Anomalous Real Parts in ω Photoproduction

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The anomalous real parts in ω photoproduction and elastic scattering from nucleons are calculated. The effect of anomalous real parts on the maximum ρ-ω phase allowed by nuclear absorption in photoproduction from complex nuclei is discussed. A large time-reversal-invariance-violating phase is needed at the vector-meson–photon vertex in order to correlate the ρ-ω phase measurements in photoproduction of lepton and pion pairs and in pion-pair production in colliding beams.

In a recent paper,[1] Bauer discusses the possibility of anomalous real parts in the photoproduction and elastic-scattering amplitudes for unstable particles. As a specific case, he considers the ρ^0 meson and finds anomalous real parts with magnitudes of the order of (30–40)% of the normal imaginary parts. In this note, we examine the anomalous real parts in ω photoproduction and elastic scattering. Any difference between the ω and ρ^0 results would contribute to the large relative ρ-ω phase seen in the lepton-pair production experiments from complex nuclei.[2]

The forward amplitude for photoproduction of a vector meson V from a nucleon is written[1]

$$f_{\gamma N\to VN}=\frac{ik}{4\pi}g_{\gamma V}\sigma_{\text{tot}}(VN)[1+i(\alpha_n+\alpha_1)], \qquad (1)$$

where k is the photon energy, $g_{\gamma V}$ is the vector-meson–photon coupling constant, and α_n is the normal real part – arising, for instance, from dispersion theory, or phenomenologically, from quark-model relations between vector-meson-nucleon amplitudes and π-nucleon amplitudes.[3] When the predominant decay mode of V is into p pions, the anomalous real part is given by

$$\alpha_1\approx p\,\frac{\Gamma_V(m_V)}{m_V}\,\frac{\sigma_{\text{tot}}(\pi N)}{\sigma_{\text{tot}}(VN)}. \qquad (2)$$

Similarly, the forward elastic V-nucleon scattering amplitude is written

$$f_{VN\to VN}=\frac{ik}{4\pi}\sigma_{\text{tot}}(VN)[1+i(\alpha_n+\alpha_2)], \qquad (3)$$

where the anomalous real part is

$$\alpha_2\approx \tfrac{1}{2}p\,\frac{d\Gamma_V(m)}{dm}\bigg|_{m=m_V}\frac{\sigma_{\text{tot}}(\pi N)}{\sigma_{\text{tot}}(VN)}. \qquad (4)$$

For the ρ^0 meson, $\alpha_1\approx +0.3$ and $\alpha_2\approx +0.4$. Since the normal real part α_n is known to be $\approx +0.2$,[3] which is also its value in π-nucleon elastic scattering, a test for the presence of anomalous real parts could be made if data existed on the phase of the ρ-production amplitude from nucleons. Although no such data exist, there are phase measurements for ρ photoproduction from complex nuclei.[4-6] The most accurate result can be found in Ref. 6, where the phase measurement was made at an average energy of 5.1 GeV using a beryllium target. The result is $\delta_\rho=11.8°\pm4.4°$.

In order to use this phase result to test for the presence of anomalous real parts, it must be modified for two reasons. As pointed out in Ref. 1, anomalous real parts can arise in production from complex nuclei due to nonresonant production from more than one nucleon. The calculation of this effect would be long and difficult and will not be attempted here. One must also remove the effect of nuclear absorption from δ_ρ, and this effect is known to depend critically on the phases of (1) and (3).[2,7] In the following, we briefly explore this phase dependence and use it as a tentative test for the presence of anomalous real parts.

