for any quark-parton model. The experiment of inelastic Compton scattering would thus be quite fruitful for understanding the structure of the hadrons.

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[1]R. P. Feynman, in *Proceedings of the Third International Conference on High-Energy Collisions at Stony Brook, 1969*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

[2]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[3]C. W. Gardiner and D. P. Majumdar, Phys. Rev. D 2, 151 (1970).

[4]D. P. Majumdar, Phys. Rev. D 3, 2869 (1971).

[5]M. Gourdin, Nucl. Phys. B29, 601 (1971).

[6]S. J. Brodsky and P. Roy, Phys. Rev. D 3, 2914 (1971).

[7]Similarly, one finds that

$$\nu W_2^{\gamma p} - \nu W_2^{\gamma n} = 2\sum_N P(N)\left\langle \sum_1^N [2(I_i^3)^3 Y_i + \tfrac{1}{2} I_i^3 Y_i^3]\right\rangle x f_N(x),$$

which yields the inequality

$$\frac{20}{27}\left\langle\frac{1}{N}\right\rangle - \frac{5}{27} \le \int_0^1 (\nu W_2^{\gamma p} - \nu W_2^{\gamma n})dx \le \frac{5}{27} - \frac{10}{27}\left\langle\frac{1}{N}\right\rangle.$$

This inequality can also be derived from Eqs. (3.4) and (3.5).

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s-Channel Analysis of πN Scattering in a Scheme of Higher Baryon Couplings

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A relativistic-hadron-coupling scheme recently proposed by Mitra and co-workers is applied to the process $\pi N \to \pi' N'$. This scheme, which is a relativistic extension of an $SU(6)\otimes O_3$ framework of $\bar{B}B_L P$ couplings, is characterized by the appearance of a relativistically invariant form factor, which is endowed with several desirable properties such as Regge universality, crossing symmetry, etc. So far three distinct choices, termed I, II, III, are available, which satisfy these criteria in varying degrees. The calculation of $\pi N \to \pi' N'$ scattering is done in a pure s-channel model where the various baryon resonances act as the propagators. The effect of other channels (t and/or u) is not considered in the calculation, in the spirit of duality. Specifically, the following processes are considered: (i) the two elastic $\pi^\pm p \to \pi^\pm p$ processes, which have a nonresonant background, and (ii) the $\pi^- p$ charge-exchange process, which does not have such a background. The quantities calculated are the following: $\sigma_T(\pi^\pm p \to \pi^\pm p)$, $\sigma_T(\pi^- p \to \pi^0 n)$, $d\sigma(0)/d\Omega(\pi^- p \to \pi^0 n)$, the polarization $P(t)$ for $\pi^- p$ charge exchange, and the sensitive dimensionless quantity $\mathrm{Im}Kf^{(-)}$. It is found that the above scheme gives a satisfactory account of the data, especially with form factor III, which fits the various details quite accurately. The role of duality in respect of the simulation of the t channel is discussed, especially in relation to the behavior of the quantities $d\sigma(0)/d\Omega$ and $\mathrm{Im}Kf^{(-)}$.

I. INTRODUCTION

In a few recent communications[1,2] it was shown how a relativistically invariant model of hadron couplings based on $SU(6)\otimes O_3$ could be constructed at the phenomenological level with a relativistically invariant form factor for each supermultiplet transition. The fair success of the model in the case of hadron decays[3-5] warrants a more stringent test in terms of the off-mass-shell extension of the form factor. For this purpose we have taken up the familiar $\pi N \to \pi' N'$ process to be studied in the light of the above model of hadron couplings. Similar studies for allied processes such as pion photoproduction[6] and vector-meson production[7] have been carried out recently with encouraging results. Apart from this, we are also interested to find out the extent of simulation of the t-channel effects by purely s-channel resonance contributions and so we are primarily concerned with the different experimentally verifiable quantities in the forward direction.

