served amplitude for $K_L^0 \to 2\gamma$; therefore, the resulting fractional change in the lower bound (1) is expected to be of a similarly small magnitude.

[6]See the review talk by J. Steinberger in *Topical Conference on Weak Interactions, CERN, Geneva, Switzerland, 1969* (CERN, Geneva, 1969), p. 291.

[7]M. Banner, J. W. Cronin, J. K. Liu, and J. E. Pilcher, Phys. Rev. 188, 2033 (1969).

[8]B. D. Hyams *et al.*, Phys. Letters 29B, 521 (1969).

[9]Apart from a misprint that gives the factor $\frac{1}{2}$ as 2, the inequality (27) is the same as the inequality (4.1) in the paper by Martin *et al.* (Ref. 5). Martin *et al.* also discussed modifications to this inequality due to other on-mass-shell intermediate states.

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Weak Radiative Decay of the Σ^+ and Λ Hyperons*

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Unitarity is found to give a reliable, model-independent lower bound on the branching ratio [rate($\Lambda \to n\gamma$)/rate($\Lambda \to$ all)] > 8.5×10^{-4}. This value is nearly as large as the experimentally determined branching ratio for the decay $\Sigma^+ \to p\gamma$. Dispersion-theoretic techniques supplemented with current algebra, PCAC, and pole models are then used to determine the real as well as imaginary parts of the amplitudes for the radiative decays of the Λ and Σ^+ hyperons. Input consists of the experimental nonleptonic decay amplitudes and pion-nucleon phase shifts. The theoretical predictions are very sensitive to the hyperon magnetic moments, and until these are better known, these results for the radiative decays can only be qualitatively compared with experiment.

I. INTRODUCTION

The radiative hyperon decays, $\Sigma^+ \to p\gamma$, $\Sigma^0 \to n\gamma$, $\Lambda \to n\gamma$, $\Xi^- \to \Sigma^-\gamma$, $\Xi^0 \to \Sigma^0\gamma$, and $\Xi^0 \to \Lambda\gamma$, are interesting because they can provide insight into the nature of the nonleptonic weak interactions. They are presumed to result from the combined effect of electromagnetic and weak interactions, so that working to lowest order in the electromagnetic interaction one has in them a probe of the nonleptonic weak interaction. Given the scarcity of experimentally accessible nonleptonic processes, each source of information on them is particularly precious. At present only $\Sigma^+ \to p\gamma$ has been seen, with a branching ratio[1]

$$(\Sigma^+ \to p\gamma)/(\Sigma^+ \to \text{all}) = (1.43 \pm 0.26)\times 10^{-3}\,.$$

The experiment of Gershwin *et al.*[1] to determine the asymmetry parameter of the decay $\Sigma^+ \to p\gamma$ has added impetus to theoretical efforts to account for these processes. The asymmetry parameter α is determined from the correlation between the final proton momentum and the polarization of the initial Σ^+. It is a measure of the relative magnitudes of the s- and p-wave amplitudes. Gershwin *et al.* found α to be $-1.03^{+0.52}_{-0.42}$. Since most theoretical models predict that α is approximately zero, this measurement is very challenging to theorists. For a summary of the predictions which various techniques have given when applied to the radiative hyperon decays, see the review article by Tanaka.[2]

The principal new result to be presented here is in fact virtually model-independent. Unitarity can be used particularly effectively because the only purely-hadronic intermediate state that is energetically accessible is the $N\pi$ state. Knowing experimentally the photoproduction and $\Lambda \to N\pi$ amplitudes enables us to give the unitarity lower limit[2a]:

$$\text{branching ratio } (\Lambda \to n\gamma)/(\Lambda \to \text{all}) \gtrsim 8.5\times 10^{-4}\,.$$

This is quite a stunning result, being only a factor of 2 smaller than the experimental branching ratio for $\Sigma^+ \to p\gamma$ given above. The corresponding unitarity lower limit for $\Sigma^+ \to p\gamma$ turns out to be "unnaturally" small, as we shall see, leading to

$$\text{branching ratio } (\Sigma^+ \to p\gamma)/(\Sigma^+ \to \text{all}) \gtrsim 6.9\times 10^{-6}\,.$$

With the incentive of a possibly large rate for $\Lambda \to n\gamma$, we proceed to make a model-dependent estimate of the real part of the amplitudes. For this we exploit our knowledge of the imaginary parts by assuming the amplitudes $\Lambda \to n\gamma$ and $\Sigma \to p\gamma$ obey unsubtracted dispersion relations in the mass squared of the initial particle. In the dispersion integral, however, we need the absorptive part of the amplitude as a function of the initial hyperon mass. Approximating the full absorptive part at all energies by the contribution of the nucleon-pion intermediate state alone, even at masses for which other hadronic intermediate states are energetically allowed, enables us to write the imaginary part as a product of photoproduction and nonleptonic decay amplitudes. The latter is only measurable at the physical value of the hyperon mass, but we assume it obeys a once-subtracted dispersion relation, thereby defining its off-mass-shell behavior.

Both the radiative and nonleptonic hyperon decay amplitudes have poles at the nucleon mass squared, whose residues depend on matrix elements of the weak Hamiltonian. These are determined by using current algebra and experimental values of the nonleptonic amplitudes.

Even in the absence of inaccuracies in the procedure outlined above, the prediction of the theory for the real parts is very uncertain due to the sensitivity of the poles of the radiative amplitudes to the hyperon magnetic moments. This is reflected in the large uncertainties quoted in Eq. (22). Nevertheless it can be seen that the theory gives order of magnitude agreement with the $\Sigma\to p\gamma$ branching ratio (tending to underestimate it) and is capable of producing large, positive or negative, values of the asymmetry parameter. The $\Lambda\to n\gamma$ predictions are less sensitive to the magnetic moments and indicate a branching ratio of about 2×10^{-3} and a positive asymmetry parameter.

The unitarity calculation is in Sec. II; the dispersion relations are in Sec. III; the determination of the matrix elements of the weak Hamiltonian is in Sec. IV; the poles of the radiative amplitudes are in Sec. V; and the numerical evaluation and conclusions are in Sec. VI. Throughout we use the Bjorken-Drell metric ($a\cdot b=a_0b_0-\vec{a}\cdot\vec{b}$) and γ-matrix conventions.

