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New Inconsistencies in the Quantization of Spin-$\frac{3}{2}$ Fields*

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The possibility of consistently quantizing a spin-$\frac{3}{2}$ field in the presence of a coupling to a scalar and a spinor field is considered. It is found that, contrary to a recent assertion, the anticommutators of the various Fermi-Dirac fields are not positive definite. Inasmuch as this inconsistency is found to be present for the case in which the $\frac{3}{2}$ field participates only linearly in the interaction, such a result demonstrates that the difficulties discovered by Johnson and Sudarshan for an electromagnetic coupling can be expected to occur for a considerably more general class of interactions.

I. INTRODUCTION

One of the most disturbing of the difficulties which currently plague quantum field theory is the absence of a consistent set of rules for the quantization of fields with spin greater than unity. While the existence of divergences in perturbation-theory calculations is doubtless of more immediate concern to practitioners of field theory than the higher-spin problem, it is clearly recognized that the divergence difficulty involves intrinsically dynamical questions which present calculational techniques are ill-equipped to handle. On the other hand, although the quantization of Fermi-Dirac fields is known to require explicit reference to the dynamics, the manner in which the form of the interaction appears is not subject to computational ambiguity (i.e., perturbation techniques are not required). Thus an inability to quantize higher-spin theories for all possible interaction terms must be interpreted as representing a somewhat mysterious but nonetheless fundamental incompatibility of higher spin with the basic ingredients of relativistic field theory.[1]

As has been shown by Johnson and Sudarshan,[2] the basic problem in the quantization of Fermi-Dirac fields of spin greater than $\frac{1}{2}$ is that one requires the existence of secondary constraints on the fields, and thus the dynamics must necessarily enter into consideration. These authors have, furthermore, shown that for the case of spin $\frac{3}{2}$, a coupling to an external electromagnetic field renders the anticommutators indefinite and thereby inconsistent with a positive definite metric. Even more disturbing perhaps is the demonstration by Velo and Zwanziger[3] that the corresponding classical wave equation possesses noncausal modes of propagation. Both of these calculations, not surprisingly, find the same threshold value for the external field at which such difficulties occur.

Recently, however, a somewhat different coupling of a spin-$\frac{3}{2}$ field has been considered[4] which differs from the Johnson-Sudarshan approach primarily in that the interaction term involves only a linear coupling of the spin-$\frac{3}{2}$ field. The assertion of that work to the effect that no inconsistency occurs in the quantization, if correct, could well provide the insight necessary to define quite precisely the conditions which must be imposed on the dynamics of higher-spin theories. In the present paper this contention is examined. It is shown that the (positive definite) anticommutators given in Ref. 4 are, contrary to the claim of that work, incompatible with the action principle and that the correct anticommutation relations for the spin-$\frac{3}{2}$ components have a form which is remarkably similar to that found by Johnson and Sudarshan for the case of a coupling to an external electromagnetic field.

In Sec. II a brief review and slight reformulation is given of the theory discussed by Nath *et al.* Sec. III presents the derivation of the commutation relations as well as a demonstration of their incompatibility with a positive definite metric. Finally, some concluding remarks are given to indicate the connection of this work to Ref. 2.

II. THE COUPLED SPIN-$\frac{3}{2}$ FIELD

The spin-$\frac{3}{2}$ field is most conveniently denoted by the 16-component Rarita-Schwinger[5] vector spinor ψ^λ, which, for the case of no interaction, is well known to be consistently described by the Lagrangian[6]

$$\mathcal{L} = \tfrac{1}{4} i\psi^\alpha \beta[g_{\alpha\beta}\gamma^\mu + W(\delta^\mu_\alpha\gamma_\beta + \delta^\mu_\beta\gamma_\alpha) - \tfrac{1}{2}(3W^2+2W+1)\gamma_\alpha\gamma^\mu\gamma_\beta]\,\partial_\mu\psi^\beta + \text{H.c.} - \tfrac{1}{2}m\psi^\alpha\beta[g_{\alpha\beta} + (3W^2+3W+1)\gamma_\alpha\gamma_\beta]\,\psi^\beta\,.$$

Under the point transformation

$$\psi'_\alpha = \psi_\alpha + \tfrac{1}{4}\mu\gamma_\alpha\gamma^\beta\psi_\beta , \tag{2.1}$$

the matrices which occur in $\mathcal{L}$ are found to have the same structure except that W is replaced by

$$W' = W(1-\mu) - \tfrac{1}{2}\mu .$$

Inasmuch as the transformation (2.1) merely mixes the spin-$\frac{1}{2}$ components while leaving the $\frac{3}{2}$ components invariant, the parameter W is without physical significance and can be chosen at will.