The model of hadron couplings considered here has been discussed extensively elsewhere,[8] but, for the sake of completeness, we outline the essential points. In this model the baryons are assumed to belong to the supermultiplet representations of $SU(6)\otimes O_3$; for example, (i) $(\underline{56}, 0^+)$, (ii) $(\underline{70}, 1^-)$, (iii) $(\underline{56}, 2^+)$, etc., which account for almost all the resonance states discovered to date in a most economical fashion. The structure can be deduced within the quark model with s-wave forces, but a discussion of the idea is not necessary for the contents of this paper. As a matter of fact, the quark model plays a rather passive role in this coupling scheme which merely assumes the existence of two kinds of supermultiplets, i.e., $[\underline{56}, 2l^+]$ and $[\underline{70}, (2l+1)^-]$, $l=0,1,2,\ldots$, together with their radial excitations. No assumption of quark dynamics is made explicitly, although the physical content of the quark recoil effect has been taken into consideration in the $(L-1)$-wave interaction. The couplings of any arbitrary spin particle, belonging to any of the above supermultiplets, with the lowest-lying baryon supermultiplet $(\underline{56}, 0^+)$ are constructed in terms of the generalized Rarita-Schwinger fields $\psi^J_{\mu_1,\mu_2,\ldots,\mu_{J-1/2}}$ of spin J,[9] the Dirac spinor ψ and multiple derivatives that are necessary for contraction. The relativistic couplings are then multiplied by certain form factors which take into account the breaking of the group $SU(6)\otimes O_3$, due to mass splitting.

Unlike the earlier cases of resonance decay widths and branching-ratio calculations, we have in our scattering problem a very different situation, where the set of intermediate particles are off the mass shell. This requires an off-mass-shell extension of the coupling structures including the form factors. From this perspective our present investigation of πN scattering is an off-mass-shell test of the model, more properly a test of the form factor.

The relativistically invariant form factors $f_L^{(\pm)}$, which have the significance of the overlap integrals of the spatial part of the initial and final wave functions, govern the dynamical aspect of the interaction. There are three different forms of form factors available presently.[1,10,11] In our investigation we will consider all three forms separately. Their parametric forms are such that they have the following features: (a) Lorentz covariance; (b) dimensionality of inverse momentum with a proper exponent, so that it produces the desired damping, even off the mass shell, as the couplings involve multiple derivatives; (c) Regge universality of the reduced coupling constant, which essentially boils down to the statement that there are only two basic "reduced" coupling constants g_0 and g_1, one each for the even- and odd-parity states, which thus effects a further reduction of the adjustable parameters; (d) the masses of the resonance states occuring in such a way that the symmetry breaking due to mass splitting is taken into account precisely by the masses themselves; (e) a simple structure $(\sim x^L)$. The form factors $f_L^{(\pm)}$(II) and $f_{LJ}^{(\pm)}$ (III) have an additional desirable feature: crossing symmetry in the momenta. Thus in this relativistically invariant coupling scheme we have essentially no adjustable parameter as the values of reduced coupling constants, mentioned above, are already fixed from well-known decay widths.

In our present investigation of πN scattering, where we make use of the above relativistic coupling scheme, we have considered only the s-channel resonance contributions to the amplitude of any particular process, in the spirit of duality[12,13] – a concept which has not yet been formulated in a precise theoretical manner. There are a number of more or less equivalent statements of duality. The sense in which we would like to take the idea of duality is the following: We would essentially decouple the background from the resonance contributions and presume that it is the contribution of the resonances that simulate the non-Pomeranchuk-type Regge trajectories, while the background generates the Pomeranchuk trajectory.[14] For $\pi^- p$ charge exchange (CEX) in the forward direction, where the quantum numbers in the t channel rule out the exchange of a Pomeranchukon, our hope is that the set of s-channel resonances generate, in the sense of the finite-energy sum rules (FESR),[15] the effect of the ρ trajectory which therefore should not be put in separately to avoid duplication.[16] We hasten to emphasize that this statement shares the ambiguity of the duality concept where it is not clear whether a partial inclusion of t- and/or u-channel effects is consistent with this concept,[17] even when one does not wish to consider an interference model.[18,19] Our expectations on the relevance of duality for the process $\pi^- p \to \pi^0 n$ are therefore limited by the above assumption, viz., the use of only s-channel resonances.

As a second aspect of this investigation we shall also make a comparative study of the sensitive dimensionless quantity $\mathrm{Im}Kf^{(-)}$ (Ref. 20) by calculating it separately in the s-channel-resonance and Regge-pole (extrapolated) models.

