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How to See the Light Cone*

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New experiments which probe the behavior of products of electromagnetic currents at short and lightlike distances are investigated. They require the detection of a massive μ pair in electron-positron annihilation or in electroproduction. The former reaction is studied in detail and the (considerable) background is discussed. It is shown that, in suitable kinematic regions, one can approach both the Bjorken-Johnson-Low limit and the scaling limit, and put to severe test many recent ideas regarding products of currents. In the Bjorken-Johnson-Low limit one can test models of equal-time commutation relations and measure the spectral functions of an axial-vector current. In the scaling region one can measure the matrix elements of the bilocal operators appearing in the light-cone expansion of the product of electromagnetic currents, and test the tensorial structure of this expansion which emerges from quark models. Finally, in the inclusive reaction, where a massive μ pair is produced in addition to any number of hadrons, the large hadronic mass limit of the cross section is completely determined in terms of a commutator of bilocal operators.

I. INTRODUCTION

Recent experimental and theoretical advances have led to increased interest in the properties of products of local current operators separated by short or lightlike distances. Much of the interest arises from the remarkable regularities discovered in the SLAC-MIT[1] experiments on deep-inelastic electron-nucleon scattering. In the so-called scaling region one is probing the commutator of two electromagnetic currents in the vicinity of the light cone; the scaling behavior observed for the structure functions has important implications for the light-cone structure of the commutator. The structure is controlled by a collection of local operators which appear in the most singular terms on the light cone; experiments of the SLAC-MIT type provide information on these theoretically interesting objects. These theoretical issues have been discussed from a variety of points of view. In particular, it has seemed an attractive idea to investigate the light-cone structure in models, with a view to abstracting general features and then abandoning the specifics of the models. This approach has been emphasized by Gell-Mann,[2] who considered the situation for the free-quark model; it has been pursued also for quark models with strong interactions mediated by SU(3) singlet mesons.[3,4] Our discussion here will be based in part on the structure implied by the quark-vector-gluon model.[3]

If current commutators have any simple features, they are likely to show up in regions where the currents are separated by short or lightlike distances. We speak, respectively, of the Bjorken-Johnson-Low[5] and the "scaling" regions. To probe such regions one wants to deal with amplitudes involving two virtual photons which are very far off the mass shell, e.g., the absorptive part of forward Compton scattering of "massive" photons on nucleons (SLAC-MIT experiments). In the present paper we consider some further examples of physical processes of this sort; in particular, we consider the processes

$$e^+ + e^- \to \mu^+ + \mu^- + X_1 + X_2 + \cdots, \tag{1}$$

where $\{X\}$ is a system of hadrons. To lowest electromagnetic order two distinct classes of Feynman graphs have to be considered. It is the graph of Fig. 1(a) that will especially interest us here. This describes coupling of the hadron system to two virtual photons and corresponds to hadron production in states which are even under charge conjugation. The graphs of Fig. 1(b) describe coupling of the hadron system to a single virtual photon, corresponding to hadron production in states which are odd under charge conjugation. Insofar as we restrict ourselves to hadron channels $\{X\}$ which are even under charge conjugation ($\{X\}$= system of neutral pions), only the graph of Fig. 1(a) need be considered. But we shall also want to contemplate the situation where one sums over all channels of given invariant mass. In this case the two classes of graphs contribute incoherently to the cross section, all interference effects between them canceling out. It is evident that the contribution to the cross section coming from the graphs of Fig. 1(b) can be computed in terms of the cross section for $e^+ + e^- \to \{X\}$, something which itself is the subject of considerable experimental and theoretical interest. So, in principle, the contribution from the graphs of Fig. 1(b) can be isolated experimentally. We are under no illusion that this will be a simple experimental task. Nevertheless, we focus here

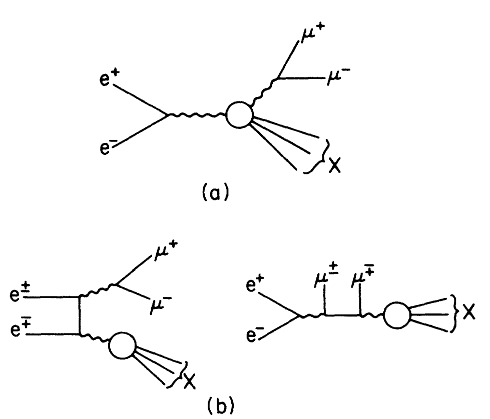

Fig. 1. Diagrams for the reaction $e^+ + e^- \to \mu^+ + \mu^- + X$.

exclusively on the cross section associated with Fig. 1(a), leaving the contributions from Fig. 1(b) to a future occasion. If the experiments are hard, at any rate the theoretical issues are worth discussing.

Let l_+ and l_- be the momenta of e^+ and e^-, k_+ and k_- the momenta of μ^+ and μ^-, and P the total momentum of the hadron system $\{X\}$. Further, define

$$\begin{aligned} &l = l_+ + l_-, \quad L = l_+ - l_-, \quad l^2 = s, \\ &k = k_+ + k_-, \quad K = k_+ - k_-, \quad Q = \tfrac{1}{2}(l+k), \end{aligned} \tag{2}$$

and observe that

$$P = l - k.$$

In computing the amplitude corresponding to the graph of Fig. 1(a), we encounter the matrix element

$$M_{\nu\mu} = i\int dx\, e^{iQ\cdot x}\langle X|\, T^*(J_\nu(\tfrac{1}{2}x), J_\mu(-\tfrac{1}{2}x))\,|0\rangle, \tag{3}$$

where J_μ is the electromagnetic current. Other factors contributing to the over-all amplitude are standard, so our theoretical attention focuses on $M_{\nu\mu}$. This matrix element depends on the four-vector Q and on the various hadron variables, in particular, the net hadron momentum P. We shall be discussing the properties of $M_{\nu\mu}$ for two different asymptotic regions: the Bjorken-Johnson-Low (BJL) limit, and the scaling limit. The important point is that both limits can be achieved *physically*. These two regions are discussed, respectively, in Secs. II and III.

