[16]Steven Weinberg, Phys. Rev. Letters 17, 616 (1966), reprinted in S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics*, Ref. 2.

[17]See the compilation by G. Ebel *et al.* (Ref. 10). The value $a_{1/2} - a_{3/2} = 0.254 \pm 0.013$, A. Donnachie and G. Shaw, Nucl. Phys. 87, 556 (1966), is representative of Panofsky-ratio values, although a value as low as $a_{1/2} - a_{3/2} = 0.245 \pm 0.001$ has been reported by C. M. Rose, Phys. Rev. 154, 1305 (1967).

[18]We use an average of the values $g_A = 1.226 \pm 0.011$ and $g_A = 1.26 \pm 0.02$ discussed by R. J. Blin-Stoyle and J. M. Freeman, Nucl. Phys. A150, 369 (1970).

[19]To be more precise, the functions $\bar{a}$, $\bar{b}$, $\bar{c}$ are regular in ν and ν_B in a neighborhood of $\nu = \nu_B = 0$. We make no assertions about analyticity in external masses. At $\nu = \nu_B = 0$ the two-pion t-channel intermediate state is formally singular in the pion mass $[O(\ln \mu^2)]$. However, this does not alter our numerical estimate. See Heinz Pagels and W. J. Pardee, Phys. Rev. (to be published).

[20]R. D. Peccei, Phys. Rev. 176, 1812 (1968).

PHYSICAL REVIEW D VOLUME 4, NUMBER 9 1 NOVEMBER 1971

Nucleon Form Factors, Vector Dominance, and Lorentz Contraction

Arthur Lewis Licht and Antonio Pagnamenta
Department of Physics, University of Illinois at Chicago Circle, Chicago, Illinois 60680
(Received 15 March 1971)

We use the notion of Lorentz contraction of a composite cluster combined with vector-meson dominance of the one-photon exchange to derive the asymptotic form of the nucleon form factors. For the three-quark model we find a t^{-2} prediction which fits the data very well at large spacelike t. Only the observed vector mesons, ρ, ω, and ϕ, are used. Our results predict deviations from the scaling laws which are directly related to the nonvanishing electric form factor of the neutron and are of the same magnitude.

I. INTRODUCTION

There are two classes of attempts to fit the electromagnetic form factors of the nucleons. The first one tries to find a simple analytic expression without giving it any theoretical justification. The best known is the dipole fit.[1] More recently we have seen a superposition of exponentials[2] and a ratio of Γ functions.[3] The data have now become accurate enough so that clear deviations from the dipole fit are evident[4] especially at high (negative) t. The second method uses dispersion relations[5] and vector-meson dominance.[6] This has not been very successful in the region of large spacelike momentum transfer. In order to get fair agreement with the data, either large negative couplings to unobserved vector mesons had to be assumed,[7] or several *ad hoc* structure parameters had to be introduced. An extensive list of references on additional work on form factors can be found in review articles.[8,9]

We propose a different approach. We assume that the nucleon at rest is a bound state of three quarks (or partons). We calculate a quark-nucleon form factor in the region of large spacelike momentum transfer by making a Lorentz transformation of the arguments of the quark wave functions.[10,11] Combining this with the dominance of only the established vector mesons, we find excellent agreement with experiment.

Our results depend to a certain extent on the choice of the quark wave function. We find the best fit for a symmetric Gaussian wave function. We have tried an antisymmetric Gaussian and an antisymmetric exponential wave function. We find that they both fit very badly at small t.

We assume that the photon couples to the quark through the known vector mesons ρ, ω, ϕ. The vector meson then couples directly to a single quark, forming the vertex shown in Fig. 1. There is also a class of vertices where the vector meson breaks up into a number of pions, which then interact with the same or with different quarks, as shown in Fig. 2.

We assume that the vertex of Fig. 2(a) is dominant, and neglect the others. It is conceivable that they might provide small corrections to our results at low momentum transfers.

An approach somewhat similar to ours has been tried by Fujimura, Kobayashi, and Namiki.[12] They use the relativistic wave function due to Takabayashi.[13] With one adjustable parameter they obtain reasonable agreement with the data. Because they have a four-dimensional harmonic oscillator wave function, they get a different dependence of the form factor on the number of constituents. In order to get a t^{-2} behavior they have to introduce an additional Lorentz factor into the already covariant meson propagator. Our results show that this is not needed. Barut[14] has proposed an interesting group-theoretical method which depends only on the Lorentz-transformation properties of the final state.

In Sec. II we briefly review our main assumptions about vector dominance, quark additivity, and wave-function contraction. In Sec. III we review in detail the application of vector dominance to nucleon form factors. In Sec. IV we derive the nucleon form factors in terms of quark form factors, using the additivity assumption. The quark form factors are derived in Sec. V, assuming that the quarks look like point Dirac particles. The results of Secs. III–V are combined in Sec. VI to give explicit expressions for the nucleon form factors. In Sec. VII we discuss the scaling laws. We show that the departure from the exact scaling is related to the nonvanishing of the neutron charge form factor. In Sec. VIII, we compare the form factors which result from a symmetric Gaussian wave function with the data. In Sec. IX we discuss some antisymmetric wave functions. Our conclusions, a discussion of the discrepancies at low momentum transfer, and some open questions are presented in Sec. X.

Our assumptions concerning the meson propagators are given in Appendix A, where we also discuss the influence of the 2π threshold and the finite width of the ρ meson. In Appendix B we derive the ϕ-ω mixing parameter from the decay rates $\omega\to 3\pi$, $\phi\to 3\pi$. In Appendix C we discuss the effect of a small anomalous quark magnetic moment on the nucleon magnetic moments.

II. REVIEW

Consider the process described by the vertex in Fig. 1. A photon becomes a vector meson which attaches itself to one of the quarks in a hadron. The hadron then changes from a quark cluster of type A, with momentum p, to a cluster of type B with momentum p'. The matrix element of the electromagnetic current for this transition is, according to the vector-dominance model,[6]

$$\langle B|J^\mu_{\rm em}|A\rangle = \sum_V c_V \Delta_V(t)\langle B|J^\mu_V|A\rangle\,. \qquad (1)$$

Here, $\Delta_V(t)$ is the propagator for the vector meson V, $\{c_V\}$ is a set of constants, and J^μ_V is the strong-current source of the vector meson V.

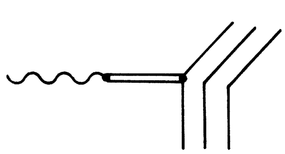

FIG. 1. Vector dominance of the photon propagator. The vector mesons ρ, ω, or ϕ are attached to a single quark. In the expression for the form factor, the sum over all three quark lines is understood.

We assume that the strong-current matrix elements depend additively[15–17] on the constituent quarks. That is

$$\langle B|J^\mu_V|A\rangle = \sum_q \langle B|J^\mu_{qV}|A\rangle\,, \qquad (2)$$

where J^μ_{qV} is the strong current carried by the quark q.

As discussed in I, the transition from A to B looks most instantaneous in the Breit frame [where $p+p'=(2p_0,\vec{0})$ has no spatial component], particularly at large momentum transfer. It seems physically reasonable, therefore, to use an impulse approximation in which the transition is regarded as instantaneous in this frame.

We evaluate the matrix elements $\langle B|J^\mu_{qV}|A\rangle$ by equating them in the $p+p'$ Breit frame to the matrix elements of the current of a pointlike quark by the use of Lorentz-boosted wave functions. It turns out that

$$\langle B|J^\mu_{qV}|A\rangle = \langle\!\langle B|J^\mu_q|A\rangle\!\rangle S_{BA}((p-p')^2)\,, \qquad (3)$$

where $\langle\!\langle B|J^\mu_q|A\rangle\!\rangle$ depends only on the momenta and the spins, and $S_{BA}(t)$ depends on the spatial wave functions of the quark clusters.