We shall also study the total cross sections for $\pi^\pm p$ elastic scattering as well as $\pi^- p$ charge-exchange scattering, and make an estimate of the Pomeranchukon effect at low energy for the two elastic processes, which is consistent with the earlier results. It is shown that for the latter process no such background is necessary, as expected.

Finally we test our model for the recoil-neutron polarization effects for the $\pi^- p$ CEX process.

In Sec. II we outline the required formalism. Results and discussions are given in Sec. III. Section IV will be concerned with summary and conclusion.

II. FORMALISM

We consider all the resonance contributions up to states of spin-parity $\frac{7}{2}^+$ in the s channel, where the participating intermediate particles are members of the supermultiplets ($\underline{56}$, 0^+), ($\underline{70}$, 1^-), and ($\underline{56}$, 2^+), which are the representations of the $SU(6)\otimes O_3$ group. Each supermultiplet transition of the form $L^P \to 0^+$ is characterized by only one free parameter: the "reduced" coupling constant g_L of the corresponding supermultiplet. Thus once we fix the masses and the half-widths of these intermediate states,[21] we are in fact left with only three independent parameters, g_L $(L=1,2,3)$.

The $\overline{B}_J N\pi$ interaction Hamiltonian when all the particles are on the mass shell has been derived on the basis of the quark model assuming broken $SU(6)\otimes O_3$ invariance for the baryons.[5] This Hamiltonian, $\mathcal{H}_I$, when modified to incorporate one off-mass-shell leg, becomes

$$\begin{aligned}\mathcal{H}_I =\, & f_L^{(+)}(q^2)\overline{\psi}^{L+1/2}_{\mu_1,\ldots,\mu_L}(x) i\gamma_5(i\not{q}) q_{\mu_1}q_{\mu_2}\cdots q_{\mu_L}\psi(x)\pi(x) + f_L^{(-)}(q^2)(-\mu^2)\left(\frac{L}{2L+1}\right)^{1/2}\overline{\psi}^{L-1/2}_{\mu_2,\ldots,\mu_L}(x) q_{\mu_2}\cdots q_{\mu_L}\psi(x)\pi(x)\\ & + f_L^{(+)}(q^2)\overline{\psi}^{L+3/2}_{\mu,\mu_1,\mu_2,\ldots,\mu_L}(x) q_\mu q_{\mu_1}\cdots q_{\mu_L}\psi(x)\pi(x) + f_L^{(+)}(q^2)\left(\frac{L+1}{2L+3}\right)^{1/2}\overline{\psi}^{L+1/2}_{\mu_1,\ldots,\mu_L}(x) i\gamma_5(i\not{q}) q_{\mu_1}\cdots q_{\mu_L}\psi(x)\pi(x)\\ & + f_L^{(-)}(q^2)\left(\frac{L(L+1)}{2(2L+1)}\right)^{1/2}\overline{\psi}^{L-1/2}_{\mu_2,\ldots,\mu_L}(x) q_{\mu_2}\cdots q_{\mu_L}\psi(x)\pi(x)\\ & + f_L^{(-)}(q^2)(-\mu^2)\left(\frac{L(L^2-1)}{2(4L^2-1)}\right)^{1/2}\overline{\psi}^{L-3/2}_{\mu_3,\ldots,\mu_L}(x) i\gamma_5(i\not{q}) q_{\mu_3}\cdots q_{\mu_L}\psi(x)\pi(x),\end{aligned} \tag{1}$$

where $f_L^{(\pm)}(q^2)$ is the form factor for L excited states, and μ is the mass of the radiation quantum.

After properly evaluating the SU(6) Clebsch-Gordan coefficients of vertices involving the particle B_J belonging to any of the three supermultiplets, we set up the S matrix for the process $\pi N \to \pi' N'$ using the above $\mathcal{H}_I$. We define the T matrix in terms of the S matrix in the usual fashion:

$$S_{fi} = \delta_{fi} - \frac{i(2\pi)^4\delta^{(4)}(P_f - P_i)\overline{u}_{N'} T u_N}{(4q_0 q_0')^{1/2}}. \tag{2}$$

