-\tfrac{5}{3}m_\pi^2), \tag{9}$$

$$\langle K_L(q_1)|\mathfrak{L}_w(0)|\pi^0(q_2)\rangle = -\langle K^+(q_1)|\mathfrak{L}_w(0)|\pi^+(q_2)\rangle \tag{10}$$

$$= -(\tfrac{1}{2})^{1/2}(g/f^2)q_1\cdot q_2, \tag{11}$$

and, finally, those allowed by $\mathfrak{L}_3 \equiv b\,\mathrm{Tr}\lambda_3(M+M^\dagger)$:

$$\langle \eta(q_1)\pi^0(q_2)|\mathfrak{L}_3(0)|\pi^+(q_3)\pi^-(q_4)\rangle = -\tfrac{1}{9}\sqrt{2}\,bf^2, \tag{12}$$

$$\langle \eta(q_1)|\mathfrak{L}_3(0)|\pi^0(q_2)\rangle = -b/\sqrt{3}. \tag{13}$$

When collected together these results give the following values to the amplitudes of Fig. 2(b1) to 2(c3) for the process $K_L(k)\to\pi^+(q_1)+\pi^-(q_2)+\pi^0(q_3)$:

$$A(\mathrm{b1}) = \frac{g}{9\sqrt{3}}b\frac{m_K^2}{m_\eta^2-m_K^2},$$

$$A(\mathrm{b2}) = -\frac{g}{3\sqrt{2}}b\frac{m_K^2}{m_\eta^2-m_K^2}\frac{m_\pi^2}{m_K^2-m_\pi^2},$$

$$A(\mathrm{b3}) = \frac{g}{36\sqrt{2}}b\frac{m_K^2}{m_\eta^2-m_K^2}\frac{3m_K^2+13m_\pi^2}{m_\eta^2-m_\pi^2},$$

$$A(\mathrm{c1}) = \frac{\sqrt{2}}{3}gb\frac{5m_\pi^2/3+(k-q_3)^2}{m_\eta^2-m_\pi^2},$$

$$A(\mathrm{c2}) = \frac{1}{3\sqrt{2}}gb\frac{m_\pi^2}{m_\eta^2-m_\pi^2},$$

$$A(\mathrm{c3}) = \frac{1}{12\sqrt{2}}gb\frac{m_\pi^2}{m_\eta^2-m_\pi^2}\frac{\tfrac{1}{3}(m_K^2-m_\pi^2)-(k-q_3)^2}{m_K^2-m_\pi^2}.$$

On general grounds one expects the A(b1) amplitude to be dominant; the above results show that some of the remaining graphs contribute significantly.

We now write $A(b)+A(c)$ in the form conventionally[20] used in $K\to3\pi$ work:

$$A = A_{\mathrm{av}}[1+\sigma(s_3-s_0)/m_\pi^2],$$

where

$$s_i=(k-q_i)^2,\quad s_0=\tfrac{1}{3}\sum_{i=1}^{3}s_i.$$

For $\epsilon_3=1/40$ we find

$$\sigma(\mathrm{b+c})=0.09, \quad A_{\mathrm{av}}(\mathrm{b+c})=2.14\times10^{-8}. \tag{14}$$

The corresponding results for the amplitudes of Fig. 2(a) are[14]

$$\sigma(\mathrm{a})=-0.24, \quad A_{\mathrm{av}}(\mathrm{a})=-71.5\times10^{-8}. \tag{15}$$

Equation (14) is changed to $\sigma(\mathrm{b+c})=0.09$, $A_{\mathrm{av}}=4.3\times10^{-8}$ if Oakes's value of ϵ_3 is used.