The spatial form factor is, in the nonrelativistic case,

$$S^0_{BA}(\vec{q}^{\,2}) = \prod_{i=1}^{n-1}\int d^3x_i\,\psi^*_B(\{\vec{x}_k\})e^{i\vec{q}\cdot\vec{x}_j}\psi_A(\{\vec{x}_k\})\,, \qquad (4)$$

where $\vec{q}=\vec{p}-\vec{p}\,'$, and ψ_A, ψ_B are the rest-frame wave functions, and n is the number of quarks. The quark positions $\vec{x}_i$, relative to the center of mass, are such that

$$\sum_{i=1}^{n}\vec{x}_i = 0\,. \qquad (5)$$

We generalize this to the relativistic case by replacing the rest-frame wave functions by properly Lorentz-contracted ones ψ^p_A, $\psi^{p'}_B$ and evaluating the integral in the $p+p'$ Breit frame. The correspondence between a moving frame and the rest frame

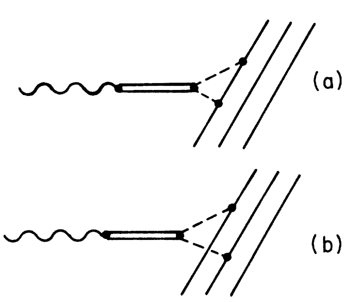

FIG. 2. (a) The diagram which is retained in the accurate calculation of the ρ propagator. (b) This diagram was neglected. Similarly, no 3π intermediate states were included in the ϕ, ω calculations.

is obtained by assuming that all the quarks move on world lines parallel to the total momentum p or p'. This amounts to neglecting the relative motion of the quarks inside the cluster. As shown in I, this implies that a Breit-frame vector $\vec{x}_i$ corresponds to the rest-frame vector

$$\vec{y}_i = \text{vector part } \{L_p^{-1}(\vec{x}_i, 0)\} = H_p \vec{x}_i \,. \tag{6}$$

We assume that an observer in the Breit frame sees the same (spinless) probability amplitude as an observer in the rest frame. Thus we identify

$$\psi_A^p(\{\vec{x}_k\}) = \psi_A(\{\vec{y}_k\}) = \psi_A(\{H_p \vec{x}_k\}) \,. \tag{7}$$

This leads to the expression for the relativistic form factor

$$S_{BA}(q^2) = \prod_{i=1}^{n-1} \int d^3x_i \psi_B^*(\{H_{p'} \vec{x}_k\}) e^{i\vec{q}\cdot\vec{x}_j} \psi_A(\{H_p \vec{x}_k\}) \,, \tag{8}$$

where the integral is to be performed in the Breit frame. Equation (8) can be written in manifestly covariant form, as the Breit-frame $\vec{q}^2$ can be expressed as a function of $q^2 = (p - p')^2$ and the masses.

For the case of the elastic electromagnetic form factors, A is the same as B. A change of variables in (8) changes the argument of S and introduces a Jacobian. It leads to the relationship

$$S(t) = \alpha^{(1-n)/2} S_{AA}^0(-t/\alpha) \tag{9}$$

between the relativistic and nonrelativistic form factors. Here $t = q^2$ is spacelike, and

$$\alpha = 1 - t/4M_A^2 \,, \tag{10}$$

and n is the number of subparticles in the cluster A.

This result has a simple physical interpretation. In the Breit frame the wave functions ψ_A^p, $\psi_B^{p'}$ have their supports Lorentz-contracted in the direction of motion. The contraction is given by the Lorentz factor $(1-\beta^2)^{1/2}$. The contraction of the support implies that the integral will be reduced by one such factor for each integration along the direction of motion. A bound state of n particles has $n-1$ independent position vectors, therefore, $n-1$ integrations. We thus expect a factor $(1-\beta^2)^{(n-1)/2}$. However, $(1-\beta^2)^{1/2}$ is $\alpha^{-1/2}$ when expressed in terms of the invariant momentum transfer t.

The contraction of the support in x space implies a corresponding expansion of the support in momentum space. For example for a Gaussian wave function the form factor is a function of $\vec{q}^2/\Delta\vec{q}^2$, where $\Delta\vec{q}^2$ is the uncertainty in momentum. The substitution $\vec{q}^2 \to \vec{q}^2/\alpha$ has the effect of replacing $\Delta\vec{q}^2$ by $\alpha\Delta\vec{q}^2$, thus increasing the dispersion in momentum.

III. VECTOR DOMINANCE

According to the usual ideas of vector dominance,[6] the electromagnetic current can be written as a sum of those meson fields which carry the same quantum numbers as the photon. The experimentally observed particles are the ρ, ω, and ϕ mesons. Hence we write

$$J_\mu^\gamma(x) = \frac{m_\rho^2}{2\gamma_\rho} \rho_\mu^{(3)}(x) + \frac{m_\omega^2}{2\gamma_\omega} \omega_\mu(x) + \frac{m_\phi^2}{2\gamma_\phi} \phi_\mu(x) \,, \tag{11}$$

where γ_ρ, γ_ω, and γ_ϕ are coupling constants. This is of the form

$$J_\mu^\gamma(x) = J_\mu^{I_3}(x) + J_\mu^S(x) \,, \tag{12}$$

where the isovector part of the current is

$$J_\mu^{I_3}(x) = \frac{m_\rho^2}{2\gamma_\rho} \rho_\mu^{(3)}(x) \tag{13}$$

and the isoscalar part is

$$J_\mu^S(x) = \frac{m_\omega^2}{2\gamma_\omega} \omega_\mu(x) + \frac{m_\phi^2}{2\gamma_\phi} \phi_\mu(x) \,. \tag{14}$$

We obtain some information about the coupling constants by requiring that for all states $|A\rangle$, $|B\rangle$,

$$\int d^3x \langle A | J_0^{I_3}(\vec{x}, 0) | B \rangle = \langle A | I_3 | B \rangle \,, \tag{15}$$

$$\int d^3x \langle A | J_0^S(\vec{x}, 0) | B \rangle = \tfrac{1}{2} \langle A | Y | B \rangle \,, \tag{16}$$

where I_3 and Y are the generators of z isospin and hypercharge.

It is assumed that the ρ couples to the isospin current J_μ^I, and that the ω and the ϕ couple to an appropriate mixture of the hypercharge current, Y_μ, and the baryon number current, N_μ.

Following Kroll, Lee, and Zumino,[6] we write

$$\begin{aligned} \langle A | \rho_\mu^i | B \rangle &= g D_\rho(t) \langle A | J_\mu^{I_i} | B \rangle \,, \\ \langle A | \omega_\mu | B \rangle &= D_\omega(t) \langle A | J_\mu^\omega | B \rangle \,, \\ \langle A | \phi_\mu | B \rangle &= D_\phi(t) \langle A | J_\mu^\phi | B \rangle \,, \end{aligned} \tag{17}$$

as well as

$$g_Y Y_\mu = \cos\theta_Y J_\mu^\phi - \sin\theta_Y J_\mu^\omega \,, \tag{18}$$

$$g_N N_\mu = \sin\theta_N J_\mu^\phi + \cos\theta_N J_\mu^\omega \,, \tag{19}$$

from which it follows that

$$\langle A | \phi_\mu | B \rangle = D_\phi(t) [\cos(\theta_Y - \theta_N)]^{-1} \times \langle A | (\cos\theta_N g_Y Y_\mu + \sin\theta_Y g_N N_\mu) | B \rangle \,, \tag{20}$$

$$\langle A|\omega_\mu|B\rangle = D_\omega(t)[\cos(\theta_Y-\theta_N)]^{-1} \times\langle A|(-\sin\theta_N g_Y Y_\mu + \cos\theta_Y g_N N_\mu)|B\rangle . \quad (21)$$

Here g, g_N, g_Y are coupling constants, θ_Y and θ_N are the ω-ϕ mixing angles for the hypercharge and the baryon number currents. The $D_V(t)$ are the vector-meson propagators with $t=(p_A-p_B)^2$. Expressions for these propagators that take into account the finite ρ width and the two-π-meson threshold are derived in Appendix A.