II. UNITARITY CALCULATION

Unitarity determines the imaginary part of the decay amplitude in terms of a sum over physical, experimentally measurable, amplitudes for intermediate states. In the case at hand one may replace the sum over intermediate states by the contribution of the $N\pi$ intermediate state only. This should be an excellent approximation since all other states energetically allowed involved leptons or photons with their much smaller amplitudes. Thus we have the imaginary part of the unknown amplitude in terms of known photoproduction and hyperon nonleptonic decay amplitudes, enabling us to obtain the following expression:

$$\operatorname{Im}A_{fi}=\tfrac{1}{2}(2\pi)^4\int\frac{d^3p'}{(2\pi)^3}\,\frac{d^3q}{(2\pi)^3}\times\delta^4(P-p'-q)\frac{m}{p_0'}\,\frac{1}{2q_0}A_{fn}^\dagger A_{ni}\,. \tag{1}$$

Here n refers to the $N\pi$ state, p' is the momentum of the intermediate nucleon, q is the momentum of the intermediate pion, and P is the initial hyperon (Y) momentum. The most general gauge-invariant expression for the radiative decay amplitude for the process in which a hyperon of momentum P decays into a nucleon of momentum p and a photon of momentum k [$Y(P)\to N(p)+\gamma(k)$] is

$$A_{Y\to N\gamma}=\bar{u}_N(p)[(-i)(a_1+a_2\gamma_5)\sigma_{\mu\nu}k^\nu\epsilon^\mu]u_Y(P)\,, \tag{2}$$

where ϵ is the photon polarization: $\epsilon^2=-1$, $\epsilon\cdot k=0$. It will prove convenient to define the Pauli-space version of Eq. (2):

$$A_{Y\to N\gamma}=\chi_N^\dagger[A_1 i\,\vec{\sigma}\cdot(\hat{k}\times\vec{\epsilon})+A_2\vec{\sigma}\cdot\vec{\epsilon}\,]\chi_Y. \tag{3}$$

Throughout, M with no subscripts is the mass of the initial hyperon, m is the nucleon mass, μ is the pion mass, and Λ or Σ refers either to the particular hyperon or its mass. In terms of these amplitudes, the rate for radiative decay is

$$w=\frac{mk_0}{M\pi}(|A_1|^2+|A_2|^2)=\frac{1}{\pi}\left(\frac{M^2-m^2}{2M}\right)^3(|a_1|^2+|a_2|^2), \tag{4}$$

and the asymmetry parameter, defined by

$$\frac{dw}{d\Omega}\propto(1+\alpha\vec{\Sigma}\cdot\hat{p}),$$

where $\vec{\Sigma}$ is the polarization of the hyperon and $\hat{p}$ is the direction of the final nucleon, is

$$\alpha=\frac{2\operatorname{Re}(A_1^*A_2)}{|A_1|^2+|A_2|^2}\qquad(-1\le\alpha\le1)\,.$$

For the amplitudes A_{ni} and A_{nf}, we take the standard definitions,

$$\begin{aligned}A_{Y\to N\pi}&=\bar{u}_N(p')i(b_1+b_2\gamma_5)u_Y(P)\\&=i\chi_N^\dagger(B_1+B_2\vec{\sigma}\cdot\hat{q})\chi_Y,\\A_{\gamma N\to N'\pi}&=\chi_{N'}^\dagger[iF_1(\vec{\sigma}\cdot\vec{\epsilon})+F_2(\vec{\sigma}\cdot\hat{q})\vec{\sigma}\cdot(\hat{k}\times\vec{\epsilon})\\&\quad+iF_3(\vec{\sigma}\cdot\hat{k})(\hat{q}\cdot\vec{\epsilon})+iF_4(\vec{\sigma}\cdot\hat{q})(\hat{q}\cdot\vec{\epsilon})]\chi_N\,.\end{aligned} \tag{5}$$

Equation (1) is then

$$\begin{aligned}\operatorname{Im}A_1&=-|\vec{q}\,|(M_{1-}^{1/2*}B_2^{1/2}+M_{1-}^{3/2*}B_2^{3/2})\\\operatorname{Im}A_2&=-|\vec{q}\,|(E_{0+}^{1/2*}B_1^{1/2}+E_{0+}^{3/2*}B_1^{3/2})\,,\end{aligned} \tag{6}$$

where M_{1-} and E_{0+} are the photoproduction multipoles given in Berends, Donnachie, and Weaver[3] (BDW), the superscripts refer to the $N\pi$ isospin state, and $|\vec{q}\,|$ is the intermediate pion momentum: $|\vec{q}\,|=186.7$ MeV/c for Σ decay and $|\vec{q}\,|=102.9$ MeV/c for Λ decay. The values of these multipoles at a center-of-mass energy equal to the Σ or Λ mass are given in Table I. The necessary nonleptonic decay amplitudes are given in Table II.

Numerically evaluating Eq. (6) we find

TABLE I. Photoproduction multipole amplitudes taken from Berends, Donnachie, and Weaver (Ref. 3). The numbers are to be multiplied by 10^{-5} and have the units MeV^{-1}. Their signs are determined by the convention selected in calculating the photoproduction Born terms.

E_γ (MeV)	Multipole	$N\pi$ final state $I=1/2$	$I=3/2$	$n\pi^+$	$p\pi^0$
283	$E_{0+}^{\gamma p}$	−9.902	−7.771	+12.571	−0.628
283	$M_{1-}^{\gamma p}$	−0.567	−4.623	+3.046	−3.325
				$n\pi^0$	$p\pi^-$
194	$E_{0+}^{\gamma n}$	+17.057	not needed	1.273	−19.990
194	$M_{1-}^{\gamma n}$	+2.465	not needed	−1.183	−3.885

$$\begin{aligned} \mathrm{Im}A_1(\Sigma) &= -32.1\ (\mathrm{MeV\,sec})^{-1/2}, \\ \mathrm{Im}A_2(\Sigma) &= -17.7\ (\mathrm{MeV\,sec})^{-1/2}, \\ \mathrm{Im}A_1(\Lambda) &= +14.8\ (\mathrm{MeV\,sec})^{-1/2}, \\ \mathrm{Im}A_2(\Lambda) &= +279.8\ (\mathrm{MeV\,sec})^{-1/2}. \end{aligned} \tag{7}$$

The contributions of these imaginary parts to the branching ratios give the lower limits quoted in Sec. I.[5]

This unitarity lower limit on the $\Lambda \to n\gamma$ branching ratio is itself substantial, in contrast to the $\Sigma^+ \to p\gamma$ case. Examining Tables I and II shows that the $\Sigma \to p\gamma$ imaginary part would be much larger if it were not for the fact that the large photoproduction amplitude is multiplied by a very small nonleptonic decay amplitude. Thus while we cannot make similar unitarity calculations for the imaginary parts of the other radiative hyperon decays, we might expect the $\Lambda \to n\gamma$ imaginary part to be typical of their order of magnitude.

