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Adler-Weisberger Theorem Reexamined*

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We consider the nature of the current-algebra constraints on the on-mass-shell amplitudes of pion-nucleon scattering at $s=m_N^2$, $t=2m_\pi^2$. We prove that, at this point, an even amplitude is determined up to corrections of order m_π^4 and that the corrections to an odd amplitude are of order m_π^2. Thus, the σ term in the even amplitude, which is of order m_π^2, can be determined, confirming the recent work of Cheng and Dashen. However, such a determination cannot be made with a mixture of even and odd amplitudes as von Hippel and Kim have attempted to do. We estimate the actual magnitude of the corrections from what should be the dominant physical mechanisms: the large nucleon size and the proximity of the strong 3-3 resonance state. These corrections are insignificant for the even amplitude. For the odd amplitude, they are separately large, about 10%, but they cancel for the most part and give a much smaller contribution. Corrections due to other resonances are insignificant compared to the 3-3 contribution. The Adler-Weisberger theorem, with our on-shell method, relates the pion-nucleon coupling constant f to the pion decay constant and an integral over the absorptive part of the physical π-N scattering amplitude; it does not involve the axial-vector coupling constant g_A. We find that this relation gives $f^2=0.077$, which is to be compared to direct evaluations which range from $f^2=0.082$ to $f^2=0.076$. We conclude that these current-algebra theorems may well be satisfied to a considerable degree of accuracy. More precise low-energy pion-nucleon scattering data are needed, however, for a definitive test.

I. INTRODUCTION AND SUMMARY

It is our purpose to reexamine critically the nature of the current-algebra constraints on the pion-nucleon scattering amplitude. We write these constraints as an identity involving a remainder term that is largely unknown except that it is explicitly bilinear in the pion momenta q', q. We always keep the pions on their mass shells and remain within the Lehmann ellipse in order that our amplitudes be unambiguously related to physical πN scattering. At the special point $\nu=\nu_B=0$ ($s=m_N^2$, $t=2m_\pi^2$) the correction due to the remainder is of order m_π^4 in an isospin-even amplitude and of order m_π^2 in a particular combination of isospin-odd amplitudes. We calculate what should be the largest contributions to the remainder at this point and find that the net correction is small. The uncertainty in the odd amplitude is formally of order m_π^2 at other points in the ν-ν_B plane, and we verify that the results obtained at $\nu=t=0$ and at threshold are in reasonable agreement with that obtained at $\nu=\nu_B=0$. However, the remainder involves the fewest unknown functions at $\nu=\nu_B=0$, and we believe results obtained there are most reliable. We shall summarize and discuss our results before presenting our work in detail.

We label the initial and final momenta of the pions by q and q' and that of the nucleons by p and p'. The energy-momentum balance reads

$$q'+p'=q+p. \tag{1}$$

We use the variables

$$\nu=-q\cdot(p'+p)/2m$$
$$=-q'\cdot(p'+p)/2m \tag{2}$$

and

$$\nu_B=q'\cdot q/2m. \tag{3}$$

Here m is the proton mass, and we shall now denote the charged-pion mass by μ. We write the amplitude for the scattering of a pion from isospin index a to a' as[1]

$$T^{a'a}=[A^{(+)}+\tfrac{1}{2}\gamma\cdot(q'+q)B^{(+)}]\delta^{a'a}$$
$$+[A^{(-)}+\tfrac{1}{2}\gamma\cdot(q'+q)B^{(-)}]i\epsilon^{a'ac}\tau_c. \tag{4}$$

Here, as throughout the paper, the nucleon spinors are omitted although we always keep the nucleons on their mass shells and tacitly assume that the amplitude appears between spinors. Since the Born term is ill defined at $\nu=\nu_B=0$, we define a set of amplitudes with it removed. With pseudoscalar coupling, the Born term contributes only to the B amplitudes, and its contribution is displayed by writing

$$B^{(+)}(\nu,\nu_B;-q'^2=-q^2=\mu^2)=\frac{g^2}{m}\frac{\nu}{\nu^2-\nu_B^2}+\bar{B}^{(+)}, \tag{5a}$$

$$B^{(-)}(\nu,\nu_B;-q'^2=-q^2=\mu^2)=\frac{g^2}{m}\frac{\nu_B}{\nu^2-\nu_B^2}+\bar{B}^{(-)}. \tag{5b}$$

With these preliminaries out of the way, we turn to discuss the current-algebra constraints.

We consider first the even amplitudes. There is no constraint on $B^{(+)}$ since, by virtue of crossing symmetry, it is already of order ν, but we do have a constraint on $A^{(+)}$:

$$A^{(+)}(\nu, \nu_B; \mu^2) = \frac{\mu^2}{F_\pi}\Sigma + \frac{g^2}{m}\left[1 + \left(\frac{\mu}{m}\right)^4 a + \left(\frac{2\nu_B}{m}\right) b + \left(\frac{\nu}{m}\right)^2 c\right]. \tag{6}$$

Here $F_\pi = 92.56$ MeV is the pion decay constant and Σ (the "σ term") is, essentially, the nucleon matrix element of the equal-time commutator of the axial-vector current with its divergence. It is a function only of $t \equiv 2(\mu^2 - m\nu_B)$. The correction terms coming from the remainder are indicated with the nonsingular, dimensionless functions a, b, and c. We note that at the special point $\nu = \nu_B = 0$, the contribution of the remainder is minimal and of order $(\mu/m)^4$. This point, where $s = m^2$, $t = 2\mu^2$, is not a physical point for pion-nucleon scattering. Nevertheless, it is within the Lehmann ellipse so that the amplitude can be computed from physical values of the pion-nucleon phase shifts. It is important that the correction term be of order $(\mu/m)^4$, not of order $(\mu/m)^2$, for otherwise the Σ term, of order $(\mu/m)^2$, could not be determined. At threshold, where $\nu = \mu$ and $\nu_B = -\mu^2/2m$, the contributions of the remainder are of the same order as the Σ term. At any rate, we have

$$A^{(+)}(0, 0; \mu^2) = \frac{\mu^2\Sigma(2\mu^2)}{F_\pi} + \frac{g^2}{m}\left[1 + \left(\frac{\mu}{m}\right)^4 a\right]. \tag{7}$$

The size of a is estimated in Sec. III. Most of its contribution comes from the $\Delta(1236)$ which is both near and strongly coupled; we find that

$$a_\Delta = 0.8\,. \tag{8}$$

Moreover, the spin-$\frac{1}{2}^\pm$ resonances do not contribute to the remainder at $\nu = \nu_B = 0$, while the spin-$\frac{3}{2}^-$ resonances give a contribution that is two orders of magnitude smaller than that of the spin-$\frac{3}{2}^+$ Δ. Hence, barring anomalously large contributions from very high mass states or large subtraction constants, we conclude that the correction is predominantly given by the Δ contribution, which is indeed of order $(\mu/m)^4$.