We now have the amplitude for $K_L\to\pi^+\pi^-\pi^0$ decay due to the Feynman graphs of Fig. 2. In order to get all the remaining $K\to3\pi$ amplitudes, we note the following relations[20]:

$$A_{\mathrm{av}}(00+)=\tfrac{1}{2}A_{\mathrm{av}}(++-), \quad A_{\mathrm{av}}(+-0)=(\tfrac{2}{3})^{1/2}A_{\mathrm{av}}(000). \tag{16}$$

Equations (16) are true in the approximation in which the H_3 term appears only once in any given diagram. This means there will be no diagram containing a $\Delta I=\frac{5}{2}$ part, and the final pions must be in isospin-1 states.[20] From $A_{\mathrm{av}}(+-0)$ we get the A_{av} for all the remaining modes. Using Eq. (16) we get $A_{\mathrm{av}}(000)$ from $A_{\mathrm{av}}(+-0)$. The other two amplitudes are obtained by noting that the graphs of Fig. 2(a) determine the $\Delta I=\frac{1}{2}$ part of $A_{\mathrm{av}}(+-0)$ from which one can get the $\Delta I=\frac{1}{2}$ part of $A_{\mathrm{av}}(++-)$. But $A_{\mathrm{av}}(++-)$ has no other part because the graphs of Figs. 2(b) and 2(c) involve η intermediate states and do not contribute to a final state with no neutral pions. The last amplitude, $A_{\mathrm{av}}(00+)$, follows from $A_{\mathrm{av}}(++-)$; again by Eq. (16).

One can now compare with experimentally determined reduced decay rates γ defined by

$$\gamma=\Gamma/\phi,$$

TABLE I. Comparison of theoretical and experimental ratios. The theory columns contain the results of the calculations described in Sec. III. Experimental numbers are from B. H. Kellett, Nucl. Phys. B26, 237 (1971). The $\Delta I=\frac{1}{2}$ rule column indicates the value of the ratio if $\mathcal{L}_w$ is strictly $\Delta I=\frac{1}{2}$ *and* there are no isospin-breaking mechanisms available.

Ratio	Theory ($\epsilon_3=\frac{1}{18}$)	Theory ($\epsilon_3=\frac{1}{40}$)	Expt.	$\Delta I=\frac{1}{2}$ rule
$\frac{\gamma(++-)}{4\gamma(00+)}$	1	1	0.955 ± 0.09	1
$\frac{\gamma(000)}{\frac{3}{2}\gamma(+-0)}$	1	1	0.94 ± 0.09	1
$\frac{\gamma(+-0)}{2\gamma(00+)}$	0.88	0.935	0.835 ± 0.11	1
$\frac{\gamma(000)}{\gamma(++-)-\gamma(00+)}$	0.88	0.935	0.835 ± 0.13	1

where ϕ is the phase space available in the process whose decay rate is Γ. We use a recent compilation of experimental results by Kellett.[11] The results[21] are shown in Table I.

We end with a comment on final-state interactions. The energy release in the $K\to3\pi$ decay is small, and the final pions move slowly. Thus, one expects final-state interactions to be non-negligible. The electromagnetic contributions have been calculated and subtracted[11] from the experimental data presented in Table I. The strong final-state interactions are not well understood and are for that reason ignored. Recent work of Mathews[22] suggests that at least a part of the $\Delta I=\frac{1}{2}$ rule violation in the $K\to3\pi$ decays comes from the strong final-state interactions. It is still possible that the smaller (and more realistic) value $\epsilon_3\simeq1/40$ can account for the data.

IV. $K\to2\pi$

If the nonleptonic weak interaction satisfies exactly the $\Delta I=\frac{1}{2}$ rule, the $K^+\to\pi^+\pi^0$ decay must be due to some isospin-breaking mechanism; if this is electromagnetism the ratio of amplitudes

$$R=\frac{A(K^+\to\pi^+\pi^0)}{A(K_S\to\pi^+\pi^-)} \tag{17}$$

is expected to be of the order of α/π. The experimental number is $R\simeq\frac{1}{22}$. There has been a number of explanations of this difficulty.[23] Riazuddin and Fayyazuddin[24] were the first to consider virtual η particles and their mixing with π^0 in this context. In an order-of-magnitude calculation they obtained good agreement with experiment while maintaining the $\Delta I=\frac{1}{2}$ rule for the weak vertex. Of the papers on this subject the work of Goyal, Li, and Segrè[25] is closest to what is done here.