Although the processes under discussion here are physically accessible, in principle, the cross sections will of course be small; the contributions of interest have, moreover, to be obtained by subtracting other contributions, which can be separately determined in principle. Another kind of process which we could consider and which bears on the issues under discussion here is

$$e + p \to e + \mu^+ + \mu^- + X. \tag{4}$$

This involves an incident spacelike (virtual) photon and an outgoing timelike photon. It can again be shown here that there is a physically achievable kinematic region which serves to probe the light cone. We are of course interested here in contributions from a graph analogous to Fig. 1(a). The analogs of Fig. 1(b) form an uninteresting "background." But unfortunately, for this reaction the two classes of graphs are coherent even when one sums over all channels. For this reason we do not pursue the above reaction any further here.

It is worth emphasizing that the amplitudes for these reactions, (1) and (4), involve two currents, in contrast to the SLAC-MIT type of experiment where it is the cross section that, via the optical theorem, is related to a two-current amplitude. Because of this, one can measure, in such experiments, the matrix elements of the local and bilocal operators that appear in the expansion of the product of electromagnetic currents at short or lightlike distances. Furthermore, via the optical theorem, one can measure matrix elements of the commutators of these local or bilocal operators, which are otherwise inaccessible since these operators do not couple directly to leptons. So, while these experiments are very difficult they are perhaps the only way of directly probing the full structure of the products of currents at short and lightlike distances.

II. BJORKEN-JOHNSON-LOW LIMIT

Here we contemplate the limit

$$Q_0 \to \infty, \quad \vec{Q} \text{ and all hadron momenta fixed}. \tag{5}$$

The limiting behavior of the matrix element $M_{\nu\mu}$ is determined by equal-time current commutators, according to the BJL expansion[5]

$$M_{\nu\mu} \xrightarrow[Q_0\to\infty]{} (\text{polynomial in } Q_0) - \frac{1}{Q_0}\int d^3x\, e^{-i\vec{Q}\cdot\vec{x}}\langle X|[J_\nu(\tfrac{1}{2}x), J_\mu(-\tfrac{1}{2}x)]|0\rangle_{x_0=0}$$
$$+\frac{i}{2Q_0^{\,2}}\int d^3x\, e^{-i\vec{Q}\cdot\vec{x}}\langle X|\{[\partial_0 J_\nu(\tfrac{1}{2}x), J_\mu(-\tfrac{1}{2}x)] - [J_\nu(\tfrac{1}{2}x), \partial_0 J_\mu(-\tfrac{1}{2}x)]\}|0\rangle_{x_0=0}\,. \tag{6}$$

The polynomial term can arise if the covariant T^* product differs from the (not necessarily covariant) time-ordered product, to which the BJL theorem applies. If the equal-time current commutators contain operator Schwinger terms (derivatives of δ functions) the T^* and T products differ, by so-called seagull terms.[6] For example, if the Schwinger term is of the form

$$[J_\nu(\tfrac{1}{2}x), J_\mu(-\tfrac{1}{2}x)]_{x_0=0} = (g_{\nu 0}g_{\mu i} + g_{\mu 0}g_{\nu i})\partial^i[S(\vec{x})\delta(\vec{x})], \tag{7}$$

then the T^* and T products differ by

$$(g_{\mu 0}g_{\nu 0} - g_{\mu\nu})S(0)\delta(x), \tag{8}$$

and the polynomial term is just

$$i(g_{\mu 0}g_{\nu 0} - g_{\nu\mu})\langle X|S(0)|0\rangle\,. \tag{9}$$

It is customary to assume that the Schwinger terms are c numbers, in which case the polynomial does not appear. This is certainly consistent with the approach we have taken elsewhere[3] to determine the short-distance behavior of current commutators in interacting quark models, where, formally, the Schwinger terms are divergent c numbers, as in the free-quark model. Of course the absence of the polynomial term is subject to experimental test for the processes under discussion. For the rest of the present discussion, however, we shall presume the absence of such terms.

In many applications of the BJL theorem, e.g., to deep-inelastic lepton scattering, the $Q_0 \to \infty$ limit is in itself unphysical. Physical implications (sum rules) are extracted through use of dispersion relations. But for the processes under present discussion, the $Q_0 \to \infty$ limit is directly physical and one is in a direct position to test various models of current commutators. That is, let E be the energy of the electron (or positron) in the center-of-mass frame of the electron pair. Hold all hadron momenta fixed and pass to the limit $E \to \infty$. In this limit

$$Q_0 \to 2E \to \infty, \qquad \vec{Q} = -\tfrac{1}{2}\vec{P}, \quad \text{fixed}. \tag{10}$$

The BJL theorem involves an expansion in inverse powers of Q_0 and makes sense only insofar as the equal-time commutators appearing in the expansion have finite matrix elements. Now suppose that the first term, at least, indeed exists in this sense. Then it follows that $M_{\nu\mu}$ has a discontinuity with respect to Q_0 which must vanish more rapidly than Q_0^{-1}. For if $M_{\nu\mu}$ behaves like Q_0^{-1} as $Q_0 \to \infty$, we see from the dispersion relation

$$M_{\nu\mu} = \frac{1}{2\pi}\int dQ_0' \frac{\mathrm{Disc}M_{\nu\mu}(Q_0')}{Q_0' - Q_0}, \tag{11}$$

that

$$\lim_{Q_0\to\infty} Q_0\,\mathrm{Disc}M_{\nu\mu} = 0\,. \tag{12}$$

In fact, if the BJL expansion exists up to terms which vanish faster than Q_0^{-n}, we have

$$\lim_{Q_0\to\infty} Q_0^{\,n}\,\mathrm{Disc}M_{\nu\mu} = 0\,; \tag{13}$$

and therefore, to order Q_0^{-n}, the phase of $M_{\nu\mu}$ is determined solely by final-state hadron interactions.

