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The (1, 8)+(8, 1) Term in $SU(3)\times SU(3)$ Symmetry Breaking and the Parameter ξ of K_{l3} Decay

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An estimate of the magnitude of the $SU(3)\times SU(3)$ symmetry-breaking term transforming as (1, 8)+(8, 1) is made using pole dominance in spectral-function sum rules. It is found that the corresponding parameter ω is probably not small as commonly believed, but approximately -1. The results are applied to K_{l3} decay, where the symmetry-breaking scheme discussed in conjunction with "weak partial conservation of axial-vector current" implies large negative values of ξ.

INTRODUCTION

Current-algebra calculations frequently require a knowledge of commutators between charges and current divergences. A systematic way to treat these so-called σ terms in the framework of the $SU(3)\times SU(3)$ current algebra has been given by Gell-Mann, Oakes, and Renner.[1] These authors propose that the Hamiltonian of strong interactions is approximately invariant under the group $SU(3)\times SU(3)$ and that the symmetry is realized through eight massless pseudoscalar mesons. The symmetry-breaking part of the Hamiltonian may be decomposed into terms that transform according to irreducible representations of $SU(3)\times SU(3)$, in particular, if exotic σ terms are excluded, according to $(3,3^*)+(3^*,3)$ and $(1,8)+(8,1)$. The $(1,8)+(8,1)$ term has been generally neglected due to the fact that it vanishes in the PCAC (partially conserved axial-vector current) approximation if one applies $SU(3)$ symmetry to the single-particle matrix elements of g_i. The latter assumption appears questionable since the violation of $SU(3)$ is probably as large as the $SU(3)\times SU(3)$ one. Such a conclusion is substantiated by the mass splitting of the octet of pseudoscalar mesons, which is of the same order as the masses themselves. To obtain an estimate on the size of the symmetry-breaking parameters, we assume meson-pole dominance for certain matrix elements but we never refer to $SU(3)$. We obtain the result that the parameter characterizing the $(1,8)+(8,1)$ breaking term is indeed not small but approximately -1, largely independent of the pole-dominance approximation.

$SU(3)\times SU(3)$ SYMMETRY BREAKING

We assume the following form for the strong-interaction Hamiltonian:

$$H(x)=H_0(x)+\epsilon_0 u_0(x)+\epsilon_8 u_8(x)+\delta_8 g_8(x)\,, \tag{1}$$

where H_0 is invariant under $SU(3)\times SU(3)$. The ϵ_0, ϵ_8, δ_8 are real parameters; $u_i(x)$ and $g_i(x)$ are scalar densities of the $(3,3^*)+(3^*,3)$ and $(1,8)+(8,1)$ representations, respectively. These densities u_i, g_i together with the pseudoscalar counterparts v_i and h_i satisfy the commutation relations

$$[F_i,u_j(x)]=if_{ijk}u_k(x),\quad [F_i,v_j(x)]=if_{ijk}v_k(x)\,, \tag{2a}$$

$$[F_i^5,u_j(x)]=-id_{ijk}v_k(x),\quad [F_i^5,v_j(x)]=+id_{ijk}u_k(x), \tag{2b}$$

$$[F_i,g_j(x)]=if_{ijk}g_k(x),\quad [F_i,h_j(x)]=if_{ijk}h_k(x)\,, \tag{2c}$$

$$[F_i^5,g_j(x)]=if_{ijk}h_k(x),\quad [F_i^5,h_j(x)]=if_{ijk}g_k(x)\,. \tag{2d}$$

The vector and axial-vector current divergences are then simply given by

$$\begin{aligned}D_i&\equiv\partial_\mu V_i^\mu(x)=\epsilon_8 f_{i8k}u_k+\delta_8 f_{i8k}g_k\,,\\ D_i^5&\equiv\partial_\mu A_i^\mu(x)=-\epsilon_0 d_{i0k}v_k-\epsilon_8 d_{i8k}v_k+\delta_8 f_{i8k}h_k\,.\end{aligned} \tag{3}$$