Substituting (12), (13), (20), and (21) into (15), we find

$$\frac{g m_\rho^2}{2\gamma_\rho} D_\rho(0)=1 \quad (22)$$

or

$$\frac{m_\rho^2}{2\gamma_\rho}=\frac{1}{gD_\rho(0)} . \quad (23)$$

Substituting (20) and (21) into (16) we obtain

$$[\cos(\theta_Y-\theta_N)]^{-1}\left(\frac{m_\omega^2}{2\gamma_\omega}D_\omega(0)\cos\theta_Y + \frac{m_\phi^2}{2\gamma_\phi}D_\phi(0)\sin\theta_Y\right)g_N=0 \quad (24)$$

as the coefficient of $\langle A|N|B\rangle$ and

$$[\cos(\theta_Y-\theta_N)]^{-1}\left(-\frac{m_\omega^2}{2\gamma_\omega}D_\omega(0)\sin\theta_N + \frac{m_\phi^2}{2\gamma_\phi}D_\phi(0)\cos\theta_N\right)g_Y=\tfrac{1}{2} \quad (25)$$

as the coefficient of $\langle A|Y|B\rangle$.

The set (24), (25) can be solved for the ϕ and ω coefficients

$$\frac{m_\omega^2}{2\gamma_\omega}=\frac{-1}{2}\frac{\sin\theta_Y}{g_Y D_\omega(0)}, \quad (26)$$

$$\frac{m_\phi^2}{2\gamma_\phi}=\frac{1}{2}\frac{\cos\theta_Y}{g_Y D_\phi(0)} . \quad (27)$$

We introduce the normalized propagators ($V=\rho,\ \omega,\ \phi$)

$$\Delta_V(t)=\frac{D_V(t)}{D_V(0)} . \quad (28)$$

Now we have for the isovector and isoscalar currents

$$\langle A|J_\mu^{I_3}|B\rangle=\Delta_\rho(t)\langle A|J_\mu^3|B\rangle , \quad (29)$$

$$\langle A|J_\mu^S|B\rangle=\tfrac{1}{2}[\Delta_\phi(t)(c_Y\langle A|Y_\mu|B\rangle + c_N\langle A|N_\mu|B\rangle)+\Delta_\omega(t)((1-c_Y)\langle A|Y_\mu|B\rangle - c_N\langle A|N_\mu|B\rangle)] , \quad (30)$$

with

$$c_Y=\frac{\cos\theta_Y\cos\theta_N}{\cos(\theta_Y-\theta_N)} , \quad (31)$$

$$c_N=\frac{g_N}{g_Y}\frac{\cos\theta_Y\sin\theta_Y}{\cos(\theta_Y-\theta_N)} . \quad (32)$$

In the quark model,[15] the hypercharge current behaves like

$$Y_\mu=\tfrac{1}{3}(\bar{\mathcal{P}}\gamma_\mu\mathcal{P}+\bar{\mathfrak{N}}\gamma_\mu\mathfrak{N}-2\bar{\lambda}\gamma_\mu\lambda) , \quad (33)$$

where $\mathcal{P}$, $\mathfrak{N}$, λ are the quark fields. The baryon current behaves like

$$N_\mu=\tfrac{1}{3}(\bar{\mathcal{P}}\gamma_\mu\mathcal{P}+\bar{\mathfrak{N}}\gamma_\mu\mathfrak{N}+\bar{\lambda}\gamma_\mu\lambda) . \quad (34)$$

The matrix elements of these currents between states which contain no λ quarks are therefore identical, i.e.,

$$\langle A|Y_\mu|B\rangle=\langle A|N_\mu|B\rangle . \quad (35)$$

This is particularly true for the nucleon form factors. In this case we have

$$\langle A|J_\mu^S|B\rangle=\tfrac{1}{2}[x\Delta_\phi(t)+(1-x)\Delta_\omega(t)]\langle A|N_\mu|B\rangle , \quad (36)$$

where

$$x=c_Y+c_N \quad (37)$$

is the parameter that determines for us the amount of ω-ϕ mixing. In Appendix B, we calculate from the observed relative rates of the decays $\phi\to 3\pi$ and $\omega\to 3\pi$ the values

$$x=+0.10\pm0.02 \quad (38)$$

or

$$x=-0.13\pm0.04 . \quad (39)$$

The data are fitted slightly better for the positive value $x=0.1$.

IV. ADDITIVITY OF QUARK AMPLITUDES

The matrix elements of a conserved current of type b (b = isospin, hypercharge, or baryon number) for a spin-$\frac{1}{2}$ hadron of type A can be expressed as

$$\langle p'\tfrac{1}{2}s_z'A|J_{\mu b}|p\tfrac{1}{2}s_zA\rangle = \bar{u}_{s_z'}(p')\left(F_b^1(t)\gamma_\mu + q^\nu\frac{\sigma^{\mu\nu}}{2M}F_b^2(t)\right)u_{s_z}(p) . \quad (40)$$

Here $q=p'-p$, $t=q^2$ and F_b^1, F_b^2 are form factors. It is more customary to use the "electric" and "magnetic" form factors defined by

$$G_b^E(t)=F_b^1(t)+\frac{t}{4M_A{}^2}F_b^2(t)\,,$$
$$G_b^M(t)=F_b^1(t)+F_b^2(t)\,. \tag{41}$$

In the $p+p'$ Breit frame the matrix element can be expressed directly in terms of these form factors as

$$\langle p'\tfrac{1}{2}s_z'A\,|(J_{b0},\vec{J}_b)|\,p\tfrac{1}{2}s_zA\rangle$$
$$=\left(G_{bA}^E(t)\phi_{s_z'}^\dagger\phi_{s_z},\ -i\,\frac{G_{bA}^M(t)}{2M_A}\vec{q}\times\phi_{s_z'}^\dagger\vec{\sigma}\phi_{s_z}\right), \tag{42}$$

where the ϕ_{s_z} are two-component spinors.

Under the assumption that each quark contributes additively, we find that

$$G^E(t)=S(t)\sum_{i=1}^{n}q^iG_i^E(t)\,, \tag{43}$$

$$G^M(t)=S(t)\sum_{i=1}^{n}\epsilon^iq^iG_i^M(t)\,, \tag{44}$$

where $S(t)$ is given in (9) and, for the ith quark, q^i is its charge, ϵ^i the expectation of its z spin, and G_i^E, G_i^M are its effective electric and magnetic form factors.

V. THE QUARK FORM FACTORS

We now make the assumption that the form factors for coupling a quark to a strong current are those of a pointlike Dirac particle[18] with an effective mass m and possibly a small anomalous magnetic moment μ_i. The form factors G_i in (43) and (44) are defined by writing the matrix elements of the current operator between quark states as

$$\langle\vec{p}s_z'i\,|(j_{b0},\vec{j}_b)|\,-\vec{p}s_zi\rangle$$
$$=g_b^i\left[G_{bi}^E(t)\phi_{s_z'}^\dagger\phi_{s_z},\ \frac{-i}{2M_A}G_{bi}^M(t)\vec{q}\times\phi_{s_z'}^\dagger\vec{\sigma}\phi_{s_z}\right]. \tag{45}$$

For point quarks this must be

$$q_b^i\left[\phi_{s_z'}^\dagger\phi_{s_z},\ -i\,\frac{1+\mu_i}{2m_i}\vec{q}\times(\phi_{s_z'}^\dagger\vec{\sigma}\phi_{s_z})\right], \tag{46}$$

where μ_i is the anomalous magnetic moment.

Thus we take

$$G_{bi}^E(t)=1 \tag{47}$$

and

$$G_{bi}^M(t)=\frac{M_A}{m_i}(1+\mu_i)\,, \tag{48}$$

both as constant.

The quark SU(6) wave functions[15] for the proton (p) or the neutron (n) with spin "up" are

$$|p\uparrow\rangle=(\tfrac{1}{3})^{1/2}\mathcal{P}_\uparrow^\dagger(\mathcal{P}_\uparrow^\dagger\mathcal{N}_\downarrow^\dagger-\mathcal{P}_\downarrow^\dagger\mathcal{N}_\uparrow^\dagger)\,|0\rangle\,, \tag{49}$$

$$|n\uparrow\rangle=(\tfrac{1}{3})^{1/2}\mathcal{N}_\uparrow^\dagger(\mathcal{P}_\uparrow^\dagger\mathcal{N}_\downarrow^\dagger-\mathcal{P}_\downarrow^\dagger\mathcal{N}_\uparrow^\dagger)\,|0\rangle\,, \tag{50}$$

where $\mathcal{P}_{s_z}^\dagger$, $\mathcal{N}_{s_z}^\dagger$ are Bose creation operators[19] for $\mathcal{P}$ or $\mathcal{N}$ quarks of spin s_z.

