Theory of the Detection of Short Bursts of Gravitational Radiation

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It is argued that the short bursts of gravitational radiation which Weber reports most probably arise from the gravitational collapse of a body of stellar mass or the capture of one collapsed object by another. In both cases the bulk of the energy would be emitted in a burst lasting about a millisecond, during which the Riemann tensor would change sign from one to three times. The signal-to-noise problem for the detection of such bursts is discussed, and it is shown that by observing fluctuations in the phase or amplitude of the Brownian oscillations of a quadrupole antenna one can detect bursts which impart to the system an energy of a small fraction of kT. Applied to Weber's antenna, this method could improve the sensitivity for reliable detection by a factor of about 12. However, by using an antenna of the same physical dimensions but with a much tighter electromechanical coupling, one could obtain an improvement by a factor of up to 250. The tighter coupling would also enable one to determine the time of arrival of the bursts to within a millisecond. Such time resolution would make it possible to verify that the radiation was propagating with the velocity of light and to determine the direction of the source.

I. INTRODUCTION

In this paper we discuss the problem of detecting short bursts of gravitational radiation. This is rather different from the detection of continuous radiation, to which most attention has been given.[1] Such bursts have been reported by Weber,[2-4] who uses detectors at Maryland and Chicago in coincidence. Analysis of the time of arrival of these bursts suggests that they may be coming from the direction of the galactic center. We shall argue that the only events which are likely to produce bursts of waves strong enough for Weber to detect and at the rate ~1 per day (reported by Weber) are the collapse of bodies of stellar mass M or the capture of one collapsed object by another. In both these cases we would expect most of the energy of the gravitational radiation to be emitted in a time τ of the order of $10^{-5}M/M_\odot$ sec, during which time the sign of the Riemann tensor components of the radiation field would reverse only a few times. In other words, we expect something like a double or triple pulse rather than a long oscillating signal. We shall analyze the response of a gravitational-wave detector to such a burst.

We show that by observing fluctuations of the phase or amplitude of the Brownian motion of the antenna, one can detect bursts of gravitational waves that would probably not have been detected if one had merely observed the rms amplitude of the detector as Weber does. Applied to a detector like Weber's, which has a very low electromechanical coupling, this method would improve the sensitivity for reliable detection by a factor of about 12. However, by using a detector consisting of two metal bars connected by a piezoelectric transducer, one could possibly improve the sensitivity by as much as 250. This way of obtaining improved sensitivity would seem to be much easier and cheaper than cooling the detector to very low temperatures as has been proposed by a number of workers.

In Sec. II we consider possible sources for the bursts that Weber reports and discuss the nature of the signals they would produce. The response to such a burst of a simple quadrupole detector is considered in Sec. III. In Sec. IV we treat the signal/noise ratio of the detector by a method of fluctuations. An alternative treatment, which gives the same answers, is given in Sec. V in terms of an equivalent circuit for the detector.

II. THE EXPECTED SIGNALS

Gravitational radiation is produced whenever massive bodies accelerate under gravitational or nongravitational forces. However, because of the weakness of the gravitational constant, the rate of energy radiated is normally very small. For example, the earth revolving around the sun radiates about 1 kW at a frequency of 3 cycles per year. The weakness of the gravitational constant also means that gravitational-wave detectors are very inefficient: A flow of 2×10^4 erg/cm^2 sec for a duration of 10 sec in a bandwidth of 0.016 Hz at 1660 Hz is needed to excite Weber's detectors to an amplitude of the same order as that of the Brownian motion of the apparatus. Weber observes the amplitude of his detectors rising above threshold within 0.5 sec of one another. If the duration of the bursts were much longer than 0.5 sec, the amplitudes of the two detectors would rise slowly and would probably not cross the threshold within 0.5 sec of each other as the initial Brownian motion of the two detectors would be different. This would be inconsistent with Weber's report that shortening the coincidence time did not significantly affect the rate of coincidences. A burst of duration less than 0.5 sec must have a bandwidth of at least 0.3 Hz and so each burst must carry an energy of at least 4×10^6 erg/cm^2. If one allowed for an equal energy in the other polarization which Weber does not observe, the total energy would be at least 8×10^6 erg/cm^2. It is not reasonable to suppose that there is gravitational radiation only in a very narrow bandwidth centered on the frequency of Weber's detector. Therefore we presume that either each burst has a much larger bandwidth, say of the order of 1000 Hz, and so carries an energy of the order of 1.2×10^{10} erg/cm^2 per polarization, or that each burst has a narrow bandwidth but that there are something like a thousand bursts at other frequencies for every one that Weber observes. In either case the energy flux is at least 10^{10} erg/cm^2 per day.