As stressed earlier, this determination of the $\Lambda \to n\gamma$ lower limit, which is virtually model-independent, should be reliable. The fact that the lower limit is so large is encouraging. It leads us, in the following sections, to use models to estimate the magnitude of the real parts of the amplitudes.

III. DISPERSION RELATION

In order to write a dispersion relation for the invariant amplitude a_1 or a_2, we need to assume it is an analytic function of the hyperon mass squared. However, a function has a unique analytic continuation only when it is defined on a dense set of points and the amplitude for a decay is only specified by nature at one point: the physical mass. Taken seriously, this analysis prevents one from using dispersion techniques for a decay amplitude at all. What we do, rather than give up the method entirely, is make an ansatz regarding the "best" choice of analytic continuation, guided by experience in scattering problems. Thus our choice of analytic continuation, for both the radiative and nonleptonic amplitudes, is defined by the dispersion relation we write for them. The validity of the particular continuation is justifiable only by its "reasonableness" and the success of the final predictions.

One expects the invariant amplitudes a_1 and a_2 to be analytic functions of the hyperon mass squared with a pole at the nucleon mass squared and a cut beginning at the $N\pi$ threshold. Assuming that it obeys an unsubtracted dispersion relation gives

$$\mathrm{Re}a_{1,2}(M^2) = \frac{R_{1,2}}{M^2 - m^2} + \frac{P}{\pi}\int_{(m+\mu)^2}^{\infty} ds' \, \frac{\mathrm{Im}a_{1,2}(s')}{s' - M^2}\,. \tag{8}$$

In this expression, $R_{1,2}$ is the residue of a_1 or a_2 at the pole and the threshold has been taken to be

TABLE II. Nonleptonic decay amplitudes taken from a review by Filthuth (Ref. 4). All the numbers are to be multiplied by 10^3 and the units are $(\mathrm{MeV\,sec})^{-1/2}$. Our definition of B_2 has the opposite sign from the corresponding amplitude used by Filthuth, and our convention on the phases of the Σ and π isotriplets lead to Σ_0^+ amplitudes of the opposite sign. The $\Delta I = \frac{1}{2}$ rule was used to determine Λ_0^0 from Λ_-^0.

	b_1	b_2	B_1	B_2
$\Lambda^0 \to p\pi^-$	+12.93 ± 0.17	−97.91 ± 2.78	12.92	−4.80
$\Lambda^0 \to n\pi^0$	−9.15	62.27	−9.14	+3.40
$\Lambda^0 \to (I=\frac{1}{2})$	−15.85	107.69	−15.83	+5.88
$\Sigma^+ \to n\pi^+$	0.17 ± 0.34	−165.51 ± 3.04	0.17	−16.22
$\Sigma^+ \to p\pi^0 (\gamma > 0)$	−13.23 ± 1.21	−99.90 ± 16.04	−13.16	−9.79
$\Sigma^+ \to p\pi^0 (\gamma < 0)$	−9.94 ± 1.55	−133.20 ± 12.14		
$\Sigma^+ \to (I=\frac{1}{2})$	7.50	192.75	7.46	18.84
$\Sigma^+ \to (I=\frac{3}{2})$	−10.90	13.97	−10.84	1.37

at the $N\pi$ intermediate state because we have already agreed to neglect nonhadronic intermediate states. Neglecting all but the $N\pi$ state in unitarity even at values of s' where other hadronic states are accessible, we see that $\mathrm{Re}\,a_{1,2}(M^2)$ depends on the nonleptonic decay amplitude at nonphysical values of the hyperon mass squared, s'.

In order to determine B_i^I for any s', we use dispersion techniques on b_i^I, the corresponding invariant amplitude. To the extent that the πN intermediate state saturates the unitarity relation for $\Sigma, \Lambda \to N\pi$, $b_i^I(s)$ has the same phase as the corresponding πN partial-wave amplitude (Fermi-Watson theorem). This knowledge of the phase of $b_i^I(s)$ for s greater than $(m+\mu)^2$, along with its analyticity properties and its value at $s = M^2$, is sufficient for determining $b_i^I(s)$ up to a polynomial in s. We can either guess the solution and verify the required properties or construct it by the N/D or Omnes-Muskhelishvili method. Here we do the former.

For both the $\Sigma^+ \to N\pi$ and the $\Lambda \to N\pi$ amplitudes the only pole in s (the variable hyperon mass squared) is at the nucleon mass squared. Both nonleptonic decay amplitudes have a right-hand cut only, beginning at $(m+\mu)^2$. Now consider an amplitude for a final state of definite orbital angular momentum and isospin, e.g., $b_1^{3/2}$, the amplitude for an s-wave, $I = \frac{3}{2}$ final state. Along the cut below the inelastic threshold it has the phase of s-wave, $I = \frac{3}{2}$, πN scattering: δ_{S31}. (The subscript S refers to s wave, the 3 to isospin $\frac{3}{2}$, and the 1 to total angular momentum $\frac{1}{2}$.) We will assume in fact that even above the inelastic threshold it continues to have the phase of elastic πN scattering. Then the nonleptonic decay amplitude $b_1^{3/2}$ as a function of the variable hyperon mass squared, s, must be

$$b_1^{3/2}(s) = \left[r_1^{3/2}\left(\frac{1}{s-m^2} - \frac{1}{M^2-m^2}\right)\exp\left(-\frac{1}{\pi}\int_{s_t}^{\infty} ds'\, \frac{\delta_{S31}(s')}{s'-m^2}\right) + b_1^{3/2}(M^2)\exp\left(-\frac{1}{\pi}\int_{s_t}^{\infty} ds'\, \frac{\delta_{S31}(s')}{s'-M^2-i\epsilon}\right) + P(s)\right] \times \exp\left(-\frac{1}{\pi}\int_{s_t}^{\infty} ds'\, \frac{\delta_{S31}(s')}{s'-s-i\epsilon}\right), \tag{9}$$

with $r_1^{3/2}$ the residue of the nucleon pole in $b_1^{3/2}(s)$. $P(s)$ is a polynomial in s which is zero at $s = M^2$.