One might well expect that the BJL expansion is at best an asymptotic one, and that it must break down beyond some order in inverse powers of Q_0. Indeed, perturbation-theory calculations indicate that the expansion generally breaks down already at the first stage.[7] On the other hand, evidence from the SLAC-MIT experiments on deep-inelastic electron scattering suggests a more optimistic possibility.[1] These experiments probe the light-cone behavior of the current commutators rather than the short-distance behavior. However, the existence of a finite scaling limit for the structure functions implies that the nucleon matrix elements are finite, at least for certain parts of the equal-time commutators $[\partial_0{}^n J_\mu, J_\nu]$; namely, the highest-spin components of the commutators. Let us recall what this means. In a previous paper[3] we have introduced the term "twist" (τ) to denote the dimension of a local operator (calculated naively) minus the highest spin contained in the operator; e.g., $\bar\psi\gamma_\mu\psi$ has (mass) dimension three and spin one, hence twist two. The light-cone behavior of current commutators receives dominant contributions from operators with the smallest twist; and the scaling behavior observed in the SLAC-MIT experiments implies that these dominant operators are of twist two. That is, the scaling results imply that the BJL limit is valid at least for the *twist-two* components of the *equal-time* commutators. Of course the BJL limit is sensitive also to operators of higher twist; i.e., it is sensitive to dimension rather than twist. It is an open question then whether, and to what order, the BJL expansion is valid; it is this question that represents one of the points of interest for the processes under discussion. For the present, we shall suppose that the expansion makes sense at least to the orders $Q_0{}^{-1}$ and $Q_0{}^{-2}$.

The equal-time commutators $[\partial_0{}^n J_\nu, J_\mu]$ are of course model-dependent. We shall adopt here the quark-gluon model, where the currents are constructed out of quark fields and the strong interactions of quarks are mediated by some combination of SU(3)-singlet mesons (vector, scalar, pseudoscalar, with fields denoted by B_μ, σ, ϕ). The electromagnetic current is

$$J_\mu(x) = \bar\psi(x) Q \gamma_\mu \psi(x), \tag{14}$$

with

$$Q = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{15}$$

the charge matrix of quarks. For the first (space-space) equal-time commutator we then have the formal result

$$[J_i(\tfrac{1}{2}x), J_j(-\tfrac{1}{2}x)]_{x_0=0} = -2i\epsilon_{0ij\mu} A^{\mu, Q^2}(0)\delta(\vec{x}), \tag{16}$$

where $A_\mu^{Q^2}$ is an axial-vector current

$$A_\mu^{Q^2}(x) = \bar\psi(x)\gamma_5\gamma_\mu Q^2\psi(x), \quad Q^2 = \tfrac{2}{9} + \tfrac{1}{3}Q. \tag{17}$$

In connection with production of 2π states, we shall also want the second commutator $[\partial_0 J_i, J_j]$ in the BJL expansion. With respect to the indices i and j this has both a symmetric and an antisymmetric part. The latter, however, is an axial-vector operator and therefore does not contribute to the 2π matrix element. So we record only the symmetric part:

$$\tfrac{1}{2}[\partial_0 J_i(\tfrac{1}{2}x), J_j(-\tfrac{1}{2}x)]_{x_0=0} + (i \leftrightarrow j) = \tfrac{1}{2}[\Theta_{ij}(0) + \Theta_{ji}(0) - 2\delta_{ij}\Theta_{kk}(0) + 2\delta_{ij}S(0)]\delta(\vec{x}), \tag{18}$$

where

$$\begin{aligned} \Theta_{ij} &= \bar\psi Q^2(i\gamma_i\overrightarrow{\partial}_j - i\gamma_i\overleftarrow{\partial}_j - 2g\gamma_i B_j)\psi, \\ S &= \bar\psi Q^2(M + g_S\sigma - ig_{PS}\phi)\psi. \end{aligned} \tag{19}$$

Here M is the quark mass, and we observe that S is a twist-four operator.

(i) For a first application, let us consider the inclusive reaction $e^+ + e^- \to \mu^+ + \mu^- + X$, summed over all hadron channels $\{X\}$ of specified four-momentum P. Let $s = l^2$ be the barycentric energy variable and consider the leading term in the limit $s \to \infty$, P_μ finite. This leading behavior is determined by the first term in BJL expansion, which gives

$$M_{ij} \to \frac{2i}{Q^2}\epsilon_{ij}{}^{\rho\sigma} Q_\rho \langle X | A_\sigma^{Q^2} | 0\rangle, \tag{20}$$

where $Q^2 \to s$ in the limit. In computing the inclusive cross section we encounter the tensor

$$\begin{aligned} (2\pi)^3 \sum_X \langle 0 | A_\sigma^{Q^2} | X\rangle\langle X | A_{\sigma'}^{Q^2} | 0\rangle \delta(P - P_x) &= \frac{1}{2\pi}\int dx\, e^{iP\cdot x}\langle 0 | [A_\sigma^{Q^2}(x), A_{\sigma'}^{Q^2}(0)] | 0\rangle \\ &= \rho_1(P^2)(P_\sigma P_{\sigma'} - P^2 g_{\sigma\sigma'}) + \rho_0(P^2) P_\sigma P_{\sigma'}, \end{aligned} \tag{21}$$

where ρ_1 and ρ_0 are, respectively, the spin-one and spin-zero weight functions in the propagator of our unusual axial-vector current $A_\mu^{Q^2}$.

In the center-of-mass frame of the electron-positron pair we write $P_\mu = (P_0, \vec{p})$ and denote by θ the angle between $\vec{p}$ and the momentum vector of the positron. In the limit $s \to \infty$, P^2 and $\vec{p}$ fixed, we find

$$d\sigma \to \frac{8\alpha^4}{3s^4} W(P^2, p, \theta)\frac{p^2}{P_0} dP^2 dp d\Omega, \tag{22}$$

where α is the fine-structure constant and

$$W = \rho_1[4P^2 + p^2(1 + \cos^2\theta)] + \rho_0 p^2(1 + \cos^2\theta). \tag{23}$$

In principle, the weight functions $\rho_1(P^2)$ and $\rho_0(P^2)$ can be separately determined at any P^2 by study of the cross section in its dependence on the variables p and $\cos\theta$. These weight functions are objects of considerable theoretical interest and deserve several comments here.