A source emitting at 1660 Hz, the frequency at which Weber observes, must presumably be undergoing collective dynamical motions at that frequency. This would seem to imply that its size was less than 100 km, which is the distance light would travel in half a cycle. Possible sources of this size might be a neutron star, a star undergoing gravitational collapse, or the capture of one collapsed object by another. A neutron star would have an available nuclear and gravitational energy equal to about 1% of its rest mass, i.e., about 2×10^{52} erg. Therefore to produce a gravitational-wave burst of 1.2×10^{10} erg/cm^2 it would have to be within a distance of 3×10^{20} cm, i.e., 100 parsecs (pc). A distance of 100 pc includes about one part in 10^4 of the galactic disk. If, therefore, sources are distributed uniformly throughout the galactic disk, ten thousand of them are using up all their available energy every day, a number which seems unreasonably high. This number is not changed if one supposes that the neutron stars emit in a narrower bandwidth; for although the sources that Weber observes can be further away, there will be a corresponding number at other frequencies which he does not observe. The only reasonable way in which neutron stars might be responsible for the bursts that Weber observes is if we happen to be untypically near a neutron star that was undergoing violent eruptions: A neutron star at 1 pc would have enough energy to produce a burst of 4×10^6 erg/cm^2 once a day for about 150 000 years.

Gravitational collapse, or the capture of one collapsed object by another, would have the advantage over neutron stars as a source of gravitational radiation that a much greater fraction of the rest-mass energy might be released. This makes the energy problem easier but the amount is still embarrassingly high.[5-7] A source at the galactic center needs an energy of about 1.4×10^{56} erg ($70M_\odot$) to produce a burst of 1.2×10^{10} erg/cm^2 at the earth.

In a collapse or a capture most of the energy would not be released until matter has fallen to near the Schwarzschild radius or one collapsed object was near to the Schwarzschild radius of the other one. Thus one would expect most of the energy of the gravitational radiation to be in a period τ of the order of the dynamical time at this stage, i.e., $\tau\sim GMc^{-3}\sim10^{-5}MM_\odot^{-1}$ sec.

The energy flux in the gravitational wave at the observer is

$$F(t)\equiv c^7(4\pi G)^{-1}\left\{\left[\int^t R_{1010}(u)du\right]^2+\left[\int^t R_{1020}(u)du\right]^2\right\}\text{ erg/cm}^2\text{ sec}\,, \tag{1}$$

where the wave is taken to be traveling in the three directions and the Riemann tensor components R_{1010} and R_{1020} represent the two states of polarization of the wave. From this formula it can be seen that for a burst of finite energy the time integrals of the Riemann tensor components over the duration of the burst must be zero. This means that the sign of the Riemann tensor components must reverse during the burst.

In the linearized theory one has the relation

$$R_{\alpha 0\beta 0}=\frac{G}{3c^6}\frac{1}{r}\frac{d^4}{dt^4}D_{\alpha\beta}\,, \tag{2}$$

where $D_{\alpha\beta}$ is the quadrupole moment of the source and r is the distance from the source.

From this formula it follows that if the quadrupole is initially and finally time-independent (as one would expect for a gravitational collapse), then not only is the time integral of the components of the Riemann tensor zero but the second and third integrals as well, i.e.,[8]

$$\int_0^\tau dt\left[\int_0^t dt'\left(\int_0^{t'} R_{\alpha 0 \beta 0}(t'')dt''\right)\right]=0\,. \qquad (3)$$

This implies that the components of the Riemann tensor must change sign at least three times during the burst. For gravitational capture, on the other hand, the quadrupole moment will initially be a quadratic function of the time and so only the first time integral of the components of the Riemann tensor will be zero, and the components of the Riemann tensor need change sign only once during the burst.

In the nonlinear theory one does not have the simple relation (2) between the source and the radiation field. Thus Eq. (3) will not in general be satisfied for a gravitational collapse, but one could expect the transition from linear to nonlinear theory to preserve the *qualitative* feature that the sign of the components of the Riemann tensor change sign at least three times during the burst.

One might therefore be able to distinguish observationally between collapse or capture by examining the wave form of the burst. The simplest models for the bursts in the two cases are shown in Fig. 1.