It is simple to verify that Eq. (9) has the required behavior. In order not to introduce additional parameters we make the assumption that $P(s)$ is zero. This is equivalent to the assumption that $b_i^I(s)$ needs only one subtraction and the integrals over the phase shifts converge.

In order to evaluate Eq. (9) one must know the residues of the poles in the nonleptonic decay amplitude, shown in Fig. 1. These depend on the matrix elements of the weak Hamiltonian between hyperon and nucleon states:

$$\langle n | H_W | \Lambda^0 \rangle \equiv -\bar{u}_n(\lambda_1 + \lambda_2\gamma_5)u_\Lambda\,.$$

and (10)

$$\langle p | H_W | \Sigma^+ \rangle \equiv -\bar{u}_p(\sigma_1 + \sigma_2\gamma_5)u_\Sigma\,.$$

Treating the pole terms as Feynman diagrams gives a definite result for the diagrams of Fig. 1. However, the Feynman diagrams make sense only for physical external particles. In particular, when continuing in the external mass squared, it is unclear how the residue of the pole should be extracted from, e.g., the expression

$$i\bar{u}_n(p')\left(g\lambda_2\frac{\not{P}-m}{P^2-m^2}\right)u_\Lambda(P)\,.$$

This is in contrast to the usual case in a scattering process where the external particles can have their physical mass and yet the total momentum squared can be varied.

In this case, the residue of the pole is

$$i\bar{u}_n(p')[g\lambda_2(\not{P}-m)]u_\Lambda(P)\,,$$

evaluated at $P^2 = m^2$. However, we are unable to evaluate it at $P^2 = m^2$ since $\not{P}u_\Lambda(P)$ is only defined when $P^2 = M^2$. If one knew that this Feynman diagram behaved purely as a pole as a function of P^2, the residue could be evaluated anywhere (in particular at $P^2 = M$), giving for the residue the constant

$$ig\lambda_2(M-m)\,.$$

Unfortunately, it is impossible to verify that the Feynman diagram does behave as a pole only, since that would require knowing the meaning of $\not{P}u_\Lambda(P)$ off the mass shell. Put differently, if one regards $ig\lambda_2(M-m)$ as the value at $P^2 = M^2$ of the function which at $P^2 = m^2$ is the residue, how does one know whether M is a constant or $\sqrt{P^2}$, the latter leading to a vanishing residue.

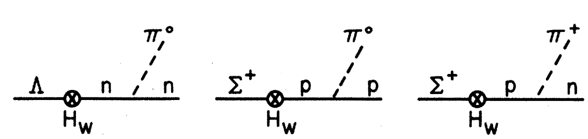

FIG. 1. Pole contributions to the nonleptonic decay amplitudes.

The prescription we give, corresponding to taking the Feynman diagram to be purely a pole in P^2, is to treat the P^2 in the denominator as the variable and take $\bar{u}_n(p')\not{P}u_\Lambda(P)$ to be fixed at its value for physical external particles. With this prescription we obtain the residues

$$r_1^{1/2}(\Lambda^0) = \sqrt{3}\, g\lambda_2(\Lambda - m), \tag{11a}$$

$$r_2^{1/2}(\Lambda^0) = -\sqrt{3}\, g\lambda_1(\Lambda + m), \tag{11b}$$

$$r_1^{1/2}(\Sigma^+) = \sqrt{3}\, g\sigma_2(\Sigma - m), \tag{11c}$$

$$r_2^{1/2}(\Sigma^+) = -\sqrt{3}\, g\sigma_1(\Sigma + m), \tag{11d}$$

$$r_1^{3/2}(\Sigma^+) = 0, \tag{11e}$$

$$r_2^{3/2}(\Sigma^+) = 0. \tag{11f}$$

IV. MATRIX ELEMENTS OF THE WEAK HAMILTONIAN

Both the pole terms in the nonleptonic amplitudes (Fig. 1) and those in the radiative amplitudes (Fig. 2) depend on the matrix elements of the weak Hamiltonian. These are parametrized by quantities called λ and σ in Eq. (10). One possibility for determining the λ's and σ's is to fit the experimental nonleptonic amplitudes by the pole terms only. The corresponding choice for the off-mass-shell behavior would be, instead of Eq. (9),

$$b_1^{3/2}(s) = b_1^{3/2}(M^2)\frac{M^2 - m^2}{s - m^2}\exp\left(-\frac{1}{\pi}\int_{s_t}^{\infty} ds'\frac{\delta_{S31}(s')}{s' - M^2} + \frac{1}{\pi}\int_{s_t}^{\infty} ds'\frac{\delta_{S31}(s')}{s' - s - i\epsilon}\right). \tag{12}$$

However, one can obtain a better fit to the experimental nonleptonic amplitudes by using a current-algebra approach first employed by Suzuki and Sugawara and then extended by others to cover the p-wave amplitudes.[6] I describe here the procedure for the second approach. Consider the amplitude of the nonleptonic hyperon decay: Hyperon (Y) of momentum P decays into a nucleon (N) of momentum p' and a pion with third component of isospin i (π^i) and momentum q. The LSZ (Lehmann-Symanzik-Zimmermann) reduction technique, upon integrating by parts and dropping the surface term, gives

$$\begin{aligned}\langle N\pi^i | H_W(0) | Y\rangle &\equiv -\bar{u}_N i(b_1 + b_2\gamma_5)u_Y \\ &= \int d^4x\, i e^{iq\cdot x}(-q^2 + \mu^2)\langle N | T(H_W(0)\varphi^i(x)) | Y\rangle .\end{aligned}$$

[Throughout, normalization factors of $(2\pi)^{-3/2}$, $1/2\omega$, etc. are suppressed, since they will be common to all terms in the final result.] Using the partial conservation of axial-vector current (PCAC) choice for the pion interpolating field and again dropping an integration-by-parts surface term, this becomes, in the limit $q \to 0$,

$$-\bar{u}(p')i(b_1 + b_2\gamma_5)u(P) = M^B_{Y\to N\pi}(q) - \frac{i}{f_\pi}\langle N | [Q_5^i(0), H_W(\vec{x})] | Y\rangle + \lim_{q\to 0}\left(-\frac{i}{f_\pi} q_\mu M^B_{Y\to NA^{i\mu}} - M^B_{Y\to N\pi}(q)\right). \tag{13}$$

The superscript B means the Born part of the amplitude, Q_5^i is the charge of the ith component of the axial-vector current, f_π is the pion decay constant (96 MeV), and $M^B_{Y\to NA^{i\mu}}$ is the pole contribution to the amplitude for a hyperon to go to a nucleon and an axial-vector current whose $SU(3)$ index is i. Each term in the last bracket is undefined as $q \to 0$; however, their sum is well defined in this limit. In arriving at Eq. (13) it is assumed that the variation of the amplitude between $q = 0$ and q physical is entirely due to the pole. This is the PCAC assumption.