The magnitude of the Σ term can be estimated in several ways. The σ model[2] (where $g = m/F_\pi$) with a σ mass near that of the ρ, gives $\Sigma \simeq g/m_\rho^2$ and $(\mu^2/F_\pi)\Sigma \simeq (g^2/m)(\mu/m)^2 \times 2$. This corresponds to a nucleon matrix element of the chiral-symmetry-breaking part of the Lagrangian

$$\langle -\mathcal{L}_{\text{S.B.}}\rangle_{\text{nuc}} = (\mu^2 m/g)\Sigma \simeq 40 \text{ MeV} \quad (\sigma \text{ model}). \tag{9}$$

In general,

$$\mu^2 F_\pi \Sigma = \tfrac{1}{3}\sum_a \langle [X_a, [X_a, -\mathcal{L}_{\text{S.B.}}]]\rangle_{\text{nuc}}\ , \tag{10}$$

where X_a are the generators of chiral transformations with an isospin normalization. If the chiral-symmetry-breaking part of the Lagrangian belongs[3] to the $(3, 3^*)\oplus(3^*, 3)$ representation of chiral $SU(3)\otimes SU(3)$,

$$-\mathcal{L}_{\text{S.B.}} = u_0 + cu_8\,, \tag{11}$$

then

$$\mu^2 F_\pi \Sigma = \tfrac{1}{3}(c + \sqrt{2})\langle u_8 + \sqrt{2}\, u_0\rangle_{\text{nuc}}\ . \tag{12}$$

The contribution of u_8 can be estimated by assuming that cu_8 taken in first order accounts for the mass splitting of the baryon octet. The Gell-Mann–Okubo formula then gives

$$\langle cu_8\rangle_{\text{nuc}} \simeq -215 \text{ MeV}\,. \tag{13}$$

The constant c can be estimated[3] from the mass splitting of the meson octet to be about

$$c \simeq -1.25\,. \tag{14}$$

Kim and von Hippel[4] have applied current-algebra techniques to conclude that

$$\langle u_0\rangle_{\text{nuc}} \simeq +215 \text{ MeV}, \tag{15}$$

and thus they obtain

$$\mu^2 F_\pi \Sigma \simeq 20 \text{ MeV} \quad (\text{K-vH}). \tag{16}$$

However, the result of Kim and von Hippel is questionable, for they work with amplitudes of definite isospin rather than with even and odd amplitudes, and they evaluate their amplitudes at threshold rather than at the special point $\nu = \nu_B = 0$. Such an analysis incurs errors of order $(\mu/m)^2$ and thus, does not furnish a reliable evaluation of the Σ term.

An alternative evaluation of $\langle u_0\rangle_{\text{nuc}}$ can be obtained if we assume that, except for a constant, the only part of the Lagrangian that breaks scale invariance is the chiral-symmetry-breaking term $u_0 + cu_8$ with a single dimension[5] d. With this hypothesis the trace of the stress-energy tensor is given by

$$-T^\mu_\mu = (4 - d)(u_0 + cu_8) + \text{const}, \tag{17}$$

and hence

$$\langle u_0 + cu_8\rangle_{\text{nuc}} = m/(4 - d)\,. \tag{18}$$

If we use the Gell-Mann – Okubo estimate for $\langle cu_8\rangle_{\text{nuc}}$ and the dimension $d = 3$ suggested by the quark model, we obtain[6]

$$\langle u_0\rangle_{\text{nuc}} \simeq 1155 \text{ MeV}, \tag{19}$$

and, with $c \simeq -1.25$,

$$\mu^2 F_\pi \Sigma \simeq 99 \text{ MeV} \quad (\text{broken dilational invariance})\,. \tag{20}$$

We should emphasize that this number is very sensitive to $c+\sqrt{2}$, which is quite small and hence very poorly determined.

Recently Cheng and Dashen[7] have computed the Σ term. They consider an amplitude

$$F(\nu, \nu_B; \mu^2) = A^{(+)}(\nu, \nu_B; \mu^2) - \nu B^{(+)}(\nu, \nu_B; \mu^2)\,. \tag{21}$$

In the limit in which first ν_B is taken to vanish and then ν is taken to vanish, only the Born term exhibited in Eq. (5a) contributes to $B^{(+)}$, and the current-algebra constraint (7) gives

$$\lim_{\nu\to 0}\left[\lim_{\nu_B\to 0} F(\nu, \nu_B; \mu^2)\right] = \frac{\mu^2}{F_\pi}\Sigma(2\mu^2) + \frac{g^2}{m}\left(\frac{\mu}{m}\right)^4 a\,. \tag{22}$$

Thus we confirm the assertion of Cheng and Dashen that the Σ term is given by such a limit and, moreover, we find that the correction term involving the dimensionless parameter a is completely negligible. Cheng and Dashen evaluate this amplitude by a dispersion relation involving a broad area subtraction and conclude[8] that

$$\mu^2 F_\pi \Sigma \simeq 95\text{–}115 \text{ MeV} \quad (\text{C-D})\,. \tag{23}$$

This range of values is certainly consistent with the simple picture of broken dilational invariance which we outlined above.

We can use the current-algebra constraint (7) to get a value for the pion-nucleon coupling constant

$$f^2 = \frac{g^2\mu^2}{16\pi m^2} = \frac{\mu^2}{16\pi m}\left[A^{(+)}(0, 0; \mu^2) - \frac{\mu^2}{F_\pi}\Sigma\right]. \tag{24}$$

If we use Adler's[9] evaluation of $A^{(+)}$ and $\mu^2 F_\pi \Sigma = 105$ MeV, we get

$$f^2 = (0.085 \pm 0.003) - 0.005 = 0.080\,, \tag{25}$$

to be compared with other determinations[10] that range from $f^2 = 0.076$ to $f^2 = 0.082$.

We turn now to the odd amplitudes. We prove in Sec. II that current algebra does not constrain the combination $A^{(-)} + \nu \overline{B}^{(-)}$, but that it does provide a constraint on the mass-shell amplitude

$$G = \nu^{-1}[A^{(-)} - \nu \overline{B}^{(-)}]\,, \tag{26}$$

namely,

$$G(\nu, \nu_B; \mu^2) = -\frac{g^2}{2m^2} + \frac{1}{2F_\pi^2}F_1^V(-(q'-q)^2) + \frac{1}{2F_\pi^2}\left[\left(\frac{\mu}{m}\right)^2 d + \left(\frac{2\nu_B}{m}\right)e + \left(\frac{\nu}{m}\right)^2 f\right]. \tag{27}$$

Here $F_1^V(t)$ is the isovector nucleon electromagnetic form factor with the normalization $F_1^V(0) = 1$, and d, e, and f are dimensionless, nonsingular functions that represent the contribution of the remainder. This is essentially the Adler-Weisberger[11] relation except that, since the scattering amplitude is evaluated with the pions on the mass shell, the axial-vector coupling constant g_A does not appear. We remark that at the point $\nu = \nu_B = 0$ the correction is of order $(\mu/m)^2$. However, this is not the only point where the remainder is formally of this order. At any point where ν is of order μ and ν_B is of order μ^2/m, the remainder is also of order $(\mu/m)^2$. In addition to the point $\nu = 0$, $\nu_B = 0$, there are two other interesting points to consider: threshold ($\nu = \mu$, $\nu_B = -\mu^2/2m$) and the point $\nu = 0$, $\nu_B = -\mu^2/2m$ ($t = 0$). However, since there is no *a priori* way to determine which of the remainder coefficients is the smaller or at what values of ν and ν_B there might be some cancellation between the various terms, the most reasonable place to evaluate Eq. (27) should be the point $\nu = 0$, $\nu_B = 0$, where only d contributes.

