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Inelastic Electron-Proton Scattering and Vector-Meson Dominance

Kazuo Fujikawa
The Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637
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A simple phenomenological picture of scaling is proposed based on generalized vector-meson dominance. This picture may provide a possible mechanism of scaling in the context of an analytic-S-matrix approach such as the dual-resonance model. The present model suggests that a very weak spectral function in the large-photon-mass region is sufficient to give the magnitude and shape of scaling-limit functions. Thus the measurement of νW_2 explains why ρ-meson dominance works in high-energy photoreactions. Experimental implications of the model, in particular the expected missing-mass distribution, are also discussed.

The existence of scaling in inelastic electron-proton scattering, predicted by Bjorken,[1] has been verified by SLAC experiments[2] for a range of the scaling parameter ω. Various models which explain scaling laws have been proposed.[3] One mystery, however, is the failure of the vector-meson dominance model (VMD) proposed by Sakurai[4,3] to show this scaling behavior, although it did predict the correct order of magnitude for νW_2. The purpose of the present note is to show that a rather weak spectral weight in the large-photon-mass region for the dispersion integral is sufficient to explain the scaling behavior; thus the measurement of scaling functions explains why ρ-meson dominance works in photoreactions. The basic idea in our model is to approximate the dispersion integral over the photon mass by an infinite number of vector-meson poles. This approximation is expected to be good for spacelike photons. By introducing the heavy mesons, we show that two of the basic properties of the parton model can be realized at the phenomenological level in a quite different manner from that of Bjorken and Paschos.[5] These basic properties of the parton model are:

(a) The mass of the particles which couple to massive photons (i.e., the mass of the partons) does not matter in the scaling limit.

(b) The partons couple to massive photons as pointlike particles.

We can incorporate these two properties into the propagator and the coupling constant of the heavy vector mesons.

The basic equation of VMD reads[4]

$$\langle A \mid j^\mu \mid B\rangle = \sum_V \frac{C_V}{{m_V}^2 - q^2} \langle A \mid J_V^\mu \mid B\rangle , \qquad (1)$$

where j^μ and J_V^μ are the electromagnetic and vector-meson currents, respectively. C_V is a constant, and q^2 and ${m_V}^2$ are the squares of the masses of the photon and the vector meson, respectively. Let us assume that heavy mesons with a mass which satisfies the relation[6]

$${m_V}^2 \sim - q^2 \qquad (2)$$

give a dominant contribution to the deep-inelastic process. We call this the "parton condition" for simplicity in the following, although it has no direct connection with the parton model. Under the present assumption we can replace ${m_V}^2$ by $-q^2$ everywhere except in the propagator in Eq. (1). Then we get the property (a) above. Equation (1) also shows that the "form factor" of the heavy meson does not fall off for $|q^2| \lesssim {m_V}^2$; this gives the property corresponding to (b). In this way we can transfer the mechanism of scaling to the strength of the coupling constant C_V and the property of the hadronic process $V+p \to$ anything. In the parton model[3,5] this has been achieved by assuming quasi-free partons. In the present model the strength of hadron interactions expressed by the Regge behavior of the forward *heavy-meson*–proton scattering amplitude combined with the parton condition determines the basic structure and the magnitude of scaling functions.[7] It is our assumption that the mechanism of scaling based on the parton condition is working in the region where a pole-dominance model like general VMD is applicable [see Eq. (5) below]. The parton condition is also the assumption that we can measure the "semilocal" structure of the spectral weight by varying q^2.

