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Regge-Pole Content of Chiral and Gauge Field Theories*

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The high-energy limits of the 4-pion and 6-pion tree-approximation amplitudes in the nonlinear chiral-invariant Lagrangian and of the ρ-ρ scattering amplitude in massive Yang-Mills theory are investigated in order to see whether Reggeization of elementary particles occurs. We find that the 4- and 6-pion amplitudes are dominated asymptotically by exchange of Regge poles with quantum numbers of the ρ and A_1 mesons, respectively. These trajectories are regarded as "ancestors," because ρ and A_1 particles are not present in the initial Lagrangian. The ρ-ρ scattering amplitude is dominated by the exchange of quantum numbers of the ρ meson, but there are two trajectories present, so that Reggeization does not occur. Nevertheless, in all amplitudes computed there is a striking regularity in the dominance of high-energy limits by the exchange of quantum numbers of the ρ and A_1 mesons.

I. INTRODUCTION

We report here results of an investigation of the high-energy behavior of tree-approximation amplitudes in chiral- and gauge-invariant meson field theories. It is known that such amplitudes beautifully embody current-algebra requirements at low energies but are too singular at high energies to satisfy sum rules.

Our motivation is to test whether the high-energy behavior, although singular, corresponds to a Reggeization of the elementary particles in the field-theory Lagrangian. It was originally shown by Gell-Mann *et al.*[1] that the spinor particle is Reggeized in a field theory of a spinor and a massive neutral vector meson. Their work critically involved higher-order loop graphs. However, Reggeization already requires a nontrivial property in the tree approximation. It requires that the leading terms have appropriate asymptotic strength and satisfy a factorization condition involving sense and nonsense channels for all values of nonasymptotic variables. It has recently[2] been shown that these conditions are satisfied for an $I=\frac{1}{2}$ spinor nucleon coupled to $I=1$ vector mesons via a Yang-Mills Lagrangian.[3] The situation in several other theories has also been investigated.[4]

We examine high-energy behavior in the nonlinear chiral-invariant Lagrangian for pions[5,6] and in Yang-Mills theory for massive vector mesons. The results can be described as follows:

(1) Complete Reggeization of external particles does not take place.

(2) Nevertheless, the leading high-energy behavior in these theories is concentrated in amplitudes of very well-defined quantum numbers.

The full significance of (1) and, especially, (2) is not clear, but we believe that a presentation and discussion of the results is justified.

In Sec. II we study nonlinear chiral theory, paying particular attention to the 6-pion amplitude, in order to test for the Reggeization of the elementary pion which occurs in intermediate states. The study of the Reggeization phenomenon in amplitudes with more than four external particles is an innovation of the present work. In chiral amplitudes we find leading asymptotic terms characteristic of ρ- and A_1-meson trajectories, not a π trajectory. This suggests that ρ and A_1 mesons should be included in the phenomenological Lagrangian from the beginning, and that one should look for Reggeization in the asymptotic limits of amplitudes with external and internal π's, ρ's, and A_1's. A closed subset of these amplitudes is the ρ-ρ scattering amplitude in which only the ρ appears internally. We test for Reggeization of this amplitude in Sec. III, using a Yang-Mills Lagrangian to describe vector-meson interactions. The factorization properties of the high-energy limit indicate that two trajectories, out of a possible four, are present, instead of the single trajectory which would mean full Reggeization. The connection of this result with the counting arguments of Mandelstam[7] is discussed. In Appendixes A and B we write the helicity amplitudes for ρ-ρ scattering in Yang-Mills theory and give the isospin analysis of the 6-pion amplitude.

II. NONLINEAR CHIRAL LAGRANGIAN

We choose the simple Lagrangian

$$\mathcal{L}=\frac{1}{2}\left(1+\frac{1}{f_\pi^2}\vec{\pi}^2\right)^{-2}(\partial_\mu\vec{\pi})^2, \tag{1}$$

which gives unique chiral-invariant S-matrix elements for processes involving only massless pions.[5]

To check for Reggeization of the pion, one must examine the asymptotic behavior of amplitudes in which the pion can be exchanged internally. The simplest such amplitude has six external pions [Fig. 1(a)] to which both pole- and contact-type Feynman graphs contribute. We express the amplitude in terms of individual isospin couplings and, by convention, take all momenta pointing inward. One isospin term is

$$A=\tfrac{1}{4}f_\pi^{-6}\delta_{a_1a_2}\delta_{a_3a_4}\delta_{a_5a_6}\bigg(6[p_1\cdot p_2+p_3\cdot p_4+p_5\cdot p_6]+\frac{(p_1+p_2-p_3)^2(p_5+p_6-p_4)^2}{(p_1+p_2+p_3)^2}$$
$$+\frac{(p_1+p_2-p_4)^2(p_3-p_5-p_6)^2}{(p_1+p_2+p_4)^2}+\frac{(p_2-p_3-p_4)^2(p_1-p_5-p_6)^2}{(p_2+p_3+p_4)^2}$$
$$+\frac{(p_1-p_3-p_4)^2(p_2-p_5-p_6)^2}{(p_1+p_3+p_4)^2}+\frac{(p_1+p_2-p_5)^2(p_3+p_4-p_6)^2}{(p_1+p_2+p_5)^2}+\frac{(p_1+p_2-p_6)^2(p_3+p_4-p_5)^2}{(p_1+p_2+p_6)^2}\bigg). \quad (2)$$

This expression is already invariant under the exchange $1\leftrightarrow 2$, $3\leftrightarrow 4$, and $5\leftrightarrow 6$ of particle momenta and isospin labels. The full amplitude contains 14 more such terms each obtained from Eq. (2) by permutation of momenta and isospin labels leading to inequivalent isospin couplings, e.g., $\delta_{a_1a_4}\delta_{a_2a_5}\delta_{a_3a_6}$. It can be verified that each isospin term separately vanishes when any pion 4-momentum $p_{i\mu}\to 0$. Thus the Adler condition[8] is satisfied.

We must calculate the asymptotic limit governed by the leading Regge pole in a single three-particle channel, and we choose the (123) channel. The Toller[9] variables, studied in detail by Bali, Chew, and Pignotti,[10] reliably ensure that the correct limit is taken. With reference to Fig. 1(b), we must specify

(1) one momentum transfer variable Q^2,

(2) two internal variables describing the 123 cluster in a standard configuration,

(3) two internal variables describing the 456 cluster in a standard configuration,

(4) three Lorentz-transformation parameters (α, ζ, γ) describing the relative orientation of the 456 and 123 clusters.

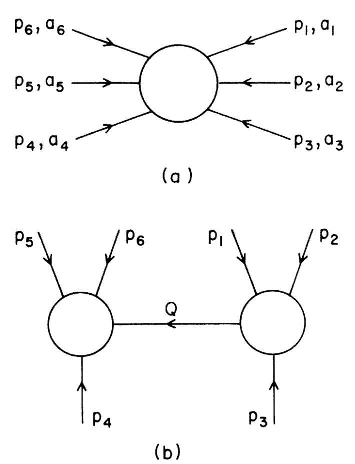

FIG. 1. The 6-pion amplitude. Momenta and isospin indices are specified in (a). The asymptotic limit of the amplitude in which a Regge pole is exchanged between the 123 and 456 clusters is illustrated in (6).