We view the $K^+\to\pi^+\pi^0$ transition as proceeding through an η intermediate state. The η-π^0 vertex is calculated from the model for H_3 and is given by Eq. (13). R is now written as[26]

$$R=\frac{\langle K^+|\mathcal{L}_w|\pi^+\eta\rangle(m_\eta^2-m_\pi^2)^{-1}\langle\eta|\mathcal{L}_3|\pi^0\rangle}{\langle K_S|\mathcal{L}_w(0)|2\pi\rangle}.$$

The ratio

$$R_\eta=\frac{\langle K^+|\mathcal{L}_w(0)|\pi^+\eta\rangle}{\langle K_S|\mathcal{L}_w(0)|2\pi\rangle} \tag{18}$$

is calculated by expanding $\mathcal{L}_w$ to third order in the fields. We find $R_\eta=-1/\sqrt{3}$. Goyal *et al.* obtain the same result using the K^*-pole model of Sakurai.[27] With this value of R_η and Eq. (13), we find $R=-\frac{1}{95}$ for Cabibbo's ϵ_3 and $R=-1/34.5$ for Oakes's ϵ_3.

Unlike the $K\to3\pi$ case, $K^+\to\pi^+\pi^0$ can proceed via pure electromagnetism. A rough estimate of this contribution to R has been made by Wallace,[13] who finds $R_{\rm EM}\simeq\frac{1}{45}$. Thus, together with our result, when Oakes's $\epsilon_3=\frac{1}{18}$ is used, the η pole in the $K^+\to\pi^+\pi^0$ decay is seen to provide an explanation of the anomalously large R.

V. RARE DECAY MODES

The K decay modes involving two photons can be simply understood as virtual weak decays of the kaon into π^0 and/or η states which then decay into 2γ's. This picture is due to Oneda.[28] Diagrams for $K_L\to2\gamma$ and $K_L\to\pi^0+2\gamma$ are shown in Figs. (3) and (4).

The transition of Fig. 4(a) involves a CP-violating vertex. We thus neglect it in comparison with Fig. 4(b). We use Eq. (9) for the weak vertices and the experimental numbers[29] for the electromagnetic vertices and obtain

$$\Gamma(K_L\to2\gamma)=\left(\frac{g}{\sqrt{2f^2}}\right)^2\left(\frac{m_K^2}{m_K^2-m_\pi^2}+\frac{R}{\sqrt{3}}\frac{m_K^2}{m_K^2-m_\eta^2}\right)^2\times(m_K/m_\eta)^3\Gamma(\eta\to2\gamma), \tag{19}$$

where R is defined by[30]

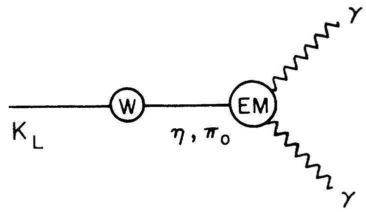

FIG. 3. Model for $K_L \to 2\gamma$ decay.

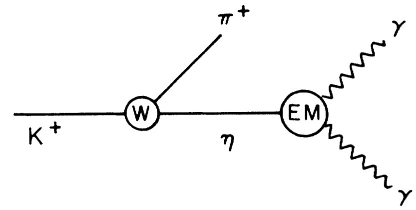

FIG. 5. Model for $K^+ \to \pi^+ + 2\gamma$.

$$\frac{\Gamma(\eta \to 2\gamma)}{\Gamma(\pi^0 \to 2\gamma)} = \frac{1}{3}\left(\frac{m_\eta}{m_{\pi^0}}\right)^3 R^2 .$$

