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Proposed *CP*- and Isospin-Violating Interaction and Symmetries of the Three-Triplet Model

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The proposition made earlier that there exists a basic nonelectromagnetic interaction at the level of electromagnetism, which violates isospin, C, CP, and T invariance "maximally," is reconsidered. It is found that the notion of this new interaction acquires several desirable features in the framework of the three-triplet model. They are: (a) the construction of partially conserved $\Delta S=\Delta Q=0$, C-even and C-odd vector currents entering into the interaction; (b) the suppression of CP violation relative to isospin violation due to the new interaction; and (c) the possibility of a large contribution from the new interaction to the $\Delta I=1$ mass differences without an appreciable contribution to the $\Delta I=2$ mass differences. The $\eta\to 3\pi$ problem does not seem to have any simple solution in the present model if we insist on a vector form for the new interaction. The possible advantages of a scalar form for the above interaction with respect to the $\eta\to 3\pi$ decay and the signs of the $\Delta I=1$ mass differences are noted. Regardless of the vector or scalar form for the interaction, the suppression of CP and T violation in the present model will have the consequences that the asymmetry parameter for the $\eta\to\pi^++\pi^-+\pi^0$ decay should be of the order of $\frac{1}{2}\%$ and that the electric dipole moment of the neutron should lie in the region of 10^{-23}–10^{-24} e cm, both of which estimates are an order of magnitude lower than the previous ones. It is emphasized that in addition to the above measurements, a search for the vector meson V^0 or alternatively the scalar meson S^0 (which mediates the new interaction) in the two-pion mass spectra would be most desirable to judge the existence of the new interaction.

I. INTRODUCTION

Recently it was proposed[1] that there exists a basic nonelectromagnetic interaction of strength similar to that of electromagnetism, which violates isospin, charge conjugation, and time-reversal invariance "maximally,"[2] while conserving parity and CT. Specifically it was suggested (in partial analogy with electromagnetism) that the new interaction (called the V^0 interaction) is of the form

$$\mathcal{L}_V=-g_V J_\mu V^0_\mu\,, \tag{1}$$

where J_μ is a $\Delta S=\Delta Q=0$ hadronic vector current with mixed isospin and charge-conjugation properties. Thus,

$$J_\mu=J_\mu^{(+)}+J_\mu^{(-)}\,, \tag{2}$$

where

$$CJ_\mu^{(\pm)}C^{-1}=\pm J_\mu^{(\pm)}\,. \tag{3}$$

The strength $g_V{}^2/4\pi$ (barring more detailed considerations) is taken to be of order 10^{-2}. The neutral vector meson V^0 (represented by the field V^0_μ) is presumed to be involved in no other basic interaction except the above and gravity. The physical mass of the V^0 is required to be greater than $3m_\pi$ (say) from considerations based on binding-energy calculations for mirror nuclei (as mentioned in I). The general motivation for proposing such an interaction and the experimental possibilities arising from the isoschizon character of V^0, as well as the C-, CP-, and T-violating consequences of the V^0 interaction, have been discussed in the earlier paper. The purpose of the present paper is to consider a number of questions which naturally arise in connection with the above idea. Some of them were raised without any solution in I. These are listed below:

(1) All currents of physical interest (vector and axial-vector) seem to be distinguished by the fact that they are either exactly conserved or conserved in the limit of a symmetry, which allows an unambiguous generation of such a current from a given Lagrangian. It is clearly desirable to demand the same of J_μ, or separately of the C-even ($J_\mu^{(+)}$) and C-odd ($J_\mu^{(-)}$) pieces of J_μ. However, it is not possible to construct a C-even $\Delta S=\Delta Q=0$ vector current in terms of spin-$\frac{1}{2}$ and or spin-0 fields in most models, which is conserved in any limit. Such is the case, for example, of a simple quark or gluon model with or without spin-zero fields.[3] One may therefore ask, what is a desirable model, if any, which will allow the construction of at least a partially conserved[4] $J_\mu^{(\pm)}$?

(2) The V^0 interaction, as proposed, is expected in general to lead to CP violation as well as isospin violation (without CP violation) in order $g_V{}^2/4\pi$

in the amplitude. However, experimentally, isospin violation is known to be of the order of a few percent in the amplitude and a significant portion of this ought to be attributed to the V^0 interaction (in view of our assumption[1] that the said interaction ought to make important contributions to the n-p and the K^+-K^0 mass difference, among other things). On the other hand, CP violation in the K^0-$\bar{K}^0$ complex is known to be of order 10^{-3} in the amplitude. Unless the K^0-$\bar{K}^0$ complex happens to be a special case,[5] one must therefore ask[6] if there is a natural mechanism to suppress C, CP, and T violation relative to isospin violation due to the V^0 interaction.

(3) We have suggested that the V^0 interaction could contribute significantly to the n-p and the K^+-K^0 mass difference; on the other hand, simple calculations of the $\Delta I = 2$ mass differences[7] (such as the π^+-π^0 mass difference), based on photon emission and absorption and low-mass intermediate-state contributions are quite successful. One may therefore ask if there is a natural way by which the V^0 interaction may make a significant contribution to the $\Delta I = 1$ mass differences without a similar contribution to the $\Delta I = 2$ mass differences.

It appears intriguing that all of the above three questions receive a satisfactory answer in the three-triplet model.[8,9] By making use of the second SU(3) degree of freedom available in this model, it is possible to construct C-even and C-odd $\Delta S = \Delta Q = 0$ vector currents in terms of spin-$\frac{1}{2}$ fields, which are conserved in the limit of a higher symmetry of the model (to be specified). Furthermore, by adopting the Han-Nambu[8] classification scheme, according to which the low-lying observed mesons and baryons are SU(3)$''$ singlets, it is possible to provide an answer in the affirmative to the last two questions raised above.

In Sec. II we discuss the necessity of a multitriplet model for the construction of conserved or partially conserved C-even $\Delta S = \Delta Q = 0$ vector current and in Sec. III we present a general form for the choice of $J_\mu^{(\pm)}$ in the *SUB* model[10] due to Cabbibo, Maiani, and Preparata.[9] The qualitative aspects of CP and isospin violation due to the choice of $J_\mu^{(\pm)}$ in the *SUB* model are discussed in Secs. IV and V. In particular, it is pointed out that CP violation is expected to be a fourth-order effect in the new interaction for the low-lying states, while isospin violation could take place, in general, in second order. It is furthermore argued that the new interaction is likely to make a significant contribution to the $\Delta I = 1$ mass differences without making appreciable contributions to the $\Delta I = 2$ mass differences. The problem of the $\eta \to 3\pi$ forbiddenness in the framework of current algebra and partial conservation of axial-vector current (PCAC) is considered in Sec. VI. It is found that no simple solution to this problem exists in the present model if we insist on the vector form for the new interaction. The possible advantages of a scalar form from this point of view and others are briefly mentioned. In Sec. VII we consider some of the special features for the production and decay mechanisms of the V^0 meson, which arise due to the SU(3)$''$ symmetry of the model. Similar comments for the scalar meson S^0 (which mediate the new interaction if it is scalar) are also included. Section VIII presents a summary with some remarks.

