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Quantum Theory of Gravitation and the Mass of the Electron*

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Gravity-modified quantum electrodynamics formulated by Salam and collaborators is applied to the electron self-energy problem in all orders of perturbation theory. It is shown that the dressed physical mass of the electron is likely to be given by the formula $m = Me^{-x/\alpha}$, where $M \equiv (\hbar c/16\pi G)^{1/2}$ is the universal quantum-gravitational mass constant, $\alpha = (137.04)^{-1}$ is the fine-structure constant, and $x = (0.362)$ is a number to be computed from the details of the theory.

INTRODUCTION

Remarkably accurate cross sections for fundamental processes involving photons, electrons, and positrons are predicted by standard renormalized quantum electrodynamics (QED) in Minkowskian space-time. In fact, the cross sections predicted by QED have yet to show even the slightest discrepancy with experiment.[1] There are, however, certain unsatisfactory theoretical features associated with QED:

(1) It has not been possible to put the theory on a rigorous mathematical basis and to justify the formal manipulation and subtraction of infinite quantities that appear in the renormalization procedure.

(2) The physical mass of the electron, $m = 9.09 \times 10^{-28}$ g, comes into the theory as an empirically prescribed quantity which cannot be predicted theoretically.

(3) Electromagnetic mass differences between the members of isospin multiplets (e.g., the $\pi^{\pm}$-π^0 mass difference of about $9m$) are predicted to be infinite by QED with the cutoff-to-infinity renormalization operation.[2]

These unsatisfactory theoretical features of QED in Minkowskian space-time may have their origin in the physical incompleteness of the theory. Namely, the role of quantum-gravitational effects, as manifest by the emission and absorption of virtual gravitons, is not taken into account in the standard QED computations of electromagnetic self-energies. In this connection it was conjectured many years ago by Weisskopf[3] that finite values for electromagnetic self-energies (and hence finite values for the QED renormalization constants) would appear in a modified version of the theory with quantum-gravitational effects included. In such a "gravity-modified QED," the universal quantum-gravitational mass constant

$$M \equiv (\hbar c/16\pi G)^{1/2} = 3.06 \times 10^{-6}\ \text{g} \qquad (1)$$

might serve as a built-in physical "cutoff parameter" in the theoretical expressions for electromagnetic self-energies. To ascertain whether quantum-gravitational effects actually engender finite theoretical expressions for electromagnetic self-energies, one must be able to perform rather sophisticated calculations with the quantum theory of general relativity in combination with QED. The quantum theory of general relativity has been advanced by the work of Dirac,[4] but the application of Dirac's complete formulation to QED self-energy calculations is still encumbered by formidable technical difficulties. Recently, an alternative and more pragmatic computational approach to the problem of quantum-gravitational effects in QED has been proposed by Salam and his collaborators.[5] Initial calculations reported by the latter authors show that to first order in the fine-structure constant $\alpha = (137.04)^{-1}$ the electron and photon self-energies are indeed finite in gravity-modified QED. For the electromagnetic self-energy of the electron, Salam and collaborators obtain the result

$$\frac{\delta m}{m} \cong \frac{3\alpha}{2\pi} \ln\left(\frac{M}{m}\right) + O(\alpha^2), \tag{2}$$

where M denotes the quantum-gravitational mass given by Eq. (1). Observe that M manifests itself as an effective invariant wave-number cutoff parameter in formula (2). If M plays this role to all orders of α in the electron mass-renormalization series and in the formulas for electromagnetic mass differences between members of isospin multiplets,[2] then the unsatisfactory theoretical features of standard renormalized QED in Minkowskian space-time are set right by the inclusion of quantum-gravitational effects.

The purpose of the present paper is to show how gravity-modified QED is likely to yield a definite and physically acceptable theoretical value for the mass of the electron. In this discussion the so-called bare mass of the electron is put equal to zero; the physical mass of the electron is supposed to be due entirely to electromagnetic and gravitational dynamical effects, as conjectured many years ago by Weisskopf.[3]