First, notice from Eq. (17), and from the relation $Q = I_3 + \frac{1}{2}Y$, that $A_\sigma^{Q^2}$ has an isovector part and an isoscalar part, where the latter is a combination of axial hypercharge and baryon currents. The spectral function $\rho_0(P^2)$ has a discrete contribution coming from the one-pion state at $P^2 = m_\pi^2$; and of course only the isovector part of $A_\sigma^{Q^2}$ contributes to this, with coefficient fixed by the PCAC (partially conserved axial-vector current) constant f_π[8]:

$$\rho_0 = (\tfrac{1}{3}f_\pi)^2\delta(P^2 - m_\pi^2) + \text{continuum}. \tag{24}$$

Thus, for the reaction $e^+e^- \to \mu^+ + \mu^- + \pi^0$ we have

$$d\sigma(e^+ + e^- \to \mu^+ + \mu^- + \pi^0) \to \frac{8\alpha^4}{3s^4}(\tfrac{1}{3}f_\pi)^2\frac{p^4}{P_0}(1+\cos^2\theta)dp\,d\Omega \tag{25}$$

in the limit $s\to\infty$, $\vec{p}$ finite.

Returning to the inclusive reaction, we next observe that the continuum of ρ_1 and ρ_0 begins at $P^2 = (3m_\pi)^2$, corresponding to production of 3π systems; since $A_\sigma^{Q^2}$ is an axial-vector operator, the 2π system does not contribute to leading order as $s\to\infty$. From G-parity considerations we see that for systems composed of an odd number of pions only the isovector part of $A_\sigma^{Q^2}$ makes a contribution; for systems composed of an even number (>4) of pions, only the isoscalar part of $A_\sigma^{Q^2}$ contributes. Although the BJL limit corresponds to $P^2 \ll s$ and $p^2 \ll s$, we can in principle contemplate experimental determination of the weight functions ρ_1 and ρ_0 for P^2 very large compared to typical hadron masses. But for $P^2\to\infty$ these weight functions are determined by the vacuum expectation value of the commutator $[A_\sigma^{Q^2}(x), A_{\sigma'}^{Q^2}(0)]$ near the light cone, $x^2\approx 0$. In the quark-gluon model, as we have discussed elsewhere,[3] this quantity is exactly the same as in the free-quark model, and, as if $A_\sigma^{Q^2}$ were a conserved current, we then expect that $\rho_0\to 0$ as $P^2\to\infty$. Moreover, in this limit we expect that $\rho_1(P^2)$ should bear a simple relation to the corresponding weight function $\rho_1^{\text{em}}(P^2)$ that arises for the propagator of the electromagnetic current. The latter weight function is of course measured by the cross section for the inclusive reaction $e^+ + e^- \to X$. In terms of expectation values of quark charges Q, we have

$$\rho_1(P^2)/\rho_1^{\text{em}}(P^2) \xrightarrow[P^2\to\infty]{} \langle Q^4\rangle/\langle Q^2\rangle = \tfrac{1}{3}, \tag{26}$$

where the equality corresponds to the usual assignment of quark charges.

(ii) As we have already noted, the amplitude $M_{\nu\mu}$ for the particular reaction

$$e^+ + e^- \to \mu^+ + \mu^- + 2\pi \tag{27}$$

receives no contribution from the first term in the BJL expansion. The second term in the expansion does contribute and is of great interest since it involves operators which contribute to inelastic electron-nucleon scattering. The SLAC-MIT experiments measure, among the rest, the diagonal nucleon matrix elements of the twist-two operator θ_{ij}. We therefore have the prospect here of measuring different matrix elements of this operator and for testing the tensor structure implied by Eq. (18). Moreover, the twist-four operator S comes into play here, whereas in the scaling limit for the SLAC-MIT experiments it makes no contribution. It is an important question whether it has finite matrix elements, as we are here assuming; i.e., whether the BJL expansion is valid at the level of twist-four operators. In the present situation, to the leading order Q_0^{-2}, it is evident from Eq. (18) that the two-pion system is produced only in states of zero isotopic spin and angular momentum $l=0$ or 2.

In order to display the structure of the matrix element let us denote the pion momenta by P_1 and P_2 and define

$$\Delta = P_1 - P_2,\quad P = P_1 + P_2. \tag{28}$$

Writing

$$\langle P_1, P_2|\Theta_{ij} + \Theta_{ji}|0\rangle = A(P^2)P_iP_j + B(P^2)\Delta_i\Delta_j \tag{29}$$

and

$$\langle P_1, P_2|S|0\rangle = P^2C(P^2), \tag{30}$$

we then have

$$M_{ij} \to \frac{1}{Q^2}[A(P_iP_j - |\vec{P}|^2\delta_{ij}) + B(\Delta_i\Delta_j - |\vec{\Delta}|^2\delta_{ij}) + CP^2\delta_{ij}], \tag{31}$$

where A, B, C depend only on the dipion invariant-mass variable P^2, and for P_μ finite and $Q_0\to\infty$, we have $Q^2\to s$. Since lepton and hadron electromagnetic currents are conserved, it is enough to specify, as we have done, the space-space components of the tensor $M_{\nu\mu}$. Although we can say nothing about the functions A, B, C, the tensor structure indicated above has a restricted form and is therefore in itself diagnostic of our underlying quark-gluon model. Computation of the differential cross section is tedious and will not be undertaken here. We merely note that, in comparison with the cross section of Eq. (22), the differential cross section for the 2π channel behaves like s^{-5} as $s\to\infty$; i.e., it falls more rapidly by one whole power of s.

