# Natural- versus Unnatural-Parity Exchange in Forward Scattering

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The assumption of one-particle (pole) exchange at the quark level for $s\to\infty$ leads to the decoupling, in the forward direction, of natural-parity particles (leading Regge trajectories) from octet-decuplet baryonic vertices. In the differential cross section of the reaction $\pi p\to\pi\Delta$ a dip is predicted at $t\simeq 0$, for high $s$. The study of this dip can lead to an estimate of the *upper* limit of the quark mass.

The question of distinguishing which particles (Regge trajectories) contribute to a given scattering process is a fundamental one in the high-energy physics of strong interactions. This problem is far from being solved.[1] While the Regge-pole model predicts that in the limit of $s\to\infty$, $t\to 0$, the trajectory with highest $\alpha$ dominates, it is known[1] that kinematic factors might change this prediction in particular cases.

It is the purpose of this paper to show that the use of the quark model can introduce new constraints into this problem. In particular, we want to put in evidence a selection rule which predicts the decoupling of reactions involving a $BB^*$ transition from the leading trajectories, in the *forward direction*. In order to emphasize the assumptions which underlie this result, we shall formulate it under the form of a theorem:

*Theorem. In the assumption of one-particle (pole) exchange in quark-quark scattering in the additivity quark model, in the limit of $s\to\infty$, $t\to 0$, natural-parity exchange does not contribute to any $BB^*$ transition.*

A corollary of this theorem is that in the high-$s$ limit, a peak in the forward-direction differential cross section for the processes of the type

$$BB\to B^*B, \tag{1}$$

$$BB\to B^*B^*, \tag{2}$$

$$MB\to M'B^*, \tag{3}$$

where $M$, $M'$ are mesons, is evidence for unnatural-parity exchange.

Our starting point in the proof of this result is based on a relation between two-body $s$-channel helicity amplitudes, derived by Cohen-Tannoudji, Salin, and Morel[2] in the assumption of an exchange with given parity $P$ and angular momentum $J$, in the limit of large $\cos\theta_t$:

$$M^s_{\lambda_3\lambda_4\lambda_1\lambda_2}\simeq P\zeta P_4P_2(-)^{S_4-S_2}(-)^{\lambda_4-\lambda_2}M^s_{\lambda_3-\lambda_4\lambda_1-\lambda_2}\,. \tag{4}$$

Here $\lambda_i$ denotes the helicities of the particles involved. $\zeta$ is the signature of the trajectory if we have Regge exchange; $\zeta=(-)^J$ if we have particle exchange. $\cos\theta_t$ is the scattering angle in the $t$ channel:

$$\cos\theta_t=\frac{2st-2t(m_1^2+m_2^2)+[t+(m_1^2-m_3^2)][t+(m_2^2-m_4^2)]}{\{[t-(m_1+m_3)^2][t-(m_1-m_3)^2][t-(m_2+m_4)^2][t-(m_2-m_4)^2]\}^{1/2}}\,. \tag{5}$$

$m_i$ are the masses of the particles in the scattering process

$$m_1+m_2\to m_3+m_4\,. \tag{6}$$

Equation (4) follows from one-particle (pole) exchange, analyticity, crossing symmetry, angular momentum, and parity conservation, in the limit

$$\cos\theta_t\gg 1 \tag{7}$$

[the neglected terms are of order $(\cos\theta_t)^{-1}$].

The proof of our theorem can now be given using the following lemma:

*Lemma. In a two-body scattering process at $\cos\theta_t\gg 1$, and $t\to 0$, helicity is conserved at each vertex separately, so that the exchanged object must be in a zero-helicity state.*

At $t\to 0$, total angular momentum conservation coincides with total helicity conservation. This means that the scattering amplitude $M_{\lambda_3\lambda_4\lambda_1\lambda_2}$ vanishes at $t\to 0$ unless

$$\lambda_3-\lambda_4=\lambda_1-\lambda_2\,. \tag{8}$$

On the other hand, from Eq. (4) it follows that $M_{\lambda_3\lambda_4\lambda_1\lambda_2}$ does not vanish unless $M_{\lambda_3-\lambda_4\lambda_1-\lambda_2}$ does.