Following Gatto,[2] we introduce the spectral representations

$$\int d^4x\, e^{iqx}\langle 0|TV_\mu^i(x)D_\mu^i(0)|0\rangle=q_\mu\int\frac{\sigma_i(x)dx}{q^2-x} \tag{4}$$

and assume that the commutation relations with the generators F_i and F_i^5, Eqs. (2a)–(2d), have a local generalization. We then obtain the spectral function sum rule:

$$S_i\equiv\int dx\,\sigma_i(x)=-f_{ikl}f_{i8k}\{\epsilon_8\langle u_l\rangle_0+\delta_8\langle g_l\rangle_0\}\,, \tag{5}$$

where $\langle\ \rangle_0$ refers to vacuum expectation values. Similarly for the axial-vector spectral function

$$\begin{aligned}S_i^5&\equiv\int dx\,\sigma_i^5(x)\\ &=\epsilon_j d_{ijk}d_{ikl}\langle u_l\rangle_0-\delta_8 f_{i8k}f_{ikl}\langle g_l\rangle_0\,.\end{aligned} \tag{6}$$

If we specialize to the case $i=1,\ldots,7$, we have

$$\begin{aligned}S_{4,5,6,7}&\equiv S=\tfrac{3}{4}(\epsilon_8\langle u_8\rangle_0+\delta_8\langle g_8\rangle_0)\,,\\ S_{1,2,3}^5&\equiv S_\pi=\tfrac{1}{3}(\sqrt{2}\,\epsilon_0+\epsilon_8)(\sqrt{2}\,\langle u_0\rangle_0+\langle u_8\rangle_0)\,,\\ S_{4,5,6,7}^5&\equiv S_K=\tfrac{1}{3}(\sqrt{2}\,\epsilon_0-\tfrac{1}{2}\epsilon_8)(\sqrt{2}\,\langle u_0\rangle_0-\tfrac{1}{2}\langle u_8\rangle_0)\\ &\quad+\tfrac{3}{4}\,\delta_8\langle g_8\rangle_0\,.\end{aligned} \tag{7}$$

We do not consider the case $i=8$ to avoid the problem of η, η' mixing. In the following we take S, S_π, and S_K to be known quantities; they can be approximately determined by using meson-pole dominance. It is therefore possible to use the three independent equations (7) to determine the parameter

$$\omega \equiv \frac{\delta_8}{\epsilon_8} \frac{\langle g_8 \rangle_0}{\langle u_8 \rangle_0}$$

characterizing the magnitude of the $(1,8)+(8,1)$ symmetry-breaking term as a function of the $(3,3^*)+(3^*,3)$ parameters ϵ_0 and ϵ_8. The result is

$$\omega = -1 - \frac{\frac{2}{3}\beta^2 S}{\frac{3}{2}S_\pi + (S_K - S - S_\pi)\times\beta}, \tag{8}$$

where

$$\beta \equiv 1 + \sqrt{2}\,\epsilon_0/\epsilon_8 .$$

At first glance this result may not seem too useful; it just expresses one unknown parameter ω in terms of another unknown parameter ϵ_0/ϵ_8. However, if we make the additional assumption that the physical world is reasonably close to the situation of massless pions, i.e., $\beta = 1+\sqrt{2}\,\epsilon_0/\epsilon_8$ is not far from the $SU(2)\times SU(2)$ limit $\beta=0$, then it follows that $\omega \approx -1$. If $SU(2)\times SU(2)$ is realized exactly, $\omega=-1$ independent of the way in which S_π goes to zero.

Figure 1 shows ω as a function β assuming pole dominance:

$$S_\pi = -m_\pi{}^2 F_\pi{}^2, \quad S_K = -m_K{}^2 F_K{}^2, \quad S = -m_\kappa{}^2 F_\kappa{}^2$$

with $m_\kappa = 1080$ MeV and $F_\kappa{}^2 = (F_K - F_\pi)^2$ given by the Glashow-Weinberg relation[3] and $F_K/F_\pi = 1.28$. The results are not sensitive to the exact values of S_π, S_K, and S as long as S is relatively small, which we expect from the approximate validity of the $SU(3)$ mass formula. It is obvious from the figure that except for the small range of values $-0.11 < \beta < -0.09$ or very large $|\beta|$, we have always ω approximately equal to -1. The solution $\omega=0$ is highly unstable under small variations of ϵ_0/ϵ_8 since it lies so close to the pole; we believe that this may be enough reason to reject this solution.