In Sec. III we estimate the size of the remainder. Here again the main contribution comes from the $\Delta(1236)$. At the point $\nu = \nu_B = 0$, the $\frac{1}{2}^\pm$ resonance contributions vanish, and a $\frac{3}{2}^-$ resonance contribution is two orders of magnitude smaller than that of the $\Delta(1236)$. At the other points of interest the other resonance contributions are still numerically insignificant. At $\nu = \nu_B = 0$ we find that

$$d_\Delta = -5.84\,, \tag{28}$$

which is a sizable correction. However, with $\nu_B = 0$, $t = 2\mu^2$, and we must take into account the deviation of $F_1^V(2\mu^2)$ from unity. This is not an insignificant effect, since the nucleon's charge radius is rather large[12]:

$$F_1^V(2\mu^2) = 1 + 4.16(\mu/m)^2\,. \tag{29}$$

Hence, barring untoward circumstances, we find

$$G(0, 0; \mu^2) = -\frac{g^2}{2m^2} + \frac{1}{2F_\pi^2}\left(1 - 1.7\frac{\mu^2}{m^2}\right). \tag{30}$$

Thus the total correction is indeed of order $(\mu/m)^2$.

We may write this as a sum rule for the pion-nucleon coupling constant,

$$f^2 = \frac{\mu^2}{16\pi F_\pi^2}(1 - 0.037) - \frac{\mu^2}{8\pi}G(0, 0; \mu^2)\,, \tag{31}$$

with the amplitude G evaluated by an unsubtracted dispersion relation. The point $\nu = 0$, $\nu_B = -\mu^2/2m$ corresponds to $t = 0$, and the dispersion integral involves simply the difference of the $\pi^+ p$ and $\pi^- p$ total cross sections. It is the well-known integral in the Adler-Weisberger relation. Adler[11] computes the value

$$-\frac{\mu^2}{8\pi}G(0, -\mu^2/2m; \mu^2) = 0.0206\,. \tag{32}$$

Adler[11] also computes the change in going from $t = 0$ to $t = 2\mu^2$ to be

$$-\frac{\mu^2}{8\pi}[G(0,0;\mu^2)-G(0,-\mu^2/2m;\mu^2)]=0.0126\,. \tag{33}$$

This is a very large correction of relative order $(\mu/m)^2$, the same formal order as the remainder corrections discussed above. It is large because the $\Delta(1236)$ makes a large contribution to the dispersion integral that is proportional to $\cos\theta$, and $\cos\theta$ differs considerably from unity at $t=2\mu^2$ since the center-of-mass momentum at the energy of this resonance is small. In fact, the $\Delta(1236)$ contributes about 0.037 to Eq. (32), which is evaluated at $\cos\theta=1$, while at the Δ mass, $\cos\theta=1.36$ for $t=2\mu^2$, and we expect a Δ contribution to the correction Eq. (33) of about $0.037\times0.36=0.013$. At any rate, adding up the number gives

$$f^2=0.0436+0.0206+0.0126=0.077\,. \tag{34}$$

This is in reasonable agreement with more direct determinations[10] that give a range of values of f^2 from 0.076 to 0.082.

We now briefly consider the two other interesting points. At $\nu=0$, $\nu_B=-\mu^2/2m$, the left-hand side of Eq. (27) is simply the Adler-Weisberger integral. Since, as we just discussed, the $\Delta(1236)$ has a large variation in going from $\nu_B=0$ to $\nu_B=-\mu^2/2m$, we should expect that the contribution to the remainder should be sizable. In fact, we find

$$(d_\Delta-e_\Delta)=+11.73\,. \tag{35}$$

We note that this remainder contribution is about twice as large as, but of opposite sign to, that in Eq. (28). Since we are at $t=0$ there is no contribution from variations of the form factor. We remark that since the numerical coefficient in Eq. (35) is so large, the correction, while formally of order $(\mu/m)^2$, is actually of order μ/m. Thus the point $\nu=0$, $\nu_B=-\mu^2/2m$ is not particularly well suited for accurate comparison with experiment, for Eq. (35) gives only an estimate of the remainder. With this cautionary remark uttered we proceed, nevertheless, to compute f^2. We have now

$$f^2=\frac{\mu^2}{16\pi F_\pi^{\,2}}(1+0.247)-\frac{\mu^2}{8\pi}G(0,-\mu^2/2m;\mu^2) \tag{36}$$

which yields, numerically,

$$f^2=0.0564+0.0206=0.077\,, \tag{37}$$

in agreement with our previous determination, Eq. (34).

Finally, let us consider the situation at threshold. We find that the correction due to the $\Delta(1236)$ is very small here. Basically this occurs because the coefficients d, e, and f are in the approximate ratio of 1:3:2 (there are, of course, higher-order terms, but these also tend to cancel). We obtain at threshold

$$\{\Delta \text{ contribution}\}=0.59\,. \tag{38}$$

The contributions from higher resonances are quite small. The function G evaluated at threshold is a familiar integral,

$$G(\mathrm{th})\equiv G(\mu,-\mu^2/2m;\mu^2)=\frac{1}{\pi}\int_\mu^\infty d\nu'\,\frac{\sigma_{\pi^-p}(\nu')-\sigma_{\pi^+p}(\nu')}{(\nu'^2-\mu^2)^{1/2}}, \tag{39}$$

which is sometimes evaluated to obtain the S-wave scattering-length combination $a_{1/2}-a_{3/2}$ via the dispersion relation

$$\left(1+\frac{\mu}{m}\right)\times\tfrac{1}{3}(a_{1/2}-a_{3/2})\mu=\frac{2f^2}{1-\mu^2/4m^2}+\frac{\mu^2}{4\pi}G(\mathrm{th}) \tag{40}$$

An accurate evaluation of the integral is difficult because it is both sensitive to threshold effects and slowly convergent. Different authors obtain values[13] varying from something like -0.030 to -0.025 in units of $8\pi/\mu^2$. If we use the value of Samaranayake and Woolcock,[14] $-(\mu^2/8\pi)G(\mathrm{th})=0.0289$, we get

$$f^2=0.0458+0.0289=0.075\,. \tag{41}$$

In view of the model-dependent cancellations which led to the very small Δ contribution (38), we do not consider this discrepancy with the other two points significant.[15]