ELECTRON MASS IN GRAVITY-MODIFIED QED

The Lagrangian density for the Maxwell, Dirac, and Einstein fields in interaction is expressed as[5]

$$\mathfrak{L} = \mathfrak{L}_{\text{grav}} + \mathfrak{L}_{\text{matter}} + \mathfrak{L}_{\text{gauge}}, \tag{3}$$

where the purely "gravitational part" is due to Einstein,

$$\mathfrak{L}_{\text{grav}} \equiv M^2 \sqrt{-g}\, R, \tag{4}$$

the "matter part" contains the Dirac and Maxwell fields,

$$\mathfrak{L}_{\text{matter}} \equiv \sqrt{-g}\, [\tfrac{1}{2} i (\bar{\psi}\gamma_a \psi_{;\mu} - \bar{\psi}_{;\mu}\gamma_a \psi) L^{\mu a} + e_0 \bar{\psi}\gamma_a A_\mu \psi L^{\mu a} - \tfrac{1}{4} g^{\mu\nu} g^{\kappa\lambda} F_{\mu\kappa} F_{\nu\lambda}], \tag{5}$$

and the "gauge part" breaks the electromagnetic and gravitational gauge symmetries,

$$\mathfrak{L}_{\text{gauge}} \equiv -\tfrac{1}{2}\sqrt{-g}\, (g^{\mu\nu} A_{\mu;\nu})^2 + 2M^2 \{\partial_\mu [(-g)^{1/4} L^{\mu a}]\} \{\partial_\nu [(-g)^{1/4} L^{\nu}{}_a]\}. \tag{6}$$

It should be noted that M appears here in place of κ^{-1}, and the bare-mass term is absent in (5) since $m_0 \equiv 0$; otherwise the notation follows Salam and collaborators, with $L^{\mu a}$ denoting their symmetrical vierbein and physical units chosen such that $\hbar$ and c equal unity. The dressed physical mass m enters the theory as a parameter in the prescribed free Lagrangian density

$$\mathfrak{L}_0 = \tfrac{1}{4}(\partial_\mu h^{\lambda\rho} \partial_\mu h^{\lambda\rho} - \tfrac{1}{2}\partial_\mu h^{\lambda\lambda} \partial_\mu h^{\rho\rho}) + \tfrac{1}{2} i (\bar{\psi}\gamma_\mu \psi_{,\mu} - \bar{\psi}_{,\mu}\gamma_\mu \psi) - m\bar{\psi}\psi - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu, \tag{7}$$

where repeated indices are understood to be contracted with respect to the Minkowskian metric tensor, $\eta^{\mu\nu} \equiv \text{diag}(+1, -1, -1, -1)$, and the free graviton field is defined as $h^{\mu\nu} \equiv M(g^{\mu\nu} - \eta^{\mu\nu})$. In order to counter the dressed physical mass term introduced in (7), the interaction Lagrangian density takes the form

$$\mathfrak{L}_{\text{int}} \equiv \mathfrak{L} - \mathfrak{L}_0 = m\bar{\psi}\psi + e_0 (\det L)^{-1} L^{\mu a} \bar{\psi}\gamma_a \psi A_\mu + (\text{terms independent of } m \text{ and } e_0 \text{ of order } M^{-1}, M^{-2}, \text{etc.}). \tag{8}$$

Neglecting the terms of order M^{-1}, M^{-2}, etc. in (8) and employing the expression derived previously for the multigraviton superpropagator,[5] the dressed electron and photon propagators can be calculated to all orders in $\alpha \equiv e^2/4\pi$, where e denotes the renormalized physical charge. Clearly, the electron mass-renormalization series generated by the interaction Lagrangian density (8) must sum to the dressed physical mass m. It is not unreasonable to assume that the quantum-gravitational mass M will manifest itself as an effective invariant wave-number cutoff to all orders of α in the electron mass-renormalization series as M manifests itself to first order in α in Eq. (2). By evoking the latter assumption, the electron mass-renormalization series can be approximated term by term. In addition to the familiar QED Feynman self-energy diagrams, there are Feynman diagrams generated by the first term in (8), with an induced renormalization-compensation effect occurring in diagrams of successive order in α. Nonetheless, it is still a simple matter to estimate all associated wave-number integrals as functions of the effective invariant cutoff parameter M and numerical prefactor constants (which are to be determined by future detailed calculations). This analytical procedure gives the form of the mass-renormalization series as

$$m \sum_{i=1}^{\infty} \alpha^i \left\{ \sum_{j=0}^{i} c_{ij} \left[\ln\left(\frac{M}{m}\right) \right]^j \right\} = m, \tag{9}$$

in which the c_{ij}'s are positive numerical constants of the order unity, e.g., $c_{10} = 3/8\pi$ and $c_{11} = 3/2\pi$. Because the empirical value $\ln(M/m) = 49.58$ is large compared to unity, the $j = i$ terms dominate the double sum in (9), and hence it follows that the quantity