We can now write, using (47) and (48) in (43) and (44),

$$G_{bA}^E(t)=g_{bA}S(t)\,, \tag{51}$$

$$G_{bA}^M(t)=\bar{g}_{bA}S(t)\,, \tag{52}$$

where

$$g_{bA}=\sum_{i=1}^{3}g_b^i\,, \tag{53}$$

$$\bar{g}_{bA}=\frac{M_A}{m}\sum_{i=1}^{3}g_b^i\epsilon_i(1+\mu_i)\,. \tag{54}$$

Using the wave functions of (49), (50) we obtain the charges shown in Table I. In the table

$$\bar{m}=\frac{M}{2m}\left(1+\frac{\mu_{\mathcal{N}}+\mu_{\mathcal{P}}}{2}\right), \tag{55}$$

$$\delta=(\mu_{\mathcal{N}}-\mu_{\mathcal{P}})/(2+\mu_{\mathcal{N}}+\mu_{\mathcal{P}})\,, \tag{56}$$

where $\mu_{\mathcal{P}}$ and $\mu_{\mathcal{N}}$ are the $\mathcal{P}$- and $\mathcal{N}$-quark anomalous magnetic moments and $M=M_A$ is the nucleon mass.

The parameter δ is evaluated from the nucleon magnetic moments in Appendix C. We find

$$\delta=+0.0243\,. \tag{57}$$

From Table I we see that the charges satisfy

$$g_{Yp}=g_{Np}\,,\quad g_{Yn}=g_{Nn}\,,$$
$$\bar{g}_{Yp}=\bar{g}_{Np}\,,\quad \bar{g}_{Yn}=\bar{g}_{Nn}\,. \tag{58}$$

According to (51) we then have

$$G_{Yp}^E(t)=G_{Np}^E(t)\,, \tag{59}$$

and, from (52),

$$G_{Yp}^M(t)=G_{Np}^M(t)\,, \tag{60}$$

and corresponding equalities for the neutron. This result also follows directly from (35).

TABLE I. The different b charges [Eqs. (51) and (52)] for proton and neutron, as computed with the wave functions (49) and 50).

b	g_{bp}	$\bar{g}_{bp}$	g_{bn}	$\bar{g}_{bn}$
I_3	$\frac{1}{2}$	$\bar{m}(\frac{5}{6}-\frac{1}{2}\delta)$	$-\frac{1}{2}$	$-\bar{m}(\frac{5}{6}+\frac{1}{2}\delta)$
Y	1	$\bar{m}(\frac{1}{3}-\frac{5}{4}\delta)$	1	$\bar{m}(\frac{1}{3}+\frac{5}{4}\delta)$
N	1	$\bar{m}(\frac{1}{3}-\frac{5}{9}\delta)$	1	$\bar{m}(\frac{1}{3}+\frac{5}{4}\delta)$

VI. NUCLEON FORM FACTORS

The nucleon matrix elements of the electromagnetic current are now, according to (29) and (36),

$$\langle A|J_\mu^\gamma|B\rangle=\Delta_\rho(t)\langle A|J_\mu^3|B\rangle+\tfrac{1}{2}\Delta_{\phi,\omega}(x,t)\langle A|N_\mu|B\rangle\,, \tag{61}$$

where we define

$$\Delta_{\phi,\omega}(x,t)=x\Delta_\phi(t)+(1-x)\Delta_\omega(t)\,. \tag{62}$$

This linear relationship between the strong and the electromagnetic currents carries over to linear relationships between the form factors. That is,

$$G_A^F(t)=\Delta_\rho(t)G_{I_3A}^F(t)+\tfrac{1}{2}\Delta_{\phi,\omega}(x,t)G_{NA}^F(t)\,, \tag{63}$$

where F denotes E or M and A is p or n for proton or neutron.

From Eqs. (51) and (52) we have

$$\begin{aligned}G_{I_3A}^E&=g_{I_3A}S(t)\,,\\ G_{NA}^E&=g_{NA}S(t)\,,\\ G_{I_3A}^M&=\bar g_{I_3A}S(t)\,,\\ G_{NA}^M&=\bar g_{NA}S(t)\,,\end{aligned} \tag{64}$$

with the constants g, $\bar g$ given in Table I.

We can now collect the above results and write for the different nucleon electromagnetic form factors

$$G_p^E(t)=\tfrac{1}{2}(\Delta_\rho+\Delta_{\phi,\omega})S(t)\,, \tag{65a}$$

$$G_p^M(t)=\bar m[(\tfrac{5}{6}-\tfrac{1}{2}\delta)\Delta_\rho+(\tfrac{1}{18}-\tfrac{5}{18}\delta)\Delta_{\phi,\omega}]S(t)\,, \tag{65b}$$

$$G_n^E(t)=-\tfrac{1}{2}(\Delta_\rho-\Delta_{\phi,\omega})S(t)\,, \tag{65c}$$

$$G_n^M(t)=\bar m[(-\tfrac{5}{6}-\tfrac{1}{2}\delta)\Delta_\rho+(\tfrac{1}{18}+\tfrac{5}{18}\delta)\Delta_{\phi,\omega}]S(t)\,. \tag{65d}$$

VII. SCALING LAWS

One of the most puzzling aspects of the experimental data is the scaling laws. It is found[20] that within the experimental error,

$$G_p^E(t)\sim G_p^M(t)/\mu_p\sim G_n^M(t)/\mu_n \tag{66}$$

for all measured spacelike t values. The data also show that $G_n^E(t)$ is very small,[21] of the order of 0.06. Somewhat more recent data[22] show small deviations from (66).

We find that these two facts are related. Our model shows that the departures from the scaling laws must be of the same order of magnitude as the departure of $G_n^E(t)/G_p^M(t)$ from zero. For, from (65) we see that

$$\frac{\mu_pG_p^E(t)}{G_p^M(t)}-1=c\,\frac{\mu_pG_n^E(t)}{G_p^M(t)}\,, \tag{67}$$

$$\frac{\mu_pG_n^M(t)}{\mu_nG_p^M(t)}-1=d\,\frac{\mu_pG_n^E(t)}{G_p^M(t)}\,, \tag{68}$$

where the numbers c and d are

$$\begin{aligned}c&=\tfrac{2}{3}(1-\tfrac{1}{3}\delta)(1-\tfrac{7}{9}\delta)^{-1}\,,\\ d&=-\tfrac{5}{6}(1-\tfrac{1}{2}\delta)(1-\tfrac{7}{9}\delta)^{-1}(1+\tfrac{1}{3}\delta)^{-1}\,,\end{aligned} \tag{69}$$

both of order of magnitude one. They are close to $c=\frac{2}{3}$ and $d=\frac{5}{6}$ which are the values for $\delta=0$.

A nonzero value for $G_n^E(t)$, therefore, implies a proportional deviation from the scaling laws. This prediction is independent of the wave-function part $S(t)$ of the form factor. It provides, therefore, a test of our vector-dominance and quark-model assumptions.

We have plotted the deviations of the proton form factors from the scaling law given by (67) in Fig. 3. Our equation predicts a positive deviation for all t, whereas the 1970 DESY data[20] show a negative deviation for $t>1$ (GeV/c)^2. If this deviation is confirmed by additional data, then on the basis of (67) we could predict that $G_n^E(t)$ should be negative in this region.[23] This would also suggest that there may be something wrong with the vector-dominance assumption, perhaps due to the presence of other vector mesons.

Figure 4 shows our prediction for the neutron magnetic-form-factor scale law. We predict about a 5% negative deviation. This is still well within the experimental error.[21] However, Eq. (68) and the proton data suggest that there may actually be a positive deviation.

