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Three-Pion Intermediate State and the $K_L^0 \to \mu^+\mu^-$ Puzzle*

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Soft-pion methods are used to derive an upper bound for the contribution of the 3π intermediate state to the imaginary part of the amplitude for $K_L^0 \to \mu^+ + \mu^-$ in terms of the experimental upper bound for the decay rate, $\pi^0 \to e^+e^-$. The numerical result obtained is insufficient to reduce significantly the (too-large) imaginary part coming from the 2γ state; that is, insufficient to resolve the $K_L^0 \to \mu^+\mu^-$ puzzle.

The recently quoted[1] experimental upper limit on the branching ratio, $\mathrm{BR}(K_L^0 \to \mu^+\mu^-/K_L^0 \to \text{all}) \leq 1.8\times10^{-9}$, has led to a situation which has been described as the $K_L^0 \to 2\mu$ puzzle. The absorptive part of the amplitude arising from the intermediate 2γ state alone produces a lower bound on the branching ratio of 6×10^{-9}, if the present experimental number for $K_L^0 \to 2\gamma$ is taken, if this reaction is assumed to conserve CP, and if ordinary quantum electrodynamics is used to calculate $2\gamma \to \mu^+\mu^-$.[2]

As pointed out by several authors,[3-5] there are a number of alternatives to simply disbelieving the new experimental result. The most attractive one would seem to be that CP violation is large in the matrix elements for $K_S^0, K_L^0 \to 2\gamma$ and for K_S^0, $K_L^0 \to \mu^+\mu^-$. Christ and Lee[4] have shown that this could lead to an escape from the difficulty mentioned above.

The next most attractive alternative is probably that CP is nearly conserved in the $K_L^0, K_S^0 \to 2\gamma$ matrix elements, but that the imaginary part coming from the 2γ intermediate state is largely canceled by imaginary parts arising from other intermediate states. To the lowest order in α these states can only be $\pi\pi\gamma$ or 3π. The consensus is that these states are very unlikely to give a significant imaginary part.[3-6] The published estimates are, however, more authoritative for the $\pi\pi\gamma$ state than for the 3π state.

In the present note we apply a new set of considerations to the estimation of a theoretical upper bound for this 3π absorptive part, $\mathrm{Im}^{(3\pi)}T(K_L^0 \to \mu^+\mu^-)$. The general idea is the following: The $K_2 \to 3\pi$ amplitude is well determined from experiment; what we want to calculate is the amplitude for the process $3\pi \to \mu^+\mu^-$. Since the pions are reasonably nonenergetic, it is plausible that soft-pion theorems could be used to relate the amplitude for $3\pi \to \mu^+\mu^-$ to that for $\pi^0 \to \mu^+\mu^-$. The connection between the hadrons and the leptons will always be through two photon lines (Fig. 1), and this kind of graph gives a minimal dependence on the lepton mass which we can determine independently of the form factors at the hadronic vertex in Fig. 1. Thus we could obtain a relation between the reactions $3\pi \to \mu^+\mu^-$ and $\pi^0 \to e^+e^-$, and finally use the experimental upper bound on the $\pi^0 \to e^+e^-$ rate to establish an upper bound for the $3\pi \to \mu^+\mu^-$ amplitude.

The main question is thus whether the requisite soft-pion theorems relating $\pi^0 \to \mu^+\mu^-$ and $3\pi \to \mu^+\mu^-$ exist. Unfortunately they do not exist as theorems within the standard framework of formal current algebra, for the reason that either process will be dominated in the soft-pion limit by Adler anomalies.[7,8] That is, the amplitudes in question would vanish in the limit of vanishing pion four-momentum, were it not for the anomalous divergences of the axial-vector current (arising from triangle graphs) in the presence of electromagnetic fields. These anomalous terms are thought to dominate the decay $\pi^0 \to 2\gamma$.

The over-all coefficient of the anomalous terms responsible for the decay $\pi^0 \to 2\gamma$ was shown by Adler to be quite model-dependent.[7] However, it has been argued, on the basis of closed-loop calculations in chiral models,[9] that a relation between $\pi^0 \to 2\gamma$ and $3\pi \to 2\gamma$ amplitudes can be derived in a model-independent way.[10]

We now make the further observation that the results of Ref. 9 relating $\pi^0 \to 2\gamma$ to $3\pi \to 2\gamma$ in the soft-pion limit are equally applicable to virtual γ's of any momentum, and hence lead to analogous relations between the $\mu^+\mu^-$ amplitudes, $\pi^0 \to \mu^+\mu^-$ and $3\pi \to \mu^+\mu^-$.