There are two possibilities. The first is that no anomalous real parts exist, i.e., $\alpha_1=\alpha_2=0$ and, therefore, the real parts of both the nucleon photoproduction and elastic-scattering amplitudes, (1) and (3), are equal to α_n. This assumption was made implicitly in Refs. 2 and 7, and the conversion of δ_ρ can then be made by reading off the curves given there. The result is a nucleon photoproduction phase $\Delta_\rho=6°\pm7°$, which corresponds to a real part $\alpha\approx+0.1\pm0.1$, consistent with $\alpha_n\approx+0.2$. The second possibility is that the anomalous real parts, $\alpha_1=+0.3$ and $\alpha_2=+0.4$, are present. In this case, the real parts of (1) and (3) are not equal, and the results of Refs. 2 and 7 must be modified accordingly.[8] When this is done, one obtains a nucleon photoproduction phase of $\Delta'_\rho=10°\pm7°$, which corresponds to a total real part in (1) of 0.2 ± 0.1, only approximately consistent with $\alpha_n+\alpha_1=0.5$. Thus the accuracy of the present data, coupled with the incomplete treatment of the anomalous real parts, makes it difficult to distinguish between the two possibilities.[9]

We proceed assuming that the anomalous real parts given by (2) and (4) exist, and examine the consequences in ω photoproduction. We restrict our attention to the dominant 3π decay mode of the ω and use $\sigma(\omega N) \approx \sigma(\rho N)$.[10] The anomalous real part for ω photoproduction from nucleons is then $\alpha_1 \approx +0.03$ and is therefore negligible. In order to calculate α_2, the mass dependence of $\Gamma_\omega(m)$ is needed, and for this we use the results of Gell-Mann, Sharp, and Wagner[11]:

$$\Gamma_\omega(m) \sim m(m-3m_\pi)^4 W(m), \tag{5}$$

where $W(m)$ is a phase-space factor which varies from unity at $m = 3m_\pi$ to a value of 3.56 at $m = m_\omega$. The mass dependence in (5) is quite rapid, and the contribution to α_2 will be large even though $\Gamma_\omega(m_\omega)/m_\omega$ is small. The results are

$$d\Gamma_\omega(m)/dm\,|_{m=m_\omega} \simeq 11.2\Gamma_\omega(m_\omega)/m_\omega$$

and

$$\alpha_2 \approx 17\,\frac{\Gamma_\omega(m_\omega)}{m_\omega}\,\frac{\sigma_{\text{tot}}(\pi N)}{\sigma_{\text{tot}}(\omega N)}, \tag{6}$$

giving $\alpha_2 \approx +0.2$. We see that since the modifications to the relation between nuclear and nucleon photoproduction phases depend on the difference $\alpha_1 - \alpha_2$,[8] they will be larger for the ω than for the ρ, even though Γ_ω/m_ω is small.

These modifications do not affect very much the maximum relative ρ-ω phase, $\delta_{\rho\omega\,\max}$, in photoproduction from complex nuclei allowed by the nuclear absorption effect,[2,7] since $\delta_{\rho\omega\,\max}$ depends on both the value of the nuclear phase δ near the nucleon phase $\Delta \approx 0°$ and the value of δ near $\Delta \approx 90°$. No matter how large the value of $\alpha_1 - \alpha_2$, δ is very little affected at the latter point.[8] The over-all effect of the above considerations is an increase in $\delta_{\rho\omega\,\max}$. For instance, for $A \approx 10$ and photoproduction at 4 GeV, $\delta_{\rho\omega\,\max}$ increases from ~35°, if no anomalous real parts are present, to ~40° if they are. This is to be compared with the relative ρ-ω phases measured in carbon (100°),[2,12] and in beryllium (41° ± 20°).[13]

In conclusion, we would like to point out that the relative ρ-ω phase measurements made in the three types of electromagnetic experiments (photoproduction of lepton pairs, photoproduction of pion pairs, and pion-pair production in colliding beams) can be made consistent with one another, and with the constraint due to nuclear absorption, if one allows time-reversal-invariance-violating nonzero phases at the vector-meson–photon vertex. The diagrams for the three types of experiments are shown in Fig. 1. Let x, y, and z be the relative ρ-ω phase for photoproduction from a complex nucleus,[14] coupling to a photon, and decay into a pion pair, respectively. If the relative phases measured in the experiments corresponding to Figs. 1(a),

