PHYSICAL REVIEW D VOLUME 4, NUMBER 8 15 OCTOBER 1971

Tachyons Without Causal Loops in One Dimension

Adel F. Antippa*
Department of Physics, Université du Québec, Trois-Rivières, Québec, Canada

and

Allen E. Everett†
Department of Physics, Tufts University, Medford, Massachusetts 02155
(Received 8 July 1971)

It is shown that, for the case of one spatial dimension, it is possible to formulate a theory of tachyons which is consistent with the special theory of relativity and involves no violation of causality. The preservation of causality requires the assumption that tachyons exist only with one sign of momentum. Although this assumption is not invariant under space inversion, it is consistent with the principle of relativity. In addition to the preservation of causality, a possible rationale for such an assumption is discussed briefly.

I. INTRODUCTION

The special theory of relativity raises two main objections against the existence of particles with speeds greater than that of light, now called tachyons. The first objection is due to the fact that an infinite amount of energy would be required to accelerate a material particle past the light barrier. This is a consequence of the energy equation[1]

$$E = m_0(1-\beta^2)^{-1/2}, \tag{1}$$

where E is the energy of the particle, m_0 its rest mass, and β its velocity. Equation (1) was derived by Einstein[2] in 1905. The second objection is due to the possibility of using tachyons to form causal loops, within the framework of the special theory of relativity. This was first pointed out by Tolman[3] in 1917 and is known as the Tolman paradox.

The problem of acceleration past the light barrier has been resolved in 1962 by Bilaniuk, Deshpande, and Sudarshan[4] (BDS) who used an elegant line of reasoning by analogy with the existence of photons, to point out an alternative procedure for the creation of tachyons. According to the BDS scheme, tachyons may be emitted and absorbed with a velocity greater than that of light, that is, in the "tachyon state." As such, the necessity of acceleration and deceleration past the light barrier as a mechanism for the creation and annihilation of tachyons is bypassed. On the other hand, the resolution of the Tolman paradox by the BDS reinterpretation principle[5] is still a controversial matter.[6,7]

Since discussions of tachyons and their connection with causality have mostly been carried out in one dimension, we would like to point out that in a strictly one-dimensional space, one can construct a model of tachyons which avoids the Tolman paradox. Our model involves the assumption that tachyons exist only with positive (or only negative) values of the momentum, just as bradyons (particles with speed less than that of light) exist only with positive energies. This means, of course, that the theory is not invariant under space inversions. We shall indicate briefly later a possible rationale for investigating the consequences of such an assumption in a theory of tachyons. Our primary purpose, however, is to point out that, given this assumption, which is in full accord with the principle of relativity, one can formulate a theory of tachyons in one dimension which does not involve violations of causality. The only additional assumption we make about the properties of tachyons is that their velocity is given by the ratio of their momentum to that of their energy, $\beta = p/E$, the same as for bradyons.

In the debate over the success or failure of the reinterpretation principle to circumvent the causal problems introduced by tachyons, many different versions of the Tolman paradox have been formulated.[6,7] The different formulations all share three main characteristics: (1) The paradox is formulated by considering the exchange of positive-energy tachyon signals moving forward in time (with respect to the emitter) in between inertial observers who are relatively in motion. (2) The essence of the paradox is traced back to the fact that, for a certain range of relative velocities of two inertial observers, a tachyon signal traveling forward in time with respect to one observer will be traveling backward in time with respect to the other observer. (3) If tachyon signals exist, then they can move along any direction in space. We will show that the paradox can be reformulated in terms of the exchange of positive- and negative-energy tachyon signals moving forward and backward in time (relative to the emitter), in between two inertial observers who are relatively at rest.[8] This reformulation simplifies and considerably clarifies the problem. As concerns the essence of the paradox, it has already been pointed out by Newton and Csonka[9] that violation of causality and the reordering of cause and effect in time are different things. We would like to emphasize this point further and note that the ordering of cause and effect in space has as much bearing on the causality question as does their ordering in time. This is because in any causal loop, information has to return to the starting point in space before there is ever a hope of violating causality.