II. NECESSITY OF A MULTITRIPLET MODEL

In order to construct a conserved or partially conserved $\Delta S = \Delta Q = 0$ C-even vector current with spin-$\frac{1}{2}$ fields, the following observation is useful. While diagonal vector combinations of the form $:\bar{\psi}_a\gamma_\mu\psi_a:$ are automatically odd under C, the antisymmetric nondiagonal combinations of the form $i(\bar{\psi}_a\gamma_\mu\psi_b - \bar{\psi}_b\gamma_\mu\psi_a)$ $(a \neq b)$ are even under C (for the same intrinsic charge parities of a and b). Furthermore, the divergence of either combination can be made to vanish under a variety of circumstances. For example, consider a simple case in which the Lagrangian is composed only of fields ψ_a, ψ_b, $\bar{\psi}_a$, $\bar{\psi}_b$ and a neutral vector-meson field Φ_μ. Let $m_a = m_b$, and let Φ_μ be coupled symmetrically to ψ_a and ψ_b in the form $g(\bar{\psi}_a\gamma_\mu\psi_a + \bar{\psi}_b\gamma_\mu\psi_b)\Phi_\mu$. Such a Lagrangian clearly leaves the three currents $\bar{\psi}\gamma_\mu\tau_i\psi$ conserved $(i = 1, 2, 3)$, where ψ is treated as an isodoublet with components ψ_a and ψ_b and τ_i are the usual Pauli matrices acting on the (a, b) indices. The currents $\bar{\psi}\gamma_\mu\tau_1\psi (= \bar{\psi}_a\gamma_\mu\psi_b + \bar{\psi}_b\gamma_\mu\psi_a)$ and $\bar{\psi}\gamma_\mu\tau_2\psi[= i(\bar{\psi}_a\gamma_\mu\psi_b - \bar{\psi}_b\gamma_\mu\psi_a)]$ necessarily carry opposite charge-conjugation properties. This is of course well known. The point that is worth noting is that in order for the currents $\bar{\psi}\gamma_\mu\tau_1\psi$ and $\bar{\psi}\gamma_\mu\tau_2\psi$ to be $\Delta S = \Delta Q = 0$ currents, it is necessary to allow the presence of more than one fermion field (such as a and b with $a \neq b$) carrying the same charge, strangeness, and other conserved quantum numbers in the Lagrangian. This is evidently not possible in a single-triplet model (such as the quark model). *We must therefore resort to a multitriplet model or a quartet model* [*as in SU(4)*] *in order to construct a conserved or partially conserved C-even current $J_\mu^{(+)}$ with spin-$\frac{1}{2}$ fields*. The latter model does not seem to be preferable, in view of other consideration to appear in this paper.

We therefore choose to construct the currents $J_\mu^{(\pm)}$ in the three-triplet model,[8,9] which appears to be the most elegant one in the former class. It is worth noting that there appears to be sufficient motivation in favor of this model: It not only preserves the usual successes of the quark model, but is favored over the quark model inasmuch as it allows one to preserve the Pauli principle for the 56-plet of SU(6), it yields the right sign and magnitude for the $\pi^0 \to 2\gamma$ decay amplitude[11] (the quark model does not), and it helps to understand the $\Delta I = \frac{1}{2}$ rule[12] for the nonleptonic hyperon decays together with current algebra and PCAC. In Sec. III, we mention briefly for completeness the main features of the *SUB* model due to Cabibbo, Maiani, and Preparata,[9] and the construction of $J_\mu^{(\pm)}$ in this model.[10,13]

III. THE *SUB* MODEL AND CONSTRUCTION OF $J_\mu^{(\pm)}$

The *SUB* model consists of nine fundamental spin-$\frac{1}{2}$ particles each with baryon number $\frac{1}{3}$ denoted by (α, i), where the Greek label runs over the indices (S, U, and B) and the Latin over the indices ($\mathcal{P}$, $\mathfrak{N}$, and λ). This allows one to define the usual SU(3) group (with the familiar operators I_3 and Y among its generators) acting on the index i and a second SU(3) group [called the SU(3)$''$ group] acting on the index α. It is presumed that the maximum useful symmetry (in the internal space) for the classification of hadrons is the SU(3)×SU(3)$''$ group, even though neither SU(3) nor SU(3)$''$ are exact symmetries. In particular, the fundamental nonet (α, i)'s are presumed to transform as the (3, 3) representation of the above group with (S, i) and (U, i) transforming as members of an SU(2)$''$ doublet and (B, i) of an SU(2)$''$ singlet. Furthermore, the low-lying baryons and mesons are assumed to be bound states of $((\alpha, i); (\beta, j); (\gamma, k))$ and $((\alpha, i); (\beta, j)^*)$, respectively, both corresponding to SU(3)$''$ singlets[14]; the SU(3)$''$ nonsinglet states are presumed to lie higher starting with, say, the 2- to 4-BeV region. The constituents (α, i) are allowed to possess integral electric charges by a modification[15] of the Gell-Mann–Nishijima formula to the form

$$Q = I_3 + \tfrac{1}{2}Y + \tfrac{1}{3}C\,, \tag{4}$$

where $C = 3Y''$ and has the eigenvalues 1, 1, and -2 for the triplets (S, i), (U, i), and (B, i), respectively.

Representing the nine constituents by the 36-component spinor field ψ, one may define, in general, vector currents in the above model by

$$J_\mu^{(\xi,\eta)}(x) \equiv \tfrac{1}{2}(\overline{\psi}(x))_{(\gamma,j)}\gamma_\mu(\rho_\xi)_{\gamma\beta}(\lambda_\eta)_{jk}(\psi(x))_{(\beta,k)} \tag{5a}$$

$$= \tfrac{1}{2}\overline{\psi}(x)\gamma_\mu(\rho_\xi \times \lambda_\eta)\psi(x) \tag{5b}$$

$$\equiv (\rho_\xi(x) \times \lambda_\eta(x))_\mu\,, \tag{5c}$$

where the λ_η's are the Gell-Mann 3×3 matrices ($\eta = 0, 1, \ldots, 8$) acting on the SU(3) indices of $\overline{\psi}$ and ψ, and the ρ_ξ's ($\xi = 0, 1, \ldots, 8$) are the analogs of the λ_η's acting on the SU(3)$''$ indices. Note that we have introduced (5c) as a shorthand which we will use later. By way of restricting the choice of the currents $J_\mu^{(\pm)}$ let us assume that the V^0 interaction conserves charm[16] C (or Y''). Thus $J_\mu^{(\pm)}$ ought to involve only[17] the charm-conserving matrices which are ρ_0, ρ_1, ρ_2, ρ_3, and ρ_8. Furthermore, they must involve only λ_0, λ_3, and λ_8 matrices in order that $J_\mu^{(\pm)}$ should satisfy $\Delta S = \Delta Q = 0$.

As a first step towards the choice of $J_\mu^{(\pm)}$, let us choose the intrinsic charge-conjugation parities of all the nine fields to be the same (see, however, remarks below). In this case, following the remarks of Sec. II, we may choose [using the shorthand (5c)]

$$J_\mu^{(+)} = a(\rho_2 \times \lambda_a)_\mu = \tfrac{1}{2}ia(\overline{S}\lambda_a\gamma_\mu U - \overline{U}\lambda_a\gamma_\mu S)\,, \tag{6}$$

where a is a constant of order unity and λ_a is an appropriate linear combination of λ_0, λ_3, and λ_8 matrices (that need not be specified for the present). The C-odd current $J_\mu^{(-)}$ may be chosen, in general, to be a sum of terms of the form

$$J_\mu^{(-)} = b(\rho_1 \times \lambda_b)_\mu + c(\rho_3 \times \lambda_c)_\mu + d(\rho_8 \times \lambda_d)_\mu + e(\rho_0 \times \lambda_e)_\mu\,, \tag{7}$$

where b, c, d, and e are constants (in general of order unity, although some of them may be zero) and λ_b, λ_c, λ_d, and λ_e are again appropriate linear combinations of λ_0, λ_3, and λ_8 (to be specified by further considerations).

