S-Wave π - π Scattering Phase Shifts*

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Using an absorptive one-pion exchange mechanism for the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$, a fit is made to experimental data at 4 GeV/c in order to extract the π - π scattering phase shifts. The analysis was carried out up to D waves in the π - π amplitude. The best results indicate a very broad I = 0 S-wave resonance with $M_R = 820$ MeV and $\Gamma = 600$ MeV.

The determination of low-energy π - π scattering phase shifts has been pursued over the last several years. Two recent works in this direction are those of Oh *et al.*¹ and of Chan *et al.*² We present an analysis similar to that of Ref. 1, but carried out on different experimental data. We study the data of Johnson *et al.*,³ and use a peripheral model with absorptive corrections to determine the π - π scattering parameters. These data have been previously analyzed^{3,4} and the improvement in the present analyses consists in the inclusion of an I=0 *D*-wave amplitude in π - π scattering. Though our analysis has been restricted to $m_{\pi\pi} < 900$ MeV, the tail of the f^0 resonance is seen to be important. The process under study is the reaction $\pi^- p$ $\rightarrow \pi^+ \pi^- n$ at 4 GeV/c incident momentum. The production mechanism was assumed to be due to a onepion exchange modified by initial- and final-state absorption. The details of this calculation are presented in a subsequent paper.⁵ A cut in t, the momentum transfer between the proton and final neutron, was made and events with $|t| < 3m_{\pi}^2$ were considered. It is hoped that with this assumption the peripheral hypothesis will be valid. Likewise, we feel that 4 GeV/c is a more desirable incident momentum for this kind of analysis than is 7 GeV/c, which was used in the analysis of Ref. 1. At 7 GeV/c the exchanges of the ω and A_2 Regge



FIG. 1. Coefficients C_i of Eq. (2) against $m_{\pi\pi}$: solid line, with D wave; dashed line, without D wave. (Normalized to total number of events.)

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FIG. 2. Treiman-Yang distribution. (Normalized to total number of events.)

trajectories may have to be taken into account. The π - π scattering amplitude was parametrized in an energy-dependent way. The P and D waves were taken to be resonance-dominated by the ρ and f^0 mesons, respectively. The positions and widths were not varied but were set at the accepted values.⁶ In the case of the P wave, a radius of interaction was included and varied in the fit. For the D wave no such radius was included as we were always on the low side of the resonance. The S wave was parametrized by an effective-range expression,

$$(q/\sqrt{s})\cot\delta_{0I}(s) = a_I + b_I q^2, \qquad (1)$$



FIG. 3. π - π phase shifts. The solid line is our solution; the dashed line is the solution of Ref. 1.

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with q and s the relative momentum and effective mass squared of the final π - π system.

Though all the distributions were fitted, the one most useful in determining the phase shifts was the $\cos\theta$ distribution, where θ is the scattering angle in the π - π center-of-mass system. At each π - π effective mass, the distribution was expanded in a power series in $\cos\theta$,

$$\frac{dN}{d\cos\theta} = \sum_{n=0}^{4} C_n \cos^n\theta.$$
 (2)

In Fig. 1 we show the experimental values of the expansion coefficients as well as the best fits with and without a *D*-wave contribution. Whereas the previous analysis of these data^{3,4} had difficulty in obtaining a good fit to the magnitude of the forward-backward asymmetry, the inclusion of the *D* wave reduces the discrepancy. We likewise present, in Fig. 2, the results for the fit to the distribution in the Treiman-Yang angle ϕ . Comparisons with other distributions are presented in Ref. 5.

The best fit to the data was attained with an S wave I=0 dominated by a broad resonance centered

at 820 MeV with a width $\Gamma = 600$ MeV. This is consistent with the findings of other groups.⁶ To recapitulate, the results we obtain for the π - π amplitudes are (the quantities in square brackets were not varied)

$$\frac{q}{\sqrt{s}} \cot \delta_{00}(s) = \frac{m_S^2 - s}{m_S \Gamma_S} \frac{q_S}{m_S},$$

$$q \cot \delta_{20}(s) = -12.5 \,\mu + 0.25 q^2 / \mu,$$

$$\frac{q^3}{\sqrt{s}} \cot \delta_{11}(s) = \frac{m_P^2 - s}{m_P \Gamma_P} \frac{q_P^3}{m_P} \frac{1 + 0.193 \,\mu^{-2} q^2}{1 + 0.193 \,\mu^{-2} q_P^2},$$
(3)

$$\frac{q^5}{\sqrt{s}}\cot\delta_{02}(s) = \frac{m_D^2 - s}{m_D \Gamma_D} \frac{q_D^5}{m_D},$$

where μ is pion mass in MeV, $m_s = 820$ MeV, $\Gamma_s = 600$ MeV, $[m_p] = 765$ MeV, $[\Gamma_p] = 125$ MeV, $[m_D] = 1264$ MeV, $[\Gamma_D] = 151$ MeV, and q_l for l = S, P, D is q at the resonance in *l*th partial wave.

A graphical presentation of these results is given in Fig. 3 together with the results of Ref. 1.

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T, *P*, and *C* Symmetries in $K_{L,S} \rightarrow \gamma \gamma$

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The Bernstein-Michel analysis of $\pi^0 \rightarrow \gamma \gamma$ is applied to the decays $K_{L,S} \rightarrow \gamma \gamma$. It is shown that, in contrast to the situation in $\pi^0 \rightarrow \gamma \gamma$, a hypothetical violation of *CP* invariance in $K_{L,S} \rightarrow \gamma \gamma$ can, in principle, be detected by a measurement of the circular polarization of *one* of the two photons.

I. INTRODUCTION

In a paper entitled "T, P, and C Symmetries in $\pi^0 \rightarrow \gamma\gamma$," Bernstein and Michel¹ obtained the necessary and sufficient conditions for the two-photon state created in this decay to be an eigenstate of P, T, and PT (or, equivalently, of CP, T, and

CPT, since the state necessarily has C = +1). Their conclusions were as follows: If the polarization wave function of the photons is

$$\psi = \alpha \vec{\epsilon} \cdot \vec{\epsilon}' + \beta \vec{\epsilon} \times \vec{\epsilon}' \cdot \hat{k}, \qquad (1)$$

then the two-photon state is an eigenstate