Following Ref. 4, one now desires to couple the Rarita-Schwinger field to a spin-$\frac{1}{2}$ field ψ and the derivative of a spin-0 field ϕ. By taking the free Lagrangian for these fields to be

$$\mathcal{L} = \tfrac{1}{4}i[\psi\beta\gamma^\mu, \partial_\mu\psi] - \tfrac{1}{2}m\psi\beta\psi + \tfrac{1}{2}[\phi^\mu\partial_\mu\phi - \phi\partial_\mu\phi^\mu] + \tfrac{1}{2}\phi^\mu\phi_\mu - \tfrac{1}{2}\mu_0{}^2\phi^2 ,$$

one notes that such a coupling can be effected by taking an interaction term of the form

$$\mathcal{L}_I = gj^\mu\phi_\mu + \tfrac{1}{2}g^2 j^\mu j_\mu , \tag{2.2}$$

where

$$j_\nu \equiv \tfrac{1}{2}[\psi^\mu, \beta\Theta_{\mu\nu}\psi]$$

and

$$\Theta_{\mu\nu} \equiv g_{\mu\nu} - \lambda\gamma_\mu\gamma_\nu .$$

The somewhat unconventional structure of (2.2), while implying the same equations of motion as

$$\mathcal{L}_I = -gj^\mu\partial_\mu\phi ,$$

has been utilized in order to avoid the explicit appearance of derivative terms in the interaction. While this has no effect upon our general conclusions, it does serve to make the theory more amenable to treatment by the action-principle formalism.

Upon performing the transformation (2.1), one readily establishes that λ has the form

$$\lambda = \tfrac{1}{2}W(1+4Z) + Z ,$$

with Z an unspecified parameter which is to be fixed by the requirement that secondary constraints appear in the theory. This is seen most simply by choosing $W=-1$ and noting that the temporal component of the field equations for ψ^μ may be written as

$$m\vec{\gamma}\cdot\vec{\psi} - (\vec{\gamma}\cdot\vec{p})(\vec{\gamma}\cdot\vec{\psi}) - \vec{p}\cdot\vec{\psi} = g(\tfrac{1}{2}-Z)\gamma^0\psi(\phi^0 + gj^0) + g(\tfrac{1}{2}+Z)\gamma^k\psi\partial_k\phi . \tag{2.3}$$

One can now define that part of ψ_k which transforms as a spin-$\frac{3}{2}$ object under spatial rotations by the relation,

$$\psi_k^{3/2} = (\delta_{kl} + \tfrac{1}{3}\gamma_k\gamma_l)\psi_l ,$$

in terms of which (2.3) assumes the form

$$(m - \tfrac{2}{3}\vec{\gamma}\cdot\vec{p})(\vec{\gamma}\cdot\vec{\psi}) - \vec{p}\cdot\vec{\psi}^{3/2} = g(\tfrac{1}{2}-Z)\gamma^0\psi(\phi^0 + gj^0) + g(\tfrac{1}{2}+Z)\gamma^k\psi\partial_k\phi . \tag{2.4}$$

In writing (2.4) it is important to note that we have retained the canonical variables ϕ^0 and ϕ, but have eliminated ϕ^k by means of the constraint equation

$$\phi^k = -\partial_k\phi - gj_k .$$

It is clearly seen from (2.4) that for $g=0$ one has a secondary constraint relating $\vec{\gamma}\cdot\vec{\psi}$ and $\psi_k^{3/2}$ at equal times. However, the general theorem of Johnson and Sudarshan requires that for a consistent quantization to be possible, this must continue to be a constraint between these same variables for nonvanishing coupling. This can be the case only if ψ^0 does not occur on the right-hand side of (2.4). Since, however,

$$j^0 = -(Z - \tfrac{1}{2})[\psi^0, \beta\psi] + (Z + \tfrac{1}{2})[\psi^k, \gamma^k\psi] ,$$

one clearly must take $Z = \frac{1}{2}$ for there to be any hope of a consistent quantization.[7] The secondary constraint thus assumes the simple form

$$(m - \tfrac{2}{3}\vec{\gamma}\cdot\vec{p})\vec{\gamma}\cdot\vec{\psi} - \vec{p}\cdot\vec{\psi}^{3/2} = g\gamma^k\psi\partial_k\phi , \tag{2.5}$$

a result which is now to be incorporated into the derivation of the commutation relations of the theory.