Thus the measurement of νW_2 in some regions of the scaling parameter is the measurement of the spectral weight for larger masses. In this way we can estimate the correction to ρ-meson dominance in photoreactions due to the heavy-mass states; this correction turns out to be small.[8]

In the following we briefly describe the basic idea of the model. Our starting equations are[4]

$$\sigma_\gamma^{(T)}(s, q^2) = \sum_V \frac{{C_V}^2}{({m_V}^2 - q^2)^2} \sigma_V^{(T)}(s) + \text{interference terms}, \qquad (3)$$

$$\sigma_\gamma^{(S)}(s,q^2)=\sum_V \frac{C_V{}^2}{(m_V{}^2-q^2)^2}\frac{(-q^2)}{m_V{}^2}\sigma_V^{(L)}(s)$$
$$+\text{interference terms}, \tag{4}$$

for

$$s\gg -q^2,\ m_V{}^2, \tag{5}$$

where $\sigma_\gamma^{(T)}(s,q^2)$ and $\sigma_\gamma^{(S)}(s,q^2)$ correspond to the transverse and longitudinal total cross sections for the process

$$\text{"}\gamma\text{"}+p\to \text{anything}. \tag{6}$$

$\sigma_V^{(T)}(s)$ and $\sigma_V^{(L)}(s)$ are the corresponding cross sections for the process

$$V+p\to\text{anything}. \tag{7}$$

Interference terms in Eqs. (3) and (4) describe the contribution due to the interference of different mesons. We neglect them in the following; qualitative features of the final result are little affected by this simplification. The condition in Eq. (5) is necessary to avoid a possible contribution from the anomalous complex singularities in the q^2 plane for fixed s (Ref. 9) and also to ensure a smooth photon mass extrapolation. Combined with the parton condition it also allows us to use the same Regge parameter for different mesons in Eqs. (8) and (9) below. Therefore our model is not valid for small values of the scaling parameter $\omega=1+s/-q^2$. We next assume that the forward heavy-meson–proton scattering amplitude has Regge behavior. Thus we get[10]

$$\sigma_V^{(T)}(s)=A\gamma(m_V)^2\frac{1}{m_V{}^2}\left(\frac{s}{m_V{}^2}\right)^{\alpha-1}, \tag{8}$$

$$\sigma_V^{(L)}(s)=B\gamma(m_V)^2\frac{1}{m_V{}^2}\left(\frac{s}{m_V{}^2}\right)^{\alpha-1}, \tag{9}$$

where A and B are constants and α is the intercept of the leading Regge trajectory, $\alpha\leqslant 1$; $\gamma(m_V)$ is a function of m_V. The parton condition demands the following relation:

$$(m_V)^0< C_V\gamma(m_V)< m_V{}^2 \tag{10}$$

for $m_V{}^2\gg m_\rho{}^2$. Thus we take the simplest choice,

$$C_V\gamma(m_V)=Cm_V, \tag{11}$$

where C is a constant or a slowly varying function of m_V. This choice automatically ensures the condition in Eq. (5) as well as scaling. However, note that the parton condition does not necessarily produce scaling. This means the main part of our conclusion is still valid even if scaling laws are weakly violated at large ω. With these assumptions we finally get

$$\sigma_\gamma^{(T)}(s,q^2)\approx f(\alpha)\left(\frac{m_\rho{}^2}{m^2}\right)\left(\frac{m_\rho{}^2}{-q^2}\right)\left(\frac{s}{-q^2}\right)^{\alpha-1}\sigma_\gamma^{(T)}(\infty,q^2=0), \tag{12}$$

$$\sigma_\gamma^{(S)}(s,q^2)\approx R\sigma_\gamma^{(T)}(s,q^2), \tag{13}$$

where

$$f(\alpha)\equiv\int_0^\infty\frac{dx}{(1+x)^2x^{\alpha-1}}\ \text{with}\ x=\frac{m_V{}^2}{-q^2}, \tag{14}$$

$$R\equiv\xi\int_{m_\rho{}^2/-q^2}^\infty\frac{dx}{x^\alpha(1+x)^2}\frac{1}{f(\alpha)},\qquad \xi=\frac{\sigma_\rho^{(L)}(\infty)}{\sigma_\rho^{(T)}(\infty)}. \tag{15}$$