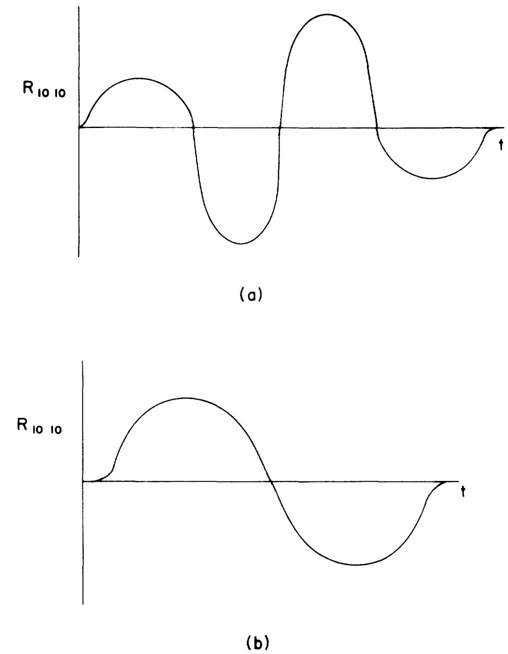

FIG. 1. Possible wave forms of the radiation field arising from (a) gravitational collapse and (b) gravitational capture.

III. THE RESPONSE OF THE DETECTOR

A simple quadrupole gravitational-wave detector consists basically of two masses m separated by a distance l and connected by a spring to give an oscillation angular frequency ω_0. The masses and the spring need not exist separately but can be combined in the form of a solid bar. The length of the bar will then be determined by the speed of sound in the material and the desired resonant frequency ω_0. The gravitational wave provides a relative force between the two masses which is proportional to their separation. Thus the equation of motion of the system is

$$\frac{d^2x}{dt^2}+\frac{\omega_0}{Q}\frac{dx}{dt}+\omega_0{}^2x=-c^2lR_{1010}\,, \qquad (4)$$

where x is the change in separation of the masses, Q is the quality factor arising from the mechanical damping, and we take the line of center of the two masses to lie in the 1 direction.

The motion induced by a gravitational burst is

$$x=A\exp[-\omega_0(2Q)^{-1}+i\omega_0]\,t\,, \qquad (5)$$

where

$$A=-c^2l(i\omega_0)^{-1}\int_0^t e^{\omega_0 u(2Q)^{-1}}R_{1010}(u)e^{i\omega_0 u}du\,. \qquad (6)$$

The duration τ of the burst we are considering will normally be much less than the damping time $Q\omega_0{}^{-1}$. This means that the motion of the detector immediately after the burst will be nearly independent of Q and will in fact be simply proportional to the Fourier components of the burst at the resonant frequency ω_0. From the discussion in Sec. II we expect the spectrum of the burst to have a maximum at a frequency of the order of $\omega_1=2\pi\tau^{-1}$ and to have a width $\Delta\omega$ of the same order as ω_1. The energy imparted to the detector by the burst, $4^{-1}m\omega_0{}^2|A|^2$, will be a maximum if the resonant frequency ω_0 is chosen to be of the same order as ω_1. Then

$$4^{-1}m\omega_0{}^2|A|^2\simeq 4\pi^3Gml^2\mathcal{E}(c^3\tau)^{-1}\,, \qquad (7)$$

where $\mathcal{E}$ is the energy per unit area. Clearly the energy imparted to the detector will be greater the greater the length l, but l is limited by the speed of sound in the material, c_s, of the detector and the desired resonant frequency ω_0, $l\leqslant c_s 2\pi\omega_0{}^{-1}$; then,

$$\tfrac{1}{4}m\omega_0{}^2|A|^2\leqslant 4\pi^3c^{-1}Gm\mathcal{E}c_s{}^2c^{-2}\tau\,. \qquad (8)$$

We expect that the energy $\mathcal{E}$ and the duration of the burst τ will both be proportional to the mass of the emitting object; thus Eq. (8) suggests that it is more favorable to construct long detectors and to look for bursts of long duration arising from the collapse of objects of large mass. On the other hand, very massive collapses probably occur less frequently and it is more difficult to isolate the effects of extraneous mechanical and electrical disturbances if the frequency is low. We would therefore suggest that about 1000 Hz is probably the optimum frequency at which to look for gravitational bursts. This is in fact about the frequency at which Weber observes. At this frequency one would be looking for bursts emitted by systems of $100M_{\odot}$.