Assuming that H_W transforms like a K^0 under isospin and that the baryon–axial-vector-current–baryon coupling is $SU(3)$-invariant, the Born terms contributing to Eq. (13) can be evaluated. This gives, e.g., for the Σ^+,

$$\begin{aligned}-\bar{u}(p')i(b_1 + b_2\gamma_5)u(P) = &-\frac{1}{f_\pi}\langle n | [Q_5^{1+i2}/\sqrt{2}, H_W(0)] | \Sigma^+\rangle \\ &- \frac{1}{f_\pi}(-ig_A/\sqrt{2})\bar{u}(p')\left\{ + \lambda_2\left(\frac{2}{3}\right)^{1/2}\frac{d}{f+d}\frac{\Sigma - m}{\Lambda + m} - \sigma_2\frac{d}{f+d}\frac{\Sigma - m}{\Sigma + m}\right. \\ &\left. + \gamma_5\left[\lambda_1\left(\frac{2}{3}\right)^{1/2}\frac{d}{f+d}\frac{\Sigma + m}{\Lambda - m} - \sigma_1\frac{d}{f+d}\frac{\Sigma + m}{\Sigma - m}\right]\right\}u(P)\end{aligned} \tag{14}$$

with $f/(f+d)$ determined from fits to hyperon β decay[7] to be +0.36.

In order to determine the commutation relations of the weak Hamiltonian with the axial-vector charges, we make use of the $SU(3)\otimes SU(3)$ current algebra proposed by Gell-Mann and a model of H_W. Two popular models of H_W are

$$H_W = J_\lambda^\dagger J^\lambda \quad \text{and} \quad H_W = d_{6ij} J_\lambda^i J^{j\lambda}.$$

J_λ is the sum of vector and axial-vector hadronic currents, which are the Cabibbo currents in the first case, and the $SU(3)$-octet current with index i or j in the second case. It is easily seen that either of the above Hamiltonians gives

$$\begin{aligned} &\langle n \| [Q_5^{1-i2}, H_W] | \Sigma^+ \rangle = 0, \\ &\langle n \| [Q_5^3, H_W] | \Lambda \rangle = \tfrac{1}{2}\bar{u}_n(\lambda_1 + \lambda_2\gamma_5)u_\Lambda, \\ &\langle p \| [Q_5^3, H_W] | \Sigma^+ \rangle = \tfrac{1}{2}\bar{u}_p(\sigma_1 + \sigma_2\gamma_5)u_\Sigma. \end{aligned} \tag{15}$$

These results may be substituted into Eq. (14) to obtain the final equations:

$$b_1(\Lambda_0^0) = \frac{-1}{2f_\pi}\left[-\lambda_1 + g_A\lambda_2\frac{\Lambda - m}{\Lambda + m} - \left(\frac{2}{3}\right)^{1/2}\frac{d}{f+d}g_A\sigma_2\frac{\Lambda - m}{\Sigma + m}\right] \tag{16a}$$

$$b_2(\Lambda_0^0) = \frac{-1}{2f_\pi}\left[-\lambda_2 + g_A\lambda_1\frac{\Lambda + m}{\Lambda - m} - \left(\frac{2}{3}\right)^{1/2}\frac{d}{f+d}g_A\sigma_1\frac{\Lambda + m}{\Sigma - m}\right], \tag{16b}$$

$$b_1(\Sigma_+^+) = \frac{g_A}{2f_\pi}\left[\sqrt{2}\,\frac{d}{f+d}\sigma_2\frac{\Sigma - m}{\Sigma + m} - \frac{2}{\sqrt{3}}\frac{d}{f+d}\lambda_2\frac{\Sigma - m}{\Lambda + m}\right], \tag{16c}$$

$$b_2(\Sigma_+^+) = \frac{g_A}{2f_\pi}\left[\sqrt{2}\,\frac{d}{f+d}\sigma_1\frac{\Sigma + m}{\Sigma - m} - \frac{2}{\sqrt{3}}\frac{d}{f+d}\lambda_1\frac{\Sigma + m}{\Lambda - m}\right], \tag{16d}$$

$$b_1(\Sigma_0^+) = \frac{-1}{2f_\pi}\left[g_A\frac{d}{f+d}\sigma_2\frac{\Sigma - m}{\Sigma + m} - \sigma_1\right], \tag{16e}$$

$$b_2(\Sigma_0^+) = \frac{-1}{2f_\pi}\left[g_A\frac{d}{f+d}\sigma_1\frac{\Sigma + m}{\Sigma - m} - \sigma_2\right]. \tag{16f}$$

Experimentally the b's have been determined. They are tabulated in Table II. Observe that $b_1(\Sigma_+^+)$ is virtually zero with small errors, allowing us to require

$$\frac{\sigma_2}{\lambda_2} = \frac{\sqrt{2}}{\sqrt{3}}\frac{\Sigma + m}{\Lambda + m}.$$

Analytically determining the best fit to the five remaining equations gives

$$\begin{aligned} \lambda_1 &= -3.22 \times 10^6\ (\text{MeV/sec})^{1/2}, \\ \lambda_2 &= +2.45 \times 10^6\ (\text{MeV/sec})^{1/2}, \\ \sigma_1 &= -7.01 \times 10^6\ (\text{MeV/sec})^{1/2}, \\ \sigma_2 &= +2.07 \times 10^6\ (\text{MeV/sec})^{1/2}. \end{aligned} \tag{17}$$

This fit gives back values for the b's listed in Table III, and the over-all goodness of the fit is not very sensitive to the choice of λ_2 and σ_2.