We can use Eq. (27) to replace $G(\mathrm{th})$ in Eq. (40) to obtain an evaluation of $a_{1/2}-a_{3/2}$ that is almost independent of f^2. We get

$$4\pi\left(1+\frac{\mu}{m}\right)\times\tfrac{1}{3}(a_{1/2}-a_{3/2})\mu=8\pi f^2\left(\frac{1}{4m^2/\mu^2-1}\right)+\frac{\mu^2}{2F_\pi^{\,2}}\left[1+0.059\left(\frac{\mu}{m}\right)^2\right] \tag{42}$$

or

$$(a_{1/2}-a_{3/2})\mu=0.243\,. \tag{43}$$

This is Weinberg's result,[16] except that he uses the Goldberger-Treiman relation to approximate F_π. Our value is to be compared with various other determinations[17] which range from about 0.245 to 0.297. We remind the reader that we cannot reliably estimate the sum of the many small resonance contributions to (42), but our result clearly favors the lower end of the experimental range.

We note that the Goldberger-Treiman formula can be expressed in terms of the pionic form factor of the nucleon, $K(t)$,

$$K(0)=(mg_A/F_\pi g)=(g_A/F_\pi)(16\pi f^2/\mu^2)^{-1/2}\,. \tag{44}$$

If we use $f^2 = 0.079$ and the recent value[18] $g_A = 1.24$, we get

$$K(0) = 0.94\,, \tag{45}$$

which is to be compared with $K(\mu^2) \equiv 1$. This variation, while substantial on the scale of $\mu^2 = 0.02$ GeV, is more reasonable than that obtained previously from $f^2 = 0.081$ and $g_A = 1.18$, which gave $K(0) = 0.88$.

We have seen that the current-algebra constraints for the pion-nucleon amplitude may well be obeyed with considerable accuracy. We would like to emphasize, however, that more precise data are needed for a definitive test. "Though I could have foreseen this from the beginning, I nevertheless did not want to withhold from the reader this spur to further efforts. Oh, that we could live to see the day when both sets of figures agree with each other." (Kepler)

II. ADLER-WEISBERGER THEOREM

We turn now to review the derivation of the current-algebra constraints. We use an isospin normalization so that, for example, the axial-vector and vector currents are related by

$$[A_a^0(\vec{r}, t), A_b^0(\vec{r}\,', t)] = i\epsilon_{abc} V_c^0(\vec{r}, t)\delta(\vec{r} - \vec{r}\,')\,. \tag{46}$$

We *define* the pion field ϕ_a by

$$\partial_\mu A_a^\mu(x) = -\mu^2 F_\pi \phi_a(x)\,, \tag{47}$$

and assume that its commutator with the axial charge density is local; i.e.,

$$[A_a^0(\vec{r}, t), \phi_b(\vec{r}\,', t)] = i\Sigma_{ab}(\vec{r}, t)\delta(\vec{r} - \vec{r}\,')\,. \tag{48}$$

The additional assumption that the symmetry-breaking term in the Lagrangian is local enables this Σ term to be written as

$$\mu^2 F_\pi \Sigma_{ab}(x) = [X_a(t), [X_b(t), -\mathcal{L}_{\text{S.B.}}(x)]]\,, \tag{49}$$

where

$$X_a(t) = \int d^3r\, A_a^0(\vec{r}, t)\,. \tag{50}$$

It follows from the Jacobi identity and the invariance of $\mathcal{L}_{\text{S.B.}}$ under isospin rotations that Σ_{ab} is symmetrical in its indices:

$$\Sigma_{ab}(x) = \Sigma_{ba}(x)\,. \tag{51}$$

Hence the nucleon matrix element can only involve isospin zero in the nucleon-antinucleon channel

$$\langle p' | \Sigma_{ab}(0) | p \rangle = \delta_{ab} \Sigma(-(p' - p)^2)\,. \tag{52}$$

The formula (10) stated in the Introduction follows immediately from Eqs. (49) and (52).

We shall write the Fourier transform of a covariant, time-ordered product between nucleon states in an abbreviated fashion; for example,

$$i(A_{a'}^{\mu'}(q') A_a^\mu(q))_+ \equiv \int d^4x\, e^{-iq'x} \langle p' | iT^*(A_{a'}^{\mu'}(x) A_a^\mu(0)) | p \rangle\,. \tag{53}$$

It should be borne in mind that $p' + q' = p + q$ and that nucleon spinors are always implicit. The commutation relations given above, along with the definition of the pion field in terms of the divergence of the axial-vector current, imply the divergence condition

$$q'_{\mu'} q_\mu i(A_{a'}^{\mu'}(q') A_a^\mu(q))_+ = i\epsilon_{a'ab} \tfrac{1}{2}(q' + q)_\nu V_b^\nu(q - q') + \mu^2 F_\pi \delta_{a'a} \Sigma(-(q' - q)^2) + (\mu^2 F_\pi)^2 i(\phi_{a'}(q') \phi_a(q))_+\,. \tag{54}$$

Here

$$V_b^\nu(k) = [\gamma^\nu F_1^V(-k^2) - i\sigma^{\nu\lambda} k_\lambda F_2^V(-k^2)] \tfrac{1}{2}\tau_b \tag{55}$$

is the nucleon matrix element of the isospin current, and we have used its conservation

$$k_\nu V_b^\nu(k) = 0 \tag{56}$$

to write the first term in Eq. (54) in terms of the symmetrical combination of pion momenta $\frac{1}{2}(q' + q)$.

In order to achieve a nonsingular expression, the pion poles must be removed from the axial-vector current matrix element. It follows from the definition (47) of the pion field that we can define an axial-vector current without a pion pole,

$$\overline{A}_a^\mu = A_a^\mu + \text{“}\partial^\mu F_\pi \phi_a\text{”}\,, \tag{57}$$

where the quotation marks indicate that the derivative is to be taken outside a time-ordered product so as not to introduce any equal-time commutator terms. Thus, if we use $\overline{A}_a^\mu(q)$ to indicate that the pion pole has been removed, we have

$$i(A_{a'}^{\mu'}(q') A_a^\mu(q))_+ = i(\overline{A}_{a'}^{\mu'}(q') \overline{A}_a^\mu(q))_+ - iq'^{\mu'} F_\pi i(\phi_{a'}(q') \overline{A}_a^\mu(q))_+ + iq^\mu F_\pi i(\overline{A}_{a'}^{\mu'}(q') \phi_a(q))_+ + q'^{\mu'} q^\mu F_\pi^2 i(\phi_{a'}(q') \phi_a(q))_+\,. \tag{58}$$