It is important to note that *both a and b must be nonzero* in order for the V^0 interaction to lead to observable C-violating effects. If, for example, b were zero, one may simply choose the intrinsic charge-conjugation parities of the S and U triplets to be opposite; in that case, the $J_\mu^{(+)}$ term [given by Eq. (6)] will correspond to a C-odd current, and so will the $J_\mu^{(-)}$ term; thus no C-violating effects could arise due to the V^0 interaction. On the other hand, if both a and b are nonzero, their interference term will lead to C violation regardless of the choice of intrinsic charge-conjugation parities of S and U.

We may now remark about the conservation of $J_\mu^{(\pm)}$. A simple possibility, which will lead to the conservation of $J_\mu^{(\pm)}$ is the following. Let the only fields entering into the Lagrangian be $\psi_{(\alpha,i)}$, $\overline{\psi}_{(\alpha,i)}$, and a neutral vector-meson field Φ_μ^0 with strong interactions between them. It is then clear that a sufficient symmetry of the above Lagrangian, which will guarantee the conservation of $J_\mu^{(\pm)}$, is the symmetry under the group U(9) consisting of all unitary transformations on the nonets $\psi_{(\alpha,i)}$ and $\overline{\psi}_{(\alpha,i)}$. If the term $d(\rho_8 \times \lambda_d)_\mu$ were absent in

Eq. (7), the corresponding sufficient symmetry would be the group U(6) consisting of all unitary transformations on the above fields for $\alpha = S, U$ and $i = \mathcal{P}, \mathfrak{N}, \lambda$. The latter symmetry will in fact ensure 36 conserved currents $J_\mu^{(\xi,\eta)}$ for $\xi = 0, 1, 2, 3$ and $\eta = 0, 1, \ldots, 8$. However, both the U(9) and the U(6) symmetries must be strongly broken (regardless of the above model) in order to conform with the observed particle spectrum.[18] In fact the highest useful symmetry for the classification of hadrons, as mentioned before, is considered to be the SU(3)×SU(3)'' group,[19] which of course may be extended to the U(3)×U(3)'' group. Such a symmetry will allow the conservation of only the 18 currents $J_\mu^{(\xi,0)}$ and $J_\mu^{(0,\eta)}$, but not of the currents $J_\mu^{(i,j)}$ (where $i \neq 0$ and $j \neq 0$).

Thus the currents $J_\mu^{(\pm)}$ for the V^0 interaction can only be partially[4] *conserved*[20] *even in the absence of terms that violate the SU(3) and SU(3)'' symmetries in the Lagrangian.* Nevertheless, as they are conserved in the limit of the higher symmetry [say the U(6) group], they may be generated unambiguously from the symmetric Lagrangian (to which one of course must add the symmetry-violating terms). This is not only desirable aesthetically, but is useful in deriving the commutation properties of the currents $J_\mu^{(\pm)}$ with each other and other currents of interest. Apart from such algebraic considerations, we will never need to use the concept of the U(6) group as a higher symmetry.

In Sec. IV we explore the consequences of the above model for isospin and CP violation.

IV. ISOSPIN AND CP VIOLATION IN THE MODEL

Let us first consider, in general, isospin violation due to the V^0 interaction without CP violation. These will, in general, arise in second order from matrix elements of the form

$$g_V^2 \int d^4p\, \Delta_{\mu\nu}(p) \int d^4x\, e^{ip\cdot x} \langle f | T(J_\mu^{(\pm)}(x) J_\nu^{(\pm)}(0)) | i \rangle . \tag{8}$$

The point to note is that the above matrix elements do not vanish, in general, due to any selection rules, even for $|i\rangle$ and $|f\rangle$ being composed of the low-lying SU(3)'' singlet states, since the products $J_\mu^+(x)J_\nu^+(0)$ and $J_\mu^-(x)J_\nu^-(0)$ contain SU(2)'' and SU(3)'' singlet pieces in them. Thus we expect that the V^0 interaction will lead, in general, to a CP-conserving, isospin-violating amplitude (even for the low-lying states) of order $g_V^2/4\pi$.

The same is not true, however, for the CP-violating amplitudes. In this case, for reasons mentioned before, we must examine whether the following matrix element is nonvanishing:

$$g_V^2(ab) \int d^4p\, \Delta_{\mu\nu}(p) \int d^4x\, e^{ip\cdot x} \times \langle f | T(J_\mu^{(2,a)}(x) J_\nu^{(1,b)}(0)) | i \rangle , \tag{9a}$$

which is equal to

$$g_V^2(ab) \int d^4p\, \Delta_{\mu\nu}(p) \int d^4x\, e^{ip\cdot x} \times \langle f | T\{(\rho_2(x)\lambda_a(x))_\mu (\rho_1(0)\lambda_b(0))_\nu\} | i \rangle , \tag{9b}$$

where we have made use of the shorthand notation defined in (5c). It is easy to see that the operator $T(J_\mu^{(2,a)}(x)J_\nu^{(1,b)}(0))$ does not contain either an SU(2)'' or SU(3)'' singlet piece in it. Thus if $|i\rangle$ and $|f\rangle$ involve only low-lying states, the above matrix element must vanish, unless we allow for appropriate SU(2)'' nonsinglet "admixture" in the states $|i\rangle$ and $|f\rangle$. To the extent that SU(2)'' is presumed to be a good symmetry, the above may serve to explain *the suppression of CP violation, in general, compared to isospin violation due to the V^0 interaction.*

We still need to consider the mechanism for SU(2)'' violation, which could lead to nonvanishing matrix elements (9a) or (9b). At this point it appears natural to *assume* that the SU(2)'' symmetry is broken by the V^0 interaction itself (and not by the strong or medium-strong interaction), even though SU(3)'' may be broken by, say, the medium-strong interaction. This is partly motivated by analogy with SU(2) versus SU(3) symmetry: The former is broken only at the level of electromagnetism[21] (and V^0 interaction, if it exists), while the latter is broken by the medium-strong interaction. Under the above assumption one may allow SU(2)'' nonsinglet admixtures of order $g_V^2/4\pi$ in the states $|i\rangle$ and $|f\rangle$. One may still ask: Do these render the matrix elements (9a) nonvanishing? In other words, can the V^0 interaction lead to CP violation for processes involving low-lying states in the fourth order of perturbation theory? The answer is in the affirmative only if we impose certain restrictions on the structure of $J_\mu^{(\pm)}$. This is discussed below.