The invariant amplitudes A and B are now defined as

$$\overline{u}_{N'}(p')Tu_N(p) = \overline{u}_{N'}(p')[-A + i\not{Q}B]u_N(p), \tag{3}$$

where

$$Q = \tfrac{1}{2}(q+q'); \tag{4}$$

p, q and p', q' are the initial and final set of nucleon and pion momenta, respectively. All the measurable quantities can now be conveniently defined in terms of A' and B, where A' has the form[22]

$$A' = A + \frac{\nu + t/4m}{1 - t/4m^2}B, \tag{5}$$

as

$$\frac{d\sigma}{d\Omega} = \left(\frac{m}{4\pi W}\right)^2\left[\left(1 - \frac{t}{4m^2}\right)|A'|^2 + \frac{t}{4m^2}\left(s - \frac{(m+\nu)^2}{1 - t/4m^2}\right)|B|^2\right], \tag{6}$$

$$\sigma_T = \operatorname{Im}A'(s, t=0)/(\nu^2 - \mu^2)^{1/2}. \tag{7}$$

With respect to $\vec{q}\times\vec{q}\,'/|\vec{q}\times\vec{q}\,'|$,

$$P = -\frac{\sin\theta}{16\pi s^{1/2}}\frac{\operatorname{Im}(A'B^*)}{d\sigma/dt}, \tag{8}$$

and

$$\frac{d\sigma}{dt} = \frac{\pi}{K^2}\frac{d\sigma}{d\Omega}, \tag{9}$$

where ν, μ, m, and W are the pion laboratory energy, pion mass, nucleon mass, and $\sqrt{s}$, respectively.

s, t are the usual Mandelstam variables. When there are n intermediate particles, we have

$$A=\sum_{j=1}^{n} A_j \quad \text{and} \quad B=\sum_{j=1}^{n} B_j,$$

where j is the intermediate-particle index.

Explicit Form of T Matrix

The process we are considering is $\pi N \to \pi' N'$ where $B_J(K)$ represents any allowed arbitrary intermediate state of spin J, mass M, and 4-momentum K. Figure 1 shows the Feynman diagram for the above process, where $N(p)$ and $N'(p')$ are the initial and final nucleons with momenta p and p', respectively, whereas $\pi(q)$ and $\pi'(q')$ represent the initial and final pions with momenta q and q', respectively. The S matrix for the process is of the form

$$S_{fi}=(-i)^2G^2\int d^4x_1 d^4x_2\langle f|\cdot \bar{\psi}_{N'}(x_2)\Gamma^{(s)} q'^{\nu_1}\cdots q'^{\nu_n} S_{F\nu_1,\dots,\nu_n}^{\mu_1,\dots,\mu_n}(x_1-x_2)_{\beta\alpha}q_{\mu_1}\cdots q_{\mu_n}\Gamma^{(s)}\psi_N(x_1)\pi_\beta(x_2)\pi_\alpha(x_1):|i\rangle, \tag{10}$$

where

$$\Gamma^{(s)}=(1, i\gamma_5 i\not{q}) \quad \text{for} \quad s=1,2 \tag{11}$$

and

$$S_{F\nu_1,\dots,\nu_n}^{\mu_1,\dots,\mu_n}(x_1-x_2)_{\beta\alpha}=\lim_{\epsilon\to 0}\frac{-i}{(2\pi)^4}\int d^4K\,\frac{-i\not{K}+M}{K^2+M^2}\,e^{iK\cdot(x_1-x_2)}\Theta_{\nu_1,\dots,\nu_n}^{\mu_1,\dots,\mu_n}(J)_K I_{\beta\alpha}$$

is the propagator for the intermediate particle $B_J(K)$. α, β are the isospin indices. $\Theta_{\nu_1,\nu_2,\dots,\nu_n}^{\mu_1,\mu_2,\dots,\mu_n}(J)_K\Lambda_+$ is the projection operator for the spin-J particle with $J-1$, $J-2$,... etc. components as we have made the replacement

$$\Theta_\mu^\nu=\left(g_\mu^\nu-\frac{P_\mu P^\nu}{P^2}\right)\to\left(g_\mu^\nu+\frac{P_\mu P^\nu}{M^2}\right). \tag{12}$$

