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Determination of f-ω-ω Coupling Constants from a Veneziano Model and the $f \to \omega + \gamma$ Decay

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A Veneziano model, previously constructed, relates the coupling of f to two vector mesons in terms of the couplings of the particle exchanged in the crossed channels. It is here used to calculate explicitly the coupling of the f meson to two ω mesons, as well as its radiative decay into $\omega + \gamma$.

In a recent paper[1] (hereafter called I), we investigated the scattering process $V_1 + \pi_\alpha \to V_2 + \pi_\beta$ in the frame of a Veneziano model. The model presented in I was constructed for isoscalar vector mesons V_1, V_2 of arbitrary mass, including photons. For the latter case, the amplitude fulfills gauge invariance in the limit $m_i \to 0$. The dual character of the model, supplemented by the requirement of absence of daughters, enabled us to relate the general coupling of f to two vector mesons in terms of the couplings of the particle exchanged in the crossed channels [see Eqs. (5.3) and (5.14)–(5.16) in I].

In this note we make use of this property tc calculate explicitly the coupling constants of the f meson to two ω mesons, as well as its radiative decay into $\omega + \gamma$.

The contribution of the f exchange in the s channel of the scattering process $V_1 + V_2 \to f \to \pi_\alpha + \pi_\beta$ [Eq. (4.4) of I] is given by

$$T_f^{\lambda_1\lambda_2} = \tfrac{1}{4} G_{V_1V_2} \Delta_\mu \Delta_\nu D^{\mu\nu\rho\sigma} \{\alpha \epsilon_\rho^{\lambda_1}(k) \epsilon_\sigma^{\lambda_2}(p) + \beta_1 [\epsilon^{\lambda_1}(k)\cdot p]\, \epsilon_\rho^{\lambda_2}(p) k_\sigma + \beta_2 [\epsilon^{\lambda_2}(p)\cdot k]\, \epsilon_\rho^{\lambda_1}(k) p_\sigma$$
$$+ \gamma [\epsilon^{\lambda_1}(k)\cdot\epsilon^{\lambda_2}(p)]\, k_\rho p_\sigma + \delta \epsilon_\rho{}^{\alpha\beta\gamma} \epsilon_\alpha^{\lambda_1}(k) k_\beta L_\gamma \epsilon_\sigma{}^{\alpha'\beta'\gamma'} \epsilon_{\alpha'}^{\lambda_2}(p) p_{\beta'} L_{\gamma'}\}, \tag{1}$$

where q_1, q_2, k, p are the momenta of the pions and vector mesons, respectively, $\Delta_\mu = (q_1 - q_2)_\mu$, $L_\mu = (q_1 + q_2)_\mu$, and $\epsilon^{\lambda_1}(k)$, $\epsilon^{\lambda_2}(p)$ are the polarization vectors of the vector mesons. $\alpha, \beta_1, \beta_2, \gamma, \delta$ are proportional to the five general coupling constants of a tensor tc two vector mesons. $D^{\mu\nu\rho\sigma}$ is the tensor meson propagator given by

$$D_{\mu\nu\rho\sigma} = [\tfrac{1}{2}(g_{\mu\rho} - L_\mu L_\rho/L^2)(g_{\nu\sigma} - L_\nu L_\sigma/L^2) + \tfrac{1}{2}(g_{\mu\sigma} - L_\mu L_\sigma/L^2)(g_{\nu\rho} - L_\nu L_\rho/L^2)$$
$$- \tfrac{1}{3}(g_{\mu\nu} - L_\mu L_\nu/L^2)(g_{\rho\sigma} - L_\rho L_\sigma/L^2)](s - M_f^2)^{-1} \tag{2}$$

and

$$G_{V_1V_2} = 4 g_{f\pi\pi} g_{fV_1V_2}. \tag{3}$$

On the other hand, $G_{V_1V_2}$ is related by duality to the coupling constants of the ρ meson, exchanged in the t and u channels, as follows:

$$G_{V_1V_2} = g_{V_1\pi\rho} g_{V_2\pi\rho}/\mu^2, \tag{4}$$

where μ is the pion mass and the coupling constants $g_{V_i\pi\rho}$ are defined through the vertex function

$$T_{V_i\pi\rho} = g_{V_i\pi\rho} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{(\rho)}^\alpha p_{(\rho)}^\beta \epsilon_{(V_i)}^\gamma p_{(V_i)}^\delta. \tag{5}$$