We normalized the constants A, B, and C at the ρ-meson values; this will give the correct order of magnitude because of the experimental fact[2] that scaling sets in at fairly small q^2. We have also replaced the summation over $m_V{}^2$ by an integration in Eqs. (3) and (4). The mass interval between two adjacent vector mesons, denoted by m^2, is about 1 BeV^2. The integrals in Eqs. (14) and (15) receive a major contribution from the region $x\sim 1$, i.e., $m_V{}^2\sim -q^2$; this is exactly what we required. The values of $f(\alpha)$ and R are shown in the following:

for $\alpha=1$:

$$f(\alpha)=1,\quad R=\xi[\ln(1-q^2/m^2)-1/(1-m^2/q^2)];$$

for $\alpha=\frac{1}{2}$:

$$f(\alpha)=\tfrac{1}{2}\pi,\quad R=\xi,$$

where $\xi=\sigma_\rho^{(S)}(\infty)/\sigma_\rho^{(T)}(\infty)$. R for $\alpha=1$ depends on q^2, and $R(q^2=4\text{ BeV}^2)\approx 2\xi$. We show νW_2, which is defined by[4]

$$\nu W_2=\frac{\nu(s-M^2)}{\pi e^2 2M}\left(\frac{-q^2}{\nu^2-q^2}\right)(1+R)\sigma_\gamma^{(T)}(q^2,s) \tag{16}$$

for $\alpha=1$ and $\frac{1}{2}$ in Fig. 1. We assumed that $R(\sim 2\xi)=0.5$, $\sigma_\gamma^{(T)}(\infty,q^2=0)=125\ \mu\text{b}$, and $m^2=2m_\rho{}^2$. It is amusing to see that the value of νW_2 in our model is about the same as predicted by Sakurai[4] and that it is not far from the experimental result.[2]

We next estimate the correction to ρ-meson dominance in photoreactions due to the heavy-

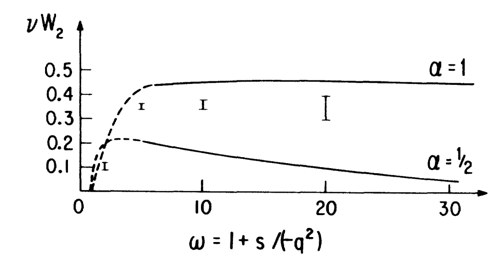

FIG. 1. νW_2 given by Eq. (16) with $R=0.5$, $\sigma_\gamma\,(s=\infty)=125\ \mu\text{b}$, $m^2=2m_\rho{}^2$, and $\alpha=1$ or $\frac{1}{2}$. Several experimental values taken from Ref. 2 are also shown with error flags to give a rough idea of the observed values of νW_2.

mass states. From Eqs. (3), (8), and (11) we get

$$\sigma_\gamma^{(T)}(s, q^2=0) \approx \sigma_{\gamma,\rho}^{(T)}(s)\left[1+\left(\frac{m_\rho^2}{m^2+m_\rho^2}\right)^2 + \left(\frac{m_\rho^2}{2m^2+m_\rho^2}\right)^2 + \cdots\right] \approx 1.2\sigma_{\gamma,\rho}^{(T)}(s) \text{ for } \alpha \approx 1, \quad (17)$$

where $\sigma_{\gamma,\rho}^{(T)}(s)$ is the value obtained by ρ-meson dominance. This shows that ρ-meson dominance is modified by about 20%.[11] This small modification of ρ-meson dominance explains why it works well in (inclusive) high-energy photoreactions.[12]

The present model suggests that the basic idea of VMD is still working in the deep-inelastic process, although the simplicity and usefulness of VMD is lost. It is the weak spectral integral that determines the magnitude and the shape of scaling-limit functions. The real test of VMD would therefore be:

(i) Measurement of R at small q^2 ($0< -q^2 < m_\rho^2$) and at large s. SLAC data[2] suggest that R is also small in this region, namely, $R \sim \xi = \sigma_\rho^{(L)}(\infty)/\sigma_\rho^{(T)}(\infty) < 0.5$.