The final form of the S matrix is

$$S_{\pi N\to\pi' N'}=-i(2\pi)^4\frac{G^2}{(4q_0q_0')^{1/2}}\,\delta^{(4)}(P_f-P_i)\bar{u}_{N'}(p')\Gamma^{(s)}\frac{-i\not{P}+M}{s-M^2}\,\Theta_{q_1',\dots,q_n'}^{q_1,\dots,q_n}(J)_P I_{\beta\alpha}\Gamma^{(s)}u_N(p)\langle\pi'|\pi_\beta\rangle\langle\pi_\alpha|\pi\rangle, \tag{13}$$

where we have used the notation

$$\Theta_{q_1',\dots,q_n'}^{q_1,\dots,q_n}(J)_P=q'^{\nu_1}\cdots q'^{\nu_n}\Theta_{\nu_1,\dots,\nu_n}^{\mu_1,\dots,\mu_n}(J)_P q_{\mu_1}\cdots q_{\mu_n}. \tag{14}$$

Explicit formulas for the Θ functions have been given.[23] It is convenient to parametrize as

$$\Theta_{q_1',\dots,q_n'}^{q_1,\dots,q_n}(J)_P=a_J+b_J\not{q}(P)\not{q}'(P).$$

Here

$$\not{q}(P)=\gamma_\mu\left(q^\mu+\frac{P^\mu(P\cdot q)}{M^2}\right)$$

and a_J, b_J are scalars given by

$$\begin{aligned}
a_{3/2}&=\tfrac{1}{3}\phi'+\tfrac{1}{5}\left(1-\frac{s}{M^2}\right)\left(\phi'-\frac{4}{3}\frac{\chi^2}{M^2}\right),\\
b_{3/2}&=-\tfrac{1}{3},\\
a_{5/2}&=\tfrac{1}{5}\phi'^2-\tfrac{1}{5}\phi^2+\tfrac{1}{7}\left(1-\frac{s}{M^2}\right)\left(\phi'^2-\tfrac{1}{5}\phi^2-\frac{8}{5}\frac{\chi^2}{M^2}\phi'-\frac{4}{5}\frac{\chi^2}{M^2}\phi\right),\\
b_{5/2}&=\tfrac{2}{5}\phi',\\
a_{7/2}&=\tfrac{1}{7}\phi'^3-\tfrac{9}{35}\phi'\phi^2+\tfrac{1}{3}\left(1-\frac{s}{M^2}\right)\left[\tfrac{1}{3}\phi'^3-\tfrac{1}{7}\phi'\phi^2-\frac{4}{7}\frac{\chi^2}{M^2}(\phi'^2+\phi\phi'-\tfrac{2}{5}\phi^2)\right],\\
b_{7/2}&=\tfrac{3}{7}(\phi'^2-\tfrac{1}{5}\phi^2),
\end{aligned} \tag{15}$$

where

$$\chi = -(m\nu + \mu^2),$$

$$\phi = -\mu^2 + \chi^2/M^2,$$

$$\phi' = \phi + \tfrac{1}{2}t.$$

The T matrix is therefore given by [comparing (2) and (13)]

$$\bar{u}_{N'}(p')Tu_N(p) = G^2\bar{u}_{N'}(p')\Gamma^{(s)}\frac{-i\not{P}+M}{s-M^2}\,\Theta^{q_1,\ldots,q_n}_{q_1',\ldots,q_n'}(J)_P I_{\beta\alpha}\Gamma^{(s)}u_N(p)\langle\pi'|\pi_\beta\rangle\langle\pi_\alpha|\pi\rangle, \qquad (16)$$

where

$$G = (\text{C.G. coefficient}) \times f_L^{(\pm)}(j). \qquad (17)$$

The expressions for G, for different intermediate particles that we have considered, are given in Table I. We are taking into account three sets of form factors $f_L^{(\pm)}(j)$ ($j=\text{I},\text{II},\text{III}$),[24] corresponding to three different parametrization schemes outlined in the introduction.

(a) *Form factor* $f_L^{(\pm)}(\text{I})$.

$$f_0^{(\pm)}(\text{I}) = g_0\mu^{-1}(M/m)^{1/2}, \qquad (18)$$

$$f_L^{(\pm)}(\text{I}) = g_L\mu^{-L-1}\left(\frac{\mu}{q_0}\right)^{L\pm1}\left(\frac{M}{m}\right)^{1/2}, \qquad (19)$$

where the reduced coupling constants are

$$\frac{g_0^{\,2}}{4\pi} = 0.03, \quad \frac{g_1^{\,2}}{4\pi} = 0.14, \quad \frac{g_2^{\,2}}{4\pi} = 0.04,$$

and

$$q_0^{-1} = 2M(s-m^2+\mu^2)^{-1}$$

is the energy of the pion in the rest frame of the intermediate particle.