We always take $Q=p_1+p_2+p_3=-p_4-p_5-p_6$ spacelike along the z axis:

$$Q=(0,0,0,\sqrt{-Q^2}). \quad (3)$$

With slight differences in convention from Bali, Chew, and Pignotti, we choose the vectors of the standard configurations as

$$\bar{p}_1=(E_1,0,0,z_1),$$
$$\bar{p}_2=(-E_2,0,-y_2,-z_2), \quad (4)$$
$$\bar{p}_3=(-E_3,0,+y_2,-z_3),$$

$$\bar{p}_6=(E_6,0,0,z_6),$$
$$\bar{p}_5=(-E_5,0,-y_5,-z_5), \quad (5)$$
$$\bar{p}_4=(-E_4,0,y_5,-z_4).$$

Energies E_i are always positive and on the mass shell. The equation $Q=\bar{p}_1+\bar{p}_2+\bar{p}_3$ gives two constraints between z_1, z_2, z_3, and y_2 leaving two independent internal variables for the 123 cluster. Similarly, the equation $-Q=\bar{p}_4+\bar{p}_5+\bar{p}_6$ leaves two independent internal variables describing the 456 cluster.

The three Lorentz parameters describe a transformation of the particular O(2, 1) subgroup of the Lorentz group which leaves the z axis invariant. We use exponential notation to write this transformation as

$$e^{-i\beta J_3}e^{-i\zeta k_2}e^{-i\alpha J_3}, \quad (6)$$

where, applied to 4-vectors, J_3 and K_2 are taken to be the 4×4 matrix representatives of generators of rotations about the z axis and boosts along the

z axis.

Invariant scalar products may be expressed in terms of the Toller variables as

$$p_i \cdot p_{i'} = \bar{p}_i \cdot \bar{p}_{i'}\,, \quad i, i' \le 3$$
$$p_j \cdot p_{j'} = \bar{p}_j \cdot \bar{p}_{j'}\,, \quad j, j' \ge 4 \tag{7}$$

$$p_j \cdot p_i = (e^{i\zeta k_2} e^{i\beta J_3} \bar{p}_j) \cdot (e^{-i\alpha J_3} \bar{p}_i), \quad i \le 3;\ j \ge 4. \tag{8}$$

We do not write these formulas in detail here.

Toller's elegant group-theoretic work tells us that the asymptotic limit of a 6-point amplitude governed by Regge singularities in the (123) channel is precisely the limit $\zeta \to \infty$ with all other Toller variables fixed.

If the leading singularity is a Regge pole at $J = \alpha(Q^2)$ with factored couplings, the Toller limit would give

$$A \approx (\cosh\zeta)^{\alpha(Q^2)} \gamma_{456}(\beta_1 z_j, y_j, Q^2) \gamma_{123}(\alpha_1 z_i, y_i, Q^2), \tag{9}$$

where we always take $i \le 3$, $j \ge 4$. It is worth pointing out that although the factorization of Regge-pole couplings to multi-particle channels is widely assumed, it has not, to our knowledge, been proven to the same level of confidence as for two-particle channels.

The three-body analog of helicity amplitudes can be defined by integration:

$$A_{mn} = \frac{1}{(2\pi)^2} \int_0^{2\pi} d\alpha \int_0^{2\pi} d\beta\, e^{i(n\alpha - m\beta)} A\,. \tag{10}$$

The index n is the number of units of angular momentum of the 123 cluster about the z axis in standard configuration, and m has a similar interpretation. It is necessary to take helicity projections of the Regge residues to discuss the question of sense and nonsense couplings of trajectories at integer values of $\alpha(Q^2)$.

In two-body amplitudes there are at most a finite number of helicity channels, but in the three-body case there are an infinite number of channels due to the helicities and internal structure of the 123 and 456 clusters. Reggeization succeeds when a single factored residue containing the pole terms of a particle in the Lagrangian can be identified in the high-energy behavior of scattering amplitudes. This is a much more stringent condition for three-body amplitudes, because an infinite number of channels are available.

In our case the residue factor $\gamma_{123}(\alpha, z_i, y_i)$ must incorporate the Bose symmetry under interchange of isospin labels and momenta among particles 123, with a similar requirement on γ_{456}. The Toller conventions would need modification in order to display this symmetry compactly. Fortunately, the requisite symmetry will be obvious from our results. Indeed, for most of our work we can keep the detailed Toller formulas (7) and (8) in the backs of our minds, and work simply with the scalar products, remembering that $p_i \cdot p_j$ becomes large in the Toller limit and $p_i \cdot p_i'$ and $p_j \cdot p_j'$ remain finite.

There are two generic types of isospin terms in the full 6-pion amplitude. Terms such as $\delta_{a_1a_2}\delta_{a_3a_4}\delta_{a_5a_6}$ are clearly pure $I = 1$ in the 123 channel. Terms such as $\delta_{a_1a_4}\delta_{a_2a_5}\delta_{a_3a_6}$ contribute to several isospins in the 123 channel. See Appendix B.

For pure $I = 1$ couplings, the Toller limit is easily calculated in terms of invariant scalar products. For example, the isospin term of Eq. (2) has the limiting behavior

$$A \sim -2f_\pi^{-6}\, \delta_{a_1a_2}\delta_{a_3a_4}\delta_{a_5a_6}\, p_3 \cdot p_4\,, \tag{11}$$

which is linear in $\cosh\zeta$. For mixed isospin couplings one finds after detailed use of Eqs. (7) and (8) that the limiting behavior is at most constant in $\cosh\zeta$, and therefore negligible compared to (11).

Alert readers may be puzzled because the Adler condition is apparently not satisfied by the leading asymptotic term (11). The puzzle can be resolved by examination of Eq. (2), which shows that because of poles, the Adler and Toller limits are not interchangeable and should only be performed in alphabetical order.

The leading Regge structure of the 6-point amplitude is now clear. Pure $I = 1$ terms are proportional to $\cosh\zeta$ in the Toller limit which indicates that there are Regge poles at $J = 1$. Similarly, Regge singularities of the mixed-isospin terms only occur for $J \le 0$.

In the language and standpoint of the Reggeization program, $J = 1$ asymptotic behavior when there are only $J = 0$ particles in the Lagrangian means that an ancestor is present. Although an ancestor at $J = 1$ does not strictly imply that Reggeization of the pion at $J = 0$ fails to take place, we chose not to study the detailed form of $J = 0$ terms because it became more interesting to investigate a possible interpretation of the ancestor, which will be discussed below.