If we invert this procedure,

$$i(\overline{A}^{\mu\prime}_{a'}(q')\phi_a(q))_+ = i(A^{\mu\prime}_{a'}(q')\phi_a(q))_+ + iq'^{\mu\prime}F_\pi(\phi_{a'}(q')\phi_a(q))_+\,, \tag{59}$$

we can compute

$$q'_{\mu'}i(\overline{A}^{\mu'}_{a'}(q')\phi_a(q))_+ = iF_\pi(q'^2+\mu^2)i(\phi_{a'}(q')\phi_a(q))_+ + i\delta_{a'a}\Sigma(-(q'-q)^2)\,. \tag{60}$$

Similarly,

$$q_\mu i(\phi_a(q')\overline{A}^\mu_a(q))_+ = -iF_\pi(q^2+\mu^2)i(\phi_{a'}(q')\phi_a(q))_+ - i\delta_{a'a}\Sigma(-(q'-q)^2)\,. \tag{61}$$

Putting all this into Eq. (54) gives an identity for the pion-nucleon scattering amplitude,

$$T_{a'a}(q',q) = (q'^2+\mu^2)(q^2+\mu^2)i(\phi_{a'}(q')\phi_a(q))_+\,, \tag{62}$$

which is not singular when the pions are on the mass shell:

$$\begin{aligned}T_{a'a} = {}& -F_\pi^{-2}i\epsilon_{a'ab}\tfrac{1}{2}(q'+q)_\nu V^\nu_b(q-q') \\ & - F_\pi^{-1}(q'^2+q^2+\mu^2)\delta_{a'a}\Sigma(-(q'-q)^2) \\ & + F_\pi^{-2}q'_{\mu'}q_\mu i(\overline{A}^{\mu'}_{a'}(q')\overline{A}^\mu_a(q))_+\,.\end{aligned} \tag{63}$$

The axial-vector current matrix element still has a nucleon pole that must be removed to get an identity that is well behaved at small pion momenta. In order to do this, we note that the nucleon matrix element of the axial-vector current

$$A^\mu_a(p'-p) = \langle p'|A^\mu_a(0)|p\rangle \tag{64}$$

obeys, in virtue of the pion-field definition (47),

$$\begin{aligned}i(p'-p)_\mu A^\mu_a(p'-p) &= \mu^2F_\pi\langle p'|\phi_a(0)|p\rangle \\ &= \mu^2F_\pi\frac{g\tau_a\gamma_5K(-(p'-p)^2)}{(p'-p)^2+\mu^2}\,.\end{aligned} \tag{65}$$

We may write the general solution to this constraint as

$$A^\mu_a(k) = \left[\left(\gamma^\mu\frac{1}{2m}+\frac{k^\mu}{k^2+\mu^2}\right)gF_\pi K(-k^2) + (\gamma^\mu k^2 - k^\mu\gamma\cdot k)H(-k^2)\right]i\gamma_5\tau_a\,. \tag{66}$$

Here we have written the covariant attached to the form factor $H(-k^2)$ in such a way that it is conserved even when the nucleons are off their mass shell. Hence it will not contribute to the contraction of the axial-vector Born term with $q'_{\mu'}q_\mu$. We note that the axial-vector coupling constant g_A is defined by

$$A^\mu_a(k=0) = g_A i\gamma_5(\tfrac{1}{2}\tau_a)\,, \tag{67}$$

and thus the Goldberger-Treiman formula (44) follows from Eq. (66). At any rate, we omit the pion-pole contribution involving $k^\mu(k^2+\mu^2)^{-1}$ from the vertex (66) to compute the axial-vector – current Born term $B^{\mu'\mu}_{a'a}$ and write

$$i(\overline{A}^{\mu'}_{a'}(q')\overline{A}^\mu_a(q))_+ = B^{\mu'\mu}_{a'a} + R^{\mu'\mu}_{a'a}\,, \tag{68}$$

with the remainder $R^{\mu'\mu}_{a'a}$ entirely nonsingular. The current-algebra identity now becomes

$$\begin{aligned}T_{a'a} = {}& -F_\pi^{-2}i\epsilon_{a'ab}\tfrac{1}{2}(q'+q)_\nu V^\nu_b(q-q') \\ & - F_\pi^{-1}(q'^2+q^2+\mu^2)\delta_{a'a}\Sigma(-(q'-q)^2) \\ & + B^{(\mathrm{PV})}_{a'a} + F_\pi^{-2}q'_{\mu'}q_\mu R^{\mu'\mu}_{a'a}\,,\end{aligned} \tag{69}$$

with

$$\begin{aligned}B^{(\mathrm{PV})}_{a'a} &= F_\pi^{-2}q'_{\mu'}q_\mu B^{\mu'\mu}_{a'a} \\ &= (g/2m)^2K(-q'^2)K(-q^2)\left[\gamma\cdot q' i\gamma_5\tau_{a'}\frac{1}{\gamma\cdot(p+q)+m}\gamma\cdot q i\gamma_5\tau_a + \gamma\cdot q i\gamma_5\tau_a\frac{1}{\gamma\cdot(p-q')+m}\gamma\cdot q' i\gamma_5\tau_{a'}\right],\end{aligned} \tag{70}$$

the Born term with pseudovector coupling.

It is sometimes convenient to use the Born term with pseudoscalar coupling rather than pseudovector coupling, since the former vanishes at large energies. Hence, we extract the pseudoscalar Born term and write

$$T_{a'a} = B^{(\mathrm{PS})}_{a'a} + \overline{T}_{a'a}\,, \tag{71}$$

where

$$B^{(\mathrm{PS})}_{a'a} = g^2K(-q'^2)K(-q^2)\left[\gamma_5\tau_{a'}\frac{1}{\gamma\cdot(p+q)+m}\gamma_5\tau_a + \gamma_5\tau_a\frac{1}{\gamma\cdot(p-q')+m}\gamma_5\tau_{a'}\right]. \tag{72}$$

With this decomposition, the current-algebra identity (69) appears as

$$\overline{T}_{a'a} = C_{a'a} + R_{a'a}, \tag{73}$$

in which

$$C_{a'a} = [(g^2/m)K(-q'^2)K(-q^2) - F_\pi{}^{-1}(q'^2 + q^2 + \mu^2)\Sigma(-(q'-q)^2)]\,\delta_{a'a} + [(g^2/2m^2)K(-q'^2)K(-q^2) - \tfrac{1}{2}F_\pi{}^{-2}F_1^V(-(q'-q)^2)]\,i\epsilon_{a'ab}\tau_b \tfrac{1}{2}\gamma(q'+q), \tag{74}$$

and

$$R_{a'a} = q'_{\mu'} q_\mu F_\pi{}^{-2}[R^{\mu'\mu}_{a'a} - \sigma^{\mu'\mu}\tfrac{1}{2}F_2^V(-(q'-q)^2)\epsilon_{a'ab}\tau_b] \equiv q'_{\mu'} q_\mu \overline{R}^{\mu'\mu}_{a'a}. \tag{75}$$