As an indication of the sensitivity of the parameters λ and σ to the model used for the nonleptonic decays, one can compare these values with the results of computing them by a pole-only model of the nonleptonic decay amplitudes. Such a model gives, instead of Eq. (16),

$$b_1(\Lambda_0^0) = g\left[\frac{\lambda_1}{\Lambda + m} + \left(\frac{2}{3}\right)^{1/2}\frac{d}{f+d}\,\frac{\sigma_2}{\Sigma + m}\right], \tag{18a}$$

$$b_2(\Lambda_0^0) = g\left[\frac{-\lambda_1}{\Lambda - m} + \left(\frac{2}{3}\right)^{1/2}\frac{d}{f+d}\,\frac{\sigma_1}{\Sigma - m}\right], \tag{18b}$$

$$b_1(\Sigma_+^+) = -\sqrt{2}\,g\left[\frac{2f+d}{f+d}\,\frac{\sigma_2}{\Sigma + m} + \left(\frac{2}{3}\right)^{1/2}\frac{d}{f+d}\,\frac{\lambda_2}{\Lambda + m}\right], \tag{18c}$$

$$b_2(\Sigma_+^+) = \sqrt{2}\,g\frac{d}{f+d}\left[\frac{\sigma_1}{\Sigma - m} - \left(\frac{2}{3}\right)^{1/2}\frac{\lambda_1}{\Lambda - m}\right], \tag{18d}$$

$$b_1(\Sigma_0^+) = -g\frac{3f+d}{f+d}\,\frac{\sigma_2}{\Sigma + m}, \tag{18e}$$

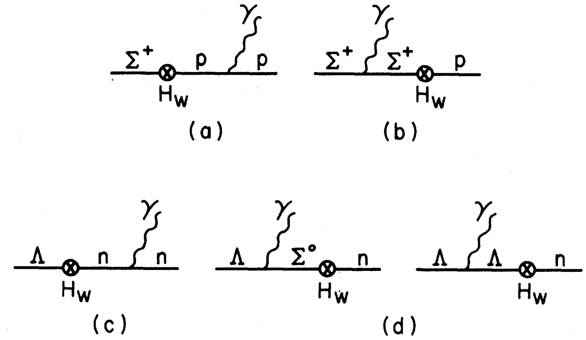

FIG. 2. Pole contributions to the radiative decay amplitudes [(a) and (c)] and contributions to the subtraction constant in the dispersion relations [(b) and (d)].

$$b_2(\Sigma_0^+) = -g\,\frac{f-d}{f+d}\,\frac{\sigma_1}{\Sigma - m}\,. \tag{18f}$$

It can immediately be seen that these equations are rather different, expecially in that there is no coupling between the parity-conserving (λ_1 and σ_1) and parity-violating (λ_2 and σ_2) quantities. Using the same input nonleptonic decay amplitudes (from Table II), we obtain the alternate solutions for matrix elements of the weak Hamiltonian which we distinguish by primes:

$$\begin{aligned}
\lambda_1' &= -3.45\times 10^6\ (\mathrm{MeV/sec})^{1/2}\,,\\
\lambda_2' &= -2.27\times 10^6\ (\mathrm{MeV/sec})^{1/2}\,,\\
\sigma_1' &= -7.25\times 10^6\ (\mathrm{MeV/sec})^{1/2}\,,\\
\sigma_2' &= +0.90\times 10^6\ (\mathrm{MeV/sec})^{1/2}\,.
\end{aligned} \tag{19}$$

As with the other method of determining these parameters, we find they are all of the same order of magnitude. This is contrary to the perfect $SU(3)$ result that λ_2 and σ_2 are identically zero, shaking our confidence in the applicability of $SU(3)$ to H_W in any guise. However, with this purely pole model we see that λ_2' has the opposite sign as λ_2. This difference leads to a substantial difference in the results for the pole contributions to the radiative decay amplitudes.

The values of the parameters given in Eq. (19) can be substituted into Eqs. (18) in order to see how well or poorly this pole model fits the hyperon decay data. The results are shown in the lower half of Table III.

V. POLE TERMS

The variable in the dispersion relation is the mass squared of the initial particle so that in the processes $\Lambda \to n\gamma$ and $\Sigma^+ \to p\gamma$ pole contributions to Eq. (8) arise from the diagrams shown in Figs. 2(a) and 2(c). The Feynman rules give, e.g., for Fig. 2(a),

$$M^B_{\Sigma\to p\gamma} = -e\bar{u}(p)\left(\frac{\mu_p}{2m}\sigma_{\mu\nu}k^\nu + i\gamma_\mu\right) \times \frac{1}{\not{P} - m}\, i(\sigma_1 + \sigma_2\gamma_5)\epsilon^\mu u(P)\,, \tag{20}$$

where e is the charge on the proton ($e^2/4\pi = \frac{1}{137}$, $e > 0$), and μ_p, μ_n, μ_Λ, and μ_Σ are the anomalous magnetic moments of proton, neutron, Λ, and Σ^+. The presence of the γ_μ terms in Eq. (20) keeps $M^B_{\Sigma\to p\gamma}$ from being gauge invariant; however, the dispersion relation is gauge invariant so the non-invariant part introduced here must be canceled by a subtraction.

Drawing a cue from Feynman theory, we see that an obvious graph contributing to the subtraction is the one shown in Fig. 2(b). Because in quantum electrodynamics gauge invariance holds order-by-order, the sum of Figs. 2(a) and 2(b) is gauge-invariant at the physical mass of the Σ. If one were interested in the amplitude $\Sigma^+ \to p\gamma$ for unphysical Σ mass, one would have to use more ingenuity to make it gauge-invariant for all values of the external mass squared. Other sources of contributions to the subtraction constant are shown in Fig. 3. These are not included for lack of information, and hopefully they are unimportant.

In the case of the Λ pole term, Fig. 2(c), the problem of gauge invariance does not arise. While the contribution of Fig. 2(d) is not needed for gauge invariance, it is still included in the spirit of approximating the subtraction by "nearby" singularities in other variables.

The sum of the diagrams in Fig. 2, evaluated for physical external mass, are the "pole" contributions to radiative hyperon decays, with some of the more important contributions to the subtraction constant included:

$$\begin{aligned}
a_1^{\mathrm{Born}}(\Lambda) &= +e\left[\frac{\lambda_1}{\Lambda - m}\left(\frac{\mu_n}{2m} - \frac{\mu_\Lambda}{2\Lambda}\right) + \frac{\sigma_1/\sqrt{2}}{\Sigma - m}\,\frac{\mu_{\Lambda\Sigma}}{\Lambda + \Sigma}\right],\\
a_2^{\mathrm{Born}}(\Lambda) &= -e\left[\frac{\lambda_2}{\Lambda + m}\left(\frac{\mu_n}{2m} + \frac{\mu_\Lambda}{2\Lambda}\right) - \frac{\sigma_2/\sqrt{2}}{\Sigma + m}\,\frac{\mu_{\Lambda\Sigma}}{\Lambda + \Sigma}\right],\\
a_1^{\mathrm{Born}}(\Sigma^+) &= +e\,\frac{\sigma_1}{\Sigma - m}\left(\frac{\mu_p}{2m} - \frac{\mu_\Sigma}{2\Sigma}\right),\\
a_2^{\mathrm{Born}}(\Sigma^+) &= -e\,\frac{\sigma_2}{\Sigma + m}\left(\frac{\mu_p}{2m} + \frac{\mu_\Sigma}{2\Sigma}\right).
\end{aligned} \tag{21}$$

TABLE III. The upper lines contain values of the nonleptonic decay amplitudes corresponding to our best fit for the weak-vertex parameters, given in Eq. (17), and the lower lines, the nonleptonic amplitudes resulting from the alternative pole-model fit given in Eq. (19). All numbers are to be multiplied by 10^3. The units are $(\mathrm{MeV\,sec})^{-1/2}$.