Of the 15 isospin couplings, nine are pure $I = 1$ and contain the leading $J = 1$ asymptotic behavior; the others are of mixed isospin type and can be ignored asymptotically. The nine $I = 1$ terms provide fully symmetrized couplings for the ancestor Regge pole. The leading asymptotic behavior of the full 6-point function can be written, using unit vectors $\epsilon(1)$, $\epsilon(2)$, etc., to describe the isospin wave functions of the individual pions as

$$A \sim -2f_\pi^{-6}\delta_{ab}g^{\mu\nu}[\vec{\epsilon}(5)\cdot\vec{\epsilon}(6)\epsilon_a(4)p_{4\mu}+\vec{\epsilon}(4)\cdot\vec{\epsilon}(6)\epsilon_a(5)p_{5\mu}+\vec{\epsilon}(4)\cdot\vec{\epsilon}(5)\epsilon_a(6)p_{6\mu}]$$
$$\times[\vec{\epsilon}(2)\cdot\vec{\epsilon}(3)\epsilon_b(1)p_{1\nu}+\vec{\epsilon}(1)\cdot\vec{\epsilon}(3)\epsilon_b(2)p_{2\nu}+\vec{\epsilon}(1)\cdot\vec{\epsilon}(2)\epsilon_b(3)p_{3\nu}], \quad (12)$$

representing the fully symmetrized factored coupling of a single isovector Regge pole at $J=1$. The Lorentz factors $p_j \cdot p_i$ indicate that the pole has normal orbital parity and therefore positive overall parity. They also show that the pole couples only to sense channels with helicities ± 1 or 0.

Therefore, the leading asymptotic behavior of the 6-pion amplitude is a term whose quantum numbers are those of an A_1 Regge trajectory without an A_1-meson pole, which would be technically described as a sense-choosing trajectory with vanishing residue at $J=1$.

For comparison we write the well-known 4-pion amplitude of Weinberg and extract its large-s, fixed-t limit

$$A(s,t) = -(2f_\pi^2)^{-1}(\delta_{a_1a_2}\delta_{a_3a_4}s+\delta_{a_1a_3}\delta_{a_2a_4}t+\delta_{a_1a_4}\delta_{a_2a_3}u)$$
$$\approx -(2f_\pi^2)^{-1}s(\delta_{a_1a_2}\delta_{a_3a_4}-\delta_{a_1a_4}\delta_{a_2a_3}). \quad (13)$$

The amplitude is dominated by the asymptotic contribution of a Regge trajectory at $J=1$ of normal parity and $I=1$ in the t channel, in short, by a ρ trajectory without a ρ-meson pole.

It is now evident that the leading asymptotic behavior of amplitudes in the nonlinear chiral theory is concentrated in amplitudes whose quantum numbers are those of ρ and A_1 trajectories, but in which there are no vector or axial-vector meson poles. We choose to interpret this as nature's way of telling us to put ρ and A_1 particles into the phenomenological Lagrangian from the beginning together with π's. We might then hope for a grander Reggeization scheme in which the tree approximation to all amplitudes involving external π's, ρ's, and A_1's is dominated by Regge trajectories on which these particles lie. In Sec. III we test this idea by examining the asymptotic behavior of the ρ-ρ scattering amplitude, where the quantum numbers permit the ρ trajectory, but not the π or A_1, to be exchanged.

III. YANG-MILLS THEORY

We choose to describe vector-meson interactions by the Yang-Mills Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g\vec{\rho}_\mu\times\vec{\rho}_\nu)^2 + \tfrac{1}{2}m^2\vec{\rho}_\mu\cdot\vec{\rho}_\nu. \quad (14)$$

It is not strictly necessary to do this in order to incorporate an SU(2)×SU(2) symmetry into the phenomenological theory, but the Yang-Mills Lagrangian brings an additional gauge symmetry, broken only by its mass term, and ensures a vector-dominated current satisfying the commutation relations of field algebra.[11] It probably also gives the best chance of success for our Reggeization scheme because Abers, Keller, and Teplitz[2] have shown that the nucleon Reggeizes when coupled to the ρ meson with a Yang-Mills Lagrangian.

Although Toller's methods can be applied to ordinary scattering amplitudes, we revert to the tried and true method of parity-conserving helicity amplitudes and seek to identify the factored residues of a ρ-meson trajectory in the $J=1$ singularity of the $I=1$ s-channel partial-wave amplitudes.

For each isospin value ($I=0,1,2$) there are 13 independent helicity amplitudes $T_{\lambda_3\lambda_4;\,\lambda_1\lambda_2}(s,t)$. After partial-wave projection one finds ten independent normal-parity $P=(-)^J$ combinations, and three independent abnormal-parity $P=(-)^{J+1}$ combinations. The ten amplitudes together form a 4×4 symmetric matrix, and it is here for $I=1$ and $J=1$ that the ρ meson resides.

The tree-approximation amplitude for ρ-ρ scattering, containing both pole- and contact-type Feynman diagrams, can be readily calculated from the Lagrangian given by Eq. (14). The notation for momenta, isospin indices, and polarization vectors is specified in Fig.(2). The amplitude is

$$T = g^2(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})\{[(p_1+p_2)^2-m^2]^{-1}[(2p_2+p_1)\cdot\epsilon(1)\epsilon_\mu(2)-(2p_1+p_2)\cdot\epsilon(2)\epsilon_\mu(1)+(p_1-p_2)_\mu\epsilon(1)\cdot\epsilon(2)]$$
$$\times g^{\mu\nu}[(2p_4+p_3)\cdot\epsilon^*(3)\epsilon^*_\nu(4)-(2p_3+p_4)\cdot\epsilon^*(3)\epsilon^*_\nu(4)+(p_3-p_4)_\nu\epsilon^*(3)\cdot\epsilon^*(4)]$$
$$+\epsilon(1)\cdot\epsilon^*(4)\epsilon(2)\cdot\epsilon^*(3)-\epsilon(1)\cdot\epsilon^*(3)\epsilon(2)\cdot\epsilon^*(4)\}$$
$$+[p_2,b,\epsilon(2)\longleftrightarrow -p_3,c,\epsilon^*(3)]+[p_2,b,\epsilon(2)\longleftrightarrow -p_4,d,\epsilon^*(4)]. \quad (15)$$

The 13 independent helicity amplitudes are written in detail in Appendix A, using faithfully the conventions of GGLMZ[1] and Jacob and Wick.[12] Calculation shows that the leading asymptotic terms are linear in z, indicating that ancestors ($J\geq 2$ Regge poles) are absent. Further, the leading linear terms are pure $I=1$ and contribute only to J-plane singularities at $J=1$ of the normal-parity partial-wave amplitudes.

Therefore, all the asymptotic action is in the ρ-meson Regge-pole segment, and to study it in detail we write the most singular parts of the normal-parity $I=1$ partial-wave amplitudes, at $J=1$, calculated from the leading asymptotic terms in z by partial-wave projection with appropriate orthogonal functions. The variables are $E=\frac{1}{2}\sqrt{s}$, $k^2=\frac{1}{4}(s-4m^2)$.