In general, the contribution of the remainder to the invariant amplitudes (4) appears in the form

$$A_R + \tfrac{1}{2}\gamma\cdot(q'+q)B_R = q'_{\mu'} q_\mu \overline{R}^{\mu'\mu}. \tag{76}$$

Now between nucleon spinors we have the identity

$$q'_{\mu'} q_\mu (-i/2m)\sigma^{\mu'\mu} = \tfrac{1}{2}\gamma\cdot(q'+q) + \nu \tag{77}$$

which we use to rewrite (76) as

$$(2\nu)^{-1}(A_R + \nu B_R) q'_{\mu'} q_\mu (-i/2m)\sigma^{\mu'\mu} + (2\nu)^{-1}(A_R - \nu B_R)[\nu - \tfrac{1}{2}\gamma\cdot(q'+q)] = q'_{\mu'} q_\mu \overline{R}^{\mu'\mu}. \tag{78}$$

Hence, current algebra does not constrain the amplitude combination $\nu^{-1}(A^{(-)} + \nu B^{(-)})$ since, by crossing symmetry, it is even and nonsingular, and its contribution to Eq. (78) can be reproduced by a remainder term. Furthermore, if we define $P = p' + p$ and use

$$q'_{\mu'} q_\mu [(-i/2m)\sigma^{\mu'\mu} - (1/4m^2)P^{\mu'}P^\mu] = \tfrac{1}{2}\nu\gamma\cdot(q'+q), \tag{79}$$

we see that, since $B^{(+)}$ must be odd in ν, current algebra does not constrain it either. Hence only $A^{(+)}$ and $G \equiv \nu^{-1}(A^{(-)} - \nu B^{(-)})$ remain as candidates for current-algebra constraints.

We can use Eq. (79) to write $B_R^{(+)}$ as the contraction of $q'_{\mu'}$ and q_μ with a nonsingular tensor. Then Eq. (76) gives

$$A_R^{(+)} = q'_{\mu'} q_\mu A^{\mu'\mu}, \tag{80}$$

with $A^{\mu'\mu}$ nonsingular. Now $A^{\mu'\mu}$ is composed of P^λ, q^λ, q'^λ, and $g^{\mu\nu}$ and is invariant under both nucleon crossing ($P \leftrightarrow -P$) and meson crossing ($q \leftrightarrow -q'$, $\mu' \leftrightarrow \mu$). It is a simple matter to write down all such terms and to perform the contraction with $q'_{\mu'}$, q_μ and obtain

$$A_R^{(+)} = q'^2 q^2 \overline{a} + \nu_B \overline{b} + \nu^2 \overline{c}, \tag{81}$$

with $\overline{a}$, $\overline{b}$, and $\overline{c}$ nonsingular.[19] Thus, with $\nu = \nu_B = 0$ and the pions on the mass shell, the amplitude $A^{(+)}$ is determined by the current-algebra constraints except for a remainder term of order μ^4. Putting everything together, we obtain formula (7), quoted in the Introduction.

The amplitude G can be treated similarly. We rewrite (78) as

$$[\nu - \tfrac{1}{2}\gamma\cdot(q'+q)]G_R = q'_{\mu'} q_\mu G^{\mu'\mu}. \tag{82}$$

By multiplying Eq. (82) from the left by $(m - \gamma p')$ and from the right by $(m - \gamma p)$ and taking the trace, we obtain a numerical relation while keeping the nucleons on their mass shells. It is again a simple matter to enumerate all possible tensor forms and thus to prove that

$$G_R = (q'^2 + q^2)\overline{d} + \nu_B \overline{e} + \nu^2 \overline{f}, \tag{83}$$

with $\overline{d}$, $\overline{e}$, and $\overline{f}$ nonsingular. This establishes the discussion in the Introduction and, again putting everything together, we obtain the formula (27).

III. REMAINDER ESTIMATE

We consider first the contribution of the $\Delta(1236)$ to the remainder term (75). The spin-$\frac{3}{2}^+$, isospin-$\frac{3}{2}$ Δ can be described by a vector-spinor ψ_a^μ. Its coupling to the nucleon (ψ) and pion (ϕ_a) can be written as

$$\mathcal{L}_{\text{int}} = F_\Delta{}^{-1}\overline{\psi}\psi_a^\mu \partial_\mu \phi_a + \text{Hermitian conjugate}. \tag{84}$$

The projection operator $P^{\mu\nu}$ for the Δ at rest is given by

$$P^{\mu 0}_{a'a} = 0 = P^{0\mu}_{a'a}, \tag{85a}$$

$$P^{k'k}_{a'a} = (\delta_{a'a} - \tfrac{1}{3}\tau_{a'}\tau_a)(1 + \tfrac{1}{3}\gamma^{k'}\gamma^k)\tfrac{1}{2}(1+\gamma^0), \tag{85b}$$

since its orthogonality to τ, γ, and p guarantees that it couples only to spin $\frac{3}{2}$ and isospin $\frac{3}{2}$. Using this projection operator, it is a simple matter to compute the decay rate Γ_Δ for $\Delta \to N\pi$:

$$\Gamma_\Delta = (1/12\pi)F_\Delta{}^{-2}(q^3/M)(m+E), \tag{86}$$

where q is the spatial center-of-mass momentum of the final state, M the Δ mass, and E the c.m. energy of the produced nucleon. For $\Gamma_\Delta = 120$ MeV, we obtain the coupling-constant value

$$F_\Delta = 65\ \text{MeV}. \tag{87}$$

The definition (47) of the pion field in terms of the divergence of the axial-vector current requires that the matrix element of this current between nucleon and Δ states have the structure,

$$\langle Np'|A_a^\mu(0)|\Delta p\rangle = \bar{u}\left\{\left[\delta_\nu^\mu - \frac{(p'-p)^\mu(p'-p)_\nu}{(p'-p)^2+\mu^2}\right](F_\pi/F_\Delta)L(-(p'-p)^2) + \cdots\right\}u_a^\nu, \tag{88}$$

where the omitted terms are divergence free for the Δ on the mass shell. Hence they do not contribute to the Δ pole in the remainder but, rather, they correspond to subtraction constants. We shall assume that the dispersion relations for the remainder converge rapidly, and shall therefore neglect such terms. Moreover, we shall suppress[20] off-mass-shell spin-$\frac{1}{2}$ contributions that occur in the Δ propagator by multiplying the vertex with the projection

$$\lambda^{\mu\nu} = g^{\mu\nu} + \tfrac{1}{4}\gamma^\mu\gamma^\nu \tag{89}$$

that reduces to the identity at the Δ pole, and write the Δ contribution to the remainder (75) as

$$R_{a'a}^{(\Delta)} = F_\Delta^{-2}L(-q'^2)L(-q^2)q'_{\mu'}q_\mu[\lambda_{\nu'}^{\mu'}\Delta_{a'a}^{\nu'\nu}(p+q)\lambda_\nu^\mu + \lambda_\nu^\mu\Delta_{aa'}^{\nu\nu'}(p-q')\lambda_{\nu'}^{\mu'}]. \tag{90}$$