III. THE COMMUTATION RELATIONS

In accord with the general formalism of the action principle,[8] the commutation relations of the theory described in Sec. II are to be determined from

$$[\chi(x), G] = \tfrac{1}{2} i\,\delta\chi(x)\,, \tag{3.1}$$

where $\chi(x)$ represents any one of the field operators ψ^μ, ψ, or ϕ. The generator G is obtained from the time-derivative terms in $\mathcal{L}$ and has the form[2]

$$G = \tfrac{1}{2}\int d^3x[i\psi_k(\delta_{kl} + \tfrac{1}{3}\gamma_k\gamma_l)\delta\psi_l + i\tfrac{2}{3}\psi_k\gamma^k\delta\gamma_l\psi_l + i\psi\delta\psi + \phi^0\delta\phi - \phi\,\delta\phi^0]\,.$$

As a consequence of (2.5) the variations which appear in the generator are not independent, but satisfy the condition,

$$(m - \tfrac{2}{3}\vec{\gamma}\cdot\vec{p})\,\vec{\gamma}\cdot\delta\vec{\psi} - \vec{p}\cdot\delta\vec{\psi}^{3/2} - g\vec{\gamma}\cdot\delta(\psi\vec{\nabla}\phi) = 0\,, \tag{3.2}$$

thereby considerably complicating the task of inferring the canonical commutation relations.

The most convenient way to handle the rather complex constraint (3.2) is by the technique of Lagrange multipliers. Thus, for the choice $\chi(x) = \phi(x)$, one writes (3.1) as

$$\int d^3x'\{[\phi(x), \tfrac{1}{2}(\phi^0(x')\delta\phi(x') - \phi(x')\delta\phi^0(x')) + \tfrac{1}{2}i\psi(x')\delta\psi(x') + \tfrac{1}{2}i\vec{\psi}^{3/2}(x')\,\delta\vec{\psi}^{3/2}(x') + \tfrac{1}{2}i\tfrac{2}{3}\vec{\psi}(x')\cdot\vec{\gamma}\delta\vec{\gamma}\cdot\vec{\psi}(x')]$$
$$- \tfrac{1}{2}i\,\delta\phi(x')\delta(\vec{x} - \vec{x}') - \tfrac{1}{2}i\Lambda(x, x')[(m - \tfrac{2}{3}\vec{\gamma}\cdot\vec{p})\vec{\gamma}\cdot\delta\vec{\psi}(x') - \vec{p}\cdot\delta\vec{\psi}^{3/2}(x') - g\vec{\gamma}\cdot\delta(\psi(x')\vec{\nabla}\phi(x'))]\} = 0 \tag{3.3}$$

and considers all variations as being independent. Thus one infers

$$\begin{aligned}
&[\phi(x), \phi(x')] = 0\,,\\
&[\phi(x), \phi^0(x')] = i\,\delta(\vec{x} - \vec{x}') + ig\vec{\nabla}'\cdot[\Lambda(x, x')\vec{\gamma}\psi(x')]\,,\\
&[\phi(x), \psi(x')] = -g\Lambda(x, x')\vec{\gamma}\cdot\vec{\nabla}\phi(x')\,,\\
&[\phi(x), \vec{\gamma}\cdot\vec{\psi}(x')] = \tfrac{3}{2}\Lambda(x, x')(m + \tfrac{2}{3}\vec{\gamma}\cdot\vec{p}')\,,\\
&[\phi(x), \psi_k^{3/2}(x')] = p_l'\Lambda(x, x')(\delta_{kl} + \tfrac{1}{3}\gamma_l\gamma_k)\,,
\end{aligned} \tag{3.4}$$

where $\Lambda(x, x')$ is to be determined such that (2.5) is satisfied. The result of this latter calculation gives $\Lambda(x, x') = 0$ and thus implies the equal-time commutation relations

$$\begin{aligned}
&[\phi^0(x), \phi(x')] = \frac{1}{i}\delta(\vec{x} - \vec{x}')\,,\\
&[\phi(x), \phi(x')] = [\phi(x), \psi(x')] = [\phi(x), \psi_k(x')] = 0\,.
\end{aligned} \tag{3.5}$$