Experimentally,[29] $R^2 \simeq 5.2$ and Eq. (19) leads to

$$\Gamma(K_L \to 2\gamma) = 1.9 \times 10^{-10} \text{ eV}.$$

The measured decay rate[29] is $\Gamma(K_L \to 2\gamma) \sim 6.3 \times 10^{-12}$ eV. It is surprising that, in spite of the cancellation occurring in Eq. (19), we get a number too large by nearly two orders of magnitude. If we take the attitude that R changes in value when the particles are off the mass shell, and, in particular, if we assume it to have its SU_3-symmetric value $R = 1$, we find $\Gamma(K_L \to 2\gamma) = 3.9 \times 10^{-11}$ eV. This is larger than Γ_{exp} by a factor of 7.5. The $\eta' \to 2\gamma$ decay mode is not known very well experimentally, but its rate could possibly[29] be considerably (by two orders of magnitude, say) larger than $\Gamma(\eta \to 2\gamma)$. In that case the η'-pole contribution could lead to a significant cancellation in Eq. (19).

Turning to the $K_L \to \pi^0 + 2\gamma$ process, we first note that a π^0 intermediate state is equally possible. As can be seen from Fig. 4(b) this would involve a

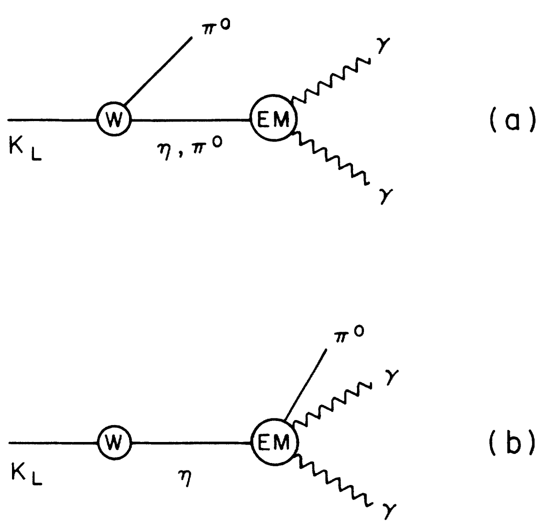

FIG. 4. Possible $K_L \to \pi^0 + 2\gamma$ transitions. (a) includes CP-violating vertex and is ignored in text. It could provide a model for $K_S \to \pi^0 + 2\gamma$ decay. EM stands for an effective electromagnetic vertex.

$\pi^0 \to \pi^0 + 2\gamma$ vertex, about which there is no phenomenological information. The η intermediate state leads simply to[31]

$$\Gamma(K_L \to \pi^0 + 2\gamma) = \left(\frac{g}{\sqrt{6}f^2}\right)^2 \left(\frac{m_K^2}{m_\eta^2 - m_K^2}\right)^2 \left(\frac{m_K}{m_\eta}\right)^3 \times \Gamma(\eta \to \pi^0 + 2\gamma) .$$

Experimentally, this process is hard to detect, and it is not listed by the Particle Data Group.

Last of all we wish to consider $K^+ \to \pi^+ + 2\gamma$. The experimental upper limit on this partial decay is 1.1×10^{-4} of the total decay rate. We assume that the dominant amplitude is that of Fig. 5.

The $K^+ \to \pi^+\eta$ vertex is given by

$$\langle K^+(k) | \mathcal{L}_w(0) | \pi^+(q_1)\eta(q_2)\rangle = -i\frac{g}{2\sqrt{3}f}(2k^2 - q_1^2 - q_2^2).$$

The off-shell amplitude for η decay into 2γ is now needed. The simplest effective Lagrangian for η decay is

$$\mathcal{L} = \gamma F_{\mu\nu}\tilde{F}^{\mu\nu}\eta,$$

where $F_{\mu\nu}$ is the electromagnetic field tensor and $\tilde{F}^{\mu\nu}$ its dual. Such a Lagrangian was first considered for the above purposes by Okubo and Sakita.[31] More recently Adler[32] has shown that $F_{\mu\nu}\tilde{F}^{\mu\nu}$ arises in perturbation theory as an anomalous part of the axial-vector divergence and leads to a good prediction for the π^0 lifetime.