If the masses of ρ, ω, and ϕ were all the same, we would have

$$\Delta_\rho(t)=\Delta_{\phi,\omega}(t)=(1-t/m_\rho^{\,2})^{-1}\,. \tag{70}$$

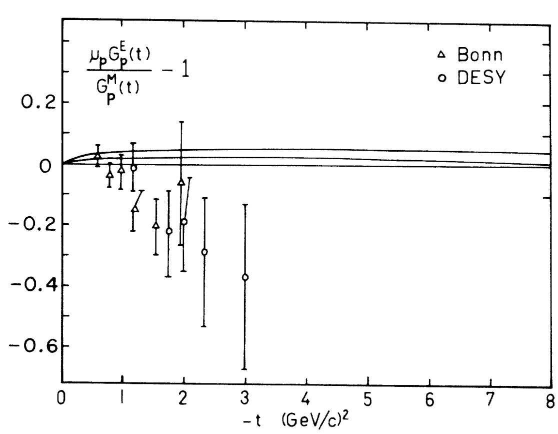

FIG. 3. The scaling law for the proton electric and magnetic form factors (67). The data are from Refs. 20 and 21.

In this limit

$$G_n^E(t)=0\,, \tag{71}$$

and the scaling laws would be exact. The departures from the scaling laws and also the nonvanishing $G_n^E(t)$ are therefore due to the meson mass splitting.

VIII. THE SYMMETRIC GAUSSIAN

A very deep but finite potential well will have wave functions that look very much like those of a harmonic-oscillator potential near the origin. If quarks were bosons in such a potential, then the nucleon's spatial wave function would be

$$\psi(\vec{x}_1,\vec{x}_2,\vec{x}_3)=N\exp[-\tfrac{1}{2}a^2(\vec{x}_1^{\,2}+\vec{x}_2^{\,2}+\vec{x}_3^{\,2})]\,, \tag{72}$$

where N is a normalization factor.

This wave function is also reasonable if the quarks are parafermions[24] of order three. It is also appropriate if they are in an antisymmetric state in some hidden quantum number, as in the three-triplet model.[25] In both of these two cases, the quarks in the known hadrons will behave like bosons.

This wave function leads to the relativistic form factor

$$S(t)=\left(\frac{1-t}{4M^2}\right)^{-1}\exp\left(\frac{t/6a^2}{1-t/4M^2}\right). \tag{73}$$

If we neglect the complications due to the different masses of the vector mesons then we would expect that

$$G_p^M(t)/\mu_p=(1-t/m_\rho^{\,2})^{-1}S(t)\,. \tag{74}$$

This crude fit approximates the data very well as shown by the curve in Fig. 4. We have plotted $G_p^M(t)/D_p^M(t)$, where $D_p^M(t)$ is the experimental dipole fit

$$D_p^M(t)=\mu_p(1-t/0.71)^{-2}\,. \tag{75}$$

We find the constant a in (73) by requiring that the curve pass through the experimental point at $t=-15.1\ (\mathrm{GeV}/c)^2$. The result is

$$1/6a^2=0.498\ (\mathrm{GeV}/c)^{-2}\,. \tag{76}$$

The solid lines in Fig. 5 show $G_p^M(t)$ relative to the dipole fit as computed from (65b) and (73). The upper curve is for $x=+0.1$, and the lower curve is for $x=-0.13$. The agreement is slightly improved over the crude fit at low t, and is not significantly affected at high t. There is a discrepancy of about 5% at $-t<3\ (\mathrm{GeV}/c)^2$, which we discuss below.

Figure 6 shows $G_n^E(t)$ from Eq. (65c), for $x=0.1$ and $x=-0.13$. The fit is reasonable considering the scatter of the experimental points. We do not fit, however, the observed slope at $t=0$,[26]

$$G'=\left.\frac{dG_n^E}{dt}\right|_{t=0}=0.05\ (\mathrm{GeV}/c)^{-2}\,. \tag{77}$$

We get

$$G'=0.25\ (\mathrm{GeV}/c)^{-2}\quad \text{for}\quad x=0.1 \tag{78}$$

and

$$G'=0.17\ (\mathrm{GeV}/c)^{-2}\quad \text{for}\quad x=-0.13\,. \tag{79}$$

It may be noted that the slope of the neutron electric form factor at $t=0$ is independent of the wave function, since

$$\left.\frac{d}{dt}G_n^E(t)\right|_{t=0}=-\frac{1}{2}\left.\left(\frac{d}{dt}\Delta_\rho(t)-\frac{d}{dt}\Delta_{\phi,\omega}(t)\right)\right|_{t=0}. \tag{80}$$

The fact that this is too low by a factor of 2 indicates that the threshold effects due to two and three π-meson states cannot be neglected.

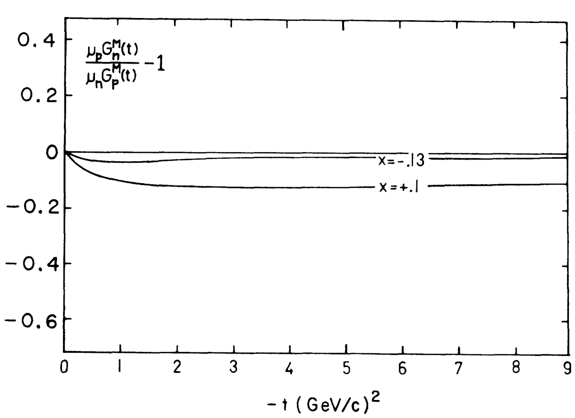

FIG. 4. The scaling law for the neutron magnetic form factor.

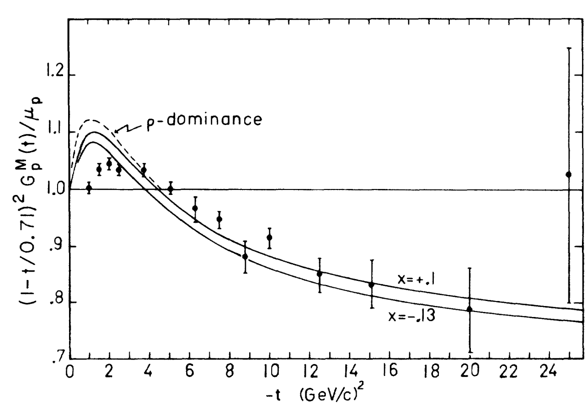

FIG. 5. Comparison of the proton magnetic form factor with the dipole fit. The dotted curve is the result of ρ dominance alone. The solid curves are our full fits with ρ, ω, ϕ for the two x values.

IX. ANTISYMMETRIC WAVE FUNCTIONS

One of the arguments used against the possibility of an antisymmetric quark wave function was that the nucleon form factors would have zeros in t.[27] No such zeros have been observed. Meyer[28] demonstrated that this argument was false by constructing an antisymmetric wave function whose corresponding form factor has no zeros. Kreps and de Swart[29] had shown numerically that the zeros of an antisymmetric wave function could be pushed out beyond present-day experiment.

We wish to point out here that since the nonrelativistic form factor $S_0(t)$ goes over into the relativistic one by (9), S_0 is used only for $\vec{q}^2 < 4M^2$. If $S_0(\vec{q}^2)$ has only zeros for $q^2 > 4M^2$ these will never show up in the relativistic form factor.

If the quarks are fermions, in a deep but finite harmonic-oscillator potential well, then the appropriate wave function is

$$\psi(\vec{x}_1, \vec{x}_2, \vec{x}_3) = N(\vec{x}_1^{\,2} - \vec{x}_3^{\,2})(\vec{x}_3^{\,2} - \vec{x}_1^{\,2}) \times \exp[-\tfrac{1}{2}a^2(\vec{x}_1^{\,2} + \vec{x}_2^{\,2} + \vec{x}_3^{\,2})]. \quad (81)$$

The resulting form factor is

$$S(t) = F(h)\exp\left(\frac{t/6a^2}{1 - t/4M^2}\right), \quad (82)$$

where, with

$$h = -t/[(1 - t/4M^2)3a^2], \quad (83)$$

$$F(h) = 1 - 63h + \tfrac{51}{20}h^2 - \tfrac{93}{70}h^3 + \tfrac{243}{560}h^4 - \tfrac{9}{112}h^5. \quad (84)$$

In Fig. 7 we have plotted several curves for $G_p^M(t)/\mu_p$ with $x = 0.1$. The parameter varied is

$$A = 1/6a^2. \quad (85)$$

The fit is very poor; the curves fall off too rapidly at large $|t|$.