$$x \equiv \alpha \ln(M/m) \tag{10}$$

must be an approximate root of the transcendental equation[6]

$$\sum_{i=1}^{\infty} c_{ii} x^i \cong 1. \tag{11}$$

The formula obtained from (10) for the mass of the electron,

$$m = Me^{-x/\alpha}, \tag{12}$$

is physically accurate if (11) yields a root at $x = 0.362$, a numerical value consistent with the positive character of the c_{ii}'s and with $c_{11} = 3/2\pi$. Theoretical confirmation of formula (12) requires a sufficient number of the c_{ii}'s to be computed, say for $i \leq k$ with $c_{kk}(0.362)^k \ll 1$, and it must be shown that $x = 0.362$ satisfies the numerically truncated form of Eq. (11),

$$\sum_{i=1}^{k} c_{ii} x^i \cong 1.$$

A noteworthy feature of (12) is the essential singularity in the formula at $\alpha = 0$, precluding an expansion of the formula about $\alpha = 0$. Also note that m tends formally to zero in the limit $\alpha \to 0$, a feature of formula (12) that may have a bearing on the massless character of the electron's chargeless (neutrino) counterpart.

SUMMARY AND CONCLUDING REMARKS

With analysis based on the Lagrangian density (3), the physical picture of an electron considered in this paper is a massless bare charge surrounded by a cloud of virtual photons, virtual electron-positron pairs, and virtual gravitons.[7] The finite physical mass of the electron emerges from the theory in formula (12), where M is the universal quantum-gravitational mass constant (1), α is the fine-structure constant, and $x = 0.362$ is a number to be computed from the details of the theory. The main assumption made in obtaining formula (12) is that the quantum-gravitational mass M manifests itself as an effective invariant wave-number cutoff to all orders of α in the electron mass-renormalization series, as it does to first order in Eq. (2). This assumption, as well as the physically motivated mathematical approximations made here and elsewhere,[5] require further study and justification within the context of the complete theory.

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[1]Recent experiments that confirm renormalized quantum electrodynamics down to distances of the order 10^{-16} cm have been reported by H. Alvensleben *et al.*, Phys. Rev. Letters 21, 1501 (1968); J. Aldins *et al.*, *ibid.* 23, 441 (1969); T. Appelquist and S. J. Brodsky, *ibid.* 24, 562 (1970).

[2]G. C. Wick and B. Zumino, Phys. Letters 25B, 479 (1967); I. S. Gerstein *et al.*, Phys. Rev. Letters 19, 1064 (1967); P. Olesen, *ibid.* 20, 525 (1968).

[3]V. Weisskopf, Phys. Rev. 56, 72 (1939). Also see the discussions by W. Pauli, Helv. Phys. Acta Suppl. 4, 69 (1956), and S. Deser, in Texas Symposium on Particle Theory, 1970 (unpublished).

[4]P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 333 (1958); Phys. Rev. 114, 924 (1959); *Recent Developments in General Relativity* (Pergamon, New York, 1962), p. 191. Hard-set divergences arise if one attempts to apply the linearized form of Dirac's theory to the electron self-energy problem.

[5]C. J. Isham, Abdus Salam, and J. Strathdee, Phys. Rev. D 3, 1805 (1971); R. Delbourgo, Abdus Salam, and J. Strathdee, Lett. Nuovo Cimento 2, 354 (1969). It is stated in the latter 1969 reference that "...the magnitude of the effective cutoff introduced by gravitation is too high to have any practical bearing on calculations of self-mass ...," a supposition contradicted by the more recent work of Salam and his collaborators.

[6]More precisely, the validity of (11) hinges on the assumption

$$\sum_{j,k=0}^{\infty} c_{j+k+1,j} x^j \alpha^k \ll \alpha^{-1},$$

as seen by putting the quantity (10) into (9) and interchanging the order of summation.

[7]To what extent other virtual particles contribute to the electron self-energy is problematical. That certain model theories involving particle fields with *negative metrics* can in fact render the electron self-energy finite (without appeal to the quantum-gravitational suppression mechanism) has been shown recently by H. Terazawa, Phys. Rev. Letters 22, 254 (1969); J. Pestieau and P. Roy, *ibid.* 23, 349 (1969); T. D. Lee and G. C. Wick, Phys. Rev. D 2, 1033 (1970).