	Λ_0^0	Σ_+^+	Σ_0^+
b_1	−17.57	0	−36.09
b_2	59.55	−162.63	−96.31
b_1	−11.93	0	−98.65
b_2	+56.90	−163.13	−109.66

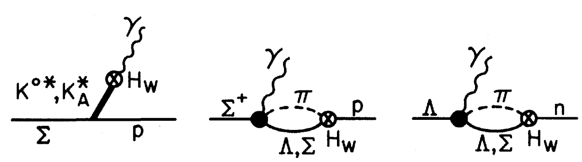

FIG. 3. Contributions to the subtraction constants for the radiative decay amplitudes which are not included.

One can immediately see from Eq. (21) how sensitive these Born contributions are to the values of the anomalous magnetic moments of the hyperons. Since μ_Λ and μ_Σ are only poorly known, and $\mu_{\Lambda\Sigma}$ (the "transition moment", i.e., the coupling of Λ-γ-Σ^+) is not measured at all, the prediction of this model for the radiative amplitudes will be very uncertain until better information is available for the magnetic moments.

VI. NUMERICAL RESULTS AND SUMMARY

Using the values of the matrix elements of the weak Hamiltonian given in Eq. (17), the dispersion integral of Eq. (8) may be numerically evaluated using as well Eqs. (6) and (9). Various approximations are necessary in order to use Eq. (9). First of all, we do not know the elastic phase shifts for s' above 4.7 GeV^2, so we cannot integrate to ∞.[8] Even if we knew them, the neglect of inelasticity would be a poor approximation at high values of s'. Explicitly varying the cutoff shows that the integrals in (9) are insensitive to it. Also, analytically integrating some function which equals the phase shift at $s' = 4.7$ GeV^2 and goes smoothly to zero indicates that if the integrals converge then they are fairly well approximated by the contribution of the region from s_t to $s' = 4.7$ GeV^2.

The value of the dispersion contribution to the radiative decay amplitudes thus obtained are given in Table IV, column 1. Since the procedure for determining the residues (r_i^I) of the nucleon poles in $Y \to N\pi$ is ambiguous (see Sec. III), we give two alternative determinations of $b_i^I(s)$ and their corresponding dispersion contribution to $\mathrm{Re}a_{1,2}$ for comparison.

One of the alternatives considered is to set the r_i^I to zero in Eq. (9). This is essentially the method of Papaioannou[5] and Contogouris and Wong,[9] who never even consider the pole terms. One could argue that in the absence of knowledge of the residues, this is the best procedure, especially if the poles are somewhat distant from the cut, since it corresponds to assuming constant off-mass-shell behavior modified by the final-state interaction. When this procedure for obtaining $b_i(s)$ is used, the values of the dispersion contribution to $\mathrm{Re}A_{1,2}$ are those given in Table IV, column 2.

The other possibility in the absence of knowledge of the residue is to assume that no subtraction is needed [i.e., $b_i^I(\infty) = 0$] and to use the physical value of the amplitude to determine the residue at the pole, i.e., use Eq. (12). The resultant values of $\mathrm{Re}A_{1,2}$ are shown in Table IV, column 3. Since we cut off the integral over $\mathrm{Im}a_i$ at a fairly low value of s ($s = 1.82$ GeV^2), the difference in $\mathrm{Re}a_i$ due to the difference between choices (2) and (3) for the b_i^I's is not great. It is unfortunate that choice (1) gives such a different result from (2) or (3). Since method (2) and (3) are somehow more "conservative," their average will be used in the final answer.

As pointed out in Sec. V, the Born contributions to the radiative decay amplitudes are very sensitive to the values of the hyperon anomalous magnetic moments. Rather than use the $SU(3)$ predictions for these moments, experimental values, when available, are used and corresponding errors are quoted. $SU(3)$ predictions are ambiguous since mass-splitting effects are important, e.g., in the $SU(3)$ limit these are equivalent:

$$\frac{\mu_{\Lambda\Sigma}}{\Lambda+\Sigma} = -\frac{3}{2}\frac{\mu_n}{2m},$$

$$\mu_{\Lambda\Sigma} = -\tfrac{3}{2}\mu_n,$$

whereas in the real world they are quite different. Since no experimental data is available on $\mu_{\Lambda\Sigma}$, the average of the above two procedures is used and the error is assigned on the basis of the quality of the corresponding $SU(3)$ predictions for μ_Λ and μ_Σ. Using the following values for the magnetic moments:

$$\frac{e\mu_n}{2m} = -1.91 \times (1.61\times 10^{-4}\ \mathrm{MeV}^{-1}),$$

$$\frac{e\mu_\Lambda}{2\Lambda} = (-0.73 \pm 0.16) \times (1.61\times 10^{-4}\ \mathrm{MeV}^{-1}),$$

$$\frac{e\mu_{\Lambda\Sigma}}{\Lambda+\Sigma} = (1.5 \pm 0.35) \times (1.61\times 10^{-4}\ \mathrm{MeV}^{-1}),$$

$$\frac{e\mu_p}{2m} = +1.79 \times (1.61\times 10^{-4}\ \mathrm{MeV}^{-1}),$$

$$\frac{e\mu_\Sigma}{2\Sigma} = (1.57 \pm 0.52) \times (1.61\times 10^{-4}\ \mathrm{MeV}^{-1}),$$

one obtains for the Born contributions to the radiative amplitudes

$$A_1^B(\Lambda) = +235 \pm 212\ (\mathrm{MeV\ sec})^{-1/2},$$

$$A_2^B(\Lambda) = +118 \pm 8\ (\mathrm{MeV\ sec})^{-1/2},$$

TABLE IV. Dispersion integral contributions to the real part of the radiative decay amplitudes, in units of $(\mathrm{MeV\ sec})^{-1/2}$. Various results correspond to the s dependence of the nonleptonic decay amplitude determined using calculated residues and a subtraction [column (1)], ignoring the pole [column (2)], and assuming no subtraction [column (3)].