III. THE SCALING LIMIT

The second asymptotic region that we wish to consider for the reaction $e^+ + e^- \to \mu^+ + \mu^- + X$ is one which is analogous to the so-called scaling limit of deep-inelastic lepton-hadron scattering. For this discussion it will be convenient to define two new quantities [see Eq. (2)]:

$$\nu = Q\cdot P = \tfrac{1}{2}(l^2 - k^2)\,, \quad \omega = \frac{Q^2}{Q\cdot P}. \tag{32}$$

In the physical region Q^2 and ν are positive; with neglect of lepton masses, we have the inequalities

$$\frac{\nu}{P^2} \geqslant \omega \geqslant 1 - \frac{P^2}{4\nu}. \tag{33}$$

For the scaling limit, we consider

$$\nu \to \infty, \quad \omega \text{ and } P^2 \text{ fixed}. \tag{34}$$

In this limit

$$\begin{aligned} &(\omega+1)\nu \to s\,, \\ &\frac{k^2}{s} \to \frac{\omega-1}{\omega+1}, \end{aligned} \tag{35}$$

where k^2 is the invariant squared mass of the muon pair, and

$$\frac{P_0}{s^{1/2}} \to \frac{1}{\omega+1}, \tag{36}$$

where P_0 is the energy of the hadron system, as measured in the laboratory frame (center of mass of the e^+, e^- pair).

Let us now consider the defining integral for $M_{\nu\mu}$, Eq. (3). For the scaling limit, we assert that the dominant contributions in the integral come from the light-cone region, $x^2 \approx 0$. This is most easily seen in the rest frame of the hadron system, where $Q_0 = \nu/\sqrt{P^2}$ and $|\vec{Q}| \approx Q_0 - \frac{1}{2}\omega\sqrt{P^2}$. For large Q_0, P^2 fixed, $M_{\nu\mu}$ receives its main contributions from the region $|x_0^2 - (\vec{x}\cdot\hat{Q})^2| \lesssim (Q^2)^{-1}$ and thus $x^2 \approx (Q^2)^{-1}$. This resembles the situation for deep-inelastic lepton-hadron scattering, where, in the scaling limit, one is also probing the light-cone structure of the product of currents. In the SLAC-MIT experiments one measures total cross sections for (virtual) photons on hadrons and thereby, via the optical theorem, the *absorptive* part of the *forward* photon-nucleon elastic amplitudes. The photons are spacelike and the scaling variable ω is restricted to the range between 0 and 1. In the situation under discussion in the present paper, one is dealing with the *full* amplitude for a virtual timelike photon to decay into a timelike photon plus hadrons. The scaling variable ω is here restricted to the range 1 to ∞. The invariant momentum transfer between incoming and outgoing photons is $(l-k)^2 = P^2$; it is positive and above the relevant hadron threshold, whereas, in the inelastic lepton-hadron scattering situation, this momentum transfer variable is zero. We thus have the opportunity in the present case to study the light-cone structure of the same product of current operators that arises in the SLAC-MIT experiments, but in new kinematical regions and for new matrix elements.

In a formal analysis based on canonical commutation relations, the behavior of a product of two currents near the light cone is controlled by those local operators in the Wilson[9] short-distance expansion which are of lowest twist. The lowest-twist operators determine the highest-spin components of the equal-time commutators of the currents. They can be collected together to give the most singular term near the light cone in an expansion in terms of bilocal operators. Recently, we have determined the formal light-cone structure in the quark-gluon model.[3] For electromagnetic currents we have

$$\begin{aligned} &[J_\nu(x), J_\mu(y)]_{\overrightarrow{(x-y)^2\approx 0}}\,[V_\mu(X,\Delta)g_{\nu\alpha} + V_\nu(X,\Delta)g_{\mu\alpha} - V_\alpha(X,\Delta)g_{\nu\mu} - i\epsilon_{\nu\mu\lambda\alpha}A^\lambda(X,\Delta)]\tfrac{1}{8}\partial^\alpha D(\Delta)\,, \\ &D(\Delta) = \frac{1}{2\pi}\epsilon(\Delta_0)\delta(\Delta^2)\,, \end{aligned} \tag{37}$$

where $X = \frac{1}{2}(x+y)$, $\Delta = \frac{1}{2}(x-y)$ and where the bilocal operators V_μ and A_μ are given by

$$\begin{aligned} &V_\mu(X,\Delta) = \tfrac{1}{2}\bar\psi(x)Q^2\gamma_\mu \exp\left[-ig\int_y^x dz^\mu B_\mu(z)\right]\psi(y) - (x \to y)\,, \\ &A_\mu(X,\Delta) = \tfrac{1}{2}\bar\psi(x)Q^2\gamma_5\gamma_\mu \exp\left[-ig\int_y^x dz^\mu B_\mu(z)\right]\psi(y) + (x \to y)\,. \end{aligned} \tag{38}$$

Here B_μ is the [SU(3) singlet] vector-gluon field, g the quark–vector-gluon coupling constant, and the integral is taken along a lightlike path from y to x. The tensor and SU(3)×SU(3) structure of the light-cone commutator leads to a variety of predictions for the structure functions of deep-inelastic lepton-hadron scattering and the nature of the light-cone singularity precisely accords with the existence of a scaling limit. The structure function $F_2(\omega)$, for example. is related to the (spinaveraged) nucleon matrix element of $V_\mu(0, \Delta)$ by

$$\langle p|\, V_\mu(0, \Delta)\,|p\rangle = \frac{p_\mu}{m}\int_{-1}^{1} d\omega \frac{F_2(\omega)}{\omega} e^{2i\omega p\cdot\Delta} + \cdots, \quad (39)$$

where the omitted term is proportional to Δ_μ.

