Bound on CP Nonconservation in $K_L \to 2\gamma$ Decay*

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The surprisingly small upper bound recently established for $K_L \to \mu^+ + \mu^-$ decay is exploited, via unitarity, to set an upper bound on K_L decay to a two-photon state with spin-parity 0^+. Special circumstances operate here to lend a fair degree of credibility to the analysis. The fraction of $K_L \to 2\gamma$ decays which proceed in this CP-nonconserving mode is less than ~0.37.

Among imaginable but not yet seen weak-interaction processes, the decay reaction $K_L \to \mu^+ + \mu^-$ has detectability features which make experimental search especially favorable, and theoretical features which are of high interest. It exemplifies the kind of process that could arise in first order from the coupling of neutral lepton and hadron currents, a sort of interaction for which there is no requirement on present phenomenology, but which might well have a natural place there. It can also be imagined to arise as a second-order effect in conventional weak interactions. One expects, formally, that such second-order effects are very small, else the usual phenomenology might be put in jeopardy. But it is not easy to be quantitative about this: Naive perturbation methods lead to divergent integrals. There is yet another mechanism for the reaction, one whose operation would seem to be indubitable: $K_L \to$ two real or virtual photons $\to \mu^+ + \mu^-$.[1] The second step of the sequence is presumably safely described by the standard electrodynamic theory of charged leptons; and as for the first step, $K_L \to 2\gamma$ decay is both expected on standard phenomenology, and seen. For this mechanism, a rough estimate would suggest that

$$\Gamma(K_L \to 2\mu)/\Gamma(K_L \to 2\gamma) \approx (1/137)^2,$$

hence

$$\text{B. R.} \equiv \Gamma(K_L \to 2\mu)/\Gamma(K_L \to \text{all}) \approx 10^{-8}.$$

This is a very tiny branching fraction indeed, and the estimate seems to leave ample room for detection of contributions from the other, more exotic mechanisms.

On the experimental side, however, the upper limit on the branching ratio has by now been pushed well below the 10^{-8} mark. From the runs in which K_L decays are allowed to take place in vacuum, Clark, Elioff, Field, Frisch, Johnson, Kerth, and Wenzel[2] obtain an upper limit which at the 90% confidence level is given by

$$(\text{B. R.})_{\text{exp}} \lesssim 1.8 \times 10^{-9}.$$

Either the estimate for the conventional mechanism is very much wrong, or there are exotic contributions which happen to add destructively – or worse. If CP-nonconserving effects can be ignored, then the absorptive part of the amplitude for $K_L \to 2\mu$ decay sets a lower bound on the modulus of the full amplitude, and unitarity relates the absorptive amplitude to a sum of contributions coming from on-mass-shell intermediate states. To lowest relevant order in the fine-structure constant, only the 2γ, 3π, and $2\pi\gamma$ intermediate states need be considered. Suppose at first that the 3π and $2\pi\gamma$ channels can be totally ignored. Then the needed $K_L \to 2\gamma$ amplitude, whose modulus is known from the observed rate for this process, is purely real; and of course the $2\gamma \to 2\mu$ amplitude is in any case reliably known, one supposes, from standard electrodynamics. One then finds[3]

$$\text{B. R.} \gtrsim 6 \times 10^{-9}.$$

Reinstatement of the 3π and $2\pi\gamma$ channels of course introduces uncertainties. Although the amplitude for $2\pi\gamma \to 2\mu$ lends itself to fairly reliable theoretical treatment, for $3\pi \to 2\mu$ the theoretical situation is more problematic. Moreover, reinstatement of the 3π channel puts the contribution of the 2γ state into doubt, since the $K_L \to 2\gamma$ amplitude itself now acquires a problematic absorptive part via the unitarity sequence $K_L \to 3\pi \to 2\gamma$. Despite all of these qualifications, however, estimates made by Martin, de Rafael, and Smith[4] suggest that it is not easy to lower the "naive" unitarity bound by much more than about 20%. Insofar as the negative experimental findings are sustained, one has the makings here of a serious crisis.