It is useful to introduce the symbolic notation

$$\rho_{1b}(x) \equiv \bar\psi(x)(\rho_1 \times \lambda_b)\psi(x) \sim [\xi_b^+(x) + \xi_b^-(x)]/\sqrt{2} , \tag{10}$$
$$\rho_{2a}(x) \equiv \bar\psi(x)(\rho_2 \times \lambda_a)\psi(x) \sim i[\xi_a^+(x) - \xi_a^-(x)]/\sqrt{2},$$

where the ~ symbol designates that the operator on its left and right transform the same way under the SU(3)'' group. The operators $\xi^+(x)$, $\xi^-(x)$, and $\xi^0(x)$ are defined to transform under the SU(2)'' group as $\pi^+(x)$, $\pi^-(x)$, and $\pi^0(x)$ do under the SU(2) group. Thus the SU(2)'' transformation of the operator in the matrix element of (9a) or (9b) is given by

$$T(\rho_{2a}(x)\rho_{1b}(0)) \sim \tfrac{1}{2}iT(\xi_a^+(x)\xi_b^+(0) - \xi_a^-(x)\xi_b^-(0) + [\xi_a^+(x)\xi_b^-(0) - \xi_a^-(x)\xi_b^+(0)]). \quad (11)$$

The first two terms on the right-hand side of (11) transform as $I''=2$, $I_3''=\pm 2$ operators and thus their matrix elements between SU(2)'' singlet states is zero; even if we allow for SU(2)'' nonsinglet admixtures in the states $|i\rangle$ and $|f\rangle$ (arising from a second-order V^0 interaction in a CP-conserving[22] way), the net contribution of the first two terms is still zero because of the relative negative sign between them. On the other hand, the two terms inside the square brackets of (11), when inserted into (9b) also cancel each other provided $\lambda_a=\lambda_b$; this follows by noting the symmetric nature of the integration, the property of $\Delta_{\mu\nu}(p)$, and the symmetry property imposed by the time-ordering operator in (9b). If $\lambda_a \neq \lambda_b$, however, the cancellation does not work in general. In this case, the two terms inside the square brackets of (11) together transform as an $I''=1$, $I_3''=0$ operator. The matrix element of this operator will be nonzero, if one admits SU(2)'' nonsinglet admixtures in the states $|i\rangle$ and $|f\rangle$ transforming as above. These could arise (in a CP-conserving way) through a second-order V^0 interaction *provided that the coefficients c and e in Eq. (7) are nonzero.*

To summarize, CP violation cannot arise under any circumstance in second order of the V^0 interaction for processes involving low-lying states, but it can arise in fourth and higher orders, provided that

$$\text{(i)}\ ab \neq 0\,, \quad \text{(ii)}\ \lambda_a \neq \lambda_b\,, \quad \text{(iii)}\ ce \neq 0\,. \quad (12)$$

Thus, the present model provides a natural mechanism for the suppression of CP violation compared to isospin-spin violation. One may account for the observed magnitudes of both by choosing to satisfy (12) together with

$$g_V^2/4\pi \simeq \tfrac{1}{30}. \quad (13)$$

The suppression of CP and T violation as mentioned above will have the following qualitative consequences. The charge asymmetry in the $\eta \to \pi^+\pi^-\pi^0$ decay is expected to be lower than what one may have expected without the present model for the V^0 interaction by about an order of magnitude. Thus the rough estimate (as discussed in Ref. 1) will suggest that the $\eta \to 3\pi$ asymmetry may lie in the region of approximately $\frac{1}{2}\%$ (in the present model). For the same reason, the electric dipole moment of the neutron will be expected to lie in the region of $10^{-23}-10^{-24}$ e cm, which is about a factor of 10 lower than the previous estimate.[1]

We next consider the problem of the $\Delta I=1$ versus the $\Delta I=2$ mass differences arising due to the V^0 interaction.

V. THE $\Delta I=1$ VERSUS THE $\Delta I=2$ MASS DIFFERENCES

Harari's[23] argument based on Regge asymptotic behavior shows that the forward scattering amplitude for the process $m+h \to m+h$ (where m stands for either the photon or the V^0 meson and h for the hadron target) satisfies a subtracted dispersion relation in the energy variable for $I=1$ and an unsubtracted dispersion relation for $I=2$, both in the t channel. Since the above amplitude directly determines the mass shifts due to second-order perturbation in the photon and V^0 interactions, one may conclude from the above that the $\Delta I=2$ mass differences ought to be well approximated by low-mass intermediate states (h') contributing to $m+h \to (h') \to m+h$, whereas the $\Delta I=1$ mass differences must receive substantial contributions from high-mass intermediate states.

As we assume all low-lying states are SU(3)'' singlets, if h is a low-lying baryon or meson, the V^0 interaction can give rise to low-mass intermediate states h' only through the SU(3)'' singlet term in $J_\mu^{(\pm)}$. This is true if either (i) SU(3)'' is a good symmetry [as good as SU(3), say] or (ii) at least SU(2)'' is a good symmetry[24] and the $d(\rho_8\times\lambda_d)_\mu$ term in Eq. (7) is absent. The only SU(3)'' singlet term in $J_\mu^{(\pm)}$ is the $e(\rho_0\times\lambda_e)_\mu$ term in Eq. (7). If we choose, however,

$$\lambda_e = \lambda_0\,, \quad (14)$$

it follows that the V^0 interaction (in second order) *cannot* contribute to mass differences through low-mass intermediate states, it can only do so through the high-mass SU(2)'' nonsinglet intermediate states. By Harari's arguments this will imply that the V^0 interaction may make a substantial contribution to the $\Delta I=1$ mass differences; however, it will *not* make a significant contribution to the $\Delta I=2$ mass differences.

As emphasized in Sec. I, this is a very desirable result. With the choice $\lambda_e=\lambda_0$ (and perhaps $d=0$)[25] the introduction of the V^0 interaction does not spoil the successes of the present $\Delta I=2$ mass-shift calculations[7] and creates a definite scope to remove the shortcomings of the $\Delta I=1$ mass-shift results.[26] We next consider the problem of the $\eta \to 3\pi$ decay in the model.

VI. THE PROBLEM OF $\eta \to 3\pi$ DECAY

It had been proposed in Ref. 1 that the contribution of the V^0 interaction could resolve the problem of the forbiddenness of the $\eta \to 3\pi$ decay based on current algebra, strong PCAC, and the usual electromagnetic interaction. A model of the C-even vector current, built out of spin-0 meson fields, had been proposed in the framework of a generalized Gell-Mann–Lévy[27] model to demonstrate that the $\eta \to 3\pi$ forbiddenness could indeed be avoided in this case. However, the main objection one may have (from an aesthetic point of view) against such a choice of J_μ^+ is that it is not conserved in any limit[3] (not even in the limit of free fields). From this point of view, we of course do prefer the choice of $J_\mu^{(\pm)}$ as presented in this paper. However, the present choice of these currents does not remove the forbiddenness of the $\eta \to 3\pi$ decay without giving up some of the traditional assumptions[28] (i.e., current algebra, PCAC, and linear extrapolation of the off-shell matrix element). This may be seen by first making an appropriate choice for the axial-vector currents a_μ^i. We choose them to be SU(3)'' singlets as is suggested by our initial assumption that pions are SU(3)'' singlets and the hypothesis of PCAC. It, of course, is also the simplest possible choice,[29] suggested by analogy with the structure of the vector currents v_μ^i, which are required[30] to be SU(3)'' singlets. Thus in the three-triplet model v_μ^i and a_μ^i may be chosen to be

$$v_\mu^i = (\tfrac{3}{2})^{1/2}\overline{\psi}(x)\gamma_\mu(\rho_0 \times \tfrac{1}{2}\lambda_i)\psi(x) \tag{15}$$

and

$$a_\mu^i = (\tfrac{3}{2})^{1/2}\overline{\psi}(x)\gamma_\mu\gamma_5(\rho_0 \times \tfrac{1}{2}\lambda_i)\psi(x)\,. \tag{16}$$