But helicity conservation for $M_{\lambda_3-\lambda_4\lambda_1-\lambda_2}$ means

$$\lambda_3+\lambda_4=\lambda_1+\lambda_2 . \qquad (9)$$

Equations (8) and (9) yield

$$\lambda_1=\lambda_3 , \quad \lambda_2=\lambda_4 , \qquad (10)$$

i.e., helicity conservation at each vertex and this proves our lemma.

A consequence of this lemma, independent of any other assumptions, is that the process

$$P+N\to V+N \qquad (11)$$

in the limit $s\to\infty$, $t\to 0$, is decoupled from natural-parity exchange. Here $P$ and $V$ are pseudoscalar- and vector-mesons, respectively. This result was obtained by Jones[3] in the frame of Regge theory and by Drell and Sullivan[4] for the particular case of pion photoproduction from a Rarita-Schwinger wave-function formalism. From the lemma above, this result follows immediately since we observe that as a consequence of parity conservation, natural-parity exchange can contribute to reactions (11) only when the helicity of $V$ does not vanish. But since $P$ has helicity zero, $V$ must also be in a zero-helicity state as a consequence of (10).

We shall now proceed to prove our theorem by applying the above considerations to the additivity quark model. We shall assume that a two-body quark-quark scattering process takes place through one-particle (pole) exchange. In order to derive the result we are interested in, one can choose either a direct way by writing the particle scattering amplitude in terms of quark scattering amplitudes[5] and then apply Eq. (4) to obtain relations between the quark helicity amplitudes, or one can use the more elegant $W$-spin formalism. We shall proceed in this last way. It is known[6] that for spin-$\frac{1}{2}$ quark forward scattering (forward has here the same quantitative meaning as in our lemma), $W$ spin is a rigorously conserved quantum number. In the quark model the $W$ assignments of $B^*$ and $B$ are, respectively, $\frac{3}{2}$ and $\frac{1}{2}$. Thus, the exchanged meson $M$ has $W=1$. From the lemma above it follows that $M$ is in a zero-helicity state $W_z=0$. But $W=1$, $W_z=0$ means in the quark model a meson with $P=(-)^{J+1}$ (unnatural parity). Hence only unnatural-parity exchange can contribute to reactions with a $BB^*$ vertex Q.E.D.[7,8]

In order to exemplify quantitatively the range of $s$ and $t$ where our theorem holds, we proceed in two steps. At first we shall investigate the limitations on $s$ and $t$ imposed by relation (4) and then the limitation on $t$ imposed by relation (8). For the sake of concreteness, let us limit ourselves to a precision of 10% in the amplitude, i.e., let us put

$$\cos\theta_t \geq 10 . \qquad (12)$$

We apply our considerations to the additivity quark model[3]; let us consider in particular the reaction

$$pp\to p\Delta^+ . \qquad (13)$$

In this case it is enough[5] to consider in Eqs. (5) and (6), $m_1=m_3=m_{\mathfrak{p}}$, $m_2=m_4=m_{\mathfrak{n}}$, where $m_{\mathfrak{p}}$ and $m_{\mathfrak{n}}$ are the masses of the protonic and neutronic quarks, respectively. From Eqs. (5) and (12) we get then for $m_{\mathfrak{p}}\simeq m_{\mathfrak{n}}\equiv m$, if $t$ is restricted to

$$t\ll m^2 , \qquad (14)$$

$$s>22m^2 . \qquad (15)$$

Relations (14) and (15) define the range where Eq. (4) is valid with a precision of 10%.

The quantitative meaning of the limit $t\to 0$, on which relation (8) is based, can be obtained by considering the $t$ dependence of the $s$-channel helicity amplitudes as given, e.g., in Ref. 2. There it is shown that for $\lambda_1\neq\lambda_3$, $\lambda_2\neq\lambda_4$, when $t\to 0$,

$$M_{\lambda_3\lambda_4\lambda_1\lambda_2}\leq (t/s_0)^{1/2}g_{\lambda_3\lambda_4\lambda_1\lambda_2} , \qquad (16)$$

where $g$ is a nonsingular function at $t=0$ and $s_0$ a constant (the equality sign applies to single helicity flip). How rapidly (16) vanishes depends thus on the value of $s_0$. In particle scattering $s_0$ is usually considered to be given by the nucleon mass ($s_0=m_N{}^2 \simeq 1\text{ GeV}^2$). In order to preserve the good results of the additivity quark model like the $\frac{2}{3}$ value for the $\pi p/pp$ ratio, it is natural to assume that the same value of $s_0$ has to be used in Reggeized quark-quark scattering.[9] In order to have 10% precision in the amplitude for relation (8) it is sufficient to have