IV. THE SIGNAL/NOISE RATIO

Assuming that extraneous electrical and mechanical disturbances can be eliminated, the main sources of noise are the Brownian movements of the detector masses, the Johnson noise associated with the electrical losses in the transducer, and the noise produced in the first stage of the amplifier. We shall discuss first the Brownian movements and show that their effect can be minimized by using a detector with a fairly high Q and observing its motion with a good time resolution.

The two masses and the spring of the detector form a simple harmonic oscillator. By the law of equipartition of energy the oscillation will have an average energy of oscillation of kT provided that it is in equilibrium with its surroundings. The amplitude x_B of the thermal oscillations will be given by

$$m\omega_0^2 x_B^2 = 4kT\ . \tag{9}$$

For $m \sim 10^6$ g, $T \sim 300\,^{\circ}K$, and $\omega_0 \sim 10^4$, this gives $\sim 4\times 10^{-14}$ cm. One might think that one could detect only bursts that induced an amplitude greater than this or equivalently that imparted to the detector an energy larger than kT. However, it is in fact possible to detect much smaller bursts.

For although the average energy of the thermal oscillation is kT in equilibrium, the *noise power*, i.e., the rate at which energy enters and leaves the thermal oscillations of the detector, is very small if Q is large (we are grateful to P. Aplin for pointing this out). This can be seen as follows: One can regard the thermal oscillations as arising from a large number of small impulses applied to the detector with random phases and amplitudes. The oscillations induced by each small impulse will die away in the damping time $Q\omega_0^{-1}$. Thus at any time the energy of the detector arises effectively from the random impulses applied in the last $Q(2\pi)^{-1}$ cycles. Since the phases of the oscillations induced by the small impulses are uncorrelated, it follows that the mean value of the energy of the oscillation induced by the small impulses in one cycle is $kT2\pi Q^{-1}$. Thus by observing the change over n cycles of the phase or amplitude of the oscillations of the detector one could detect a gravitational burst that imparted to the detector in that time an energy greater than $2\pi nkTQ^{-1}$.[9] Obviously, using a smaller value of n gives a better time resolution.

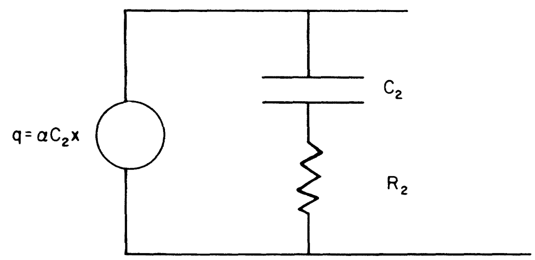

FIG. 2. The equivalent circuit of the transducer.

The most practical way to observe the motions of the detector seems to be to use a piezoelectric transducer[1] whose output is fed into a high-impedance amplifier. A piezoelectric transducer can be represented electrically as a charge generator in parallel with a capacitance and a resistance in series (Fig. 2). Here C_2 is the electrical capacitance of the transducer, α is its voltage output per unit change of separation x of the two masses of the detector, and $R_2 = (\omega_0 C_2)^{-1}\tan\delta$ represents the electrical losses in the transducer, where $\tan\delta$ is the "dissipation factor."

The Johnson noise of the resistor produces a mean-square voltage in a bandwidth $\Delta\omega$ of

$$V_T^2 = 2\pi^{-1}kTR_2\Delta\omega = 2\pi^{-1}kT\tan\delta\,\Delta\omega(\omega_0 C_2)^{-1}\ . \tag{10}$$

This may be compared to the mean-square voltage produced by the Brownian movements of the detector masses

$$V_B^2 = \alpha^2 x_B^2\ , \tag{11}$$

where x_B is given by (9). Thus,

$$V_B^2 = 4\alpha^2 kT(m\omega_0^2)^{-1}\ . \tag{12}$$

By observing fluctuations over n cycles, one can detect against the Brownian noise a burst that imparts to the detector an energy greater than $2\pi nkTQ^{-1}$. For such a burst the signal output voltage V_S satisfies $V_S^2 \geq 2\pi n V_B^2 Q^{-1}$. In order to detect a signal against both the transducer noise and the Brownian noise, one needs

$$V_S^2 \geq V_T^2 + 2\pi n Q^{-1} V_B^2\ .$$

The greatest sensitivity, therefore, is obtained by choosing the value of n for which