$$\begin{aligned}
T_{1,1;1,1}&=\tfrac{1}{6}g^2[k^2(s-m^2)]^{-1}m^2(4E^2-19m^2)\delta_{J,1},\\
T_{1,1;0,1}&=\tfrac{1}{6}g^2[k^2(s-m^2)]^{-1}2Em(4E^2-19m^2)\delta_{J,1},\\
T_{1,1;0,0}&=\tfrac{1}{3}g^2[k^2(s-m^2)]^{-1}(4E^4-7E^2m^2-12m^4)\delta_{J,1},\\
T_{0,1;0,1}&=\tfrac{2}{3}g^2[k^2(s-m^2)]^{-1}E^2(4E^2-19m^2)\delta_{J,1},\\
T_{0,1;0,0}&=\tfrac{2}{3}g^2[k^2(s-m^2)]^{-1}\frac{4E}{m}(4E^4-7E^2m^2-12m^4)\delta_{J,1},\\
T_{0,0;0,0}&=\frac{2g^2}{3m^2}[k^2(s-m^2)]^{-1}(4E^6-3E^4m^2-8E^2m^4-8m^6)\delta_{J,1},\\
T_{1,1;1,-1}&=\frac{5g^2}{\sqrt{6}}\frac{m^2}{k^2}(J-1)^{-1/2},\\
T_{1,-1;0,1}&=\frac{10g^2}{\sqrt{6}}\frac{Em}{k^2}(J-1)^{-1/2},\\
T_{1,-1;0,0}&=\frac{2g^2}{\sqrt{6}}\frac{1}{k^2}(E^2+4m^2)(J-1)^{-1/2},\\
T_{1,-1;1,-1}&=g^2\frac{1}{k^2}(8E^2-3m^2)(J-1)^{-1}.
\end{aligned}\tag{16}$$

These ten amplitudes are the elements of a 4×4 symmetric matrix in the helicity channels 11, 01, 00, and −11 of which the first three are "sense" and the last is "nonsense."

In the Reggeization program one looks for the factorization properties of the amplitude in Eq. (16) among the helicity channels, interpreting the Kronecker $\delta_{J,1}$ as

$$\delta_{J,1}\approx-\frac{\alpha-1}{J-\alpha},\tag{17}$$

which is the approximate form for a Regge pole near $\alpha=1$.

The partial-wave amplitudes of Eq.(16) can be expressed as the sum of factored couplings of two Regge trajectories, one sense-choosing and one nonsense-choosing. We use the conventions of GGLMZ to present the results, writing

$$T_{\lambda_3,\lambda_4;\lambda_1,\lambda_2}=g^2\frac{\gamma^{(s)}_{\lambda_3,\lambda_4}\gamma^{(s)}_{\lambda_1,\lambda_2}}{J-\alpha}+g^2\frac{\gamma^{(n)}_{\lambda_3,\lambda_4}\gamma^{(n)}_{\lambda_1,\lambda_2}(\alpha-1)(\alpha+2)}{J-\alpha}+O(g^4)\tag{18a}$$

for sense-sense amplitudes of the helicity channels 11, 01, or 00,

$$(J-1)^{-1/2}(J+2)^{-1/2}\,T_{\lambda_3,\lambda_4;1,-1}=g^2\frac{\gamma^{(s)}_{\lambda_3,\lambda_4}\gamma^{(s)}_{1,-1}}{J-\alpha}+g^2\frac{\gamma^{(n)}_{\lambda_3,\lambda_4}\gamma^{(n)}_{1,-1}}{J-\alpha}+O(g^4)\tag{18b}$$

for sense-nonsense amplitudes, and

$$T_{1,-1;1,-1}=g^2\frac{(\gamma^{(s)}_{1,-1})^2(\alpha-1)(\alpha+2)}{J-\alpha}+g^2\frac{(\gamma^{(n)}_{1,-1})^2}{J-\alpha}+O(g^4)\tag{18c}$$

for the nonsense-nonsense amplitude. In the tree approximation $\alpha=1+O(g^2)$.

The residue factors are

$$\begin{aligned}
\frac{\gamma^{(s)}_{1,1}}{(\alpha-1)^{1/2}}&=\left[\frac{3m^4}{k^2(s-m^2)}\right]^{1/2},\\
\gamma^{(n)}_{1,1}&=\left(-\frac{m^2}{18k^2}\right)^{1/2},\\
\frac{\gamma^{(s)}_{1,0}}{(\alpha-1)^{1/2}}&=\frac{2E}{m}\left[\frac{3m^4}{k^2(s-m^2)}\right]^{1/2},\\
\gamma^{(n)}_{1,0}&=\frac{2E}{m}\left(-\frac{m^2}{18k^2}\right)^{1/2},\\
\frac{\gamma^{(s)}_{0,0}}{(\alpha-1)^{1/2}}&=\left[\frac{4}{3}\frac{1}{k^2(s-m^2)}\right]^{1/2}(E^2+2m^2),\\
\gamma^{(n)}_{0,0}&=\frac{2E^2}{m^2}\left(-\frac{m^2}{18k^2}\right)^{1/2},\\
(\alpha-1)^{1/2}\gamma^{(s)}_{1,-1}&=\left(\frac{2}{3}\frac{s-m^2}{k^2}\right)^{1/2},\\
\gamma^{(n)}_{1,-1}&=\left(-\frac{m^2}{k^2}\right)^{1/2}.
\end{aligned}\tag{19}$$

The leading asymptotic behavior of the ρ-ρ scattering amplitude is not dominated by a single Regge

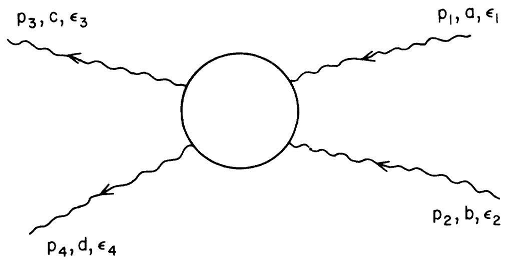

FIG. 2. The ρ-ρ scattering amplitude with momenta p_i, polarization vectors ϵ_i, and isospin components specified.

trajectory of the ρ particle. There is a second trajectory with the same quantum numbers present. The desired Reggeization of the ρ particle does not occur, although it fails by a slim margin.

It is traditional in studies of the Reggeization phenomenon to compare the results of explicit calculation with the prediction of the Mandelstam counting argument.[7] In this argument one counts the number of Castillejo-Dalitz-Dyson- (CDD) pole and subtraction parameters in the sense-sense amplitudes, and the number of threshold ($s = 4m^2$) and nontrivial conspiracy ($s = 0$) conditions. If the number of conditions exceeds the number of parameters, then the Froissart-Gribov partial-wave amplitude analytically continued from the higher value of J coincides with the $J = 1$ amplitude calculated from the Lagrangian, if both amplitudes are unitary.

In the case of ρ-ρ scattering with three sense and one nonsense channels at $J = 1$, there are four CDD-pole parameters and four subtraction constants. In an L-S basis one can count ten threshold conditions. There are two conspiracy conditions. Therefore, the theory should Reggeize if the amplitudes are unitary.

Of course tree-approximation amplitudes are not unitary, and the Mandelstam counting argument does not strictly apply. However, evidence has accumulated[4] indicating that failure of the Reggeization of spinor-vector amplitudes in the tree approximation is connected with violation at large energy of the unitarity bound on partial waves. Previous work has also suggested that when the violation of the unitarity bound is confined to a few partial waves, there are fewer trajectories than the number of helicity channels available.

The Yang-Mills model also has this feature. In Eq.(16) one can see that $T_{0,0;0,0} \sim E^2$ and $T_{0,1;0,0} \sim E$ for $J = 1$ in violation of the unitarity bound. It can also be shown that higher partial waves are bounded for large E. The occurrence of two trajectories in this model in association with a mild violation of the unitarity bound agrees with previous work.