For the pions on the mass shell, we have

$$L(\mu^2) = 1. \tag{91}$$

A permissible Δ propagator can be obtained by a straightforward extension of the rest-frame spin-$\frac{3}{2}$ projection operator (85) to a form that is valid in an arbitrary frame. Such an extension can be accomplished using an operator that is transverse on the mass shell,

$$t^{\mu\nu}(p) = g^{\mu\nu} + p^\mu p^\nu/M^2, \tag{92}$$

with the result

$$G_{ab}^{\mu\nu}(p) = (\delta_{ab} - \tfrac{1}{3}\tau_a\tau_b)[t^{\mu\nu}(p) + \tfrac{1}{3}t^{\mu\lambda}(p)\gamma_\lambda t^{\nu k}(p)\gamma_k]\frac{M-\gamma\cdot p}{p^2+M^2}. \tag{93}$$

However, this propagator contains contact terms that are exhibited by the algebraic reduction

$$G_{ab}^{\mu\nu}(p) = \Delta_{ab}^{\mu\nu}(p) - (\delta_{ab} - \tfrac{1}{3}\tau_a\tau_b)(1/3M^4)p^\mu p^\nu(M - \gamma\cdot p), \tag{94}$$

where

$$\Delta_{ab}^{\mu\nu}(p) = (\delta_{ab} - \tfrac{1}{3}\tau_a\tau_b)\left\{[g^{\mu\nu} + (2/3M^2)p^\mu p^\nu + (1/3M)(p^\mu\gamma^\nu - \gamma^\mu p^\nu) + \tfrac{1}{3}\gamma^\mu\gamma^\nu]\frac{M-\gamma\cdot p}{p^2+M^2}\right\}. \tag{95}$$

We note that the contact term $p^\mu p^\nu(M - \gamma\cdot p)$ shares the crossing symmetry of the propagator, and thus $\Delta_{ab}^{\mu\nu}(p)$ is crossing symmetric. We shall use the propagator $\Delta_{ab}^{\mu\nu}(p)$ rather than $G_{ab}^{\mu\nu}(p)$ since it vanishes more rapidly for large p.

It is now straightforward to compute the $\Delta(1236)$ contribution to $A_R^{(+)}(0,0;\mu^2)$ and $G_R(\nu,\nu_B;\mu^2)$. We find

$$A_R^{(+)}(0,0;\mu^2) = \frac{2}{9F_\Delta{}^2}\left[\frac{2M+m}{M^2(M^2-m^2)}\right]\mu^4, \tag{96}$$

and

$$G_R(\nu,\nu_B;\mu^2) = -\frac{2}{9F_\Delta{}^2M^2}\left\{\frac{(M+m)^2}{(M^2-m^2+2m\nu_B)^2 - 4m^2\nu^2}\left[(M-m)(M+2m)\mu^2 + \frac{3M^3 - m^2M + 2m^3}{M+m}(2m\nu_B) + 2m^2\nu^2 + Y\right] + Z\right\}, \tag{97}$$

where

$$Y = \frac{1}{(M+m)^2}\{(4\nu^2\mu^2 - 4\nu^2\nu_B m)m^2 + \tfrac{1}{2}\mu^4(M^2+m^2+4mM) + 2m\nu_B[\mu^2(mM+m^2) + \tfrac{1}{2}\mu^4 + m\nu_B(7M^2 - 2mM - 3m^2 + 2m\nu_B - 2\mu^2)]\}, \tag{98}$$

and

$$Z = \tfrac{1}{2}\mu^2 - \tfrac{3}{4}m\nu_B . \qquad (99)$$

Evaluating these expressions gives the results quoted in the Introduction. We should note that the $\gamma^\mu\gamma^\nu$ terms in the projection (89) give no contribution to $A_R^{(+)}(0, 0; \mu^2)$. Their contribution to $G_R(\nu, \nu_B; \mu^2)$, given by Z, is numerically insignificant at the points of interest.

The contributions of higher resonances to the remainder can be obtained in a similar manner. Individually, the contribution of each of these resonances is much less than that of the $\Delta(1236)$. Their coupling constants $1/F_{N^*}$ are smaller than $1/F_\Delta$, since the allowable phase space for decay into πN is increased, and the partial widths are, in general, smaller than $\Gamma_\Delta = 120$ MeV. This reduction in the coupling constant is already of an order of magnitude for resonances around 1500 – 1600 MeV. The $\frac{3}{2}^-$ resonance contributions are further suppressed by angular momentum; the 1^+ axial-vector current connects $\frac{1}{2}^+$ to $\frac{3}{2}^+$ in the S wave, but can couple $\frac{1}{2}^+$ to $\frac{3}{2}^-$ only with P or higher waves. This is explicitly exhibited by the amplitudes (96), (97), for the parity-reversed $\frac{3}{2}^-$ contribution is obtained by the substitution $M \to -M$ and is obviously much smaller.

It is elementary to verify that any contribution to $T^{a'a}$ of the form

$$\tau^{a'}\tau^{a}\gamma\cdot q'[F(\nu, \nu_B) + \gamma\cdot(p+q)H(\nu, \nu_B)]\gamma\cdot q + (q' \leftrightarrow -q,\ a' \leftrightarrow a) \qquad (100)$$

gives a contribution to G or $A^{(+)}$ which vanishes at $\nu = \nu_B = 0$, provided only that F and H are regular at $\nu = \nu_B = 0$. Thus we conclude, without invoking any particular model, that the $\frac{1}{2}^\pm$ resonances do not contribute at $\nu = \nu_B = 0$.

We find that the addition of the lowest pion-nucleon resonances, $N(1470)\ \frac{1}{2}^+$, $N(1520)\ \frac{3}{2}^-$, $N(1535)\ \frac{1}{2}^-$, $\Delta(1650)\ \frac{1}{2}^-$, and $\Delta(1670)\ \frac{3}{2}^-$, does not affect, at the point $\nu = \nu_B = 0$, the estimates presented in Sec. I. At other points it is a simple matter to estimate that their contributions are not significant compared to the one of the $\Delta(1236)$, except for threshold where the $\Delta(1236)$ contribution itself is very small. There the contribution of all these resonances is comparable to the one of the $\Delta(1236)$.

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[1]We use the metric with signature $(-+++)$ and Dirac matrices defined by $\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}$, $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$.

[2]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960), reprinted in S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).

[3]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[4]F. von Hippel and J. Kim, Phys. Rev. D 1, 151 (1970).

[5]M. Gell-Mann, Hawaii Summer School Notes, 1969 (unpublished); J. Ellis, P. H. Weisz, and B. Zumino, Phys. Letters 34B, 91 (1971).

[6]This relation has been derived independently by R. J. Crewther, Phys. Rev. D 3, 3152 (1971).