Our general argument is as follows. The existence of positive-energy bradyons moving forward in time to the exclusion of negative-energy bradyons moving backward in time is consistent with the principle of relativity. Furthermore, positive-energy bradyon signals are not sufficient for the formation of causal loops due to their unidirectional motion in time. In a one-dimensional space, the existence of positive-momentum tachyons to the exclusion of negative-momentum tachyons is likewise consistent with the principle of relativity. Furthermore, even though positive-momentum tachyons can move forward and backward in time, they do not form causal loops due to their unidirectional motion in space. In a three-dimensional space rotational invariance is a consequence of Lorentz invariance because the special Lorentz transformations must be supplemented by space rotations in order to complete the Lorentz group. As such the existence of tachyons with positive momentum along some space axis to the exclusion of tachyons with negative momentum along that axis violates rotational, and thus Lorentz, invariance. In a one-dimensional space on the other hand, the special Lorentz transformations do form a group and Lorentz invariance is decoupled from space-inversion invariance.

Having outlined the line of reasoning which resolves the Tolman paradox in a one-dimensional space, we will now proceed to a detailed proof of the above considerations.

II. GENERAL RESULTS

The discussion and results of this section are restricted to a one-dimensional space. Our purpose is to derive the minimal behavior and range of values of the energy and momentum of tachyons which is consistent with the principle of relativity, subject to the assumption that the velocity of a tachyon is equal to the ratio of its momentum to its energy,

$$\beta = p/E \ . \tag{2}$$

A. Preliminary Results

Consider two events e_1 and e_2, consisting of the passage of a particle at two points in a two-dimensional space-time. As viewed from two inertial coordinate systems K_1 and K_2, the space and time separations of these two events are $(\Delta x)_1$, $(\Delta t)_1$ in K_1 and $(\Delta x)_2$, $(\Delta t)_2$ in K_2. Thus the velocities β_1 and β_2 of the particle relative to K_1 and K_2, respectively, are given by

$$\beta_i = (\Delta x)_i/(\Delta t)_i\ , \quad i = 1, 2\ . \tag{3a}$$

If the particle has energy E_i and momentum p_i relative to the inertial coordinate system K_i, then according to Eq. (2)

$$\beta_i = p_i/E_i\ . \tag{3b}$$

Let K_2 have a velocity β relative to K_1, with $|\beta| < 1$; then the Lorentz transformations give

$$\begin{aligned} (\Delta x)_1 &= \gamma(\Delta x)_2\left(1 + \beta\frac{(\Delta t)_2}{(\Delta x)_2}\right), \\ (\Delta t)_1 &= \gamma(\Delta t)_2\left(1 + \beta\frac{(\Delta x)_2}{(\Delta t)_2}\right) \end{aligned} \tag{4a}$$

and

$$p_1 = \gamma p_2\left(1 + \beta\frac{E_2}{p_2}\right), \quad E_1 = \gamma E_2\left(1 + \beta\frac{p_2}{E_2}\right), \tag{4b}$$

where

$$\gamma = (1 - \beta^2)^{-1/2}\ . \tag{5}$$

From Eqs. (3a), (3b), (4a), and (4b) we obtain

$$\frac{(\Delta x)_1(\Delta x)_2}{(\Delta x)_2{}^2} = \frac{p_1 p_2}{p_2{}^2} = \gamma[1 + (\beta/\beta_2)] \tag{6a}$$

and

$$\frac{(\Delta t)_1(\Delta t)_2}{(\Delta t)_2{}^2} = \frac{E_1 E_2}{E_2{}^2} = \gamma(1 + \beta\beta_2)\ . \tag{6b}$$

Equations (3a), (6a), and (6b) lead to the law of addition of velocities,

$$\beta_1 = \frac{\beta + \beta_2}{1 + \beta\beta_2}\ , \tag{7}$$

from which we obtain

$$1 - \beta_1{}^2 = \frac{(1 - \beta^2)(1 - \beta_2{}^2)}{(1 + \beta\beta_2)^2}\ . \tag{8}$$

Using Eqs. (7) and (8) and remembering that $|\beta| < 1$, we make the following conclusions[10]:

(i) If the particle is a tachyon in K_2, then it is also a tachyon in K_1. Similarly if the particle is a bradyon in K_2, then it is also a bradyon in K_1. That is $|\beta_2| \lesseqgtr 1 \Rightarrow |\beta_1| \lesseqgtr 1$.