We fix the value of the constant γ from the experimental decay rate of $\eta \to 2\gamma$. It is straightforward now to write down the amplitude. Integrating over the two independent variables, conveniently chosen to be the energies of the two photons, one obtains

$$\Gamma(K^+ \to \pi^+ + 2\gamma) = 2.2 \times 10^{-12} \text{ eV}.$$

The experimental limit on the partial rate is

$$\Gamma_{\text{exp}}(K^+ \to \pi^+ + 2\gamma) < 5.5 \times 10^{-12} \text{ eV}.$$

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[1]S. Coleman and S. L. Glashow, Phys. Rev. 134B, 671 (1964).

[2]H. Harari, Phys. Rev. Letters 17, 1303 (1966).

[3]S. Okubo, Phys. Rev. Letters 18, 256 (1967).

[4]K. G. Wilson, Phys. Rev. 179, 1499 (1969).

[5]N. Cabibbo and L. Maiani, Phys. Letters 28B, 131 (1968).

[6]R. Gatto, G. Sartori, and M. Tonin, Phys. Letters 28B, 128 (1968).

[7]R. J. Oakes, Phys. Letters 29B, 683 (1969).

[8]S. Bose and A. Zimerman, Nuovo Cimento 43A, 1165 (1966); R. Ramachandran, *ibid.* 47A, 669 (1967); Y. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. 161, 1612 (1967).

[9]R. Dashen, Phys. Rev. 183, 1245 (1969).

[10]H. Osborn and D. J. Wallace, Nucl. Phys. B20, 23 (1970).

[11]B. H. Kellett, Nucl. Phys. B26, 237 (1971).

[12]R. D. Mathews, LRL Report No. UCRL-20247 (unpublished).

[13]D. J. Wallace, Nucl. Phys. B12, 245 (1969).

[14]J. A. Cronin, Phys. Rev. 161, 1483 (1967).

[15]See J. A. Cronin, Ref. 14, and references quoted therein.

[16]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[17]See, however, D. Greenberg, Phys. Rev. 178, 2190 (1969), who obtains good results with just these intermediate states.

[18]C. Bouchiat, J. Nuyts, and J. Prentki, Phys. Letters 3, 156 (1963).

[19]D. G. Sutherland, Phys. Letters 23, 384 (1966).

[20]R. E. Marshak, Riazuddin, and C. Ryan, *Theory of Weak Interactions in Particle Physics* (Interscience, New York, 1969).

[21]There are two independent slope-parameter ratios, only one of which is changed from the $\Delta I = \frac{1}{2}$ rule predictions by our calculation. The uncorrected (i.e., the $\Delta I = \frac{1}{2}$ rule) value of $\sigma(00+)/\sigma(+-0)$ is 1. This is changed to 1.05 by our work. The experimental number is 1.25. Thus, although the change is to the better, it is not a significant change.

[22]See Ref. 12 and references quoted therein.

[23]N. Cabibbo, Phys. Rev. Letters 12, 62 (1964); J. J. Sakurai, Phys. Rev. 156, 1508 (1967); L. J. Clavelli, *ibid.* 160, 1384 (1967); J. Schechter, *ibid.* 161, 1660 (1967); S. Okubo, R. E. Marshak, and V. S. Mathur, Phys. Rev. Letters 19, 407 (1967).

[24]Riazuddin and Fayyazuddin, Phys. Rev. 129, 2337 (1963).

[25]A. Goyal, L-F. Li, and G. Segrè (unpublished).

[26]It is easily seen that $\langle K_S|\mathcal{L}_w|\pi^+\pi\rangle = \langle K_S|\mathcal{L}_w|2\pi^0\rangle$. We thus use the charged and neutral modes interchangeably.

[27]J. J. Sakurai, see Ref. 23.

[28]S. Oneda, Phys. Rev. 158, 1541 (1967).

[29]Particle Data Group, Rev. Mod. Phys. 42, 87 (1970).

[30]The $(m_K/m_\eta)^3$ factor is due to phase space.

[31]S. Okubo and B Sakita, Phys. Rev. Letters 11, 50 (1963).

[32]S. L. Adler, Phys. Rev. 177, 2426 (1969).