(b) *Form factor* $f_L^{(\pm)}(\text{II})$.

$$f_L^{(\pm)}(\text{II}) = g_L\{1, m_\pi m_\rho^{-1}\}f_q m_\pi^{-1}[\sigma(m_A/abc)^{1/2}]^L,$$

where the first and second quantities within the curly brackets correspond to $(L+1)$- and $(L-1)$-wave interactions. f_q is the qqP coupling constant $[\sim(0.03\times4\pi)^{1/2}]$. The scale factor σ has the value 1.8 ± 0.1.

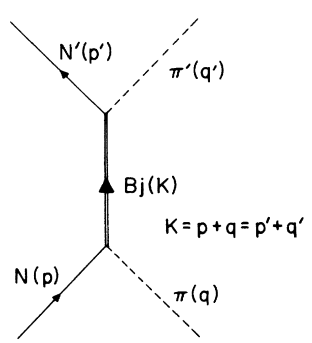

FIG. 1. Feynman diagram for the process $\pi N \to \pi' N'$ in the s channel.

$$a = (-2P\cdot p)^{1/2} = (s+m^2-\mu^2)^{1/2},$$

$$b = (-2P\cdot q)^{1/2} = (s+\mu^2-m^2)^{1/2}, \qquad (21)$$

$$c = (-2p\cdot q)^{1/2} = (s-\mu^2-m^2)^{1/2},$$

and

$$g_0 = g_2 = \cdots = g_{2l}, \quad g_0^{\,2} = 1,$$

$$g_1 = g_3 = \cdots = g_{2l+1}, \quad g_1^{\,2} = 0.62\pm0.1. \qquad (22)$$

(c) *Form factor* $f_{LJ}^{(\pm)}(\text{III})$.

For this form factor we have separate parame-

TABLE I. The resonances considered in the calculation together with the corresponding SU(6) C.G. coefficients and geometrical factors. The energies and widths are given in MeV.

Intermediate states	G
$P_{33}(1238, 101)$	$\sqrt{8}f_0$
$P_{13}(1863, 335)$	$\frac{5}{3}(-\sqrt{3})f_2(\frac{2}{5})^{1/2}$
$F_{37}(1940, 245)$	$\sqrt{8}f_2$
$S_{31}(1630, 160)$	$(\frac{1}{3})^{1/2}f_1(\frac{1}{3})^{1/2}$
$S_{11}(1715, 323)$	$\frac{1}{3}\sqrt{8}(-\sqrt{3})f_1(\frac{1}{3})^{1/2}$
$D_{15}(1675, 238)$	f_1
$S_{11}(1525, 127)$	$f_1(\frac{1}{3})^{1/2}$
$P_{11}(938)$	$\frac{5}{3}(-\sqrt{3})f_0$
$F_{15}(1690, 102)$	$\frac{5}{3}(-\sqrt{3})f_0$
$F_{35}(1880, 250)$	$\sqrt{8}f_2(\frac{3}{7})^{1/2}$
$P_{31}(1905, 300)$	$\sqrt{8}f_2(\frac{1}{5})^{1/2}$
$D_{33}(1670, 225)$	$(\frac{1}{3})^{1/2}f_1$
$D_{13}(1515, 115)$	$\frac{1}{3}\sqrt{8}(-\sqrt{3})f_1$
$D_{13}(1675, 101)$	$f_1(\frac{2}{5})^{1/2}$
$P_{11}(1470, 260)$	$\frac{5}{3}(-\sqrt{3})f_0'$

trization for $(L+1)$-wave and $(L-1)$-wave interactions, and a moderate J dependence.