For the reactions $e^+ + e^- \to \mu^+ + \mu^- + X$, we will be concerned with the matrix elements $\langle X|\, V_\mu(0, \Delta)\,|0\rangle$ and $\langle X|\, A_\mu(0, \Delta)\,|0\rangle$. As an immediate example, consider the one-pion channel: $e^+ + e^- \to \mu^+ + \mu^- + \pi^0$. We have already discussed this process in the BJL limit. For the scaling limit, noting that only the bilocal axial vector A_μ contributes, we have

$$M_{\nu\mu} \to \epsilon_{\nu\mu\lambda\alpha}\int dx\, e^{iQ\cdot x}\langle P|A^\lambda(0, x)\,|0\rangle\theta(x_0)\partial^\alpha D(x). \quad (40)$$

We have expressed $M_{\nu\mu}$ in terms of the retarded commutator of the electromagnetic currents. Now define $G(\omega) = G(-\omega)$ by

$$\langle P|A_\lambda(0, \Delta)\,|0\rangle = P_\lambda \int_{-\infty}^{\infty} d\omega\, G(\omega) e^{-i\omega P\cdot\Delta} + \cdots, \quad (41)$$

where the omitted term is proportional to Δ_λ and is irrelevant for the scaling limit. We then find

$$M_{\nu\mu} \to -i\epsilon_{\nu\mu\lambda\alpha}P^\lambda Q^\alpha \frac{1}{\nu}\int_{-1}^{1} d\omega' \frac{G(\omega')}{\omega' - \omega}. \quad (42)$$

The dispersion integral runs over $|\omega| < 1$ only, since the absorptive part of $M_{\nu\mu}$ vanishes outside this region:

$$G(\omega) = 0, \quad |\omega| > 1.$$

A simple argument goes as follows.[10] For general ν and ω, the absorptive part may be written

$$\begin{aligned} \mathrm{Im} M_{\nu\mu} &= \pi\epsilon_{\nu\mu\lambda\alpha}P^\lambda Q^\alpha \frac{1}{\nu} G(\nu, \omega) \\ &= \frac{1}{2}\int dx\, e^{iQ\cdot x}\langle P|\,[J_\nu(\tfrac{1}{2}x), J_\mu(-\tfrac{1}{2}x)]\,|0\rangle. \end{aligned} \quad (43)$$

When ν is large compared to the pion mass, $G(\nu, \omega)$ has support properties expressed by

$$G(\nu, \omega) = 0 \quad \begin{cases} \nu > 0, & \omega < -1 \\ \nu < 0, & \omega > 1. \end{cases} \quad (44)$$

Moreover $G(\omega, \nu)$ is symmetric under crossing: $G(\omega, \nu) = G(-\omega, -\nu)$. In the scaling limit $\nu \to \infty$, $G(\omega, \nu) \to G(\omega)$, where $G(\omega)$ is given by Eq. (41). But we can also consider the limit $\nu \to -\infty$, where G approaches the same function $G(\omega)$. So $G(\omega)$ is symmetric in ω and it vanishes for $|\omega| > 1$. In the Appendix we derive the same result more generally using the Jost-Lehmann-Dyson[11] representation for matrix elements of the current commutator.

Next let us turn to the *inclusive* reaction $e^+ + e^- \to \mu^+ + \mu^- + X$, where we sum over all hadron channels of specified momentum P and consider the limit $\nu \to \infty$, P^2, ω fixed. To compute the cross section in this limit we replace the retarded commutator in $M_{\nu\mu}$ by its leading light-cone approximation, so that

$$\begin{aligned} \sum_X M^*_{\nu'\mu'} M_{\nu\mu}\delta(P - P_X) &\equiv M_{\nu'\mu';\nu\mu} \\ &= \sum_X \Big[\int dx\, e^{iQ\cdot x}\langle X|\,\Theta_{\nu\mu\alpha}(0, \tfrac{1}{2}x)\,\theta(x_0)\partial^\alpha D(x)\,|0\rangle \\ &\qquad \times \int dz\, e^{-iQ\cdot z}\langle X|\,\Theta_{\nu'\mu'\alpha'}(0, \tfrac{1}{2}z)\,\theta(z_0)\partial^{\alpha'} D(z)\,|0\rangle^*\Big]\delta(P - P_X), \end{aligned} \quad (45)$$

where the bilocal operator $\Theta_{\mu\nu\alpha}(0, \Delta)$ multiplies $\partial^\alpha D$ in Eq. (37). We shall now argue that the tensor structure of the quantity appearing in Eq. (45) has the same form as it would have if the sum Σ_X ran over only two *one-particle* channels, scalar and pseudoscalar, each with momentum P.

This is best seen in the "infinite-momentum frame" $P_0 \to \infty$, where we take

$$\begin{aligned} Q_\mu &= (\omega P_0, 0, 0, 0), \\ P_\mu &= (P_0, 0, 0, P_0 - P^2/2P_0). \end{aligned} \quad (46)$$

Now consider a particular state X consisting of N hadrons, with momenta p_i. We write

$$p_i = \alpha_i P + p_i^T, \quad (47)$$

where

$$P\cdot p_i^T = Q\cdot p_i^T = 0$$

and $\sum_1^N \alpha_1 = 1$. In general, the hadrons also bear spin labels, which, however, we suppress. Near the light cone, we encounter matrix elements of the bilocal vector and axial-vectors, e.g.,

$$\int dx\, e^{iQ\cdot x}\langle X|\, V_\mu(0, \tfrac{1}{2}x)\,|\,0\rangle\theta(x_0)\partial_\nu D(x)\,. \tag{48}$$

But

$$\langle X|\, V_\mu(0, \tfrac{1}{2}x)|\,0\rangle = P_\mu f(x\cdot P,\, x\cdot p_i^T,\, p_i^T\cdot p_j^T,\, \alpha_i,\, P^2) + \epsilon_{\mu\alpha\beta\gamma}P^\alpha g^{\beta\gamma}(x\cdot P,\, x\cdot p_i^T, \ldots) + \cdots\,, \tag{49}$$

where the omitted terms can be ignored in the limit $P_0 \to \infty$ since they depend on the components of P_μ only through $x\cdot P$ and P^2. In the final sum over states, the two terms retained above do not interfere and they make contributions with the same tensor structure. So it will be enough for us tc consider the first term alone. Writing

$$f(x\cdot P,\, x\cdot p_i^T, \ldots) = \int d\omega \prod_i d\beta_i \tilde{f}(\omega, \beta_i, \ldots)\exp\Big[-i(\omega P + \sum_j \beta_j p_j^T)\cdot \tfrac{1}{2}x\Big], \tag{50}$$

we carry out the integration in Eq. (48) and find the result

$$P_\mu(Q_\nu - \tfrac{1}{2}\omega P_\nu)\frac{1}{\nu}\int_{-1}^{1}\frac{d\omega'}{\omega' - \omega - i\epsilon}\int \prod_i d\beta_i \tilde{f}(\omega', \beta_i, \alpha_i, p_i^T\cdot p_j^T, P^2)\,. \tag{51}$$