$$V_T^2 = 2\pi n V_B^2 Q^{-1}\ .$$

This gives

$$n^2 = Q\tan\delta\,(8\pi^3\beta)^{-1}\ , \tag{13}$$

where $\beta=\alpha^2C_2(m\omega_0^2)^{-1}$ represents the proportion of elastic energy of the detector that can be extracted electrically from the transducer in one cycle. Thus one can detect gravitational waves which impart to the detector an energy of

$$(2\tan\delta)^{1/2}(\pi\beta Q)^{-1/2}kT.$$

For Weber's detector, $Q=10^5$, $\beta=5\times10^{-6}$, and $\tan\delta=5\times10^{-3}$.[10] Thus the optimum value of n is 600. This would give a time resolution of about 0.4 sec (about what Weber uses) and would enable one to detect bursts which imparted to the detector an energy of about $\frac{1}{12}kT$. For a broad spectrum burst of the type we have considered this would correspond to an energy flow in one polarization $\mathcal{E}$ of 10^9 erg/cm^2. If, however, instead of observing fluctuations in the phase or amplitude of the oscillations of the detector one observes only whether the amplitudes of these oscillations increase suddenly (as Weber does), one could detect bursts of this intensity only if the phase of the oscillations induced by the burst coincided with the phase of the Brownian oscillations of the detector. This would mean that the probability of observing such a burst on a single detector would be about 1 in 4, and the probability of obtaining a coincidence of two detectors would be about 1 in 16. If, therefore, the bursts that Weber reports correspond to an energy of $\frac{1}{12}kT$, there must be 16 of them for each one that Weber observes, and the total energy flow must still be greater than 10^{10} erg/cm^2 per day.

In order to obtain greater sensitivity one would like to make β nearer unity. In fact, $\beta=\kappa^2\gamma$, where γ is the fraction of the elastic energy of the detector that is actually in the piezoelectric material and κ is a piezoelectric coupling constant which depends only on the material of the transducer. Values of κ range from 0.1 for quartz to about 0.6 for lead zirconate titanate. In order to make β larger, one wants to use a material like lead zirconate titanate and make γ higher. One way of doing this would be to use the transducer as a spring connecting two metal bars (Fig. 3). (This configuration was suggested by P. Aplin.) By suitably choosing the cross-sectional area of the transducer one can arrange that γ is 0.2 and so obtain a value of β of 0.07. Since the mechanical losses of the piezoelectric material are likely to be much greater than those in the metal bars, the mechanical Q of the detector would be the mechanical Q of the piezoelectric material divided by γ. One can obtain lead zirconate titanate with a Q of about 1000 and so the effective Q might be about 5000, although it would probably be rather lower because of mechanical losses in the cement joining the transducers to the bar, etc. The optimum value

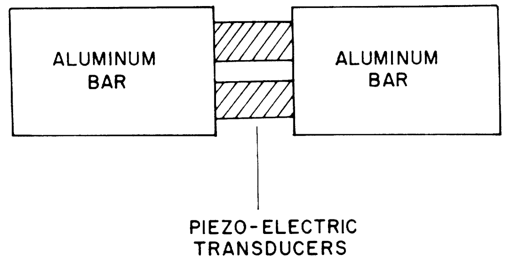

FIG. 3. A possible configuration for a detector with a high value of β, i.e., a tight electromechanical coupling.

of n for such a detector would be about unity. This would give a time resolution of about 10^{-3} sec and enable one to detect bursts which imparted in that time to the detector an energy greater than $\frac{1}{250}kT$. For $m\sim10^6$ g and an effective value of l of about 100 cm,[11] this would correspond to an energy flow in one polarization of 4×10^7 erg/cm^2 in 1 msec at the earth, and to a total energy emitted in one polarization of at least $0.2M_\odot$ if the source is at the galactic center. Of course, to be sure that the observed fluctuations of phase or amplitude of the oscillations of the detector are really due to gravitational bursts and not simply due to larger than average input of noise energy, one would have to set a threshold of several times the rms noise fluctuation in phase and amplitude. If

$$B\exp[-\omega_0(2Q)^{-1}+i w_0 t]$$

represents the oscillations of the detector induced by the noise energy entering in one cycle, then B will be distributed according to

$$P(z)dz=z_0^{-1}e^{-z/z_0}dz, \tag{14}$$

where $z=|B|^2$, $z_0=|B_0|^2$, and B_0 is the rms amplitude of the oscillation induced in one cycle. A threshold for $|B|^2$ of $25|B_0|^2$, therefore, would be exceeded by chance less than once a year. Such a threshold would enable one to detect sources at the galactic center which emitted at least $5M_\odot$ in gravitational radiation.