IV. CONCLUSIONS

The Reggeization program, which is the theme and leitmotiv of this paper, never fully succeeds in any of the amplitudes calculated here. Nevertheless, the leading high-energy behavior of the tree approximation in chiral- and gauge-invariant Lagrangians seems to be very simply described in the language of Regge poles, since exchanges of very well-defined quantum numbers, those of ρ and A_1 mesons, are dominant. This regularity would not be expected merely on the basis of momentum power-counting arguments in the interaction Lagrangian.

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APPENDIX A

After applying constraints due to parity and time-reversal invariance, and the scattering of identical particles, one finds that there are 13 independent helicity amplitudes for ρ-ρ scattering. The amplitudes listed below are calculated from Eq. (15) of the text. We choose s as the direct channel so that $s = 4E^2$, $t = 2k^2(z-1)$, and $u = -2k^2(z+1)$ with $k^2 = E^2 - m^2$ and z the cosine of the scattering angle. We also use the notation $y = (1-z^2)^{1/2}$.

Each amplitude is expressed as

$$T_{\lambda_3,\lambda_4;\,\lambda_1,\lambda_2} = (\delta_{ac}\,\delta_{bd} - \delta_{ad}\,\delta_{bc})g^2 T^s_{\lambda_3,\lambda_4;\,\lambda_1,\lambda_2} + (\delta_{ab}\,\delta_{cd} - \delta_{ad}\,\delta_{bc})g^2 T^t_{\lambda_3,\lambda_4;\,\lambda_1,\lambda_2} + (\delta_{ac}\,\delta_{bd} - \delta_{ab}\,\delta_{cd})g^2\, T^u_{\lambda_3,\lambda_4,\,\lambda_1,\lambda_2}.$$

The subamplitudes are listed below with subscripts omitted for simplicity.

$T_{1,1;1,1}$:

$$T^s = -\frac{4k^2 z}{s - m^2} - z\,,$$

$$T^t = \frac{-1}{t - m^2}\left\{8k^2y^2 + \tfrac{1}{2}(1+z)^2[\,2E^2 + k^2(1+z)]\right\} + \frac{1}{4}(1-z)^2 - 1\,,$$

$$T^u = \frac{1}{u-m^2}\left\{8k^2y^2 + \tfrac{1}{2}(1-z)^2[2E^2 + k^2(1-z)]\right\} - \frac{1}{4}(1+z)^2 + 1;$$

$T_{1,1;-1,-1}$:

$$T^s = -\frac{4k^2z}{s-m^2} + z,$$

$$T^t = \frac{-1}{t-m^2}\,\frac{1}{2}(1-z)^2[2E^2 + k^2(1+z)] + \frac{1}{4}(1+z)^2 - 1,$$

$$T^u = \frac{1}{u-m^2}\,\frac{1}{2}(1+z)^2[2E^2 + k^2(1-z)] - \frac{1}{4}(1-z)^2 + 1;$$

$T_{0,1;0,1}$:

$$T^s = \frac{-1}{s-m^2}\,\frac{8k^2E^2}{m^2}(1+z) - \frac{1}{2}(1+z),$$

$$T^t = \frac{1+z}{t-m^2}\left(\frac{k^4}{m^2}(3-z) - \frac{2E^4}{m^2}z + \frac{k^2E^2}{m^2}(9z-8)\right) - \frac{1}{2}\,\frac{E^2}{m^2}y^2,$$

$$T^u = \frac{1}{u-m^2}\left(-2\frac{k^4}{m^2}y^2 - \frac{2E^4}{m^2}y^2 + \frac{k^2E^2}{m^2}(1+z)(1-5z)\right) + \frac{1}{2}\,\frac{1}{m^2}(k^2 - E^2z)(1+z);$$

$T_{0,1;0,-1}$:

$$T^s = \frac{-1}{s-m^2}\,\frac{8k^2E^2}{m^2}(1-z) + \frac{E^2}{m^2}(1-z) - \frac{1}{2}(1-z),$$

$$T^t = \frac{1-z}{t-m^2}\left(-\frac{2E^4}{m^2}z - \frac{k^4}{m^2}(1+z) + \frac{k^2E^2}{m^2}z(5-z)\right) + \frac{1}{2}\,\frac{E^2}{m^2}y^2,$$

$$T^u = \frac{1}{u-m^2}y^2\frac{1}{m^2}[2(E^4+k^4) + k^2E^2(z-7)] + \frac{1}{2}\,\frac{1}{m^2}(k^2 - E^2z)(1-z);$$

$T_{1,1;0,1}$:

$$T^s = \frac{-1}{s-m^2}\,\frac{8k^2E}{\sqrt{2}\,m}y - \frac{1}{\sqrt{2}}\,\frac{E}{m}y,$$

$$T^t = \frac{-1}{t-m^2}\,\frac{1}{\sqrt{2}}y\left(\frac{4k^2E}{m}(1-3z) + \frac{E}{m}(1+z)[2E^2 + k^2(1+z)]\right) - \frac{1}{2\sqrt{2}}\,\frac{E}{m}y(1-z),$$

$$T^u = \frac{-1}{u-m^2}\,\frac{1}{\sqrt{2}}\,y\left(+\frac{4k^2E}{m}(1+3z) + \frac{E}{m}(1-z)[2E^2 + k^2(1-z)]\right) - \frac{1}{2\sqrt{2}}\,\frac{E}{m}y(1+z);$$

$T_{1,1;0,-1}$:

$$T^s = \frac{1}{s-m^2}\,\frac{8k^2E}{\sqrt{2}m}y - \frac{1}{\sqrt{2}}\,\frac{E}{m}y,$$

$$T^t = \frac{1}{t-m^2}\,\frac{y}{\sqrt{2}}\left(\frac{4k^2E}{m}(1-z) - \frac{E}{m}(1-z)[2E^2 + k^2(1+z)]\right) - \frac{1}{2\sqrt{2}}\,\frac{E}{m}y(1+z),$$

$$T^u = \frac{1}{u-m^2}\,\frac{y}{\sqrt{2}}\left(\frac{4k^2E}{m}(1+z) - \frac{E}{m}(1+z)[2E^2 + k^2(1-z)]\right) - \frac{1}{2\sqrt{2}}\,\frac{E}{m}y(1-z);$$

$T_{1,1;0,0}$:

$$T^s = \frac{-1}{s-m^2}\,\frac{4k^2}{m^2}z(2E^2 + m^2),$$

$$T^t = \frac{1}{t-m^2}\left(\frac{2}{m^2}y^2(E^4+k^4) + \frac{k^2E^2}{m^2}(1-z)(z^2 - 8z - 1)\right) + \frac{1}{m^2}(E^2+k^2) - \frac{1}{2}\,\frac{E^2}{m^2}y^2,$$

$$T^u = \frac{1}{u-m^2}\left(-\frac{2}{m^2}y^2(E^4+k^4) + \frac{k^2E^2}{m^2}(1+z)(1-8z-z^2)\right) - \frac{1}{m^2}(E^2+k^2) + \frac{1}{2}\,\frac{E^2}{m^2}y^2;$$