The equal-time commutator relevant for the $\eta \to 3\pi$ decay is given by

$$\begin{aligned}[a_0^i(\vec{x},t), J_\mu^{(\xi,\eta)}(\vec{y},t)] &= \tfrac{1}{2}[a_0^i(\vec{x},t), \overline{\psi}(\vec{y},t)\gamma_\mu(\rho_\xi \times \lambda_\eta)\psi(\vec{y},t)] \\ &= i f_{i\eta\alpha}\overline{\psi}(x)\gamma_\mu\gamma_5(\rho_\xi \times \tfrac{1}{2}\lambda_\alpha)\psi(x)\delta^3(\vec{x}-\vec{y})\,. \end{aligned} \tag{17}$$

Because of the $f_{i\eta\alpha}$ coefficients, the isospin structure of these commutators is the same as that of the commutators of a_0^i with v_μ^i. It therefore follows that the contribution of the V^0 interaction to the $\eta \to 3\pi$ decay is forbidden[31] for the same reasons[28] as that of electromagnetism. It is possible that the solution to this problem may lie in giving up some of the traditional assumptions for the treatment of the $\eta \to 3\pi$ decay. For instance, it may arise through the hypothesis of weak PCAC[32] and/or through the dependence of the off-shell $\eta \to 3\pi$ matrix element on squares of four-momenta of the pions, etc.

As an additional interesting possibility, however, we are tempted to mention that there is a simple solution to the $\eta \to 3\pi$ problem through the new interaction if we do not insist that it be vector or axial vector in nature. For instance, let us assume that it is a scalar Yukawa-type interaction of the form

$$\mathcal{L}_S = -g_S J(x) S^0(x)\,, \tag{18}$$

where

$$J(x) = \sum_{\xi,\eta} C_{\xi\eta}\overline{\psi}(x)(\tfrac{1}{2}\lambda_\xi \times \tfrac{1}{2}\rho_\eta)\psi(x)\,. \tag{19}$$

$S^0(x)$ denotes the field of a scalar meson, which plays the same role as the V^0 meson in all respects[33] except that it has spin zero. The coefficients $C_{\xi\eta}$ are chosen in accordance with the corresponding choice for $J_\mu^{(\pm)}$ [Eqs. (6) and (7)] so that the scalar interaction $\mathcal{L}_S$ has the same C, P, T, SU(3), and SU(3)'' properties as the vector interaction $\mathcal{L}_V$. Thus the discussion on C, CP, and isospin violation of the previous sections are *unaltered* by choosing the scalar form of the interaction. It is easy to see, however, that the relevant equal-time commutator for the $\eta \to 3\pi$ decay treated in second-order perturbation in the scalar interaction $\mathcal{L}_S$ has $d_{i\xi\alpha}$ coefficients instead of the $f_{i\xi\alpha}$ coefficients of Eq. (17). This *allows*[34] the $\eta \to 3\pi$ decay under the usual assumptions mentioned before.

There appears to be an added motivation for the scalar form of the new interaction as opposed to the vector form. The self-energy due to the former treated in second-order perturbation theory is negative[35] and opposite in sign to that due to the latter. This may have interesting implications on the signs of the $\Delta I = 1$ mass differences, for which the photon contribution seems to be inadequate. These considerations as well as the consideration of the $\eta \to 3\pi$ decay based on the scalar interaction will be taken up in a subsequent note. In Sec. VII we consider the production and decay mechanisms of the V^0 meson; the major differences in these respects for the scalar S^0 meson are also mentioned.

VII. PRODUCTION AND DECAY OF THE V^0 MESON IN THE PRESENT MODEL

The selection rules arising from the SU(3)'' symmetry of the present model lead to certain important differences as regards the production and especially the decay mechanisms of the V^0 meson from those discussed in Ref. 1. First we note that the V^0 meson may be produced in a variety of reactions with cross sections of order $g_V^2/4\pi$ compared to that of the ω^0 meson (say) provided that the SU(3)'' singlet term $e(\rho_0\times\lambda_e)_\mu$ is present in $J_\mu^{(-)}$ given by Eq. (7). It is already asserted from considerations of Sec. IV that $e\neq 0$. Thus we expect production of V^0 in order g_V in the amplitude by reactions such as (see *Note added in proof*)

$$\begin{aligned}
&\text{(a)}\quad \pi+N\to N+V^0\,,\\
&\text{(b)}\quad K^-+p\to\Lambda+V^0\,,\\
&\text{(c)}\quad \bar{p}+p\to\pi^++\pi^-+V^0\,,\\
&\text{(d)}\quad d+d\to \mathrm{He}^4+V^0\,,\\
&\text{(e)}\quad e^-+e^+\to V^0\,,\ \text{etc.}
\end{aligned}\tag{20}$$

Since the e term in Eq. (7) is required to be isoscalar [see Eq. (14)], a real V^0 cannot effectively be coupled to the two-pion system to first order in g_V and zeroth order in electromagnetism (as V^0 has spin 1). Thus V^0 cannot be produced peripherally (unlike the ρ^0 meson) in reaction (a); however, it can be produced via two-pion exchange in (a) and by a host of other mechanisms *similar* to those for the SU(3) singlet component of the ω^0 or the ϕ^0 meson.

The V^0 may decay to a variety of hadronic systems depending upon its mass. Some of its important decay channels are (see also *Note added in proof*)

$$\begin{aligned}
&\text{(a)}\quad V^0\to\pi^++\pi^-\\
&\text{(b)}\quad\quad\ \to\pi^0+\gamma\\
&\text{(c)}\quad\quad\ \to\pi^++\pi^-+\pi^0\\
&\text{(d)}\quad\quad\ \to K^++K^-\\
&\text{(e)}\quad\quad\ \to K_L+K_S\\
&\text{(f)}\quad\quad\ \to\pi^++\pi^-+\gamma\,,\ \text{etc.}
\end{aligned}\tag{21}$$

In general, the amplitude for the above decays are of order g_V, because of the presence of the SU(3)'' singlet term $e(\rho_0\times\lambda_e)_\mu$ in Eq. (7). However, the above term being isoscalar [see Eq. (14)] can lead to the decay mode (a) in Eq. (21) only through effective isospin violation. Thus we expect that the matrix element for the decay mode (a) to be of order[36]

$$M(V^0\to\pi^++\pi^-)\sim O(g_V)[O(e^2/4\pi)+O(g_V{}^2/4\pi)]\,.\tag{22}$$

This will clearly lead to a rather small partial width for the $V^0\to\pi^++\pi^-$ decay. We may estimate this width on the one hand from the observed width of the $\rho^0\to\pi^++\pi^-$ decay with an appropriate phase-space factor. This yields

$$\Gamma(V^0\to\pi^++\pi^-)\sim(g_V{}^2/4\pi)^3\Gamma(\rho^0\to\pi^++\pi^-)\times\left(\frac{m_\rho}{m_V}\right)^2\left(\frac{m_V{}^2-4m_\pi{}^2}{m_\rho{}^2-4m_\pi{}^2}\right)^{3/2}\sim 2\ \text{keV}\quad(\text{for } m_V\simeq 4m_\pi)\,,\tag{23a}$$

where we have used $g_V{}^2/4\pi\simeq 3\times10^{-2}$ and $\Gamma(\rho^0\to\pi^++\pi^-)\sim 120$ MeV. Alternatively, one may estimate the above width from that of the isospin-violating $\omega^0\to\pi^++\pi^-$ decay with the appropriate phase-space factor Y, as in (23a). This yields