A less trivial result is found for the choice $\chi(x) = \phi^0(x)$. Here the commutation relations are found to be

$$\begin{aligned}
&[\phi^0(x), \phi(x')] = \frac{1}{i}\delta(\vec{x} - \vec{x}')\,,\\
&[\phi^0(x), \phi^0(x')] = ig\vec{\nabla}'\cdot[\Lambda(x, x')\vec{\gamma}\psi(x')]\,,\\
&[\phi^0(x), \psi(x')] = -g\Lambda(x, x')\vec{\gamma}\cdot\vec{\nabla}\phi(x')\,,\\
&[\phi^0(x), \vec{\gamma}\cdot\vec{\psi}(x')] = \tfrac{3}{2}\Lambda(x, x')(m + \tfrac{2}{3}\vec{\gamma}\cdot\vec{p}')\,,\\
&[\phi^0(x), \psi_k^{3/2}(x')] = p_l'\Lambda(x, x')(\delta_{kl} + \tfrac{1}{3}\gamma_l\gamma_k)\,,
\end{aligned} \tag{3.6}$$

where all operator products on the right-hand side are understood to be symmetrized. A straightforward calculation of $\Lambda(x, x')$ in this case yields

$$\begin{aligned}
&[\phi^0(x), \phi^0(x')] = \tfrac{1}{3}g^2\,\mathrm{Tr}\,\vec{\gamma}\cdot\vec{\nabla}\frac{1}{m^2 - \frac{2}{3}g^2(\nabla\phi)^2}[\psi, \psi]\vec{\gamma}\cdot\vec{\nabla}\delta(\vec{x} - \vec{x}')\,,\\
&[\phi^0(x), \psi(x')] = -\tfrac{2}{3}ig^2\vec{\gamma}\cdot\vec{\nabla}\psi\frac{1}{m^2 - \frac{2}{3}g^2(\nabla\phi)^2}(\vec{\gamma}\cdot\vec{\nabla}\phi)\delta(\vec{x} - \vec{x}')\,,\\
&[\phi^0(x), \vec{\gamma}\cdot\vec{\psi}(x')] = ig\vec{\gamma}\cdot\vec{\nabla}\psi\frac{1}{m^2 - \frac{2}{3}g^2(\nabla\phi)^2}(m - \tfrac{2}{3}\vec{\gamma}\cdot\vec{p})\delta(\vec{x} - \vec{x}')\,,\\
&[\phi^0(x), \psi_k^{3/2}(x')] = -\tfrac{2}{3}g\vec{\gamma}\cdot\vec{\nabla}\psi\frac{1}{m^2 - \frac{2}{3}g^2(\nabla\phi)^2}\nabla_l(\delta_{kl} + \tfrac{1}{3}\gamma_l\gamma_k)\delta(\vec{x} - \vec{x}')\,.
\end{aligned} \tag{3.7}$$

The calculational technique having now been displayed, it is sufficient merely to present the results one obtains for the remaining commutators. Taking $\chi(x)=\psi(x)$ there follows

$$\begin{aligned}
&[\psi(x),\phi^0(x')] = -\tfrac{2}{3}ig^2(\vec\gamma\cdot\vec\nabla\phi)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}\psi\vec\gamma\cdot\vec\nabla\delta(\vec x-\vec x'),\\
&\{\psi(x),\psi(x')\} = \frac{1}{1-\frac{2}{3}(g^2/m^2)(\nabla\phi)^2}\delta(\vec x-\vec x'),\\
&\{\psi(x),\vec\gamma\cdot\vec\psi(x')\} = g(\vec\gamma\cdot\vec\nabla\phi)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}(m-\tfrac{2}{3}\vec\gamma\cdot\vec p)\delta(\vec x-\vec x'),\\
&\{\psi(x),\psi_k^{3/2}(x')\} = \tfrac{2}{3}ig(\vec\gamma\cdot\vec\nabla\phi)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}\nabla_l(\delta_{kl}+\tfrac{1}{3}\gamma_l\gamma_k)\delta(\vec x-\vec x').
\end{aligned}\tag{3.8}$$