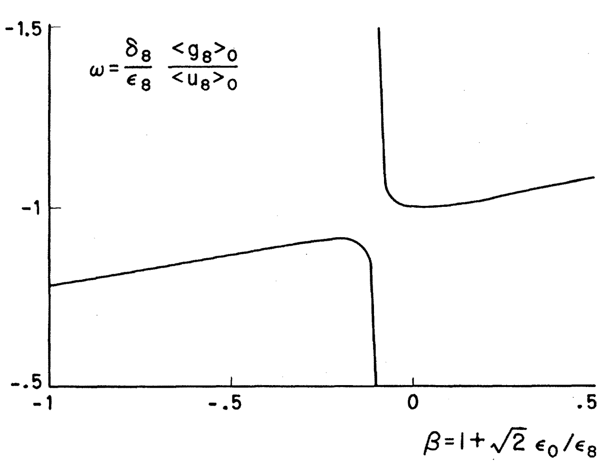

FIG. 1. The $(1,8)+(8,1)$ symmetry-breaking parameter ω as a function of $\beta = 1+\sqrt{2}\,\epsilon_0/\epsilon_8$ for $F_\kappa = 0.28\,F_\pi$.

APPLICATION TO K_{l3} DECAY

The usefulness of the results of the previous section to the determination of current-algebra σ terms is not as straightforward as in the case where the $(1,8)+(8,1)$ symmetry breaking is absent. As an illustration we choose the example of K_{l3} decays. The relevant commutators

$$[D_3^5, F_{4+i5}] = -[(\tfrac{2}{3})^{1/2}\epsilon_0 + (\tfrac{1}{3})^{1/2}\epsilon_8] \times \tfrac{1}{2}(v_4 + iv_5),$$
$$[D_{4+i5}, F_3^5] = [D_3^5, F_{4+i5}] - \tfrac{1}{2}D_{4+i5}^5 \tag{9}$$

and the axial-vector divergence

$$D_{4+i5}^5 = -[(\tfrac{2}{3})^{1/2}\epsilon_0 - \tfrac{1}{2}(\tfrac{1}{3})^{1/2}\epsilon_8](v_4 + iv_5) - \tfrac{1}{2}\sqrt{3}\,\delta_8(h_5 - ih_4) \tag{10}$$

are no longer proportional to each other. Since we are ultimately interested in vacuum-single-particle matrix elements, we need an expression for

$$\langle 0 | h_5 - ih_4 | K^- \rangle .$$

For this purpose we take the vacuum expectation value of Eq. (2d) and saturate with single-particle intermediate states to obtain

$$F_K \langle 0 | h_{5-i4} | K^- \rangle = \sqrt{3}\,\langle g_8 \rangle_0 . \tag{11}$$

Combining this result with Eq. (7), we can relate the current-algebra σ term to the matrix element of the axial-vector divergence

$$\langle 0 | [D_{4+i5}, F_3^5] | K^- \rangle = \frac{3}{4}\frac{1}{\beta - \frac{3}{2}} \langle 0 | D_{4+i5}^5 | K^- \rangle + \frac{\beta}{\beta - \frac{3}{2}} \frac{1}{F_K} \frac{S\omega}{1+\omega} . \tag{12}$$

That is, in addition to the usual result, we get a term proportional to the parameter ω characterizing the $(1,8)+(8,1)$ symmetry violation. To discuss the K_{l3} form factors we employ Brandt and Preparata's "weak" version of PCAC[4] [without adopting their additional assumption of a symmetry-breaking scheme $SU(3)\times SU(3) \to SU(3) \to SU(2)$]. These authors argue that the amplitude

$$M(q^2, p^2) = \frac{1}{F_\pi m_\pi{}^2}(m_\pi{}^2 - q^2) \times \int dx\, e^{iqx} \langle 0 | T D_3^5(x) D_{4-i5}(0) | K^+(p) \rangle \tag{13}$$

has a smooth extrapolation from the soft-pion point to the mass shell (rather than the vector-current amplitude itself). From Eq. (13) one derives the low-energy theorem:

$$M(0,m_K^2)=m_K^2\{f_+(0,m_K^2)+f_-(0,m_K^2)\}$$
$$=\frac{1}{F_\pi}\int dx\,\delta(x_0)\langle 0\,|[A_0^3(x),D_{4-i5}(0)]\,|K_+\rangle\ . \tag{14}$$