$T_{0,1;0,0}$:

$$T^s = \frac{1}{s-m^2}\frac{4\sqrt{2}\,k^2E}{m^3}y(2E^2+m^2) - \sqrt{2}\,\frac{k^2E}{m^3}y\,,$$

$$T^t = \frac{1}{t-m^2}\sqrt{2}\;yE[2E^4z + k^2E^2(4-9z+z^2) + k^4(5z-3)] - \frac{1}{\sqrt{2}}\frac{E}{m^3}y(k^2+E^2z)\,,$$

$$T^u = \frac{1}{u-m^2}\sqrt{2}\;yE[-2E^4z + k^2E^2(4+9z+z^2) - k^4(5z+3)] - \frac{1}{\sqrt{2}}\frac{E}{m^3}y(k^2-E^2z)\,;$$

$T_{0,0;0,0}$:

$$T^s = \frac{-1}{s-m^2}\frac{4k^2}{m^4}z(2E^2+m^2)^2 + \frac{4k^2E^2}{m^4}z\,,$$

$$T^t = \frac{1}{t-m^2}\left(8\frac{k^2E^2}{m^4}(1-z)(3+z)(k^2-E^2z) - 8\frac{k^2E^2}{m^4}(1-z)^2[2k^2+E^2(1+z)] - 2\frac{1}{m^4}(k^2-E^2z)^2[2E^2+k^2(1+z)]\right)$$
$$-2\frac{k^2E^2}{m^4}(1-z) - \frac{E^4}{m^4}y^2\,,$$

$$T^u = \frac{-1}{u-m^2}\left(8\frac{k^2E^2}{m^4}(1+z)(3-z)(k^2+E^2z) - 8\frac{k^2E^2}{m^4}(1+z)^2[2k^2+E^2(1-z)] - 2\frac{1}{m^4}(k^2+E^2z)^2[2E^2+k^2(1-z)]\right)$$
$$+2\frac{k^2E^2}{m^4}(1+z) + \frac{E^4}{m^4}y^2\,;$$

$T_{1,-1;0,1}$:

$$T^s = 0\,,$$

$$T^t = \frac{1}{t-m^2}\frac{y}{\sqrt{2}}(1-z)\frac{E}{m}[k^2(3-z) - 2E^2] + \frac{1}{2\sqrt{2}}\frac{E}{m}y(1-z)\,,$$

$$T^u = \frac{1}{u-m^2}\frac{y}{\sqrt{2}}(1-z)\frac{E}{m}[2E^2 - k^2(3+z)] - \frac{1}{2\sqrt{2}}\frac{E}{m}y(1-z)\,;$$

$T_{1,-1;0,0}$:

$$T^s = 0\,,$$

$$T^t = \frac{1}{t-m^2}\frac{y^2}{m^2}[k^2E^2(5-z) - 2(E^4+k^4)] + \frac{1}{2}\frac{E^2}{m^2}y^2\,,$$

$$T^u = \frac{1}{u-m^2}\frac{y^2}{m^2}[-k^2E^2(5+z) + 2(E^4+k^4)] - \frac{1}{2}\frac{E^2}{m^2}y^2\,;$$

$T_{1,1;1,-1}$:

$$T^s = 0\,,$$

$$T^t = \frac{1}{t-m^2}\frac{1}{2}y^2[k^2(3-z) - 2E^2] + \frac{1}{4}y^2\,,$$

$$T^u = \frac{1}{u-m^2}\frac{1}{2}y^2[2E^2 - k^2(3+z)] - \frac{1}{4}y^2\,;$$

$T_{1,-1;1,-1}$:

$$T^s = 0\,,$$

$$T^t = \frac{-1}{t-m^2}\frac{1}{2}(1+z)^2[2E^2+k^2(1+z)] + \frac{1}{4}(1+z)^2\,,$$

$$T^u = \frac{1}{u-m^2}\frac{1}{2}(1+z)^2[2E^2+k^2(1-z)] - \frac{1}{4}(1+z)^2\,.$$

APPENDIX B

The most general amplitude for the scattering of three particles of isospin 1 and z components a_1, a_2, and a_3 into three particles of isospin 1 and z components a_4, a_5, and a_6 is $(\delta_{a_1,a_2}\equiv\delta_{12})$

$$\begin{aligned}M_{456;123}=&A_1\delta_{65}\delta_{43}\delta_{21}+A_2\delta_{64}\delta_{53}\delta_{12}+A_3\delta_{63}\delta_{54}\delta_{21}\\&+A_4\delta_{65}\delta_{42}\delta_{31}+A_5\delta_{64}\delta_{52}\delta_{31}+A_6\delta_{62}\delta_{54}\delta_{31}\\&+A_7\delta_{65}\delta_{41}\delta_{32}+A_8\delta_{64}\delta_{51}\delta_{23}+A_9\delta_{61}\delta_{54}\delta_{23}\\&+A_{10}\delta_{63}\delta_{52}\delta_{41}+A_{11}\delta_{62}\delta_{53}\delta_{41}+A_{12}\delta_{63}\delta_{51}\delta_{42}\\&+A_{13}\delta_{62}\delta_{51}\delta_{43}+A_{14}\delta_{61}\delta_{53}\delta_{42}+A_{15}\delta_{61}\delta_{52}\delta_{43},\end{aligned}$$

where $A_1,\ldots,A_{15}$ are isoscalars. We wish to find the combinations of $A_1,\ldots,A_{15}$ which correspond to states of total isospin 0, 1, 2, 3.

We find the total isospin T by adding together two of the isospin vectors to give an isospin vector t and then adding the third isospin vector to t to get T. For example,

$$\vec{t}_1+\vec{t}_2=\vec{t}\,,$$
$$\vec{t}+\vec{t}_3=\vec{T}\,.$$

Squaring the two preceding equations gives

$$t(t+1)=4+2\vec{t}_1\cdot\vec{t}_2\,,$$
$$T(T+1)=t(t+1)+2+2\vec{t}\cdot\vec{t}_3\,.$$

We find

$$\begin{aligned}T=0:&\quad \vec{t}_1\cdot\vec{t}_2=-1,\quad \vec{t}\cdot\vec{t}_3=-2;\\T=1:&\quad \vec{t}_1\cdot\vec{t}_2=-2,\quad \vec{t}\cdot\vec{t}_3=0,\\&\qquad\qquad=-1,\qquad\quad=-1,\\&\qquad\qquad=+1,\qquad\quad=-3;\\T=2:&\quad \vec{t}_1\cdot\vec{t}_2=-1,\quad \vec{t}\cdot\vec{t}_3=+1,\\&\qquad\qquad=+1,\qquad\quad=-1;\\T=3:&\quad \vec{t}_1\cdot\vec{t}_2=+1,\quad \vec{t}\cdot\vec{t}_3=+2.\end{aligned}$$

We can use this to construct projection operators for the various states. Since a definite value of T corresponds to a definite value of $\vec{t}_1\cdot\vec{t}_2+\vec{t}\cdot\vec{t}_3$, we can form a projection operator out of $\vec{t}_1\cdot\vec{t}_2+\vec{t}_1\cdot\vec{t}_3+\vec{t}_2\cdot\vec{t}_3$ to project out states of total isospin. Then to project out the states within a given T value we can form projection operators out of $\vec{t}_1\cdot\vec{t}_2$ to operate on the initial particles and $\vec{t}_4\cdot\vec{t}_5$ to operate on the indices corresponding to the final particles.