The choice $\chi(x)=\vec\gamma\cdot\vec\psi(x)$ yields

$$\begin{aligned}
&[\vec\gamma\cdot\vec\psi(x),\phi^0(x')] = ig(m+\tfrac{2}{3}\vec\gamma\cdot\vec p)\gamma_i\psi(x)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}\nabla_i\delta(\vec x-\vec x'),\\
&\{\vec\gamma\cdot\vec\psi(x),\psi(x')\} = g(m+\tfrac{2}{3}\vec\gamma\cdot\vec p)(\vec\gamma\cdot\vec\nabla\phi)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}\delta(\vec x-\vec x'),\\
&\{\vec\gamma\cdot\vec\psi(x),\vec\gamma\cdot\vec\psi(x')\} = -\tfrac{3}{2}\left[(m+\tfrac{2}{3}\vec\gamma\cdot\vec p)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}(m-\tfrac{2}{3}\vec\gamma\cdot\vec p)-1\right]\delta(\vec x-\vec x'),\\
&\{\vec\gamma\cdot\vec\psi(x),\psi_k^{3/2}(x')\} = (m+\tfrac{2}{3}\vec\gamma\cdot\vec p)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}p_l(\delta_{kl}+\tfrac{1}{3}\gamma_l\gamma_k)\delta(\vec x-\vec x'),
\end{aligned}\tag{3.9}$$

while $\chi(x)=\psi_k^{3/2}(x)$ implies

$$\begin{aligned}
&[\psi_k^{3/2}(x),\phi^0(x')] = \tfrac{2}{3}g(\delta_{kl}+\tfrac{1}{3}\gamma_k\gamma_l)\nabla_l\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}\gamma_i\psi\nabla_i\delta(\vec x-\vec x'),\\
&\{\psi_k^{3/2}(x),\psi(x')\} = \tfrac{2}{3}g(\delta_{kl}+\tfrac{1}{3}\gamma_k\gamma_l)\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}(\vec\gamma\cdot\vec\nabla\phi)\delta(\vec x-\vec x'),\\
&\{\psi_k^{3/2}(x),\vec\gamma\cdot\vec\psi(x')\} = -(\delta_{kl}+\tfrac{1}{3}\gamma_k\gamma_l)p_l\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}(m-\tfrac{2}{3}\vec\gamma\cdot\vec p)\delta(\vec x-\vec x'),\\
&\{\psi_k^{3/2}(x),\psi_l^{3/2}(x')\} = (\delta_{km}+\tfrac{1}{3}\gamma_k\gamma_m)\left(\delta_{mn}+\tfrac{2}{3}p_m\frac{1}{m^2-\frac{2}{3}g^2(\nabla\phi)^2}p_n\right)(\delta_{nl}+\tfrac{1}{3}\gamma_n\gamma_l)\delta(\vec x-\vec x').
\end{aligned}\tag{3.10}$$

It is readily verified that the above sets of commutators are entirely consistent with each other (e.g., the $\{\vec\gamma\cdot\vec\psi,\psi\}$ result is consistent with that obtained for $\{\psi,\vec\gamma\cdot\vec\psi\}$) and that this consequently represents the desired solution of the constrained variational problem.

An examination of the form of $\{\psi(x),\psi(x')\}$ strongly suggests the indefinite nature of the commutation relations. This can be made more precise by introducing the real function $f(x)$ and defining

$$U=\exp\left[i\int\phi^0(x)f(x)d^3x\right],\tag{3.11}$$

which for sufficiently regular $f(x)$ is a unitary operator. Upon considering

$$\langle 0|U\{\psi(x),\psi(x')\}U^\dagger|0\rangle=\left\langle 0\left|\left[1-\frac{2}{3}\frac{g^2}{m^2}(\nabla\phi-\nabla f)^2\right]^{-1}\right|0\right\rangle\delta(\vec x-\vec x'),\tag{3.12}$$

one trivially sees that for suitably large f the anticommutator necessarily becomes negative and consequently incompatible with a positive definite metric.