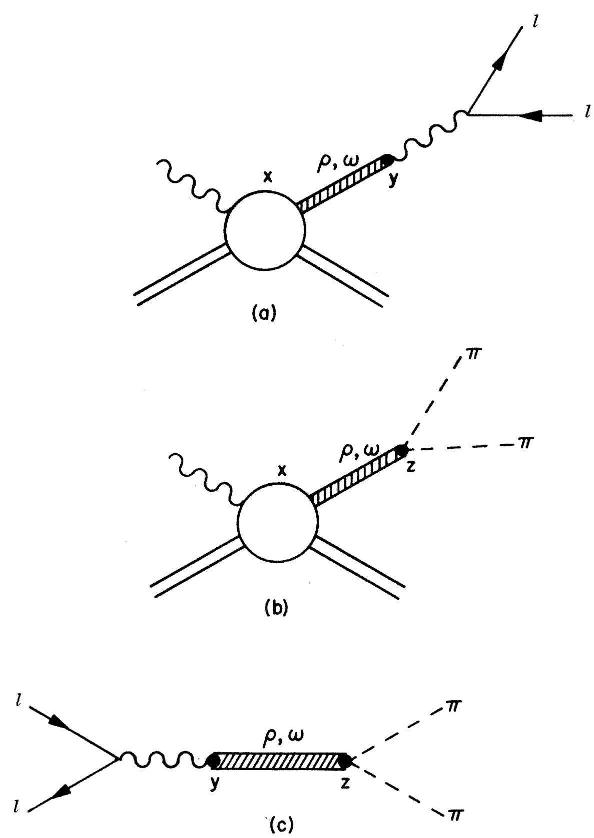

FIG. 1. Diagrams for the three types of electromagnetic experiments in which the relative ρ-ω phase has been measured. (a) Photoproduction of lepton pairs. (b) Photoproduction of pion pairs. (c) Pion-pair production in colliding beams.

1(b), and 1(c) are denoted by P_l, P_π, and P_c, then

$$x = \tfrac{1}{2}(P_l + P_\pi - P_c),$$
$$y = \tfrac{1}{2}(P_c + P_l - P_\pi), \tag{7}$$
$$z = \tfrac{1}{2}(P_c + P_\pi - P_l).$$

If we take $P_l \approx 100°$,[2,12] $P_\pi \approx 105°$,[15] and $P_c \approx 165°$,[16] we obtain

$$x \approx 20°, \quad y \approx 80°, \quad z \approx 85°.$$

The relative ρ-ω production phase x agrees with the constraint $\delta_{\rho\omega\,\max}$ discussed above. The relative phase for decay into two pions, z, is in fair agreement with the resonance-overlap prediction of ~110°,[17] which does not contain possible modifications due to unknown direct-decay amplitudes. There is now, however, an additional large relative phase y in the vector-meson–photon coupling which must be included in order that all three types of electromagnetic experiments be consistent.

[1]T. Bauer, Phys. Rev. Letters 25, 485 (1970); 25, 704(E) (1970).

[2]The ρ-ω phase data are compiled in Fig. 3 of G. Greenhut and R. Weinstein, Phys. Letters 33B, 363 (1970).

[3]G. K. Greenhut, Phys. Rev. D 2, 1915 (1970).

[4]J. G. Asbury *et al.*, Phys. Letters 25B, 565 (1967).

[5]D. R. Earles *et al.*, Phys. Rev. Letters 25, 129 (1970).

[6]H. Alvensleben *et al.*, Phys. Rev. Letters 25, 1377 (1970).

[7]J. Pumplin and L. Stodolsky, Phys. Rev. Letters 25, 970 (1970).

[8]In Ref. 2, the necessary modifications are obtained by first letting Δ be the phase in V-nucleon elastic scattering, i.e., $\Delta=\tan^{-1}(\alpha_n+\alpha_2)$. Then, for a given point (δ,Δ) in Fig. 2 of Ref. 2, the nucleon photoproduction phase is $\Delta'=\tan^{-1}(\tan\Delta+\alpha_1-\alpha_2)$ and the nuclear photoproduction phase is $\delta'=\delta+\Delta'-\Delta$. We are assuming that only the anomalous real parts due to photoproduction and elastic scattering from single nucleons enter into the phase for photoproduction from complex nuclei. It is only these single-nucleon amplitudes that enter into the formalism of Refs. 2 and 7. [See S. D. Drell and J. S. Trefil, Phys. Rev. Letters 16, 552 (1966); 16, 832(E) (1966).]