This has the same tensor structure that we find for a single- (scalar) particle state of momentum P. Proceeding in the same way for the axial-vector bilocal operator and employing the full structure of Eq. (37), we obtain finally

$$\nu^2 M_{\nu'\mu';\nu\mu} \to g_{PS}(\omega, P^2)\epsilon_{\nu'\mu'\rho'\sigma'}\epsilon_{\nu\mu\rho\sigma}P^{\rho'}P^{\rho}Q^{\sigma'}Q^{\sigma}$$
$$+ g_S(\omega, P^2)\,[P_{\nu'}P_{\mu'}\omega - (P_{\nu'}Q_{\mu'} + Q_{\nu'}P_{\mu'}) + \nu g_{\nu'\mu'}]\,[P_\nu P_\mu\omega - (P_\nu Q_\mu + Q_\nu P_\mu) + \nu g_{\nu\mu}], \tag{52}$$

where g_S and g_{PS} are unknown functions of P^2 and the scaling variable ω. We may observe here that, to leading order in the scaling limit, $\nu\to\infty$, ω, P^2 finite, the above expression is compatible with current conservation, which requires that $k_\nu M_{\nu\mu} = M_{\nu\mu}l_\mu = 0$. Recalling that $l = Q + \frac{1}{2}P$, $k = Q - \frac{1}{2}P$, we see that these requirements are indeed satisfied to leading order. For this reason, it is not necessary here to employ the elaborate projection-operator methods which we have discussed elsewhere to enforce current conservation in leading order.[3]

We may now record the differential cross section for the inclusive reaction $e^+ + e^- \to \mu^+ + \mu^- + X$ in terms of the scaling-limit functions $g_S(\omega, P^2)$ and $g_{PS}(\omega, P^2)$. We find

$$d\sigma \to \frac{16\pi^3\alpha^4}{3s^2}\,\frac{1+\cos^2\theta}{(\omega-1)(\omega+1)^2}\,[g_S(\omega, P^2) + g_{PS}(\omega, P^2)]\,d\omega\, dP^2 d\Omega\,, \tag{53}$$

where θ is the angle between $\vec{1}_+$ and $\vec{P}$, the three-momentum vectors of the incident positron and the outgoing hadron system, as measured in the center-of-mass (laboratory) frame of the electron-positron pair.

In the scaling limit the cross section for the inclusive reaction $e^+ + e^- \to \mu^+ + \mu^- + X$ is determined by the bilocal operators which represent the current commutator on the light cone. Using completeness, let us rewrite Eq. (45) in the form

$$M_{\nu'\mu';\nu\mu} \to \frac{1}{(2\pi)^4}\int dz dx dx'\, \theta(x_0)\,\theta(x_0')\exp(-iP\cdot z - iQ\cdot x' + iQ\cdot x)$$
$$\times\langle 0|\,[\theta^\dagger_{\nu'\mu'\alpha'}(\tfrac{1}{2}z, \tfrac{1}{2}x'),\ \theta_{\nu\mu\alpha}(-\tfrac{1}{2}z, \tfrac{1}{2}x)]|\,0\rangle\,\partial^{\alpha'}D(x')\partial^{\alpha}D(x)\,. \tag{54}$$

Evidently, what is measured in the scaling limit is some sort of spectral function for the commutator of the bilocal operators. This resembles the situation for the BJL limit, where the inclusive reaction measures the spectral function of the local axial-vector current. Concerning the latter spectral function, we saw in turn that the large-P^2 behavior is determined by the light-cone commutator of the axial-vector currents. For the scaling limit it is therefore natural to inquire whether the large P^2 behavior is determined by the light-cone structure of the commutator of the bilocal operators. We are inquiring about the functions $g_S(\omega, P^2)$ and $g_{PS}(\omega, P^2)$ in the limit $P^2 \to \infty$, ω fixed. We shall argue that the limit exists and that the ω dependence of the limiting functions $g_S(\omega)$ and $g_{PS}(\omega)$ can be fully determined. We are concerned with the ordered limits

$$\lim_{P^2\to\infty;\,\omega\text{ fixed}}\ \lim_{\nu\to\infty;\,\omega,P^2\text{ fixed}} M_{\nu'\mu';\nu\mu}\,. \tag{55}$$

Let us examine the nature of this double limit. The first is achieved by letting $Q_\mu \to\infty$ along some direction, keeping P_μ and

$$\omega^{-1} = P_\mu \frac{Q^\mu}{Q^2} = P\cdot n \tag{56}$$

fixed. We have defined $n^\mu = Q^\mu/Q^2$ to be a vector with finite components in the scaling limit and we observe that it becomes lightlike in the limit:

$$n^2 = 1/Q^2 \to 0\,. \tag{57}$$

In Eq. (54) we see that the dominant contributions in the scaling limit come from integration regions where $Q\cdot x$ and $Q\cdot x'$ are bounded. But this means that x^μ and x'^μ must be essentially proportional to n^μ and therefore x and x' are essentially lightlike. In asserting this much, we are only recapitulating the argument that the light-cone structure of the current commutators is probed in the scaling limit. If we now let $P^2\to\infty$, still keeping ω fixed, we must let P^μ approach an infinite vector along a direction parallel to n^μ, so that $\omega^{-1} = n\cdot P$ can remain fixed. Therefore, in this second limit the dominant contributions will come from the region where $n\cdot z \approx 1/P^2$, i.e., from z^μ essentially parallel to n^μ. Thus, for our ordered double limit the main contributions will come from the region in which z, x, and x' are collinear points on a *lightlike* ray. But in our formal treatment of the quark-gluon model, we have recently shown that it is just for this situation that the commutators of the bilocal operators close on themselves, to leading order in light-cone singularities.[3] Thus, consider the four space-time points x, y, z, t in the region where $(x-t)^\mu$ and $(y-t)^\mu$ are proportional to $(z-t)^\mu$ and $(z-t)^2\approx 0$. Then we find, for example, that