[7]T. P. Cheng and R. Dashen, Phys. Rev. Letters 26, 594 (1971).

[8]Recently G. Höhler, H. P. Jakob, and R. Strauss [Phys. Letters 35B, 445 (1971)] have extracted the Σ term by a different method than that of Cheng and Dashen (Ref. 7). They use a subtracted dispersion relation, with the subtraction constant being the Σ term. Their evaluation, over a wide range of energies, yields a Σ term which is roughly half that obtained by Cheng and Dashen. It should be noted, however, that their dispersion relation evaluated for energies near threshold gives values that are much more compatible with that of Cheng and Dashen. Since the broad-area-subtraction method emphasizes this region, this is heartening, but it points out the need for better low-energy measurements. (L. S. B. thanks Roger Dashen for an informative conversation on this work.)

[9]S. L. Adler, Phys. Rev. 137, B1022 (1965), Eq. (48).

[10]G. Ebel and H. Pilkuhn, Nucl. Phys. B17, 1 (1970).

[11]S. L. Adler, Phys. Rev. 140, B736 (1965); W. I. Weisberger, *ibid.* 143, 1302 (1966). These papers are reprinted in S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics*, Ref. 2.

[12]L. H. Chan, K. W. Chen, J. R. Dunning, Jr., N. F. Ramsey, J. K. Walker, and Richard Wilson, Phys. Rev. 141, 1298 (1966).

[13]J. Hamilton, Phys. Letters 20, 687 (1966), obtains values for f^2 and $a_{1/2} - a_{3/2}$ which require that $-(\mu^2/8\pi)G(\text{th}) = 0.0307 \pm 0.0014$. G. Höhler, J. Baacke, and R. Strauss, Phys. Letters 21, 223 (1966), obtain $-(\mu^2/8\pi)G(\text{th}) = 0.0264 \pm 0.003$. More recently, G. Höhler, H. P. Jakob, and R. Strauss [Karlsruhe report (unpublished)] obtain $-(\mu^2/8\pi)G(\text{th}) = 0.0245$.

[14]V. K. Samaranayake and W. S. Woolcock, report submitted to the Fifth International Conference on Elementary Particles, Lund, Sweden, 1969 (unpublished).

[15]Our numbers for all three points differ consistently from those of Höhler *et al.* (Karlsruhe report, Ref. 13), primarily because we use the Adler evaluation of G at $\nu = t = 0$. Using their numbers throughout gives reasonably self-consistent values for f^2 (= 0.068, 0.074, and 0.071 at $\nu = \nu_B = 0$, $\nu = t = 0$, and threshold, respectively), but these values are about 10% too small. We note, incidentally, that if we use Adler's value for $G(\nu = t = 0)$ and Höhler's corrections to extrapolate to threshold we obtain $-(\mu^2/8\pi)G(\text{th}) = 0.282$, in agreement with Samaranayake and Woolcock.

[16]Steven Weinberg, Phys. Rev. Letters 17, 616 (1966), reprinted in S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics*, Ref. 2.

[17]See the compilation by G. Ebel *et al.* (Ref. 10). The value $a_{1/2} - a_{3/2} = 0.254 \pm 0.013$, A. Donnachie and G. Shaw, Nucl. Phys. 87, 556 (1966), is representative of Panofsky-ratio values, although a value as low as $a_{1/2} - a_{3/2} = 0.245 \pm 0.001$ has been reported by C. M. Rose, Phys. Rev. 154, 1305 (1967).

[18]We use an average of the values $g_A = 1.226 \pm 0.011$ and $g_A = 1.26 \pm 0.02$ discussed by R. J. Blin-Stoyle and J. M. Freeman, Nucl. Phys. A150, 369 (1970).

[19]To be more precise, the functions $\bar{a}$, $\bar{b}$, $\bar{c}$ are regular in ν and ν_B in a neighborhood of $\nu = \nu_B = 0$. We make no assertions about analyticity in external masses. At $\nu = \nu_B = 0$ the two-pion t-channel intermediate state is formally singular in the pion mass $[O(\ln \mu^2)]$. However, this does not alter our numerical estimate. See Heinz Pagels and W. J. Pardee, Phys. Rev. (to be published).

[20]R. D. Peccei, Phys. Rev. 176, 1812 (1968).

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Nucleon Form Factors, Vector Dominance, and Lorentz Contraction

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We use the notion of Lorentz contraction of a composite cluster combined with vector-meson dominance of the one-photon exchange to derive the asymptotic form of the nucleon form factors. For the three-quark model we find a t^{-2} prediction which fits the data very well at large spacelike t. Only the observed vector mesons, ρ, ω, and ϕ, are used. Our results predict deviations from the scaling laws which are directly related to the nonvanishing electric form factor of the neutron and are of the same magnitude.

I. INTRODUCTION

There are two classes of attempts to fit the electromagnetic form factors of the nucleons. The first one tries to find a simple analytic expression without giving it any theoretical justification. The best known is the dipole fit.[1] More recently we have seen a superposition of exponentials[2] and a ratio of Γ functions.[3] The data have now become accurate enough so that clear deviations from the dipole fit are evident[4] especially at high (negative) t. The second method uses dispersion relations[5] and vector-meson dominance.[6] This has not been very successful in the region of large spacelike momentum transfer. In order to get fair agreement with the data, either large negative couplings to unobserved vector mesons had to be assumed,[7] or several *ad hoc* structure parameters had to be introduced. An extensive list of references on additional work on form factors can be found in review articles.[8,9]

We propose a different approach. We assume that the nucleon at rest is a bound state of three quarks (or partons). We calculate a quark-nucleon form factor in the region of large spacelike momentum transfer by making a Lorentz transformation of the arguments of the quark wave functions.[10,11] Combining this with the dominance of only the established vector mesons, we find excellent agreement with experiment.

Our results depend to a certain extent on the choice of the quark wave function. We find the best fit for a symmetric Gaussian wave function. We have tried an antisymmetric Gaussian and an antisymmetric exponential wave function. We find that they both fit very badly at small t.

We assume that the photon couples to the quark through the known vector mesons ρ, ω, ϕ. The vector meson then couples directly to a single quark, forming the vertex shown in Fig. 1. There is also a class of vertices where the vector meson breaks up into a number of pions, which then interact with the same or with different quarks, as shown in Fig. 2.

We assume that the vertex of Fig. 2(a) is dominant, and neglect the others. It is conceivable that they might provide small corrections to our results at low momentum transfers.

An approach somewhat similar to ours has been tried by Fujimura, Kobayashi, and Namiki.[12] They use the relativistic wave function due to Takabayashi.[13] With one adjustable parameter they obtain reasonable agreement with the data. Because they have a four-dimensional harmonic oscillator wave function, they get a different dependence of the form factor on the number of constituents. In order to get a t^{-2} behavior they have to introduce an additional Lorentz factor into the already covariant meson propagator. Our results show that this is not needed. Barut[14] has proposed