In effective-Lagrangian language these results for the production of the $\mu^+\mu^-$ state, analogous to the results of Ref. 9 for the 2γ state, are

$$\mathcal{L}_{\text{eff}} = C[\pi_3 + (6f^2)^{-1}\pi_3(\vec{\pi}\cdot\vec{\pi}) + \text{higher powers of } \pi] \times \bar{\psi}_\mu \gamma_5 \psi_\mu \ . \tag{1}$$

Here we have used a definition of the pion field such that the chiral variation of the pion field is given by $\delta_{\text{chiral}}\vec{\pi}=\delta\vec{\omega}(f^2-\vec{\pi}^2)^{1/2}$ (σ-model gauge). This $\mathcal{L}_{\text{eff}}$ must be used in conjunction with the pion Lagrangian from the nonlinear σ model,

$$\mathcal{L}_0=\tfrac{1}{2}[\partial_\mu\vec{\pi}\cdot\partial_\mu\vec{\pi}+\partial_\mu(f^2-\vec{\pi}^2)^{1/2}\partial_\mu(f^2-\vec{\pi}^2)^{1/2}]\ . \tag{2}$$

We have assumed that it is a good approximation to take the pion as massless in evaluating matrix elements, but of course when we integrate over the 3π phase space we shall use the physical pion mass.

The basis for Eq. (1) is a consideration of the closed-nucleon- (or -quark-) loop graphs for the process (neutral pions)$\rightarrow 2\gamma$. In chiral models, we would formally have expected $\delta_{\text{chiral}}\mathcal{L}_{\text{eff}}=0$, under the chiral transformation generated by $Q_3^{(5)}$. This would lead to vanishing of the amplitudes in the soft-pion limit. However, the closed-loop contribution gives a nonvanishing amplitude (Adler anomaly) for which the above criterion is replaced by

$$\delta_{\text{chiral}}\mathcal{L}_{\text{eff}}=\text{(a term independent of the pion field)}.$$

This, coupled with the $\Delta I<3$ rule of second-order electromagnetism, leads to the form of the dependence of (1) on the pion field. Further details are given in Ref. 9. To the lowest order in the momenta, $\bar{\psi}_\mu\gamma_5\psi_\mu$ is the only leptonic invariant of interest.

The amplitude for $3\pi\rightarrow\mu^+\mu^-$ will be given by a combination of the single-pion-pole term arising from (1) and (2) and the direct 3π term from (1). As noted in Ref. 9, and also by Abers and Fels,[11] the $3\pi^0$ amplitude obtained in this way will vanish. However, the $\pi^+\pi^-\pi^0$ amplitude will be nonvanishing in the soft-pion limit and will be given by

$$T(\pi^+\pi^-\pi^0\rightarrow\mu^+\mu^-)=C\frac{1}{f^2}\left(\frac{1}{3}-\frac{(k_++k_-)^2}{(k_++k_-+k_0)^2}\right)\bar{u}(\mu^-)\gamma_5 v(\mu^+)\ . \tag{3}$$

The next step is to relate the coefficient C to the decay rate $\pi^0\rightarrow e^+e^-$. To do this we note that in the result of the calculation of the photon-loop graph of Fig. 1 there will necessarily be a factor of the lepton mass multiplying the γ_5 invariant. This can be seen by considering both the external and internal leptons in Fig. 1 to be massless, in which case the helicities of the outgoing lepton and antilepton must be opposite, which is impossible in a spin-zero decay. This leads us to write the matrix element for $\pi^0\rightarrow e^+e^-$ as

$$T(\pi^0\rightarrow e^+e^-)=C(m_e/m_\mu)\bar{u}(e^-)\gamma_5 v(e^+). \tag{4}$$

Of course, there could be additional lepton-mass dependence, but we estimate it to be unimportant in the case of interest to us, which is the case of a very high effective cutoff in the loop integral of Fig. 1, such as to give rise to a rather large total amplitude.