$$\begin{aligned}[V_\mu(\tfrac{1}{2}(x+y),\ \tfrac{1}{2}(x-y)),\ V_\nu(\tfrac{1}{2}(z+t),\ \tfrac{1}{2}(z-t))] &= \bar\psi(x)\gamma_\mu\gamma_\alpha\gamma_\nu Q^4 \exp\left[-ig\int_t^x B_\mu(u)du^\mu\right]\psi(t)\partial^\alpha D(y-z)\\ &\quad - \bar\psi(z)\gamma_\nu\gamma_\alpha\gamma_\mu Q^4 \exp\left[-ig\int_y^z B_\mu(u)du^\mu\right]\psi(y)\partial^\alpha D(x-t)\,.\end{aligned} \tag{58}$$

In the present situation we are interested in the vacuum expectation values of such commutators. These are easily gotten on the basis of the formal procedures discussed in Ref. 3. We then find

$$\begin{aligned}\lim_{P^2\to\infty;\,\omega\text{ fixed}}\ \lim_{\nu\to\infty;\,P^2,\omega\text{ fixed}} M_{\nu'\mu';\nu\mu} &\to \langle Q^4\rangle \int dz dx dx' \exp(-iP\cdot z - iQ\cdot x' + iQ\cdot x)\,\theta(x_0)\,\theta(x_0')\\ &\quad\times[\mathrm{Tr}(\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta\gamma_{\mu'}\gamma_\gamma\gamma_{\nu'}\gamma_\delta)\partial^\alpha D(x)\partial^\beta D(z+\tfrac{1}{2}(x+x'))\partial^\gamma D(x')\partial^\delta D(z-\tfrac{1}{2}(x+x'))\\ &\quad-(\mu\leftrightarrow\nu,\ x\to -x) - (\mu'\leftrightarrow\nu',\ x'\to -x') + (\mu\leftrightarrow\nu,\ \mu'\leftrightarrow\nu',\ x\to -x,\ x'\to -x')]\,.\end{aligned} \tag{59}$$

This complicated-looking expression, it may be noticed on careful inspection, is just what one would find for the inclusive cross section in the *free*-quark model, where all the hadron channels X are composed simply of a pair of massless quarks. This is clearly a kind of parton-model result.[12] The integral is most simply evaluated by standard Feynman methods, and one finds

$$\begin{aligned} g_S(\omega, P^2) &\xrightarrow[P^2\to\infty]{} g_S(\omega) = \langle Q^4\rangle \int_{-1}^{1} \frac{dz}{\pi}\,\frac{z^2(1-z^2)}{(\omega^2-z^2)^2}\,,\\ g_{PS}(\omega, P^2) &\xrightarrow[P^2\to\infty]{} g_{PS}(\omega) = \langle Q^4\rangle \int_{-1}^{1} \frac{dz}{\pi}\,\frac{\omega^2(1-z^2)}{(\omega^2-z^2)^2}\,.\end{aligned} \tag{60}$$

We have thus determined the complete ω dependence of the "spectral" functions for large P^2. The double limit affords the possibility of a dramatic test of the ideas on the light-cone structure of current commutators discussed in Ref. 3.

APPENDIX

In this Appendix we exhibit the Jost-Lehmann-Dyson (JLD)[11] representation of the matrix element $M_{\nu\mu}$ in the scaling limit. In particular, we will see that the discontinuity in Q_0 vanishes for the scaling limit, with $|\omega|>1$. For the matrix element to a particular hadron state X, momentum P, the (unsubtracted) JLD representation is given by

$$M_{\nu\mu}=\int_R d\vec{u}dK^2\frac{\phi_{\nu\mu}(\vec{u},K^2,P,Q,\ldots)}{Q_0{}^2-(\vec{Q}+\vec{u})^2-K^2+i\epsilon}, \tag{A1}$$

where the weight-function tensor $\phi_{\nu\mu}$ depends in an unspecified way on $\vec{u}$, K^2, P, and other variables of state X; it is a polynomial in the components of Q^μ. We are in the hadron rest frame, $\vec{P}=0$. The integration domain is given by

$$\begin{aligned}&|\vec{u}|<\tfrac{1}{2}\sqrt{P^2},\\ &K>\mu-(\tfrac{1}{4}P^2-|\vec{u}|^2)^{1/2},\end{aligned} \tag{A2}$$

where μ is the smallest mass of contributing intermediate states. In the scaling limit, we have

$$Q_0=\frac{\nu}{\sqrt{P^2}}\to\infty,\quad \omega=\frac{Q^2}{Q_0\sqrt{P^2}}\quad\text{fixed}, \tag{A3}$$

and, choosing $\vec{Q}$ along the three-axis,

$$Q_3\simeq\frac{\nu}{\sqrt{P^2}}-\tfrac{1}{2}\omega\sqrt{P^2}. \tag{A4}$$

In the scaling limit

$$M_{\nu\mu}\to\frac{1}{\nu}\int_R d\vec{u}dK^2\frac{\phi_{\nu\mu}}{\omega-2u_3/\sqrt{P^2}+i\epsilon}. \tag{A5}$$

Writing

$$\frac{2u_3}{\sqrt{P^2}}=\omega',\quad |\omega'|<1,$$

we see that

$$M_{\nu\mu}\to\frac{1}{\nu}\int_{-1}^{1}d\omega'\frac{\tilde{\phi}_{\nu\mu}(\omega',\ldots)}{\omega'-\omega-i\epsilon}. \tag{A6}$$

This clearly establishes the result that if scaling holds, then the functions $g_S(\omega,P^2)$ and $g_{PS}(\omega,P^2)$ have no ω discontinuities outside the interval $-1<\omega<1$.

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