(ii) At large s and ω, some fraction (depending on ξ) of the final-state particles will correspond to the longitudinal ρ-meson-proton scattering almost independently of q^2 ($\gg m_\rho^2$).[13] This is the test of the gauge condition implemented by Sakurai.[4] The test of our model would be:

(iii) If the Regge-like behavior of νW_2 observed at large ω (Refs. 2 and 3) is confirmed to be a genuine one, it is a partial support for the parton condition. If mesons of small mass dominate the process independently of q^2, it would be difficult to get a well-defined Regge behavior with $\alpha < 1$.

(iv) It is expected that we observe an increasingly heavier fireball corresponding to projectile fragments when we increase $|q^2|$ at large s and *large* ω, or a heavier missing mass (up to $\sim|q^2|$) for larger q^2 at large fixed s (Ref. 14) if only the final-state proton is observed. The transverse momentum distribution of final-state particles with respect to the incident "photon" could also be distinct from those observed in purely hadronic processes.

In the present note we have studied the possible mechanism of scaling by utilizing heavy particles instead of neglecting the mass of the particles involved. What we have found is a consistent picture of the Regge-like behavior, scaling at fairly small q^2, good agreement with measured values of νW_2, and the success of ρ-meson dominance in (inclusive) photoreactions.

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[1]J. D. Bjorken, Phys. Rev. 179, 1545 (1969).

[2]E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High-Energy Physics, Kiev, U.S.S.R., 1970.

[3]For a list of earlier references and the present status of theoretical understanding of the deep-inelastic process, see J. D. Bjorken, in Lectures at the Scottish Universities Summer School (unpublished).

[4]J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969). See also C. F. Cho, G. J. Gounaris, and J. J. Sakurai, Phys. Rev. 186, 1734 (1969). We follow the notation in this reference except the difference in the metric; $q^2 < 0$ corresponds to a spacelike vector in *our* metric.

[5]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[6]Experiments also suggest the heavier mesons may be more important for larger momentum transfers. See L. Mo, in *High Energy Collisions,* Third International Conference held at State University of New York, Stony Brook, 1969, edited by C. N. Yang, J. A. Cole, M. Good, R. Hwa, and J. Lee-Franzini (Gordon and Breach, New York, 1969). Equation (2) is the naive identification, small distances ~ heavier mesons.

[7]This model was motivated by the observation made by Rittenberg and Rubinstein based on the dual-resonance model. They suggest that the forward heavy-meson–proton amplitude may scale by itself at high energies; V. Rittenberg and H. R. Rubinstein, Nucl. Phys. B28, 184 (1971).

[8]Our model should be regarded as a correction to ρ-meson dominance due to the heavy-mass states.

[9]A. suri, Phys. Rev. Letters 26, 208 (1971).

[10]We assume that $\sigma_V^{(L)}(s)/\sigma_V^{(T)}(s)=$const. This is the result suggested by Rittenberg and Rubinstein (Ref. 7). They also show that $\gamma(m_V)=$const, and $\alpha < 1$ if one uses the dual-resonance model.

[11]This correction mainly depends on the measured values of νW_2 and the parton condition combined with scaling even at fairly small q^2. If one includes interference terms the theoretical value for νW_2 would be increased, and the constants in Eqs. (8), (9), and (11) are forced to be smaller; the result in Eq. (17) is thus not so sensitive to the specific assumption we made about the interference terms.

[12]D. O. Caldwell *et al.*, Phys. Rev. Letters 25, 609 (1970); 25, 613 (1970).

[13]The violation of scaling by a logarithmic term in R for $\alpha=1$ in our model is the result of the assumption $\sigma_V^{(L)}(s)/\sigma_V^{(T)}(s)=$const.

[14]If one uses the gluon model (vector-meson–quark–quark coupling) and calculates the process "γ" $+ q \to q + q + \bar{q}$, the invariant mass of the quark-antiquark pair, which could be converted into a multipion system via final-state interactions, has these characteristics. See also J. D. Bjorken, SLAC Report No. SLAC-PUB-905, 1971 (unpublished). Experimental data do not contradict this behavior; see D. E. Andrews *et al.*, Phys. Rev. Letters 26, 38 (1971).