V. THE EQUIVALENT CIRCUIT

One can also discuss the signal/noise ratio in terms of an equivalent electrical circuit for the detector.[12] This may be derived as follows. The equivalent circuit of the transducer contains a charge generator $q=\alpha C_2 x$, where x obeys Eq. (4). This equation is the same as that for the displaced charge q in a series LCR circuit with a voltage generator V if $L_1C_1=\omega_0^{-2}$, $Q=R_1^{-1}L_1^{1/2}C_1^{-1/2}$, and $V=-c^2lR_{1010}(\alpha C_2L_1)^{-1}$. The remaining relation between L, C, and R is specified by requiring that the energy in the LCR circuit is equal to the energy of the motion of the detector. This gives

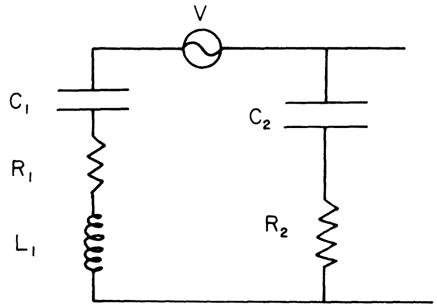

FIG. 4. The equivalent circuit of the detector and transducer.

$$L_1 = (\omega_0^2 \beta C_2)^{-1},$$
$$C_1 = \beta C_2,$$
$$R_1 = (Q\omega_0 \beta C_2)^{-1},$$

and

$$V = -c^2 l R_{1010}(2\omega_0)^{-1} m^{1/2}(\beta C_2)^{-1/2}.$$

The equivalent circuit of the detector and transducer is therefore given by Fig. 4.[13]

We shall assume that the output of this equivalent circuit is fed into a high-impedance amplifier with gain A and that the amplifier output is divided in the ratio Z_3/Z_4 (Fig. 5). In this figure Z_1 represents the impedance of the series $L_1C_1R_1$ and Z_2 represents the impedance of the series C_2R_2. The impedances Z_3 and Z_4 are chosen as $Z_3 = D(Z_1 + Z_2)^{-1}$ and $Z_4 = HZ_2^{-1}$, where D and H are constant with $D >> H$. The Z_3, Z_4 circuit "undoes" the effect of the resonance of the detector and gives an output signal voltage

$$V_S = AHD^{-1} V$$
$$= -AHD^{-1}c^2 l R_{1010}(Z\omega_0)^{-1} m^{1/2}(\beta C_2)^{-1/2}.$$

It is not necessary to use such an inverse circuit but it is convenient for discussing the signal/noise ratio. The impedances Z_3 and Z_4 could be realized physically by a parallel LCR circuit and a parallel LR circuit respectively, though one might need to use a superconducting inductance in Z_3 to obtain a sufficiently high Q. It would probably be more convenient to simulate the Z_3, Z_4 circuit electronically.

Superimposed on the output signal voltage will be the Johnson noise produced by R_1 which represents the Brownian noise of the detector. This will produce at the output a flat noise spectrum with a mean-square voltage

$$V_B^2 = (AHD^{-1})^2 2kT\pi^{-1}(Q\omega_0\beta C_2)^{-1}$$

per unit bandwidth. The transducer noise produced by the resistance R_2 will give at the output a mean-square voltage of

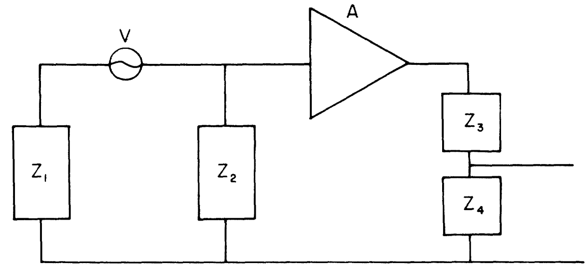

FIG. 5. The equivalent circuit for discussing the signal/noise ratio.

$$V_T^2 = (AHD^{-1})^2 2kT\pi^{-1}\tan\delta\,(\omega_0 C_2)^{-1}|Z_1|^2|Z_2|^{-2}$$
$$= (AHD^{-1})^2 2kT\pi^{-1}\tan\delta\,(\omega_0 C_2)^{-1}(\beta\omega_0)^{-2}$$
$$\times[\omega^2 Q^{-2} + \omega_0^{-2}(\omega^2 - \omega_0^2)^2]$$

per unit bandwidth. This has a sharp minimum at the resonant frequency ω_0. The noise produced by the amplifier will have a rather similar spectrum at the output. Using modern techniques, it seems possible to reduce the amplifier noise below the transducer noise and it will be neglected.