This result thus contradicts the assertion of Nath *et al.* to the effect that the Johnson-Sudarshan effect does not occur for a linear coupling of the spin-$\frac{3}{2}$ field. In fact, the result of Ref. 2 for the anticommutator of the spin-$\frac{3}{2}$ components,

$$\{\psi_k^{3/2}(x),\psi_l^{3/2}(x')\}=(\delta_{km}+\tfrac{1}{3}\gamma_k\gamma_m)\left(\delta_{mn}+\tfrac{2}{3}\Pi_m\frac{1}{m^2-\frac{2}{3}eq\vec\sigma\cdot\vec H}\Pi_n\right)(\delta_{nl}+\tfrac{1}{3}\gamma_n\gamma_l)\delta(\vec x-\vec x'),\tag{3.13}$$

where $\vec\Pi=(\vec p-eq\vec A)$ differs from the result found here primarily in that the electromagnetic coupling has an inconsistency to first order in the external field, while the present calculation (for unquantized ϕ) encounters the same dilemma in second order. Such a result is, of course, intuitively quite reasonable.

The present paper thus provides an extension of the work of Johnson and Sudarshan in two main respects. First, it weakens the coupling of the spin-$\frac{3}{2}$ field by considering the case in which it appears linearly in the interaction, and secondly, the effect of a quantized boson field is included. The negative result found here suggests that conventional techniques for the avoidance of the higher-spin difficulties appear to offer little hope of success at the present time.

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[1]One refers here, of course, to special relativity as this remark is not valid for the case of Galilean-invariant theories of arbitrary-spin particles [cf. C. R. Hagen, Commun. Math. Phys. 21, 219 (1971)].

[2]K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N.Y.) 13, 126 (1961).

[3]G. Velo and D. Zwanziger, Phys. Rev. 186, 1337 (1969).

[4]L. M. Nath, B. Etemadi, and J. D. Kimel, Phys. Rev. D 3, 2153 (1971).

[5]W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).

[6]We take $\hbar = c = 1$ and $g^{\mu\nu} = (1, 1, 1, -1)$. All fields will be taken to be Hermitian, and we consequently employ a Majorana representation for the Dirac algebra such that $\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}$.

[7]This value of Z coincides with that obtained in Ref. 4 by a somewhat different line of reasoning.

[8]J. Schwinger, Phys. Rev. 91, 713 (1953).

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Fourier Transforms in the Nonpolynomial Harmonic-Oscillator Model*

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Problems associated with the Lee-Zumino approach to nonpolynomial interactions are shown to exist even after employing their special method for calculating the Fourier transforms of Green's functions.

INTRODUCTION

In an earlier paper,[1] hereafter referred to as I, we examined a method suggested by Lee and Zumino[2] (LZ) for treating nonpolynomial interactions in the context of a perturbed harmonic-oscillator model. While the results obtained strongly suggested that the method suffered certain problems, the investigation concerned itself entirely with configuration space. In that a crucial part of the LZ technique deals with methods for evaluating Fourier transforms of Green's functions, our examination cannot be considered complete. In the present paper, we attempt to remedy this defect to a limited extent. It should be noted at the outset that substantial portions of this work are essentially pedagogical in intent, and we shall not apologize for taking time to reiterate the results of other authors.

The paper is arranged as follows: In Sec. I, we explain the harmonic-oscillator model and discuss the problems which arise in going from the time variable to frequency via Fourier transforms. In particular, we explore the relation between the "true" Fourier representation of certain Green's functions, and that used by LZ. We shall see that the "true" Fourier transform suffers from certain anomalies which are removed by the LZ modification. In particular, the LZ Fourier transform has satisfactory analyticity properties. In Sec. II, we examine certain aspects of the LZ Green's functions in ω space, and find that, unfortunately, certain disagreeable features still persist. While none of the anomalies we shall discover are necessarily fatal to the LZ method, their existence does little to strengthen one's faith in its reasonableness.

I. THREE TRANSFORM PRESCRIPTIONS

Let us consider the Hamiltonian

$$H = H_0 + \lambda U ,$$