(ii) For $|\beta_2| < 1$, as β takes all values in the open interval $R_1 = (-1, +1)$, β_1 also takes all values in R_1. For $|\beta_2| > 1$, as β scans R_1, β_1 scans

$R_2 = (-\infty, -1) \cup (+1, +\infty)$.

(iii) For $|\beta_2| > 1$, $\beta_1\beta_2 < 0$, if and only if $\beta\beta_2 < -1$, while for $|\beta_2| < 1$, $\beta_1\beta_2 < 0$, if and only if $\beta/\beta_2 < -1$.

Furthermore, Eqs. (6a) and (6b), above, lead to the following conclusions:

(iv) $(\Delta x)_1(\Delta x)_2$ and $p_1 p_2$ are either both positive or both negative. Similarly $(\Delta t)_1(\Delta t)_2$ and $E_1 E_2$ are either both positive or both negative.

(v) For bradyons, $|\beta_2| < 1$, the observers in K_1 and K_2 always agree as to the sign of the energy and direction of travel in time. But, for $\beta/\beta_2 < -1$, they disagree as to the sign of the momentum and direction of travel in space.

(vi) For tachyons, $|\beta_2| > 1$, the observers in K_1 and K_2 always agree as to the sign of the momentum and direction of travel in space. But, for $\beta\beta_2 < -1$, they disagree as to the sign of the energy and direction of travel in time.

To derive Lorentz-invariant tachyon properties from the above results, we make use of the principle of relativity.

B. Applications of the Principle of Relativity

We will consider the consequences of the application of the principle of relativity to two problems that are intimately connected. First, if the existence of a certain particle with definite energy and momentum is postulated, what other particles must be simultaneously postulated in order to satisfy the principle of relativity? Second, what requirements does the principle of relativity place on the variation with velocity of the energy, momentum, and direction of motion in space-time of the particle?

Let an observer in an inertial coordinate system K be capable of producing the event e. Suppose that an observer in another inertial coordinate system views this as event f. Then the principle of relativity guarantees the observer in K the capability of producing the event f. By varying the relative velocity β of the two observers, a whole spectrum of events $F_e \equiv \{f(\beta)\,; 0 \leq |\beta| < 1\}$ becomes available to the observer in K.[11] By repeated application of the principle of relativity each event $f(\beta)$ generates a set of events $G_\beta \equiv \{g_\beta(\beta')\,;\ 0 \leq |\beta'| < 1\}$. Owing to the group property of the Lorentz transformations, $G_\beta = F_e$ for $0 \leq |\beta| < 1$. Thus F_e is complete and closed in the sense that each element of F_e generates F_e. Thus the capability of the observer in K to produce the event e guarantees him, and consequently every other inertial observer also, the capability of producing the set of events F_e. Equally important is the fact that the principle of relativity does not guarantee any inertial observer the possibility of observing an event not belonging to F_e. When the event e is the observation of a particle with definite properties, the set of events F_e generated by it gives the minimum range of properties of that particle consistent with the principle of relativity.

We now combine the principle of relativity with results (i) – (vi) to deduce tachyon and bradyon properties. Result (v) applied to bradyons yields well-known consequences. First it tells us that if a bradyon has positive energy in one Lorentz frame, it does so in every frame, so that it is consistent with the principle of relativity to postulate the existence of only positive-energy bradyons. Secondly, it tells us that the time ordering of two events on a bradyon world line is invariant. For example, if its detection follows its creation in one Lorentz frame, it will in every frame. On the other hand, the sign of the momentum, and the spatial ordering of two events on the world line of a bradyon may differ for different observers. One can summarize this by saying that if the event e is the observation of a bradyon with positive energy and momentum, then the set F_e is the observation of bradyons with all possible positive energies, and positive and negative momenta. Such particles will "travel" only forward in time, but both forward and backward in space.