Thus the decay rate for $\pi^0\rightarrow e^+e^-$ determines the constant C which we then use to calculate the three-pion absorptive part in $K_L^0\rightarrow\mu^+\mu^-$, using (3). We have done this calculation with a linear matrix element for the process $K_L^0\rightarrow 3\pi$,[12] and we obtain the result

$$|\text{Im}^{(3\pi)}T(K_L^0\rightarrow\mu^+\mu^-)|=7.17\left[\frac{\Gamma(\pi^0\rightarrow e^+e^-)}{\Gamma(\pi^0\rightarrow\gamma\gamma)}\right]^{1/2}|\text{Im}^{(2\gamma)}T(K_L^0\rightarrow\mu^+\mu^-)|\ . \tag{5}$$

If we take an early experimental value[13] for the upper limit on $\pi^0\rightarrow e^+e^-$,

$$\frac{\Gamma(\pi^0\rightarrow e^+e^-)}{\Gamma(\pi^0\rightarrow\gamma\gamma)}\leq 5\times10^{-4},$$

we find

$$|\text{Im}^{(3\pi)}T(K_L^0\rightarrow\mu^+\mu^-)|\leq 0.16\,|\text{Im}^{(2\gamma)}T(K_L^0\rightarrow\mu^+\mu^-)|\ . \tag{6}$$

No doubt this bound could be lowered considerably if better experimental data were available for the decay $\pi^0\rightarrow e^+e^-$. With the upper limit given by (6) we can obtain at best a total absorptive part for $K_L^0\rightarrow\mu^+\mu^-$ 30% less than that arising from the 2γ intermediate state alone. So we conclude that the three-pion state cannot be the solution to the $K_L^0\rightarrow\mu^+\mu^-$ puzzle. It could be said that our conclusions support CP-violating theory.[4]

Christ and Lee, in a footnote to Ref. 4, have given a simple model for estimating the effects of the 3π intermediate state in the $K_L^0\rightarrow\mu^+\mu^-$ decay, also leading to the conclusion that it is negligible. Their calculation was of the 3π absorptive part in $K_L^0\rightarrow 2\gamma$, using the pole model,

$$K_L^0\rightarrow 3\pi\rightarrow\pi^0\rightarrow 2\gamma\ ,$$

and the result was a small fraction (10^{-3}) of the measured K_L^0 amplitude. Another conclusion of

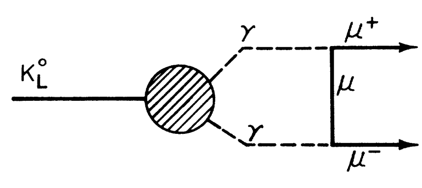

FIG. 1. The decay $K_L^0\rightarrow\mu^+\mu^-$ goes through two photons. The graph has an unknown hadronic vertex, but we assume that the coupling to the μ mesons is through ordinary electrodynamics.

the same model would be that the 2γ absorptive part in $3\pi \to \mu^+\mu^-$ is very small compared to the total $3\pi \to \mu^+\mu^-$ amplitude required to resolve the $K_L^0 \to \mu^+\mu^-$ puzzle. These conclusions are in complete accord with ours. However, we believe that our considerations are more conclusive, for the following reasons:

(a) If the current-algebraic foundations are correct, our calculation is the complete answer in the soft-pion limit, and is not dependent on a selection of graphs.

(b) We determine a bound on the complete amplitude for $3\pi \to \mu^+\mu^-$ and not merely the imaginary part from the 2γ state. Although it is reasonable that the real part should be of the order of the imaginary part, it could in principle be one million times as large.

(c) In view of the considerations of Refs. 9 and 11, the pion-pole model is particularly unsuited to the present problem. Two thirds of the decay $K_L^0 \to 3\pi$ is into the state $3\pi^0$, yet the pole graphs for $3\pi^0 \to 2\gamma$ are exactly canceled by the direct graphs in the soft-pion limit. In the case of the $\pi^+\pi^-\pi^0$ amplitude the cancellation is incomplete, but the pole model still gives a considerable overestimate of the three-pion absorptive part of $K_L^0 \to 2\gamma$.

Note Added in Proof. Adler, Lee, Treiman, and Zee have shown that the effective Lagrangian approach of Ref. 9 is inadequate for discussion of the reaction $\gamma + \gamma \to \pi^+\pi^-\pi^0$. These authors show that a $\gamma 3\pi$ vertex with the second γ absorbed on an external pion line must be included for consistency, and that the coefficient $\frac{1}{3}$ in Eq. (3) must be modified as well. Our conclusions for the $3\pi^0$ state are unaltered; the $3\pi^0$ matrix elements to $\mu^+\mu^-$ vanish in the soft-meson limit (or more exactly, the coefficient of $\bar{\mu}\gamma_5\mu$ vanishes). Therefore the contribution of the $3\pi^0$ intermediate state to the $K_L \to \mu^+\mu^-$ imaginary part remains much less than the amount estimated in this note for the contribution of the $\pi^+\pi^-\pi^0$ state and therefore is of no significance to the puzzle. The $\pi^+\pi^-\pi^0$ state must be reestimated, however, incorporating the modifications of Adler *et al.* We anticipate no large increase in the final result.

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