$g_{f\pi\pi}$ is defined in the $f \to \pi + \pi$ transition amplitude $T_{f\to\pi\pi}$ by

$$T_{f\to\pi\pi} = g_{f\pi\pi} \tau^{\mu\nu} \Delta_\mu \Delta_\nu, \tag{6}$$

and finally the transition amplitude $T_{f\to V_1V_2}$ is

$$T_{f\to V_1V_2} = g_{fV_1V_2} \tau^{\mu\nu} [\alpha \epsilon_\mu^{V_1} \epsilon_\nu^{V_2} + \beta_1 (\epsilon^{V_1}\cdot p) \epsilon_\mu^{V_2} k_\nu + \beta_2 (\epsilon^{V_2}\cdot k) \epsilon_\mu^{V_1} p_\nu + \gamma (\epsilon^{V_1}\cdot\epsilon^{V_2}) p_\mu k_\nu + \delta \epsilon_\mu{}^{\alpha\beta\gamma} \epsilon_\alpha^{V_1} k_\beta L_\gamma \epsilon_\nu{}^{\alpha'\beta'\gamma'} \epsilon_{\alpha'}^{V_2} p_{\beta'} L_{\gamma'}], \tag{7}$$

where $\tau_{\mu\nu}$ in (6) and (7) is the polarization tensor of the f meson.

The partial decay width for f decaying into particles x and y is

$$\Gamma_{f\to xy} = \frac{(2\pi)^4}{5\times 8M_f E_x E_y}\int \sum_{\lambda_f,\lambda_x,\lambda_y} |T_{f\to xy}|^2 \delta(M_f - E_x E_y)\delta^3(\vec{p}_x+\vec{p}_y)\frac{d^3p_x\, d^3p_y}{(2\pi)^6}$$

$$= \frac{|p|}{40\pi M_f^{\ 2}}\sum_{\lambda_x,\lambda_y}|T_{f\to xy}|^2 . \tag{8}$$

Using the experimental value[2] $\Gamma_{f\to 2\pi} = 151$ MeV, one obtains from (6) and (8)

$$g_{f\pi\pi}^{\ \ 2}/4\pi = 4/M_f^{\ 2} . \tag{9}$$

Inserting this value into (3) and (4), one has

$$(g_{fV_1V_2}M_f)^2 = 8(\mu G_{V_1V_2})^2 . \tag{10}$$

In order to calculate $G_{\omega\omega}$ and $G_{\omega\gamma}$ from (4), we use $g_{\omega\rho\pi}^{\ \ 2}/4\pi = 0.6$, which corresponds to a width for $\omega\to 3\pi$ of 11 MeV, and $g_{\rho\pi\gamma}^{\ \ 2}/4\pi = \frac{1}{9}g_{\omega\pi\gamma}^{\ \ 2}/4\pi = 0.02\alpha$ (which corresponds to a width for $\rho\to\pi\gamma$ of 0.13 MeV), where α is the fine-structure constant. In Table I we list the different coupling constants of f to two vector mesons in the general case as well as for the two particular cases discussed here.

For the decay $f\to\omega+\gamma$ one has

$$\sum_{\lambda_f,\lambda_\gamma,\lambda_\omega}|T_{f\to\omega\gamma}|^2 = g_{f\omega\gamma}^{\ \ 2}\{3\alpha'^2 + (k^2/m^2)[\alpha' + \tfrac{1}{2}\beta'(M+m)^2][\alpha' + \tfrac{1}{2}\beta'(M-m)^2] + \tfrac{1}{3}(\gamma' - \alpha')^2\}, \tag{11}$$

where

$$\alpha' \equiv \alpha - \delta(Mk)^2, \quad \beta' \equiv \beta_2 + \tfrac{1}{2}\delta(M^2+m^2), \quad \gamma' \equiv \gamma - \delta M^2 ,$$