$$\Gamma(V^0\to\pi^++\pi^-)\sim O(g_V{}^2/4\pi)\Gamma(\omega^0\to\pi^++\pi^-)\times Y\sim(2.5\ \text{keV})\times(1\text{–}3)\quad(\text{for } m_V\simeq 4m_\pi)\,,\tag{23b}$$

where we have used $\Gamma(\omega^0\to\pi^++\pi^-)\sim(0.15\ \text{MeV})\times(1\text{–}3)$.[37] More appropriately, one may estimate the above via the intermediate step $V^0\to\omega^0\to\pi^++\pi^-$ with allowance for the lack of enhancement[38] of the off-mass-shell $\omega^0\to\pi^++\pi^-$ amplitude. This leads to[39]

$$\Gamma(V^0\to\pi^++\pi^-)\sim g_V{}^2\left(\frac{m_\omega{}^2}{m_V{}^2-m_\omega{}^2}\right)^2\times(\tfrac{1}{50})\Gamma(\omega^0\to\pi^++\pi^-)\times Y\sim(3\ \text{keV})\times(1\text{–}3)\quad(\text{for } m_V\simeq 4m_\pi)\,,\tag{23c}$$

where the factor $\frac{1}{50}$ takes account of the said enhancement factor.[38] The estimates (23a)–(23c) are sufficiently close to each other for our purpose and suggest that the partial width $\Gamma(V^0\to\pi^++\pi^-)$ is expected to be of the order of several keV. Note that the above estimate is incorrect if the term $d(\rho_8\times\lambda_d)_\mu$ is present in Eq. (7) and λ_d contains λ_3 in it. In this case the amplitude for $V^0\to\pi^++\pi^-$ decay is expected to be of order

$$M(V^0\to\pi^++\pi^-)\sim O(g_Vd)O(\epsilon)\,,\tag{24}$$

where ϵ denotes a parameter to characterize medium-strong SU(3)'' breaking [similar to that for SU(3) breaking]. If $d\sim 1$ and $\epsilon\sim\frac{1}{10}$, one may expect $\Gamma(V^0\to\pi^++\pi^-)$ to be about an order of magnitude higher than the estimates (23). In either case, the width is considerably smaller than that mentioned in Ref. 1. Because of this the radiative decay $V^0\to\pi^0+\gamma$ is expected to be a competing mode compared to the $\pi^++\pi^-$ mode. The amplitude for the radiative decay is of order g_Ve; its width may thus be estimated from that of the observed $\omega^0\to\pi^0+\gamma$ decay to yield

$$\Gamma(V^0\to\pi^0+\gamma)\sim(g_V{}^2/4\pi)\Gamma(\omega^0\to\pi^0+\gamma)\times\left(\frac{m_V{}^2-m_\pi{}^2}{m_\omega{}^2-m_\pi{}^2}\right)^3\left(\frac{m_\omega}{m_V}\right)^3\sim 9\ \text{keV}\quad(\text{for } m_V\simeq 4m_\pi)\,.\tag{25}$$

The $V^0\to\pi^++\pi^-+\pi^0$ decay is of order g_V in the amplitude; we estimate its width from that of the $\omega\to\pi^++\pi^-+\pi^0$ decay to yield

$$\Gamma(V^0 \to \pi^+ + \pi^- + \pi^0) \sim (g_V^2/4\pi)\Gamma(\omega \to \pi^+ + \pi^- + \pi^0) \times \left(\frac{m_V - 3m_\pi}{m_\omega - 3m_\pi}\right)^4 \left(\frac{m_V}{m_\omega}\right) \frac{W(m_V)}{W(m_\omega)} \sim (\tfrac{1}{5}, 2, 300)\ \text{keV} \quad (\text{for } m_V \simeq m_K,\ m_\eta, m_\omega), \qquad (26)$$

where $W(m)$ is an integral arising in the phase-space factor $[W(m_\pi) = 3.56,\ W(3m_\pi) = 1]$. The modes (d) and (e) in (21) are of order g_V in the amplitude. They are expected to be important decay modes (compared to the three-pion mode) for sufficiently heavy V^0 ($m_V > 1400$ MeV say). The radiative mode $V^0 \to \pi^+ + \pi^- + \gamma$ decay is of order $g_V e$ in the amplitude. It is, however, expected to be suppressed compared to the $\pi^0\gamma$ mode due to angular momentum barrier and phase-space factors.

In summary, the present model suggests that for sufficiently light V^0 ($3m_\pi < m_V < 500$ MeV, say), the radiative mode (b) in (21) as well as the $\pi^+ + \pi^-$ mode are expected to be its dominant decay modes. (See, however, *Note added in proof.*) In this case the V^0 width is expected to be of the order of, say, 10 – 100 keV. For moderately heavy V^0 ($700 < m_V < 1400$ MeV, say), the $\pi^+ + \pi^- + \pi^0$ mode is expected to be the dominant decay mode; the associated width should lie in the region of 100 keV to several hundred keV depending upon its mass. For still heavier V^0, the $K\bar{K}$ modes begin to be as important as the three-pion mode.

Thus, in the low-mass region, a search for a very narrow peak in the $\pi^+ + \pi^-$ mass spectrum (with no accompanying peaks in the charged two-pion and $2\pi^0$ system) is still the best way to look for the V^0 meson. We emphasize (as mentioned in the note added in proof in I) that an excellent candidate consistent with the above properties does exist in the 480-MeV mass region.[40] A careful search with high statistics and high resolution in this region will indeed be very desirable to shed more light on the existence of this object.

We next remark on the possibility that the new interaction is scalar rather than vector in form (as suggested at the end of Sec. VI). In this case the basic features for the production of the scalar meson S^0 are the *same* as those for the V^0 meson. However, there are marked differences in their decay mechanisms, assuming that the same combination of $\rho_\xi \times \lambda_\eta$ matrices enter to the scalar density $J(x)$ as to the currents $J_\mu^{(\pm)}(x)$. The S^0 meson can decay to *both* $\pi^+ + \pi$ and $\pi^0 + \pi^0$ systems in the S state and therefore with $I = 0$ in order g_V for the amplitude (rather than g_V^3). Thus the partial width $\Gamma(S^0 \to \pi + \pi)$ is expected to be of the order of an MeV rather than several keV. Furthermore the 3π decay mode is forbidden by parity. Hence the two-pion mass spectrum[41] is again the best way to look for S^0 as it is for V^0. The presence or absence of a peak in the $2\pi^0$ system and/or spin-parity determination based on angular distribution could of course easily distinguish between the S^0 and V^0.

Finally, an additional remark is worth noting. The choice of the interaction as suggested in the present model has the interesting consequence that the V^0 meson (or alternatively the S^0 meson) would *act effectively as an isoscalar*[42] *in its production and decays* involving the low-lying SU(3)″ singlet states, even though it is coupled to mixed isospin densities. In other words, the isochizon character of the V^0 or the S^0 meson cannot explicitly be revealed without involving the high-lying SU(3)″ nonsinglet states. This makes the identification of the V^0 or the S^0 meson slightly less apparent than what is expected in general (see Ref. 1). However, they should be distinguishable from the normal strongly interacting mesons by virtue of their extremely narrow widths, as discussed here.