$$t/s_0\leq 10^{-2} . \qquad (17)$$

The conclusion of this theorem, besides its theoretical interest,[10-13] might also have practical implications. Thus, e.g., on the basis of our selection rule one expects for values of $s$ satisfying relation (11) a dip in the forward direction in the reaction

$$PB\to PB^* \qquad (18)$$

($P$ is a pseudoscalar meson), since only natural parity can be exchanged in the upper vertex. The position of this dip will define the precise value of $s_0$ in Eq. (17) and, what is more challenging, if the assumptions on which our theorem is based hold, this dip will become pronounced only from a given $s$ upwards. This $s$ provides through relation (11) an estimate of the *upper* limit of the quark mass. This point seems of special interest since the present data provide us in general only with *lower* limits. If we choose in Eq. (17) $s_0 \simeq 1$ GeV$^2$, this dip should occur at $t \leq 10^{-2}$ GeV$^2$. The precision of present experiments is not sufficient to compare this prediction with data and given the above implications, experimentalists are urgently invited to improve this situation.

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[5]E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 2, 105 (1965) [Soviet Phys. JETP Letters 2, 65 (1965)]; H. J. Lipkin, F. Scheck, and M. Stern, Phys. Rev. 152, 1375 (1966); C. Itzykson and M. Jacob, Nuovo Cimento 48A, 909 (1967).

[6]H. Harari and H. J. Lipkin, Phys. Rev. 140, B1617 (1965); H. J. Lipkin in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966,* edited by A. Perlmutter, J. Wojtaszek, E. C. G. Sudarshan, and B. Kurşunoğlu (Freeman, San Francisco, 1966), p. 97; H. J. Lipkin and S. Meshkov, Phys. Rev. 143, 1269 (1966).

[7]The proof above is based on one-particle (pole) exchange and on the assumptions of the additivity quark model in which the particle scattering process is due to the scattering of one quark from the projectile with one quark from the target. Once we relax these last assumptions and consider the possibility that two or three quarks interact simultaneously, the conditions under which our theorem holds are changed. It can be shown (Ref. 8) that in reactions (1) and (2) in which all three quarks interact simultaneously, the amplitude vanishes at $t \to 0$ in only two cases: (a) when all three quarks exchange natural parity, (b) when all three quarks exchange unnatural parity. The vanishing of the scattering amplitude at $t = 0$ is also connected with the famous conspiracy-cut issue.[1] These effects have been invented in order to explain the peaks in the differential cross sections at $t = 0$, for processes due to pion (kaon) exchange. In a quark model with multiple scattering, these effects can be obtained by maintaining one-particle (pole) exchange at quark level, but by allowing simultaneous quark interactions with multiple-Regge exchange. A concrete model along these lines is given in Ref. 8 for antibaryon-baryon scattering, where simultaneous quark-quark interactions are assumed to be the dominant mechanism.

[8]R. M. Weiner, Phys. Rev. D 4, 813 1971.

[9]I am indebted to Dr. L. Horwitz and Dr. A. Pagnamenta for instructive discussions on this point.

[10]Recently, in view of the difficulties encounted by the duality approach in $\bar{B}B$ scattering, Kugler (Ref. 11) has put forward the hypothesis that the leading trajectories, which are all of natural parity, do not contribute to reactions $\bar{B}B \to \bar{B}^*B$. Futhermore, experimental data (Ref. 12) in charge-exchange reactions involving a $BB^*$ transition show that even at the highest available lab energies ($p_L \simeq 24$ GeV/$c$) the pion exchange dominates, although the leading trajectories are also allowed to contribute. It would be interesting to investigate the connection between the theorem proven above and these two facts. See Refs. 12 and 13.

[11]M. Kugler, Phys. Letters 32B, 106 (1970).

[12]G. Wolf, Phys. Rev. 182, 1538 (1969); A. Ma *et al.*, Phys. Rev. Letters 24, 1031 (1970).

[13]Our selection rule favors, in general, unnatural-parity exchange and not only $\pi$ exchange. One would have thus to consider perhaps also the $A_1$, $B$, etc., as possible candidates for the reactions discussed above.