Meyer[28] has considered the wave function

$$\psi(\vec{x}_1, \vec{x}_2, \vec{x}_3) = R^{-6}(\vec{x}_1^{\,2} - \vec{x}_2^{\,2})(\vec{x}_3^{\,2} - \vec{x}_1^{\,2})(\vec{x}_2^{\,2} - \vec{x}_3^{\,2})f(R), \quad (86)$$

where

$$R = (\vec{x}_1^{\,2} + \vec{x}_2^{\,2} + \vec{x}_3^{\,2})^{1/2} \quad (87)$$

and

$$f(R) = NR^{-3/2}e^{-bR/2}. \quad (88)$$

This is totally antisymmetric, but it yields the form factor

$$S_0(\vec{q}^2) = \tfrac{32}{5}(y + 8)^{-8}G(y), \quad (89)$$

with

$$G(y) = 19y^4 + 48y^3 + 88y^2 + 80y + 40 \quad (90)$$

and

$$y = (1 + 2\vec{q}^2/3b^2)^{1/2} - 1, \quad (91)$$

which, as Meyer showed, has no zero. After the relativistic corrections are made, this becomes

$$S(t) = (1 - t/4M^2)^{-1}S_0(-t/(1 - t/4M^2)), \quad (92)$$

and we obtain the curves shown in Fig. 8. Here $x = 0.1$ and the parameter varied is

$$A = 2/3b^2. \quad (93)$$

Agreement with the data is very poor.

With the nonrelativistic form factor and with an intermediate ρ meson, Meyer can find a b such that he finds a very good fit to G_p^M/μ_p. We can only conclude that this is an accident, as the inclusion of relativistic corrections destroys his fit.

X. CONCLUSION

We have found a fit to the proton magnetic form factor which is justified by physical principles. It

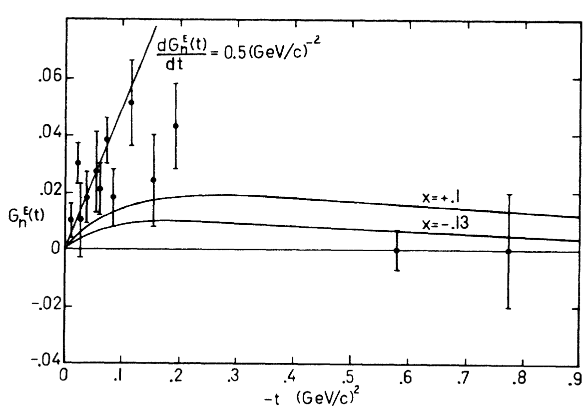

FIG. 6. The electric form factor of the neutron. The data are from Ref. 22.

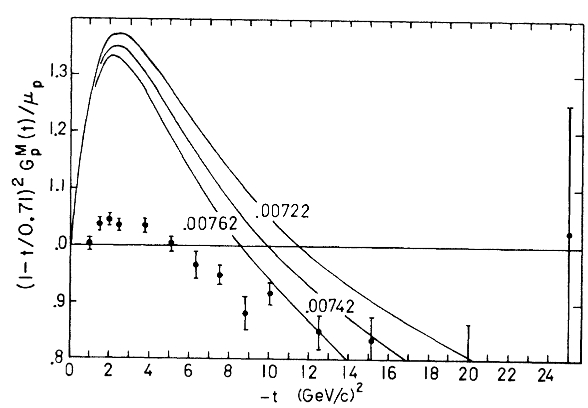

FIG. 7. Comparison of the data for the proton magnetic form factor with the antisymmetric Gaussian model. The parameter varied is $A = 1/6a^2$, Eq. (85).

shows an asymptotic behavior which is in agreement with experiment and is quite accurate for spacelike momentum transfers greater than 3 $(\text{GeV}/c)^2$. This fit requires only one parameter not given by independent data. This parameter, the size of the proton quark core, can be fitted using a single point at high momentum transfer. It determines the coefficient of the t^{-2} behavior.

We have made the assumption that the nucleons are composed of n subparticles. We find best agreement with the data for nucleons made of three boson-like quarks bound in a harmonic-oscillator-like potential. These quarks interact with the photon through only the known vector mesons ρ, ω, and ϕ.

Our curves for the symmetric Gaussian model deviate somewhat from the data at low t. In this region, which corresponds to large values of x, of course, our impulse approximation should be expected to break down, as the internal motion of the quarks becomes significant. Also, the wave function could have an exponential tail for large x and this could contribute here.

Several other factors could also cause small corrections. More complicated intermediate states, where the π mesons attach to different quarks, as shown in Fig. 2(b), could contribute. The region near $t=0$ is quite close to 2π and 3π thresholds of our propagators. In Appendix A we have investigated the influence of the 2π threshold in the ρ propagator on the $t=0$ slope of G_p^M.

We can rule out, on the basis of our fits, the possibility that the quarks are fermions in an harmonic-oscillator potential. We can also rule out that the quarks are fermions with Meyer's wave function – it is not possible at present to rule out completely the presence of yet unknown vector mesons.[30] These mesons, if introduced with positive couplings, will not change the t^{-2} behavior but

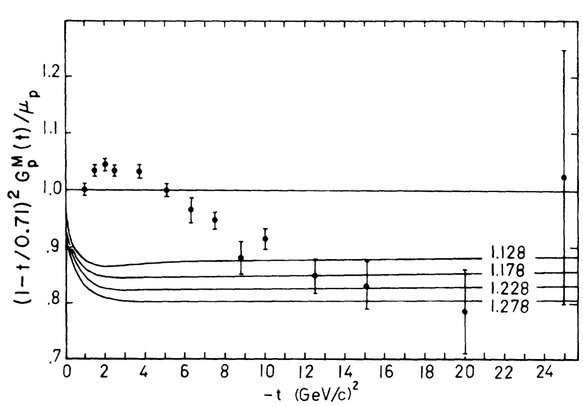

FIG. 8. Comparison of the data with the result of Meyer's wave function after relativistic corrections are applied. The parameter varied is $A=2/3b^2$, Eq. (93).

they might contribute at small t.

The Lorentz factor $(1-t/4m_\pi^2)^{-1/2}$ in our expression for the form factors leads to a nearby singularity and therefore to a poor prediction for the π-meson electromagnetic form factor. Vector dominance leads to too steep a falloff if applied to the inelastic form factors W_1, W_2 of the proton. These and related problems are under investigation.

APPENDIX A: THE VECTOR-MESON PROPAGATORS

The ω and ϕ mesons have very small widths. Therefore, especially in the region of negative t, we may take

$$D_\omega(t)=(m_\omega^2-t)^{-1}, \tag{A1}$$

$$D_\phi(t)=(m_\phi^2-t)^{-1}. \tag{A2}$$

The ρ, however, has a very large width. Moreover, its 2π threshold is close to the $q^2<0$ region. We must, therefore, construct a more realistic approximation to the physical propagator.

The vector-meson propagator is actually a tensor of the form

$$D_{\mu\nu}(t)=g_{\mu\nu}D(t)-p_\mu p_\nu E(t), \tag{A3}$$

with invariant functions $D(t)$, $E(t)$. Only part D can contribute to the couplings to a conserved current. This function has a branch point at $t=4m_\pi^2$ with a cut along the positive real axis. For very large t we expect the propagator to look like a simple pole

$$D(t)\to t^{-1}, \quad \text{as } |t|\to\infty. \tag{A4}$$

At the threshold, since the spin of the ρ is one, we expect the discontinuity to go like $(t-4m_\pi^2)^{3/2}$.