In the present note, although we shall do nothing to resolve the crisis, we wish to observe that one can extract from it some useful information bearing on CP nonconservation. There are two aspects to CP nonconservation in the neutral K-meson system. For one thing, it is known that the states K_S and K_L are not quite pure with respect to CP, whatever may be the origin of this impurity. For another, there is the possibility that various *mass-shell* decays of K^0 and $\overline{K}^0$ may be CP-nonconserving. Let us continue to ignore the small CP impurity of K_L, and similarly ignore the nearly forbidden 2π channels in K_L decay. But, allowing for the possibility that there may be substantial CP-nonconserving effects in the electromagnetic interactions of hadrons,[5] let us contemplate a CP-nonconserving decay of K_L into the $^3P_0$ state of the muons (in the preceding discussion it was the $^1S_0$ state that was in question). Similarly, of course, we allow for CP-nonconserving decays into 2γ and $2\pi\gamma$ states with spin-parity 0^+.[6] The absorptive amplitude for $K_L \to 2\mu(^3P_0)$ decay can now receive no contribution from a 3π intermediate state, since three pions cannot form a system with spin-parity 0^+. Moreover, the amplitude for $K_L \to 2\gamma(0^+)$ now has no absorptive part to lowest relevant electromagnetic order, since again there is no 3π contribution. But this means that the 2γ contribution to the absorptive amplitude for $K_L \to 2\mu(^3P_0)$ can be reliably expressed in terms of the *modulus* of the $K_L \to 2\gamma$ (0^+) amplitude.[7] Let us suppose for a moment that the $2\pi\gamma$ state can be ignored. We then get a lower (unitarity) bound on the rate for the CP-nonconserving decay $K_L \to 2\gamma(0^+)$, or turning this around, an upper limit on the $2\gamma(0^+)$ rate proportional to the $2\mu(^3P_0)$ rate. But the Berkeley experiment sets an upper limit on the latter. We hereby learn that

$$\frac{\Gamma(K_L \to 2\gamma(0^+))}{\Gamma(K_L \to 2\gamma(\text{all}))} \leq 0.37,$$

i.e., the rate into the CP-nonconserving 0^+ state is at most 37% of the net $K_L \to 2\gamma$ rate.[8]

It remains only to argue that the $2\pi\gamma$ contribution to the absorptive $K_L \to 2\mu(^3P_0)$ amplitude is negligible on the above scale. We are aided by the fact that the $K_L \to 2\pi\gamma$ branching ratio has an upper limit, 4×10^{-4},[9] which is already quite small. In the unitarity equation we need the amplitudes for $K_L \to 2\pi\gamma(0^+)$ and $2\pi\gamma \to 2\mu$. To lowest relevant electromagnetic order, the latter has only one unknown element, the pion electromagnetic form factor, f_π. As for the $K_L \to 2\pi\gamma$ process, we maximize the unitarity contribution by supposing the two-pion system to be purely p wave. Let K, q_+, q_-, and k be, respectively, the K_L, π^+, π^-, and γ momenta; ϵ_μ, the photon polarization vector; M and μ, the kaon and pion masses; $\omega = -K\cdot k/M$, the photon energy in the kaon rest frame. Define

$$q = q_+ + q_-, \quad Q = q_+ - q_-.$$

The $K_L \to 2\pi\gamma$ amplitude then has the form

$$A(2\pi\gamma) = ef(\omega)[(q\cdot k)(Q\cdot\epsilon) - (Q\cdot k)(q\cdot\epsilon)]$$

and the decay rate to the 0^+ state is

$$\Gamma(2\pi\gamma) = \frac{\alpha M}{12\pi^2}\int_0^{\omega_0} d\omega\,|f(\omega)|^2\omega^3\left(\frac{\omega_0-\omega}{\omega_1-\omega}\right)^{3/2}(\omega_1-\omega),$$

where

$$\omega_0 = \tfrac{1}{2}M\left(1-\frac{4\mu^2}{M^2}\right), \quad \omega_1 = \omega_0 + \frac{2\mu^2}{M}.$$