Suppose now that the output of the circuit in Fig. 5 is fed into a filter of bandwidth $\Delta\omega$. If the signal is of the form suggested in Sec. 2, i.e., a burst of one to three cycles, its Fourier transform will have a maximum at a frequency ω_1 of the order of $2\pi\tau^{-1}$ and a half-width of the same order. Therefore, if the filter pass band is centered at ω_1, the *amplitude* of the transmitted signal will be $V_S\Delta\omega\omega_1^{-1}$ and the *power* will be $V_s^2(\Delta\omega)^2\omega_1^{-2}$ for $\Delta\omega \le \omega_1$. This behavior distinguishes short bursts from continuous incoherent radiation where the power is proportional to $\Delta\omega$. It is the reason why it is desirable to use a fairly large value of $\Delta\omega$, i.e., good time resolution.

If the resonant frequency ω_0 is chosen to be equal to ω_1,[14] the filter will transmit a Brownian noise power approximately equal to

$$(AHD^{-1})^2 2kT\pi^{-1}(Q\omega_0\beta C_2)^{-1}\Delta\omega$$

and a transducer noise power

$$(AHD^{-1})^2 kT(3\pi)^{-1}\tan\delta\,\beta^{-2}C_2^{-1}(\Delta\omega)^3\omega_0^{-3}.$$

The optimum value of $\Delta\omega$ will be the smaller of ω_0 and the value for which the transmitted noise power equals the transmitted transducer noise power. This gives

$$(\Delta\omega\omega_0^{-1})^2 = 6\beta(Q\tan\delta)^{-1}$$

which agrees almost exactly with Eq. (13). With this value of $\Delta\omega$ one could detect against the noise a short burst in which the amplitude of R_{1010} was of the order of

$$4\sqrt{2}\,\omega_0^2 kT(2\beta)^{1/2}(c^4 ml^2)^{-1}(3\pi^2\tan\delta)^{-1/2}.$$

This would correspond to an energy imparted to the detector of

$$2kT(2\tan\delta)^{1/2}(3\pi^2\beta Q)^{-1/2},$$

which agrees well with the estimate in Sec. IV.

VI. CONCLUSION

We have shown that by using an antenna with tight electromechanical coupling one could detect sources at the center of the Galaxy which emitted bursts carrying an energy of about $5M_\odot$. This energy still seems rather high, but it is considerably less than that needed to explain the events reported by Weber. One could increase the sensitivity by somewhat lowering the threshold or using several detectors in coincidence. The short time resolution of these detectors would make it possible to measure the difference in the time of arrival of a burst at two stations separated by more than 300 km. A chain of four stations would enable one to determine the direction and velocity of the signal. This would enable one to eliminate seismic disturbances which travel with the velocity of sound and would provide a test of general relativity, since there are other theories in which gravitational radiation travels with a different velocity from that of electromagnetic waves.[15]

ACKNOWLEDGMENT

We are greatly indebted to P. Aplin, who aroused our interest in the problem of the detection of short bursts of gravitational radiation and who suggested many of the ideas in this paper.

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[10] These values can be deduced from the equivalent circuit given in J. Weber, Lett. Nuovo Cimento 4, 653 (1970).

[11] The effective value of l is about $2\pi^{-1}$ times the total length of the detector.

[12] J. Weber, Lett. Nuovo Cimento 4, 653 (1970).

[13] The resonant frequency of this circuit is ω_0 when the output of the transducer is short circuited. When it is not short circuited the resonant frequency is $(1+\beta)^{1/2}\omega_0$.

[14] Taking $\omega_0 < \omega_1$ improves the signal/(Brownian noise) ratio but makes the signal/(transducer noise) ratio worse. One cannot make $\omega_0 > \omega_1$ without reducing the length of the detector.

[15] S. Deser and B. E. Laurent, Ann. Phys. (N.Y.) 50, 76 (1968).