Defining $\langle 456|123\rangle=\delta_{63}\delta_{52}\delta_{41}$ and using for example

$$\langle 456|\vec{t}_1\cdot\vec{t}_2|123\rangle=-\delta_{63}\epsilon_{j41}\epsilon_{j52}\,,$$

we can write all projection operators in terms of Kronecker δ's.

After projecting completely, the amplitude $M_{456;123}$ in Eq.(1) is written as the sum of a $T=0$ amplitude, a $T=3$ amplitude, four $T=2$ amplitudes, and nine $T=1$ amplitudes, each multiplied by the corresponding isospin projection operator

$$\begin{aligned}M_{456;123}=&M_0(P_0)_{456;123}+M_3(P_3)_{456;123}\\&+\sum_{i,j=1}^{2}M_2^{(i)(j)}(P_2^{(i)(j)})_{456;123}\\&+\sum_{i,j=0}^{2}M_1^{(i)(j)}(P_1^{(i)(j)})_{456;123}\,.\end{aligned}$$

For $T=2$ the index means t_4 and t_5 are added together to give $t=1$ while $i=2$ implies t_4 and t_5 are added to give $t=2$. The index j refers to the same thing for t_1 and t_2. For $T=1$ the index $i=0$, 1, or 2 ($j=0$, 1, or 2) identifies the states with $t=0$, 1, or 2 for the final (initial) particles.

The projection operators are normalized by

$$P_0^{\,2}=P_0\,,$$
$$P_3^{\,2}=P_3\,,$$
$$\sum_{n=1}^{2}P_2^{(i)(n)}P_2^{(n)(j)}=P_2^{(i)(j)},\quad i,j=1,2$$
$$\sum_{n=0}^{2}P_1^{(i)(n)}P_1^{(n)(j)}=P_1^{(i)(j)},\quad i,j=0,1,2$$

where we require $(P_T^{(i)(j)})_{456;123}=(P_T^{(j)(i)})_{123;456}$. The normalized projection operators are

$$(P_0)_{456;123}=\tfrac{1}{6}(\delta_{63}\delta_{52}\delta_{41}+\delta_{62}\delta_{51}\delta_{43}+\delta_{61}\delta_{53}\delta_{42}-\delta_{63}\delta_{51}\delta_{42}-\delta_{62}\delta_{53}\delta_{41}-\delta_{61}\delta_{52}\delta_{43})\,,$$

$$\begin{aligned}(P_3)_{456;123}=\tfrac{1}{30}(&5\delta_{63}\delta_{52}\delta_{41}+5\delta_{62}\delta_{51}\delta_{43}+5\delta_{61}\delta_{53}\delta_{42}+5\delta_{63}\delta_{51}\delta_{42}+5\delta_{62}\delta_{53}\delta_{41}+5\delta_{61}\delta_{52}\delta_{43}-2\delta_{63}\delta_{54}\delta_{12}\\&-2\delta_{64}\delta_{52}\delta_{13}-2\delta_{65}\delta_{41}\delta_{23}-2\delta_{65}\delta_{42}\delta_{13}-2\delta_{65}\delta_{43}\delta_{12}-2\delta_{64}\delta_{51}\delta_{23}-2\delta_{64}\delta_{53}\delta_{12}\\&-2\delta_{62}\delta_{54}\delta_{13}-2\delta_{61}\delta_{54}\delta_{23})\,,\end{aligned}$$

$$\begin{aligned}(P_2^{(1)(1)})_{456;123}=\tfrac{1}{24}(&4\delta_{63}\delta_{52}\delta_{41}+4\delta_{63}\delta_{51}\delta_{42}-4\delta_{63}\delta_{54}\delta_{12}-2\delta_{62}\delta_{51}\delta_{43}-2\delta_{62}\delta_{53}\delta_{41}-2\delta_{61}\delta_{53}\delta_{42}-2\delta_{61}\delta_{52}\delta_{43}\\&+2\delta_{65}\delta_{43}\delta_{12}+2\delta_{64}\delta_{53}\delta_{12}+2\delta_{62}\delta_{54}\delta_{13}+2\delta_{61}\delta_{54}\delta_{23}-\delta_{65}\delta_{41}\delta_{32}-\delta_{65}\delta_{42}\delta_{31}-\delta_{64}\delta_{51}\delta_{32}-\delta_{64}\delta_{52}\delta_{31})\,,\end{aligned}$$

$$\begin{aligned}(P_2^{(1)(2)})_{456;123}=\tfrac{1}{24}\sqrt{3}\,(&2\delta_{62}\delta_{51}\delta_{43}+2\delta_{62}\delta_{53}\delta_{41}-2\delta_{61}\delta_{53}\delta_{42}-2\delta_{61}\delta_{52}\delta_{43}+\delta_{64}\delta_{52}\delta_{31}\\&+\delta_{65}\delta_{42}\delta_{31}-\delta_{64}\delta_{51}\delta_{32}-\delta_{65}\delta_{41}\delta_{32}+2\delta_{61}\delta_{54}\delta_{32}-2\delta_{62}\delta_{54}\delta_{31})\,,\end{aligned}$$

$$(P_2^{(2)(1)})_{456;123} = \tfrac{1}{24}\sqrt{3}\,(2\delta_{61}\delta_{53}\delta_{42} + 2\delta_{62}\delta_{53}\delta_{41} - 2\delta_{62}\delta_{51}\delta_{43} - 2\delta_{61}\delta_{52}\delta_{43} + \delta_{64}\delta_{51}\delta_{32} + \delta_{64}\delta_{52}\delta_{31} - \delta_{65}\delta_{42}\delta_{31} - \delta_{65}\delta_{41}\delta_{32} + 2\delta_{65}\delta_{43}\delta_{21} - 2\delta_{64}\delta_{53}\delta_{21}),$$

$$(P_2^{(2)(2)})_{456;123} = \tfrac{1}{24}(4\delta_{63}\delta_{52}\delta_{41} - 4\delta_{63}\delta_{51}\delta_{42} + 2\delta_{62}\delta_{53}\delta_{41} + 2\delta_{61}\delta_{52}\delta_{43} - 2\delta_{62}\delta_{51}\delta_{43} - 2\delta_{61}\delta_{53}\delta_{42} + 3\delta_{64}\delta_{51}\delta_{32} + 3\delta_{65}\delta_{42}\delta_{31} - 3\delta_{64}\delta_{52}\delta_{31} - 3\delta_{65}\delta_{41}\delta_{32}),$$

$$(P_1^{(0)(0)})_{456;123} = \tfrac{2}{9}\delta_{63}\delta_{54}\delta_{21},$$

$$(P_1^{(0)(1)})_{456;123} = \frac{1}{6\sqrt{3}}(\delta_{61}\delta_{54}\delta_{32} - \delta_{62}\delta_{54}\delta_{31}),$$

$$(P_1^{(0)(2)})_{456;123} = \frac{1}{36\sqrt{5}}(3\delta_{62}\delta_{54}\delta_{31} + 3\delta_{61}\delta_{54}\delta_{32} - 2\delta_{63}\delta_{54}\delta_{21}),$$

$$(P_1^{(1)(0)})_{456;123} = \frac{1}{6\sqrt{3}}(\delta_{65}\delta_{43}\delta_{21} - \delta_{64}\delta_{53}\delta_{21}),$$