Exactly analogous results follow from result (vi) for tachyons, but with the roles of momentum and energy, and of space and time, interchanged. Thus a tachyon with positive momentum in one Lorentz frame will have positive momentum in every frame, though the sign of its energy may be different for different observers. Likewise, the spatial, but not the temporal, ordering of events along the world line of a tachyon is invariant. Hence, if a tachyon is detected at a more positive value of x than it is created at in one Lorentz frame, the same statement will hold in every Lorentz frame, although there will be some frames in which its detection occurs at an earlier time than its creation. Hence the set F_e corresponding to the observation of a tachyon of positive energy and positive momentum is the observation of tachyons with all possible positive momenta and positive and negative energies. Such particles travel only forward in space, but forward and backward in time.

Another way of looking at these results is the following: On comparing result (iii) with results (v) and (vi), we find that a bradyon can be thought of as reversing the sign of its velocity by reversing its direction of motion in space, while a tachyon reverses the sign of its velocity by reversing its direction of motion in time.

We can summarize these results as follows. Consider the two-dimensional space-time diagram shown in Fig. 1. Consider a particle leaving the origin at $t=0$ with constant velocity, so that its world line is a straight line. The light cone divides

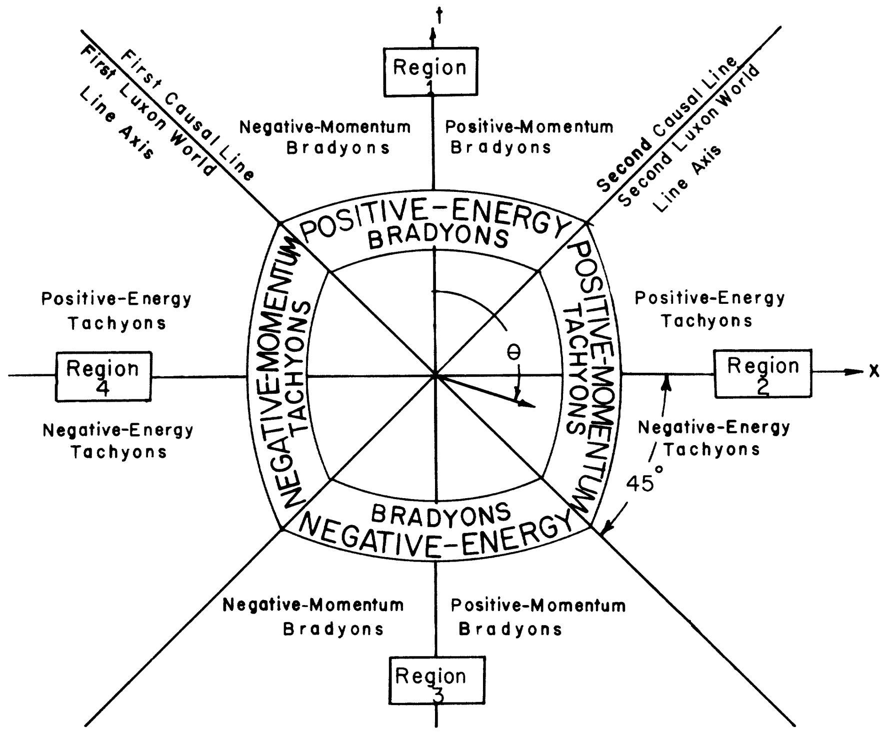

FIG. 1. The light cone is bounded by the two causal lines and separates two-dimensional space-time into four independent regions.

space-time into four regions containing, respectively, the world lines of positive-energy bradyons, positive-momentum tachyons, negative-energy bradyons, and negative-momentum tachyons.[12] We call the two sides of the light cone, i.e., the world lines of luxons (particles moving with the speed of light), causal lines. If the postulated event e is the observation of a particle whose world line runs from the origin into one of these four regions, then the consequent set of particles F_e is precisely all those whose world lines run into the corresponding region. That is each region is generated by any one of its particles and postulating any one or more of the four regions to the exclusion of all the rest satisfies the principle of relativity. Put in a negative way this means that postulating the existence of a particle belonging to one region does not imply the existence of a particle belonging to a different region.