The simplest function which satisfies these requirements is

$$D^{-1}(t)=t-a+\lambda t\int_c^\infty\left(\frac{t'-c}{t'}\right)^{3/2}\frac{dt'}{t'(t'-t)}, \tag{A5}$$

with free constants a and λ and $c=4m_\pi^2$. We evaluate a and λ by requiring that, near $t=m_\rho^2$,

$$D^{-1}(t)=A(t-m_\rho^2+i\Gamma_\rho m_\rho), \tag{A6}$$

where $\Gamma_\rho=\Gamma$ is the ρ width. Defining L by

$$D^{-1}(t)=t-a+\lambda L(t), \tag{A7}$$

we find

$$L(x)=\tfrac{2}{3}+2x^2+x^3\left(\ln\frac{1-x}{1+x}+i\pi\right), \tag{A8}$$

where

$$x=\left(\frac{t-c}{c}\right)^{1/2}, \tag{A9}$$

$$a = m_\rho^2 + \lambda L(x(m_\rho^2)), \tag{A10}$$

$$\lambda = \frac{\Gamma}{\pi x^3/m_\rho - \Gamma B'}\bigg|_{t=m_\rho^2}, \tag{A11}$$

and

$$B' = \frac{d}{dt}\,\mathrm{Re}L(x(t)). \tag{A12}$$

Note that

$$D(0) = -1/a. \tag{A13}$$

Thus, for the ρ meson, we have

$$\Delta(t) = \frac{a}{a - t - \lambda L(x(t))}. \tag{A14}$$

We get from the values[31] $\Gamma_\rho = 120$ MeV, $M_\rho = 760$ MeV

$$a = 0.569\ (\mathrm{GeV}/c)^2 \tag{A15}$$

$$\lambda = 0.0336\ (\mathrm{GeV}/c)^2. \tag{A16}$$

The slope at $t=0$ can also be computed; we find good agreement with the dipole fit. Unfortunately, this slope then becomes flatter very quickly. A comparison with the small-t data is shown in Fig. 9.

APPENDIX B

In this appendix we will evaluate the ω-ϕ mixing parameter x [Eq. (36)] from the decay rates of ω, ϕ into three π mesons.

The parameter x is given by (31), (32), and (37) as

$$x = \frac{\cos\theta_Y}{\cos(\theta_Y - \theta_N)}\left(\cos\theta_N + \frac{g_N}{g_Y}\sin\theta_Y\right). \tag{B1}$$

Let $\Phi_\mu^V(x)$ denote the field operator of either the ϕ or the ω meson and let

$$J_\mu^V(x) = (\Box + m_V^2)\Phi_\mu^V(x) \tag{B2}$$

be its source. By standard LSZ (Lehmann-Symanzik-Zimmermann) reduction techniques,

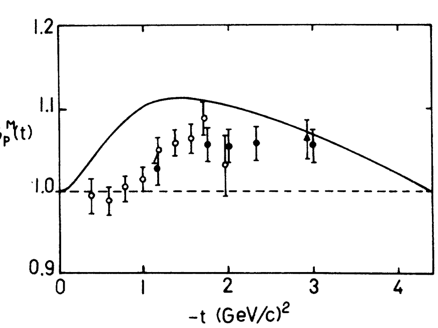

FIG. 9. The behavior of the data and our fit for very small t values where the 2π threshold is important.

we find that the decay rate $V \to 3\pi$ is given by

$$R(V\to 3\pi) = \frac{1}{3}\frac{(2\pi)^{-5}}{2M_V}\prod_{i=1}^{3}\int\frac{d^3p_i}{2\omega_i}\delta^{(4)}(q_V - \textstyle\sum p_i) \times |\langle 0|\vec{J}^V(0)|3\pi\rangle|^2, \tag{B3}$$

where

$$q_V = (M_V, \vec{0}), \quad \omega_i^2 = \vec{p}_i^{\,2} + m_\pi^2.$$

From (20) and (21) we see that

$$\langle 0|J_\mu^\omega|3\pi\rangle = (-g_Y\sin\theta_N + g_N\cos\theta_Y)\frac{\langle 0|N_\mu|3\pi\rangle}{\cos(\theta_Y - \theta_N)}, \tag{B4}$$

$$\langle 0|J_\mu^\phi|3\pi\rangle = (g_Y\cos\theta_N + g_N\sin\theta_Y)\frac{\langle 0|N_\mu|3\pi\rangle}{\cos(\theta_Y - \theta_N)}. \tag{B5}$$

They are of the form

$$\langle 0|J_\mu^V|3\pi\rangle = A_V\langle 0|N_\mu|3\pi\rangle, \tag{B6}$$

where we have used the fact that

$$\langle 0|Y_\mu|3\pi\rangle = \langle 0|N_\mu|3\pi\rangle, \tag{B7}$$

as no λ quarks are present in either the $|0\rangle$ or the $|3\pi\rangle$ state.

The matrix element $\langle 0|N_\mu|3\pi\rangle$ should be a pseudovector orthogonal to $\sum p_i$. The simplest form is

$$\langle 0|N_\mu|3\pi\rangle = \epsilon_{\mu\alpha\beta\gamma}p_1^\alpha p_2^\beta p_3^\gamma F(p_i\cdot p_j), \tag{B8}$$

F being a scalar function of the invariant inner products. We assume that F is slowly varying and replace it by a constant. Note that

$$\langle 0|\vec{N}|3\pi\rangle = -[\omega_1\vec{p}_2\times\vec{p}_3 + \omega_2\vec{p}_3\times\vec{p}_1 + \omega_3\vec{p}_1\times\vec{p}_2]. \tag{B9}$$

For our purpose we need only to estimate the integral in (B3) as a function of M_V. Let

$$\vec{p}_i = M_V\vec{v}_i, \tag{B10}$$

$$m_\pi^2 = 0.02 M_V^2, \tag{B11}$$

then

$$|\langle 0|\vec{N}|3\pi\rangle|^2 \approx (M_V^3)^2, \tag{B12}$$

with a fractional error of the order of $|0.02 - m_\pi^2/M_V^2|$, which is negligible. We find for the mass dependence

$$R(V\to 3\pi) \approx M_V^{9-4+6-3-1}|A_V|^2 \tag{B13}$$

to within $O(0.02 \pm m_\pi^2/M_V^2)$, and thus

$$\frac{R(\phi\to 3\pi)}{R(\omega\to 3\pi)} = \left(\frac{M_\phi}{M_\omega}\right)^7\left|\frac{\cos\theta_N + (g_Y/g_N)\sin\theta_Y}{-\sin\theta_N + (g_Y/g_N)\cos\theta_Y}\right|^2. \tag{B14}$$

This becomes, with a little algebra,

$$\frac{R(\phi\to 3\pi)}{R(\omega\to 3\pi)}=\left(\frac{M_\phi}{M_\omega}\right)^7\tan^2\theta_Y\frac{x^2}{(1-x)^2}, \tag{B15}$$

where x is given in (36). Let

$$y=\left[\left(\frac{M_\omega}{M_\phi}\right)^7\frac{R(\phi\to 3\pi)}{R(\omega\to 3\pi)}\tan^{-2}\theta_Y\right]^{1/2}, \tag{B16}$$

and, from (26) and (27)

$$\tan^2\theta_Y=\left(\frac{\gamma_\phi}{\gamma_\omega}\right)^2. \tag{B17}$$

Now $(M_\omega/M_\phi)^7=0.159$ and from Ref. 17 (p. 19)

$$\gamma_\omega^{-2}/4\pi=4.0\pm0.9, \tag{B18}$$

$$\gamma_\phi^{-2}=3.1\pm0.7, \tag{B19}$$

hence

$$\left(\frac{\gamma_\phi}{\gamma_\omega}\right)^2=0.73\pm0.28. \tag{B20}$$

From the Particle Data Tables,[31] we have

$$R(\omega\to 3\pi)=(0.87\pm0.04)(12.7\pm1.2)=11.6\pm1.15, \tag{B21}$$

$$R(\phi\to 3\pi)=(0.181\pm0.049)(3.9\pm0.4)=0.705\pm0.207. \tag{B22}$$

We find

$$y=\left(\frac{0.159}{0.732}\,\frac{0.705}{11.06}\right)^{1/2}=(0.0139)^{1/2}$$

$$=\pm0.12\pm0.03. \tag{B23}$$

Since $x=y/(1+y)$ we have either

$$x=+0.10\pm0.02 \tag{B24}$$

or

$$x=-0.13\pm0.04. \tag{B25}$$

APPENDIX C

In this appendix we evaluate the parameter δ, Eq. (56), in terms of the ratio of the nucleon magnetic moments.