$$(P_1^{(1)(1)})_{456;123} = \tfrac{1}{6}(\delta_{65}\delta_{41}\delta_{32} + \delta_{64}\delta_{52}\delta_{31} - \delta_{65}\delta_{42}\delta_{31} - \delta_{64}\delta_{51}\delta_{32}),$$

$$(P_1^{(1)(2)})_{456;123} = \frac{1}{12\sqrt{15}}(2\delta_{65}\delta_{43}\delta_{21} - 2\delta_{64}\delta_{53}\delta_{21} + 3\delta_{64}\delta_{52}\delta_{31} + 3\delta_{64}\delta_{51}\delta_{32} - 3\delta_{65}\delta_{42}\delta_{31} - 3\delta_{65}\delta_{41}\delta_{32}),$$

$$(P_1^{(2)(0)})_{456;123} = \frac{1}{36\sqrt{5}}(3\delta_{65}\delta_{43}\delta_{21} + 3\delta_{64}\delta_{53}\delta_{21} - 2\delta_{63}\delta_{54}\delta_{21}),$$

$$(P_1^{(2)(1)})_{456;123} = \frac{1}{12\sqrt{15}}(2\delta_{61}\delta_{54}\delta_{23} - 2\delta_{62}\delta_{54}\delta_{31} + 3\delta_{65}\delta_{42}\delta_{31} + 3\delta_{64}\delta_{52}\delta_{31} - 3\delta_{65}\delta_{41}\delta_{32} - 3\delta_{64}\delta_{51}\delta_{32}),$$

$$(P_1^{(2)(2)})_{456;123} = \tfrac{1}{90}(-6\delta_{65}\delta_{43}\delta_{21} - 6\delta_{64}\delta_{53}\delta_{21} - 6\delta_{62}\delta_{54}\delta_{31} - 6\delta_{61}\delta_{54}\delta_{32} + 4\delta_{63}\delta_{54}\delta_{21} + 9\delta_{65}\delta_{42}\delta_{31} + 9\delta_{64}\delta_{52}\delta_{31} + 9\delta_{65}\delta_{41}\delta_{32} + 9\delta_{64}\delta_{51}\delta_{32}).$$

The isospin amplitudes are

$$M_0 = A_{10} - A_{11} - A_{12} + A_{13} + A_{14} - A_{15},$$

$$M_3 = A_{10} + A_{11} + A_{12} + A_{13} + A_{14} + A_{15},$$

$$M_2^{(1)(1)} = 2A_{10} - A_{11} + 2A_{12} - A_{13} - A_{14} - A_{15},$$

$$M_2^{(1)(2)} = \sqrt{3}\,(A_{11} + A_{13} - A_{14} - A_{15}),$$

$$M_2^{(2)(1)} = \sqrt{3}\,(A_{11} - A_{13} + A_{14} - A_{15}),$$

$$M_2^{(2)(2)} = 2A_{10} + A_{11} - 2A_{12} - A_{13} - A_{14} + A_{15},$$

$$M_1^{(0)(0)} = \tfrac{1}{2}(3A_1 + 3A_2 + 9A_3 + A_4 + A_5 + 3A_6 + A_7 + A_8 + 3A_9 + 3A_{10} + A_{11} + 3A_{12} + A_{13} + A_{14} + A_{15}),$$

$$M_1^{(0)(1)} = \sqrt{3}\,(-A_4 - A_5 - 3A_6 + A_7 + A_8 + 3A_9 - A_{11} - A_{13} + A_{14} + A_{15}),$$

$$M_1^{(0)(2)} = 2\sqrt{5}\,(A_4 + A_5 + 3A_6 + A_7 + A_8 + 3A_9 + A_{11} + A_{13} + A_{14} + A_{15}),$$

$$M_1^{(1)(0)} = \sqrt{3}\,(3A_1 - 3A_2 + A_4 - A_5 + A_7 - A_8 - A_{11} + A_{13} - A_{14} + A_{15}),$$

$$M_1^{(1)(1)} = \tfrac{3}{4}(-2A_4 + 2A_5 + 2A_7 - 2A_8 + 2A_{10} + A_{11} - 2A_{12} - A_{13} - A_{14} + A_{15}),$$

$$M_1^{(1)(2)} = \tfrac{1}{2}\sqrt{15}\,(-2A_4 + 2A_5 - 2A_7 + 2A_8 - A_{11} + A_{13} - A_{14} + A_{15}),$$

$$M_1^{(2)(0)} = 2\sqrt{5}\,(3A_1 + 3A_2 + A_4 + A_5 + A_7 + A_8 + A_{11} + A_{13} + A_{14} + A_{15}),$$

$$M_1^{(2)(1)} = \tfrac{1}{2}\sqrt{15}\,(2A_4 + 2A_5 - 2A_7 - 2A_8 - A_{11} - A_{13} + A_{14} + A_{15}),$$

$$M_1^{(2)(2)} = \tfrac{5}{2}(A_4 + A_5 + A_7 + A_8) + \tfrac{1}{4}(6A_{10} + A_{11} + 6A_{12} + A_{13} + A_{14} + A_{15}).$$

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Hadron Bootstrap Hypothesis*

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A discussion is given of the conjecture that classical space-time properties prescribe a unique S matrix which approximates strong-interaction phenomena.

I. INTRODUCTION

An esthetically compelling speculation is that the laws of nature might be determined uniquely by requirements of self-consistency or, phrased more picturesquely, by a "bootstrap." This paper puts foward and analyzes a "partial bootstrap" conjecture that has for some time been the subject of informal discussion, but that heretofore has not found its way into research publication. The conjecture is the following: *Quantum superposition, when expressed through a nontrivial S matrix, can achieve compatibility with the real (classical) world in only one possible way — close to the way exhibited by nature for hadrons.* Recent progress by Stapp and collaborators[1-3] in clarifying the relation between the S matrix and classical space-time suggests that the moment may be ripe for systematic analysis of this uniqueness conjecture.

From the standpoint of hard science, the complete bootstrap idea is inadmissible because science requires the *a priori* acceptance of certain language-defining concepts, so that "questions" can be formulated and experiments performed to give "answers." The role of theory is to provide a set of rules for predicting the results of experiment, but rules necessarily are formulated in a language of accepted ideas. Among currently unquestioned notions prerequisite to the conduct of science are:

(1) Three-dimensional space and a unidirectional time, with an associated cause-effect event structure; the existence of suitable measuring rods and clocks is corollary.

(2) The arrangement of macroscopic matter into blobs of sufficiently well-defined shape and permanency that the isolated system or "object" concept becomes meaningful.

(3) The existence of weak long-range interactions like electromagnetism and gravity that allow "measurements" to be made upon "objects" without the objects losing identity; the observer's integrity must also be preserved.

The foregoing detailed prerequisites may deceptively be summarized by the single term, "measurement," but the concept of measurement, on which hard science is based, is admissible only because of certain special attributes of nature, attributes that a complete bootstrap theory would have to explain as necessary components of self-consistency. It is in this sense that the idea of a complete bootstrap, while not obviously foolish, is intrinsically unscientific.

Although natural philosophy eventually will no doubt identify a framework more general than