III. THE TOLMAN PARADOX

To study the consequences of the postulated existence of a particular kind of particle, for example, a tachyon with positive energy and positive momentum, it is possible to proceed by one of two completely equivalent procedures. The first is to place different observers in different inertial systems, provide each of them with the postulated particle, and work out the consequences. This is the method that is usually used in analyzing the Tolman paradox. The second method is to put all observers in the same inertial frame and provide each of them with the set of events F_e which is generated by e, the observation of the particle.

Consider two or more inertial observers who are relatively at rest, and the four regions of particles shown in Fig. 1. As proved in Sec. II, the only requirement that the principle of relativity imposes on the spectrum of particles available to these observers is that this spectrum contain regions of particles as a whole. It is easy to see that the existence of particles in any one region, or any two adjacent regions, is not sufficient for the formation of causal loops. On the other hand, any other combination of regions makes the formation of causal loops possible. If only one of the four regions is postulated then the motion of the particles available to the observers is either unidirectional in time or unidirectional in space.[13] In either case when such particles are used as signals, information can never be returned to the point of origin in space-time, and consequently no causal loops can be formed. In the case of two adjacent regions, the inertial observers have available to them particles that can move forward and backward in space and others that can move forward and backward in time. But all of the particles move only forward (or only backward, as the case may be) along the space-time direction defined by the luxon world line separating the two regions. Again the unidirectionality of all the particles along the luxon world-line axis prevents the formation of causal loops. To illus-

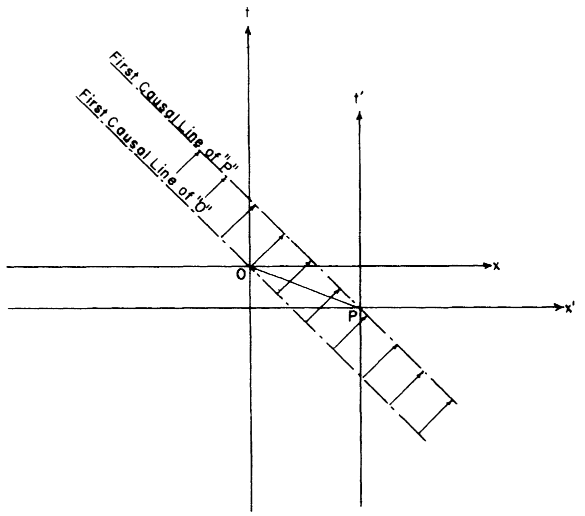

FIG. 2. Transfer of information by use of tachyons between two observers O and P. P can send signals only to those space-time points lying above and to the right of the first causal line through P, and hence cannot return information to O.

trate this, consider the model we are discussing, in which one supposes that one has available particles in regions 1 and 2, i.e., positive-energy bradyons and positive-momentum tachyons. Let us refer to Fig. 2. Suppose an observer located at the origin in space-time sends a signal with a tachyon beam which is detected by an observer at the space-time point P. The only space-time points to which the observer at P can send a signal are those located to the right and above the dashed line, i.e., the first causal line, through P. In particular, the observer at P cannot send a signal which will be detected at $x=0$ and $t\leqslant 0$, and hence no causal loop is possible. Loosely speaking, in using bradyons to make up for the space lost by tachyons, more time is lost than space is gained because for bradyons $(\Delta t)^2>(\Delta x)^2$, and similarly in using tachyons to make up for the time lost by bradyons, more space is lost than time gained, because for tachyons $(\Delta x)^2>(\Delta t)^2$.

On the other hand, if there are also negative-momentum tachyons, i.e., particles in region 4, then it is perfectly possible for the observer at P to use such a particle to send a signal which would be received at $x=0$ and $t<0$. This is, of course, just another statement of the well-known difficulties[3,4,6,7] with the possibility of paradoxical, causality-violating behavior which arise in a theory which includes tachyons. What we have now seen is that the possibility of violations of causality depends explicitly on the assumption, which is made tacitly and automatically in previous discussions of the problem, that if one has tachyons at all, then one has tachyons with both signs of the momentum. As we have seen, in a one-dimensional universe this is not required by the principle of relativity.