The nucleon magnetic form factors, evaluated at $t=0$, give the magnetic moments

$$\mu_p=G_p^M(0), \tag{C1}$$

$$\mu_n=G_n^M(0). \tag{C2}$$

From Eq. (65) we find

$$\mu_p=\overline{m}(1-\tfrac{7}{9}\delta), \tag{C3}$$

$$\mu_n=-\tfrac{2}{3}\overline{m}(1+\tfrac{1}{3}\delta). \tag{C4}$$

The ratio μ_p/μ_n would be exactly $-\frac{3}{2}$ if the $\mathcal{P}$ and $\mathfrak{N}$ quarks had no anomalous magnetic moments. Calculating δ from the observed μ_p/μ_n we find that

$$\delta=+0.0243. \tag{C5}$$

It is, of course, possible that the deviation of μ_p/μ_n from $-\frac{3}{2}$ is due not to the quark anomalous magnetic moments, but to a contribution at $t=0$ from intermediate multipion states. This remains a problem to be settled. The effect on the form factors is small, less than 1%.

---

[1]R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

[2]T. T. Wu and C. N. Yang, Phys. Rev. Letters 20, 1213 (1968).

[3]R. Jengo and E. Remiddi, Lett. Nuovo Cimento 44, 309 (1969).

[4]P. N. Kirk *et al.*, SLAC Report No. SLAC-PUB-656 (unpublished); D. H. Coward *et al.*, Phys. Rev. Letters 20, 292 (1968).

[5]Y. Nambu, Phys. Rev. 106, 1366 (1957); W. R. Frazer and J. R. Fulco, *ibid.* 17, 1603 (1960).

[6]N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967); H. Joos, in *Special Problems in High Energy Physics*, edited by P. Urban (Springer, Vienna, 1967), p. 320.

[7]M. Goitein, J. R. Dunning, and R. Wilson, Phys. Rev. Letters 18, 1018 (1967); King-yuen Ng, Phys. Rev. 170, 1435 (1968); T. Massam and A. Zichichi, Nuovo Cimento 44A, 309 (1966).

[8]J. G. Rutherglen, in the *International Symposium on Electron and Photon Interactions at High Energies, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970). See also Ref. 21.

[9]T. Appelquist and J. R. Primack, Phys. Rev. D 1, 1144 (1970).

[10]A. L. Licht and A. Pagnamenta, Phys. Rev. D 2, 1150 (1970) (hereafter referred to as I).

[11]A. L. Licht and A. Pagnamenta, Phys. Rev. D 2, 1156 (1970) (hereafter referred to as II).

[12]K. Fujimura, T. Kobayashi, and M. Namiki, Progr. Theoret. Phys. (Kyoto) 43, 73 (1970); *ibid.* 44, 193 (1970).

[13]T. Takabayashi, Phys. Rev. 139, B1381 (1965).

[14]A. O. Barut, Phys. Rev. 161, 1464 (1967), and in *Coral Gables Conference on Fundamental Interactions at High Energy II*, edited by A. Perlmutter, G. J. Iverson, and R. M. Williams (Gordon and Breach, New York, 1970), p. 199.

[15]R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50A, 617 (1967).

[16]E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma Redaktsiyu 2, 105 (1965) [Soviet Phys. JETP Letters 2, 65 (1965)].

[17]H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).

[18]A. Pagnamenta, *Symmetries and Quark Models* (Gordon and Breach, New York, 1970), p. 291.

[19]O. W. Greenberg and M. Resnikoff, Phys. Rev. 163,

1844 (1967).
[20]W. Bartel *et al.*, Phys. Letters 33B, 245 (1970); Ch. Berger *et al.*, *ibid.* 28B, 276 (1968).
[21]E. Lohrmann, in *Proceedings of the Lund International Conference on Elementary Particles*, edited by G. von Dardel (Berlingska, Lund, Sweden, 1970), p. 11.
[22]Our neutron data are taken from the compilation of G. Weber, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, Stanford, California, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968), p. 59.
[23]A negative $G_n^E(t)$ for $1<-t<4$ $(\mathrm{GeV}/c)^2$ is also suggested by an analysis of the neutron data by C. R. Schumacher and I. M. Engle, ANL-HEP Report No. 7032 (1970).
[24]O. W. Greenberg, Phys. Rev. Letters 13, 598 (1964).
[25]M. Y. Hahn and Y. Nambu, Phys. Rev. 139, B1006 (1965).
[26]V. E. Krohn and G. R. Ringo, Phys. Rev. 148, 1303 (1960).
[27]A. N. Mitra and R. Majumdar, Phys. Rev. 150, 1194 (1966).
[28]R. F. Meyer, Lett. Nuovo Cimento 2, 76 (1969).
[29]R. E. Kreps and J. J. de Swart, Phys. Rev. 162, 1729 (1967).
[30]H. Högaasen and A. Frisk, Phys. Letters 22, 90 (1966); R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters 18, 259 (1967).
[31]Particle Data Group, Rev. Mod. Phys. 42, 87 (1970).

PHYSICAL REVIEW D VOLUME 4, NUMBER 9 1 NOVEMBER 1971

Decay Correlations of Heavy Leptons in $e^+ + e^- \to l^+ + l^-$*

Yung-Su Tsai
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 22 July 1971)

Assuming that leptons heavier than muons exist in nature, we consider their decay modes and the correlations between the decay products of l^+ and l^- in the colliding-beam experiment: $e^+ + e^- \to l^+ + l^-$. Far above the threshold, the helicities of l^+ and l^- tend to be opposite to each other. Near the threshold the directions of spins of l^+ and l^- prefer to be parallel to each other, and the sum of the two spins prefers to be either parallel or antiparallel to the direction of the incident electron. Because the parity conservation is violated maximally in the decays of l^+ and l^-, the angular distributions of decay products depend strongly on the spin orientation of the heavy leptons. Since the spins of l^+ and l^- are strongly correlated in the production, we found a strong correlation between the energy-angle distributions of the decay products of l^+ and l^-. The decay widths of l^- into channels $\nu_l \bar{\nu}_e e^-$, $\nu_l \bar{\nu}_\mu \mu^-$, $\nu_l \pi^-$, $\nu_l K^-$, $\nu_l \rho^-$, $\nu_l K^*$, $\nu_l A_1$, $\nu_l Q$, and ν_l + hadron continuum as functions of the mass of l^- are estimated.

I. INTRODUCTION

Since muons exist in nature for no apparent reason, it is possible that other heavy leptons may also exist in nature. If one discovers heavy leptons, one may be able to understand why muons exist and obtain some clue as to why the ratio of the muon mass to the electron mass is roughly $m_\mu/m_e \approx 210$. Searches for these leptons have been attempted in the past,[1,2] and no doubt people will be looking for these particles in the $e^+ + e^-$ colliding-beam experiments[2] ($e^+ + e^- \to l^+ + l^-$), pair photoproduction experiments[3] ($\gamma + z \to l^+ + l^- + z^*$), and neutrino experiments from the electron machine[4] ($\nu_l + z \to l^- + z^*$). We have made extensive calculations for these cross sections. This paper deals mainly with the decay correlations in the reaction, $e^+ + e^- \to l^+ + l^-$.

We assume that if heavy leptons exist the leptonic current in the usual current-current effective Lagrangian[5] of the weak interaction is given by

$$J^\lambda_{\text{lept}} = \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\mu + \bar{\nu}_e \gamma^\lambda (1-\gamma_5) e + \bar{\nu}_l \gamma^\lambda (1-\gamma_5) l,$$

and the electromagnetic interaction of the heavy lepton is exactly like that of an electron or a muon. The major difference between the heavy lepton and the muon is that, whereas the muon is lighter than any strongly interacting particle, the heavy lepton, if it exists, is expected to be heavier than the K meson; and hence the heavy lepton decays[6] into hadrons in addition to electron and muon.

In the electromagnetic *scattering* of an electron, it is well known that at high energies $[(m/E) \to 0]$ the helicity of the electron remains the same during the scattering, whereas at low energies $[(m/E) \to 1]$ the direction of the spin with respect to a fixed coordinate system in space is preserved during the scattering.[7] In Sec. IV we show that analogous things happen in the reaction $e^+ + e^-$