The $2\pi\gamma$ contribution to the absorptive amplitude for $K_L \to 2\mu(^3P_0)$ decay is given by

$$\operatorname{Im}A(K_L \to 2\mu)|_{2\pi\gamma} = \frac{e\alpha m}{24\pi^2 v}\int_0^{\omega_0} d\omega\,\omega\left(\frac{\omega_0-\omega}{\omega_1-\omega}\right)^{3/2} \times f^*(\omega)f_\pi(\omega)\left[\alpha - \beta\left(1-\frac{2\omega}{M}\right)\right],$$

where m is the muon mass, v is the muon velocity (in the kaon rest frame),

$$\alpha = \ln\left|\frac{1+v}{1-v}\right|, \quad \beta = \frac{1}{v^2}\left(\ln\left|\frac{1+v}{1-v}\right| - 2v\right),$$

and f_π is the pion electromagnetic factor (it depends on the variable q^2, but q^2 is a simple function of ω and we are regarding f_π as a function of ω). For f_π it seems reasonably safe to adopt a ρ-meson-dominance approximation. From the experimental upper bound on $\Gamma(2\pi\gamma)$,[9] we can then get an upper bound on $\operatorname{Im}A|_{2\pi\gamma}$, or rather, we could do so if the functional form were known for $f(\omega)$. We have considered two choices: f = constant and f proportional to f_π. The results are not very different. The unitarity contribution of the $2\pi\gamma$ state is very small on the scale of present interest: Taken alone, this contribution corresponds at most to a $K_L \to 2\mu(^3P_0)$ rate two orders of magnitude below the experimental upper bound on $K_L \to 2\mu$ decay.

It seems therefore that the upper bound we give for CP-nonconserving $K_L \to 2\gamma$ decay should be reliable, to within at most about 20% – insofar as the Berkeley results are sustained. This bound is not, to be sure, at a level which is especially restrictive for models which envisage an electromagnetic origin of CP nonconservation. But it has seemed to us worth noting, as a by-product of the $K_L \to 2\mu$ crisis.

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[8]We have used $\Gamma(K_L \to \gamma\gamma)/\Gamma(K_L \to \text{all}) = 5\times 10^{-4}$, an informal average of $(4.68 \pm 0.65)\times 10^{-4}$ [M. Banner, J. W. Cronin, J. K. Liu, and J. E. Pilcher, Phys. Rev. Letters 21, 1103 (1968)] and $(5.3 \pm 1.5)\times 10^{-4}$ [R. Arnold, I. A. Budakov, D. C. Cundy, G. Myatt, F. Nezrick, G. H. Trilling, W. Venus, H. Yoshiki, B. Aubert, P. Heusse, E. Nagy, and C. Pascaud, Phys. Letters 28B, 56 (1968)].

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Note on the Most Probable Number of Partons in Parton Models*

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From analysis of the inelastic e-p scattering data it is found that an important feature of the parton probability function $P(N)$ is that it should be peaked around some small number of partons. The most probable number of partons is prognosticated to be around 4.5. Contrary to the usually accepted forms of $P(N)$, where it decreases uniformly from a maximum, we find that the data also allow a behavior of $P(N)$ where it rises to a maximum first and then decreases with increase of N. It is conjectured that there is a connection between this peaking and the observed peaking of the multiplicity distribution in inelastic collisions.

In a previous paper[1] (herein referred to as I) we discussed some parton models, and showed how some versions would fit the data on inelastic e-p scattering and certain others would not. In particular, three main qualitative points were made.

(i) The parton concept can give a quantitative description of inelastic e-p scattering in the approximation that scale invariance is satisfied.

(ii) The average charge squared per parton is very small (~0.04), which rule out any reasonable possibility of associating *individual* quarks with partons.

(iii) The minimum number of partons N_0 plays an important role in parton models, and we found that in our formulation we could obtain good fits only by choosing $N_0 = 4$.

In this paper we report on more extensive studies on the nature of the parton probability function $P(N)$ which show that $P(N)$ should be peaked near a small number of partons, like four or five, and not necessarily at the number $N = N_0$, which is the usually accepted form.

We have arrived at these conclusions by studying a large class of parton models which can fit the inelastic e-p scattering data. In the parton-model[2] picture, the important concepts are $P(N)$, the probability of finding N partons in a nucleon, $f_N(x)$ the probability of finding a parton with longitudinal fraction x of the proton's four-momentum, and $\langle \sum_1^N Q_i^2 \rangle$ the average value of the sum of the squared charges of the partons in a configuration of N partons. To describe a general parton model, we first