We see that each causal line separates two-dimensional space-time into two sections, each of which contains two adjacent regions. Particles whose world lines all lie on the same side of a causal line belong to two adjacent regions and hence cannot be used to form causal loops. The causal line replaces the concept of the light cone in the sense that it divides the "absolute past" from the "absolute future."

Finally, let us indicate briefly why a theory with only positive-momentum tachyons might be of interest (apart from the fact that it does indeed, as we have seen, avoid the usual difficulties with causality). Parker[14] has discussed tachyons, also in the case of only one spatial dimension, in terms of a generalization of the Lorentz transformation which includes transformations to superluminal reference frames, i.e., reference frames with velocities greater than c relative to the "ordinary" subluminal reference frames with which we are familiar. The same generalized Lorentz-transformation equations have been arrived at independently by one of us from a somewhat different viewpoint.[15] Letting p' and E' be the momentum and energy of a particle in a superluminal reference frame, the resulting expression for its momentum, p, in a subluminal frame is

$$p=(\sinh\alpha)p'+(\cosh\alpha)E', \quad -\infty<\alpha<\infty \tag{9}$$

where α is related to the relative velocity of the two reference frames.[14] In a superluminal frame, a tachyon is an "ordinary" particle with speed less than that of light. It is therefore plausible to assume that, in such a frame its energy $E'>0$, just as the energy of a bradyon >0 in a subluminal frame. If $E'>0$, it then follows from (9) that $p>0$.

IV. CONCLUSION

It has been shown that a certain class of tachyons (positive-momentum tachyons) can exist in a one-dimensional space without violating either the principle of relativity or the principle of causality.

---

*Supported in part by the National Research Council of Canada.

†Supported in part by the U. S. Atomic Energy Commission.

[1]We use units such that the velocity of light in a vacuum is unity.

[2]A. Einstein, Ann. Physik 17, 891 (1905). An English translation is to be found in Einstein *et al.*, *The Principle*

of Relativity (Dover, New York, 1923).

[3]R. C. Tolman, *The Theory of Relativity of Motion* (Univ. of California Press, Berkeley, 1971), pp. 54–55.

[4]O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. 30, 718 (1962); G. Feinberg, Phys. Rev. 159, 1089 (1967); O. M. P. Bilaniuk and E. C. G.Sudarshan, Phys. Today 22 (No. 5), 43 (1969).

[5]According to this principle, a negative-energy tachyon with momentum p and traveling backward in time is to be interpreted as a positive-energy tachyon with momentum $-p$ and traveling forward in time.

[6]O. M. P. Bilaniuk, S. L. Brown, B. S. DeWitt, W. A. Newcomb, M. Sachs, E. C. G. Sudarshan, and S. Yoshikawa, Phys. Today 22 (No. 12), 47 (1969).

[7]W. B. Rolnick, Phys. Rev. 183, 1105 (1969); F. A. E. Pirani, Phys. Rev. D 1, 3224 (1970); G. A. Benford, D. L. Book, and W. A. Newcomb, *ibid.* 2, 263 (1970); Paul L. Csonka, Nucl. Phys. B21, 436 (1970).

[8]Among the different formulations of the Tolman paradox, that of DeWitt in Ref. 6 comes nearest to reducing the problem to one of exchange of signals between two observers who are relatively at rest.

[9]R. G. Newton, Phys. Rev. 162, 1274 (1967); Paul L. Csonka, *ibid.* 180, 1266 (1969).

[10]Results (i) and (ii) are also clear from Fig. 6 of the first article in Ref. 4.

[11]Note that $f(0)=e$.

[12]We are here making use of the result obtained previously that negative energy implies backward motion in time, while positive energy implies forward motion in time. This, in addition to the analogous correlation of momentum to motion in space, makes possible superimposing the (x,t) and (p,E) diagrams as done in Fig. 1.

[13]The projection of the world line of these particles on either the time axis (for bradyons) or the space axis (for tachyons) is constantly increasing in absolute value.

[14]Leonard Parker, Phys. Rev. 188, 2287 (1969).

[15]Adel F. Antippa, Université du Québec a Trois-Rivières Report No. UQTR-TH-1, 1970 (unpublished).