This gives, using Eq. (12) and the pole-dominance approximation, the result

$$[f_+(0,m_K^2)+f_-(0,m^2)]$$
$$=\frac{F_K}{F_\pi}\frac{3}{2}\frac{1}{\frac{3}{2}-\beta}\left[1-\frac{2}{3}\frac{F_\kappa^2 m_\kappa^2}{F_K^2 m_K^2}\frac{\omega}{1+\omega}\beta\right]. \tag{15}$$

Referring to Fig. 1 we note that the second term in the bracket is generally >0 (except in the immediate vicinity of the pole), i.e., the (1, 8)+(8, 1) symmetry-breaking term reduces the right-hand side of Eq. (15) to the extent of even making it negative. Smoothness and the Ademollo-Gatto theorem in connection with Eq. (15) then imply that the decay parameter $\xi=f_-/f_+$ is large and negative. Figure 2 shows ξ as a function β for two different values of the κ-decay constant $F_\kappa=0.28\ F_\pi$ and $F_\kappa=0.5F_\pi$ together with the experimental result of the X_2 group[5] $\xi=-0.58\pm0.13$. Owing to the prevailing uncertainty in the experimental value of ξ and in F_κ it is difficult to make quantitative predictions.

The inclusion of the (1, 8)+(8, 1) term into the $SU(3)\times SU(3)$-breaking part of the Hamiltonian therefore constitutes a simple mechanism to remove the discrepancies between theory and experiment in K_{l3} decay data. This mechanism does not require a large $SU(2)\times SU(2)$ symmetry breaking. It lends new support to the idea expressed in Ref. 1 that

$$SU(3)\times SU(3)\rightarrow SU(2)\times SU(2)\rightarrow SU(2)$$

may be the symmetry-breaking patterns realized in nature.

Added note. It is interesting to compare our result for ξ with the perturbation-theoretical arguments of Dashen and Weinstein.[6] The additional term in Eq. (15) is easily seen to be of order $SU(3)$ times $SU(2)\times SU(2)$ breaking; therefore, the general

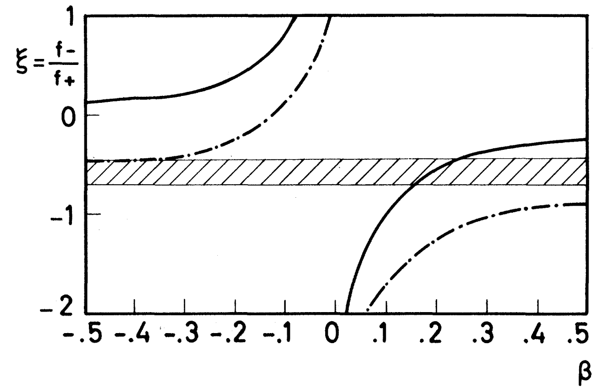

FIG. 2. $\xi=f_-/f_+$ as a function of $\beta=1+\sqrt{2}\,\epsilon_0/\epsilon_8$ for $F_\kappa=0.5\,F_\pi$ (solid line) and $F_\kappa=0.28F_\pi$ (dot-dash line). The shaded area represents the experimental result of the X_2 group.

theorem of Ref. 6 is satisfied. We have constructed a possible mechanism that allows us to obtain a value of ξ large enough to explain the data without formally violating the theorem. Compared to the model of Brandt and Preparata[4] our approach has the advantage that we introduce consistently a comparatively large $SU(3)$ breaking. Their assumption on the breaking of $SU(3)$ is rather unorthodox, as pointed out by Weinstein[7]; it ought to be much smaller than the $SU(2)\times SU(2)$ breaking while $SU(3)$ is badly violated at the same time by renormalization constants of the "fields" u_K and u_π.

To give a numerical example: for $S\approx 0$, we obtain $\omega\approx -1$, $\beta\approx -0.37$, and $\xi\approx -0.58$.

After this manuscript was submitted for publication the author became aware of a paper by Arnowitt, Friedman, and Nath[8] in which it was also proposed that an (8, 1)+(1, 8) symmetry-breaking term may explain large negative values of ξ.

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