VIII. SUMMARY AND REMARKS

In summary, we note that the notion of the new interaction proposed earlier[1] acquires several desirable features in the framework of the three-triplet model. They are (a) the construction of partially conserved currents $J_\mu^{(\pm)}$ with spin-$\frac{1}{2}$ fields, (b) the suppression of CP and T violation, and (c) the possibility of a large contribution from the new interaction to the $\Delta I = 1$ mass differences without appreciable contribution to the $\Delta I = 2$ mass differences.

While we were motivated to consider the three-triplet model to provide a basis for the new interaction primarily by the question of conservation of the vector currents $J_\mu^{(\pm)}$, it was noted in Sec. VI that a scalar form could be preferable to the vector form from considerations of the $\eta \to 3\pi$ decay and the signs of the $\Delta I = 1$ mass differences. Thus the consequences of the scalar interaction appear to be worth studying in more detail. One must of course still adopt the three-triplet model, first, in order to have $\Delta S = \Delta Q = 0$ scalar densities with spin-$\frac{1}{2}$ fields having opposite charge-conjugation parities and, second, to maintain the desirable results on CP and isospin violation of Secs. IV and V.

The strength of the new interaction as suggested here appears to be intermediate between the so-scalled medium strong and the electromagnetic interaction. This suggests that isospin violation may be dominated in general by the contribution from the proposed new interaction except in those cases where the long-range nature of the photon interaction is an important consideration. That is the case, for example, for binding-energy differences between mirror nuclei. The same is also true for the $\Delta I = 2$ mass differences if we adopt a basic quark or *SUB* picture. This is because the latter can arise only from the exchange of the photon and the V^0 (or the S^0 meson) between the quarks; by contrast the $\Delta I = 1$ mass differences can arise also through self-energy diagrams of the quarks. The latter diagrams are *not* sensitive to the range of the interaction, while the former are. We believe that such considerations could be useful to suggest whether the photon is the only isoschizon or not (ignoring the weak schizons).

From the point of view of experimental tests, the suppression of CP and T violation in the present model will have the consequences that the asymmetry parameter for the $\eta \rightarrow \pi^+ + \pi^- + \pi^0$ decay should be of the order of $\frac{1}{2}\%$ and the electric dipole moment of the neutron should lie in the region of $10^{-23}-10^{-24}$ e cm, both of which are an order of magnitude lower than the previous estimates.[1] We wish to emphasize that in addition to these measurements a search for the meson V^0 (or alternatively for the scalar meson S^0 in the two-pion mass spectra through extremely narrow peaks would be most desirable to judge on the existence of the new interaction.

Note added in proof. In the text we have missed emphasizing that if the term $d(\rho_8 \times \lambda_d)_\mu$ is present in $J_\mu^{(-)}$ and if λ_d contains λ_0 in it, then the leptonic modes [i.e., e^-e^+ and $\mu^-\mu^+$ modes] could turn out to be the important decay modes of the V^0 meson, especially for low-mass V^0. This is because, in the above case, the V^0 can be coupled directly to the charm part of the photon [see Eq. (7)] in order $e(g_V d)$. Denoting the effective V^0-γ interaction by the gauge-invariant form $F_{\mu\nu}(\partial_\mu V_\nu^0 - \partial_\nu V_\mu^0)$ with a strength $e(g_V d)/2$, the partial width $\Gamma(V^0 \rightarrow e^+e^-)$ is given by $\frac{1}{3}(\alpha^2 m_V)(4\pi g_V{}^2 d^2) \simeq d^2(25 \text{ keV})$ for $m_V \simeq 4m_\pi$ and $g_V{}^2/4\pi \simeq \frac{1}{30}$. Thus the leptonic modes would compete favorably with the $\pi^+\pi^-$ and $\pi^0\gamma$ decay modes [see estimates in Eqs. (22)–(25)] and may even be the dominant modes for low-mass V^0. By the same consideration, the V^0 may be produced easily by the e^-e^+ colliding-beam experiments with appropriate e^-e^+ invariant mass. As an additional consequence, the effective V^0-γ interaction, as mentioned above, will make a significant contribution to the anomalous magnetic moment of the muon, if the V^0 is light. The above considerations are, of course, not relevant for the scalar S^0 meson.

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[1]J. C. Pati, Phys. Rev. D 2, 2061 (1970). This paper will be referred to as I.

[2]By maximal violation we mean that the conserving and violating parts of the matrix elements for physical processes are in general comparable, as is the case with isospin violation for photonic processes and parity violation for weak processes.

[3]Note that the C-even current $d_{ijK}(\pi_j \partial_\mu \pi_K - \sigma_j \partial_\mu \sigma_K)$ (with $i = 3 + 8/\sqrt{3}$) proposed in I is not conserved even in the free-field limit with or without zero bare masses.

[4]By partial conservation of a current we mean that it is conserved in the limit of a symmetry which would be exact in the absence of certain terms in the Lagrangian. As examples: The familiar vector currents v_μ^i ($i = 1, \ldots, 8$) and the axial-vector currents a_μ^i ($i = 1, \ldots, 8$) are partially conserved in the above sense if we admit both electromagnetic and weak interactions into the Lagrangian.

[5]In addition to the K^0-$\bar{K}^0$ complex, the upper limit on the electric dipole moment of the neutron ($d_n = 4 \times 10^{-23}$ e cm) also seems to be consistent with an effect of order 10^{-3} rather than a few times 10^{-2} for T violation.

[6]Note that this question also arises if electromagnetism violates C, CP, and T "maximally" as proposed by J. Bernstein, G. Feinberg, and T. D. Lee [Phys. Rev. 139, B1650 (1965)] and S. Barshay [Phys. Letters 17, 78 (1965)].

[7]S. K. Bose and R. E. Marshak, Nuovo Cimento 25, 529 (1962); R. H. Socolow, Phys. Rev. 137, B1221 (1965).

[8]M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1964). A. N. Tavkhelidze *et al.*, *High Energy Physics and Elementary Particles* (International Atomic Energy Agency, ICTP, Trieste, 1965), p. 763.

[9]N. Cabibbo, L. Maiani, and G. Preparata, Phys. Letters 25B, 132 (1967).

[10]The main differences between the Han-Nambu model (Ref. 8) and the *SUB* model (Ref. 9) are emphasized in a recent paper by J. C. Pati and C. H. Woo, Phys. Rev. D 3, 1173 (1971).

[11]S. Adler, Phys. Rev. 177, 2426 (1969); S. Okubo, *ibid.* 179, 1629 (1969).

[12]J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920 (1971).

[13]Similar considerations can be extended to the Han-Nambu model (Ref. 8) as well.

[14]Han and Nambu have proposed a mechanism, which could naturally lead to the ordering of states according to the dimension of their SU(3)″ representation.

[15]The modification is of course necessary only if the fundamental fields carry integral charges.

[16]This can of course be checked precisely if we observe stable or semi-stable charmed particles in the high-mass

region. See discussion on this by J. C. Pati and C. H. Woo, Phys. Rev. D 3, 1173 (1971).

[17]This is true only with $V_\mu^0 = V_\mu^{0\dagger}$.

[18]This is because if either the U(9) or the U(6) group is a good symmetry in any sense, one will expect to see low-lying SU(3)″ or SU(2)″ nonsinglet partners of the observed so-called SU(3)″ singlet states, all of which belong to common U(9) or U(6) representations. This is clearly far from the observed spectrum.