[9]The second possibility is more nearly compatible with the large real part (~0.45) needed in (3) in order to make the ρ-photon coupling constant measured in photoproduction from complex nuclei agree with that measured in colliding-beam experiments [J. Swartz and R. Talman, Phys. Rev. Letters 23, 1078 (1969)].

[10]A. Silverman, *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 71.

[11]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[12]P. J. Biggs *et al.*, Phys. Rev. Letters 24, 1197 (1970).

[13]H. Alvensleben *et al.*, Phys. Rev. Letters 25, 1373 (1970).

[14]This analysis does not take into account the possible production phase in the coupling between the incoming photon and a virtual vector meson into which it materializes before scattering off the target. If such a phase were present, it would presumably enter in photoproduction both from nucleons and from complex nuclei.

[15]P. J. Biggs *et al.*, Phys. Rev. Letters 24, 1201 (1970).

[16]J. E. Augustin *et al.*, Lett. Nuovo Cimento 2, 214 (1969).

[17]See, for instance, D. Horn, Phys. Rev. D 1, 1421 (1970).

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Comment on a Rigorous Bound for the $f_+(0)$ Form Factor of K_{l3} Decay

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We derive a rigorous bound on the form factor $f_+(0)$ in the form $|f_+(0)|\le[16/(m_K-m_\pi)\times(\sqrt{m_K}+\sqrt{m_\pi})][\pi\Delta(0)/3(m_K+m_\pi)]^{1/2}$, where $\Delta(0)$ is the propagator of the divergence of the strangeness-changing current at zero momentum transfer. Using the estimate of Mathur and Okubo for $\Delta(0)$, we get $|f_+(0)|\le 1.0$ and $|f_K/f_\pi|\le 1.28$.

Recently, we have derived a bound on the K_{l3} decay parameter $\xi\equiv f_-(0)/f_+(0)$,[1] where the form factors $f_\pm(t)$ are defined by

$$\langle\pi^0(p)|V_\mu^{K^+}(0)|K^+(k)\rangle=\tfrac{1}{2}[(k+p)_\mu f_+(t)+(k-p)_\mu f_-(t)],\tag{1}$$

with $t=(p-k)^2$.

In this note we want to point out that an upper bound can also be established for $f_+(0)$ by using the method of Meiman.[2] Here we consider the matrix element of the divergence of the current, which is related to $f_\pm(t)$ by

$$\begin{aligned}\tfrac{1}{2}d(t)&=\langle\pi^0(p)|i\partial_\mu V_\mu^{K^+}(0)|K^+(k)\rangle\\&=\tfrac{1}{2}[(m_K{}^2-m_\pi{}^2)f_+(t)+tf_-(t)].\end{aligned}\tag{2}$$

In Ref. 1 it is shown that $d(t)$ is bounded by the spectral function of the propagation function of the divergence of the current, i.e.,

$$\rho(t)\ge\frac{\pi}{(2\pi)^3}\frac{1}{2t}[(t-t_0)(t-t_1)]^{1/2}\tfrac{3}{4}|d(t)|^2\tag{3}$$

for $t\ge t_0$, where $\rho(t)$ is defined by

$$\begin{aligned}\Delta(t)&=\int d^4x\,e^{iq\cdot x}\langle 0|(\partial_\mu V_\mu^{K^+}(x)\partial_\lambda V_\lambda^{K^-}(0))_+|0\rangle\\&=\int_0^\infty\frac{\rho(t')dt'}{t'-t},\end{aligned}\tag{4}$$

with $q^2=t$, and where

$$t_0=(m_\pi+m_K)^2,\quad t_1=(m_K-m_\pi)^2.$$

From (3) and (4), we have

$$\begin{aligned}\Delta(0)&=\int_{t_0}^\infty\frac{\rho(t)dt}{t}\\&\ge\frac{\pi}{(2\pi)^3}\int_{t_0}^\infty\frac{dt}{2t^2}[(t-t_1)(t-t_0)]^{1/2}\tfrac{3}{4}|d(t)|^2.\end{aligned}\tag{5}$$