	(1)	(2)	(3)
$\mathrm{Re}A_1(\Lambda^0)$	+124.5	−98.9	−54.5
$\mathrm{Re}A_2(\Lambda^0)$	+382.8	+181.6	+88.2
$\mathrm{Re}A_1(\Sigma^+)$	+21.7	+17.8	+6.7
$\mathrm{Re}A_2(\Sigma^+)$	+175.8	−4.8	+4.0

$$A_1^B(\Sigma^+) = -247 \pm 583 \text{ (MeV sec)}^{-1/2},$$

$$A_2^B(\Sigma^+) = -131 \pm 20 \text{ (MeV sec)}^{-1/2}.$$

The uncertainties reflect the uncertainties given above for the magnetic moments.

Using for the dispersion contribution the average of Table IV columns 2 and 3, and the imaginary parts given in Sec. II, one finally finds

$$\text{branching ratio } (\Sigma \to p\gamma) \approx (3.4 \pm 12.5) \times 10^{-4},$$

$$\alpha(\Sigma \to p\gamma) \approx +0.8^{+0.2}_{-1.1},$$

$$\text{branching ratio } (\Lambda \to n\gamma) = (1.9 \pm 0.8) \times 10^{-3},$$

$$\alpha(\Lambda \to n\gamma) = +0.5 \pm 0.4.$$

The uncertainties shown should not be understood in the same sense as statistical errors, but are meant to give an indication of the sensitivity of the physical quantities to the magnetic moments. The measurements[1] give

$$\text{branching ratio } (\Sigma \to p\gamma) = (1.43 \pm 0.26) \times 10^{-3},$$

$$\alpha(\Sigma \to p\gamma) = -1.03^{+0.52}_{-0.42}.$$

Thus this result is a factor of 3 too small on the Σ rate, a problem common to most models (unless the magnetic moments are chosen most favorably), and the asymmetry parameter is so sensitive to the magnetic moments that there is no definite prediction. Nonetheless, a large negative result for α is certainly within range of this theory. This is in contrast to previous work,[10] which has $\alpha \approx 0$ by virtue of approximately incorporating $SU(3)$ which gives σ_2 and $\lambda_2 \approx 0$. The parameters in $\Lambda \to n\gamma$ decay are less sensitive to uncertainties in the magnetic moments.

The importance of this work is less its detailed predictions than the following qualitative results:

(i) The unitarity contribution, and thereby the dispersion contribution, can be substantial and must not be casually neglected.

(ii) A theory not incorporating $SU(3)$, instead obtaining matrix elements of the weak Hamiltonian from hyperon nonleptonic decay data, can give large values of α for $\Sigma \to p\gamma$ decay.

(iii) There is a firm lower bound on the branching ratio for $\Lambda \to n\gamma$:

$$\text{branching ratio } (\Lambda \to n\gamma)/(\Lambda \to \text{all}) \gtrsim 8.5 \times 10^{-4}$$

and in fact the branching ratio may be several times this large. The asymmetry parameter of this decay is probably nonzero and positive.

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[1]L. K. Gershwin, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Galtieri, T. S. Mast, F. T. Solmitz, and R. D. Tripp, Phys. Rev. 188, 2077 (1969).

[2]K. Tanaka, Ohio State University report, 1970 (unpublished). In particular, see R. E. Behrends, Phys. Rev. 111, 1691 (1958), who first discussed the structure of the radiative decay amplitudes, and G. Calcucci and G. Furlan, Nuovo Cimento 21, 679 (1961), J. C. Pati, Phys. Rev. 130, 2097 (1963), and R. H. Graham and S. Pakvasa, *ibid.* 140, B1144 (1965), who did early pole-model calculations like those of Sec. V.

[2a]*Note added in proof*: It has been brought to my attention that V. I. Zakharov and A. B. Kaidalov, Yad. Fiz. 5, 369 (1967) [Sov. J. Nucl. Phys. 5, 259 (1967)], using similar methods, estimated the unitarity lower limit on the Λ radiative decay rate.

[3]F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. B4, 1 (1967). There are three consecutive articles by these authors dealing with various aspects of the subject.

[4]H. Filthuth, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN, Geneva, 1969).

[5]G. M. Papaioannou, Phys. Rev. 178, 2169 (1969), does for $\Sigma \to p\gamma$ the unitarity calculation described here, also intending to carry out the dispersion calculation. He claims to use the same photoproduction and nonleptonic decay amplitudes as used here; however, his tables of amplitudes do not bear this out, and he gets a much larger unitarity contribution to $\Sigma \to p\gamma$ than we do.

[6]M. Suzuki, Phys. Rev. Letters 15, 986 (1965); H. Sugawara, *ibid.* 15, 997 (1965); L. S. Brown and C. M. Sommerfield, *ibid.* 16, 751 (1966); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* 16, 380 (1966); S. Obuko, Ann. Phys. (N.Y.) 47, 361 (1968); C. Itzykson and M. Jacob, Nuovo Cimento 48, 655 (1967); A. Kumar and J. Pati, Phys. Rev. Letters 18, 1230 (1967).

[7]N. Brene, R. Veje, M. Roos, and C. Cronström, Phys. Rev. 149, 1288 (1966).

[8]Phase shifts are taken from the compilation of πN partial-wave amplitudes of the Particle Data Group at Berkeley [LRL Report No. UCRL-20030 πN (unpublished)]. We use the "CERN theoretical" fit to the S_{11}, S_{31}, P_{11}, and P_{31} phase shifts from threshold (1.158 GeV^2) to 4.7 GeV^2. This fit is due to A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters 26B, 161 (1968), who smooth the experimental fit by requiring it to have the energy dependence specified by a dispersion relation.

[9]A. P. Contogouris and How-sen Wong, Nucl. Phys. 40, 34 (1963).

[10]The work of M. A. Ahmed, Nuovo Cimento 58A, 728 (1968), giving $\alpha(\Sigma \to p\gamma) = -0.6$ has been shown to be incorrect by L. R. Ram Mohan, Phys. Rev. D 3, 785 (1971); L. Heiko and J. Pestieau, Lett. Nuovo Cimento 1, 347 (1971), and B. Holstein, Nuovo Cimento (to be published).