[19]Interactions of the following type, for example, break both U(9) and U(6) symmetries while preserving SU(3) and SU(3)″ symmetries: (a) $g(\bar\psi\gamma_\mu\rho_j\times 1)M_\mu^j$ and (b) $f(\bar\psi\gamma_\mu(1\times\lambda_k)\psi)\Phi_\mu^k$, where M_μ^j and Φ_μ^k are an octet of vector mesons ($j=1,\ldots,8$; $k=1,\ldots,8$) transforming as $(\underline{1},\underline{8})$ and $(\underline{8},\underline{1})$, respectively, under the SU(3)×SU(3)″ group. Note that the g interaction was proposed by Han and Nambu (Ref. 8) as a mechanism to explain the high mass of SU(3)″ nonsinglet states.

[20]It is easy to check that the addition of mesonic terms to the components $J_\mu^{(\xi,\eta)}$ corresponding to terms in the Lagrangian of the type mentioned in Ref. 19 cannot alter the nonconservation of $J_\mu^{(\xi,\eta)}$.

[21]Note that the choice of the electric-charge operator as $Q = I_3 + \frac{1}{2}Y$ or $Q = I_3 + \frac{1}{2}Y + Y''$ implies that electromagnetic interaction preserves SU(2)″ symmetry.

[22]We want the admixtures to be induced by a CP-conserving interaction as one is interested in an effective CP violation through the matrix element (9b).

[23]H. Harari, Phys. Rev. Letters 17, 1303 (1966).

[24]This is consistent with our assumption in Sec. IV that the SU(2)″ symmetry is broken by the V^0 interaction and perhaps by other weaker interactions (but not by the strong and medium-strong interactions).

[25]As mentioned already, it is not necessary to choose $d=0$, if SU(3)″ is a fairly good symmetry.

[26]See, for example, D. Gross and H. Pagels, Phys. Rev. 172, 1381 (1968); R. Chanda, *ibid.* 188, 1988 (1969); M. Elitzur and H. Harari, Ann. Phys. (N.Y.) 56, 81 (1970).

[27]M. Lévy, Nuovo Cimento 52A, 23 (1967).

[28]For a clear discussion of these points see J. S. Bell and D. G. Sutherland, Nucl. Phys. B4, 315 (1968).

[29]Note that in principle there are additional possibilities for the choice of the axial-vector current, if we only demand that it must satisfy Gell-Mann's SU(3)×SU(3) algebra. The allowed choices are $a_\mu^i = \bar\psi\gamma_\mu\gamma_5(\text{“}\rho\text{”}\times\frac{1}{2}\lambda i)\psi$, where “$\rho$” is either $(\frac{3}{2})^{1/2}\rho_0$ or $-\frac{1}{3}(1+2\sqrt{3}\rho_8)$ or $\frac{1}{3}(1\pm 3\rho_3 - \sqrt{3}\rho_8)$. See S. Okubo (Ref. 11) for a discussion of these choices.

[30]Note that the vector currents v_μ^i contributing to the familiar weak and electromagnetic processes are required to be SU(3)″ singlets. This follows by noting that they are conserved and lead to generators of the SU(3) and SU(2) group [at least those which satisfy the CVC (conservation of vector current) hypothesis], so that one can calculate the basic strength of the corresponding transitions together with the familiar notion of universality of hadron-lepton couplings. This does not allow for any choice for v_μ^i other than the SU(3)″ singlet structure [Eq. (15)] unlike the situation with a_μ^i (see Ref. 29).

[31]We have considered the possibility that a_μ^i is not a pure SU(3)″ singlet with the hope that this may provide a simple solution to the $\eta\to 3\pi$ problem. In particular, we considered the choice “ρ” $=\frac{1}{3}(1\pm 3\rho_3 - \sqrt{3}\rho_8)$ as mentioned in Ref. 29. In this case one may not assume strong PCAC; however, one may still assume PCAC in the sense of dominance of dispersion relations for matrix elements involving $\partial_\mu a_\mu^i$ by the pion pole in the region of interest (i.e., $p_\pi^2\to 0$). With this choice, one obtains $d_{i\eta\alpha}$ coefficients (in addition to the $f_{i\eta\alpha}$ terms) on the right side of Eq. (17). The former has the right isospin structure to allow the $\eta\to 3\pi$ decay; however, it turns out to have the wrong SU(2)″ structure, so that the $\eta\to 3\pi$ decay is now forbidden because of SU(2)″ selection rules.

[32]R. A. Brandt, M. Goldhaber, C. A. Orzalesi, and G. Preparata, Phys. Rev. Letters 24, 1517 (1970).

[33]In other words $S^0(x)$ is not involved in any other basic interaction (except $\mathcal{L}_S$ and gravity). Its coupling constant g_S is of the same order as g_V (i.e., $g_S^2/4\pi \sim g_V^2/4\pi \sim \frac{1}{30}$, as in this paper) and its mass is required to be greater than $3m_\pi$ for the same reasons as is discussed in Ref. 1 for the V^0 meson.

[34]The question of the slope of the $\eta\to\pi^+ + \pi^- + \pi^0$ decay and the connection of the $\eta\to 3\pi$ amplitude with the $\eta\to\pi^0$ transition mass will be discussed in more detail in a subsequent note with M. Reiner.

[35]This is based on representing the fermion-boson vertex by γ_μ and 1 for the vector and scalar interactions, respectively. See, for example, S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1961), p. 523.

[36]Note that the order-g_V^3 term in Eq. (22) could also arise through SU(3)″ nonsinglet terms in $J_\mu^{(\pm)}$ which would allow for appropriate SU(2)″ violations, of order $g_V^2/4\pi$, in the matrix elements.

[37]See, for example, G. Goldhaber, talk presented at the Philadelphia Conference on Meson Spectroscopy, LRL Report No. UCRL-19850, 1970 (unpublished).

[38]The enhancement of the on-mass-shell $\omega^0\to\pi^+\pi^-$ amplitude arises due to the intermediate step $\omega^0\to\rho^0\to\pi^+\pi^-$ with $m_\omega \sim m_\rho$. The above leads to the Breit-Wigner propagator $(p_\omega^2 - m_\rho^2 + im_\rho\Gamma_\rho)^{-1}$. The enhancement is found by putting $p_\omega^2 = m_\omega^2$ and comparing with the value for $p_\omega^2 \neq m_\omega^2$. This gives (for $p_\omega^2 = 16m_\pi^2$) a ratio $\simeq 7$.

[39]The strength of the $V^0\to\omega^0$ vertex is represented by $g_V m_\omega^2$.

[40]L. Dubal and M. Roos, Nucl. Phys. B12, 146 (1969). I wish to thank Professor Roos for further communication on the status of the structure in the $\pi^+\pi^-$ system at 480 MeV.

[41]Note that the 480-MeV $\pi^+ + \pi^-$ peak (Ref. 40) mentioned before is a candidate for the S^0 meson as much as it is for the V^0 meson. In the note added in proof of Ref. 1, it is argued that considerations based on the K_L-K_S mass difference disfavor the $I=J=0^+$ possibility for this object; the argument, however, was oriented against a normal strongly interacting $I=J=0^+$ meson with a width of the order of 20 MeV (consistent with the resolution of Ref. 40). It does not apply to the S^0 meson, since $\Gamma(S^0\to\pi+\pi)$ is expected to be of the order of 1 MeV.

[42]This is because only the SU(3)″ singlet term $e(\rho_0\times\lambda_e)_\mu$ of Eq. (7) is relevant for such processes, and $\lambda_e = \lambda_0$ from the considerations of Sec. V [see Eq. (14)].