and

$$M \equiv M_f , \quad m \equiv m_\omega .$$

From (8), (11), and the values given in Table I, we calculate the corresponding partial decay width:

$$\Gamma_{f\to\omega\gamma} = 80\ \text{keV} \tag{12}$$

TABLE I. The values of the f-vector-vector coupling constants [for definition, see Eq. (7)]. Column A contains the five general independent coupling constants. In B are the values for f-ω-ω (here, because of the identity $V_1 = V_2$, there are only four independent coupling constants, with $\beta_1 = \beta_2$). In C are the values for f-ω-γ (in this case there are only three independent coupling constants, since gauge invariance imposes $\beta_1\, k\cdot p + \alpha = 0$, $\beta_2 + \gamma = 0$). The numbers pertaining to α, β_1, β_2, γ, and $\delta M_f^{\ 2}$ listed in the table are exact only to within an order of $2\mu^2/m^2$.[a] This has an effect of $\approx 5\%$ in the rate.

	A	B	C
$(g_{fV_1V_2}M_f)^2$	$8(\mu G_{V_1V_2})^2$	432	10.5×10^{-2}
α	$k\cdot p - 4k^2$	$0.61M_f^{\ 2}$	$-8.0\times 10^{-2}M_f^{\ 2}$
β_1	$1 - 2m_2^{\ 2}/M_f^{\ 2}$	0.25	0.25
β_2	$1 - 2m_1^{\ 2}/M_f^{\ 2}$	0.25	1
γ	-1	-1	-1
$\delta M_f^{\ 2}$	-4	-4	-4
$k\cdot p$	$(M_f^{\ 2} - m_1^{\ 2} - m_2^{\ 2})$	$0.11M_f^{\ 2}$	$0.3M_f^{\ 2}$
$4k^2$	$\dfrac{[M_f^{\ 2} - (m_1+m_2)^2][M_f^{\ 2} - (m_1-m_2)^2]}{M_f^{\ 2}}$	$-0.5M_f^{\ 2}$	$0.4M_f^{\ 2}$

[a] The origin of this inaccuracy is that the coupling constants were determined at the point $\alpha_s = 2$ [Eq. (5.14) of I], which for a degenerate ρ-f trajectory with a slope $\epsilon' = [2(m_\rho^{\ 2} - \mu^2)]^{-1}$ corresponds to a squared mass $M_f^{\ 2} = 90\mu^2$ instead of the physical value $M_f^{\ 2} = 81\mu^2$.

and

$$\Gamma_{f\to\omega\gamma}/\Gamma_{f\to 2\pi}=5\times 10^{-4}\,. \tag{13}$$

In concluding, we should like to mention that Renner has recently calculated these couplings[3] by using a model based on the assumption of tensor-meson dominance for the energy-momentum tensor and vector-meson dominance for the electromagnetic current. It should be noted that, although the two calculations are based on different models, the value of the partial decay width $\Gamma_{f\to\omega\gamma}$ obtained by Renner is 0.20 MeV. In particular, the two couplings $\mathcal{G}_1$ and $\mathcal{G}_3$ derived from tensor-meson dominance[3] are very close to the corresponding ones in our model, while for those derived from using vector-meson dominance the agreement is not as good.

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[1]N. Levy and P. Singer, Phys. Rev. D 3, 1028 (1971).

[2]Particle Data Group, Phys. Letters 33B, 1 (1970).

[3]B. Renner, Nucl. Phys. B30, 634 (1971).

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$$\mathcal{G}_1=\tfrac{1}{4}\gamma' g_{fV_1V_2}M_f\,,$$

$$\mathcal{G}_2=\tfrac{1}{4}\delta M_f{}^2 g_{fV_1V_2}M_f\,,$$

$$\mathcal{G}_3=-\tfrac{1}{4}\beta'_2 g_{fV_1V_2}M_f\,,$$

$$\mathcal{G}_4[-M_f{}^2+\alpha(k^2+k'^2-2M_V{}^2)]=-\tfrac{1}{2}\alpha' g_{fV_1V_2}M_f.$$