(i) For an $(L+1)$-wave interaction,

$$f_{LJ}^{(+)}(\mathrm{III}) = C_L^{(+)}\left(\frac{x_F(4Mm)^{1/2}J^{1/4}}{-P^2-p^2-q^2}\right)^{L+1}. \tag{23}$$

(ii) For an $(L-1)$-wave interaction,

$$f_{LJ}^{(-)}(\mathrm{III}) = C_L^{(-)}\left(\frac{x_F(4Mm)^{1/2}J^{1/4}}{-P^2-p^2-q^2}\right)^{L}\frac{m_A^{\,2}}{m_{A_1}}, \tag{24}$$

where the reduced coupling constants have the values

$$C_0^{(+)} = C_2^{(+)},\quad C_{2l}^{(\pm)} = C_2^{(\pm)},$$

$$C_0^{(+)2} = 4\pi\times 1.3,\quad C_2^{(-)2} = 4\pi\times 0.035,\quad C_{2l+1}^{(\pm)} = C_1^{(\pm)},$$

$$C_1^{(+)2} = 4\pi\times 1.3,\quad C_1^{(-)2} = 4\pi\times 0.035. \tag{25}$$

m_A is the mass of the A meson, the counterpart [in the sense of partial conservation of axial-vector current (PCAC)] of the π meson. P, p, and q have their usual meaning.

The scale factor x_F is found to be 1.25. As we are considering the $\pi N\to\pi' N'$ case, the Van Royen - Weisskopf factor is unity.

The quantity $f^{(-)}$ appearing in the dimensionless quantity $\mathrm{Im}Kf^{(-)}$ is defined as $f^{(-)} = \sigma_T(\pi^- p\to\pi^- p) - \sigma_T(\pi^+ p\to\pi^+ p)$. The relation connecting f and the invariant amplitude A' is $f = (4\pi m)^{-1}\sqrt{s}\,A'$. Assuming charge independence of the interaction we can readily deduce the expression

$$\mathrm{Im}Kf^{(-)} = -\sqrt{2}\,(4\pi s^{1/2})^{-1} mK\,\mathrm{Im}A'_{\mathrm{CEX}}(t=0). \tag{26}$$

In the first case A' is represented as a sum of the s-channel resonances and we make use of the relation (5). In the second case $\mathrm{Im}Kf^{(-)}$ has been calculated in terms of t-channel ρ-Regge exchange. Here the amplitude A' is parametrized as[25]

$$A' = C(t)[\tan\tfrac{1}{2}\pi\alpha_\rho(t) + i](\nu/\nu_0)^{\alpha_\rho(t)}$$

from where we deduce the expression for $\mathrm{Im}Kf^{(-)}$ as

$$\mathrm{Im}Kf^{(-)} = C_0[\alpha_\rho(0)+1]K(\nu/\nu_0)^{\alpha_\rho(0)}. \tag{27}$$

Here, the scale factor ν_0 is set at 1 GeV. The parameters C_0 and $\alpha_\rho(0)$ have the values 2.3 mb GeV/c and 0.56 ± 0.01, respectively, which are consistent with the best fit of the data in the ranges $|t|\leq 1.4$ (GeV/c)^2 at $p_\pi = 5.85$, 13.3, and 18.2 GeV/c and $|t|\leq 0.8$ (GeV/c)^2 at $p_\pi = 9.8$ GeV/c.

III. RESULTS AND DISCUSSIONS

A. Dependence of Total Cross Section on Energy for the Processes $\pi^\pm p\to\pi^\pm p$

As the first case, we consider the two elastic scattering processes $\pi^\pm p\to\pi^\pm p$, where we apply our direct-channel-resonance model. It is conjectured that, inasmuch as the s-channel resonances are capable of generating only the ρ-Regge-pole effects in the low-energy region for processes like $\pi N\to\pi N$, any other Reggeon that might be exchanged in the high-energy region for these processes would also have a marked effect in the low-energy domain.[14] It would manifest itself as a systematic deviation from the experimental curve. It is therefore interesting to investigate how the total cross sections (σ_T) for the elastic processes $\pi^\pm p\to\pi^\pm p$ behave as a function of $\sqrt{s}$ in the low-energy region in this s-channel-resonance model. The motivation for choosing σ_T's for study is because they project maximum t-channel effects.

Figure 2 shows the theoretical curves (inclusive of background) and the experimental curves for both the processes $\pi^\pm p\to\pi^\pm p$, together with the background. The experimental curves for the processes $\pi^- p\to\pi^- p$ and $\pi^+ p\to\pi^+ p$ are labeled as II and IV, respectively. The theoretical curves which are drawn under two different conditions, namely,

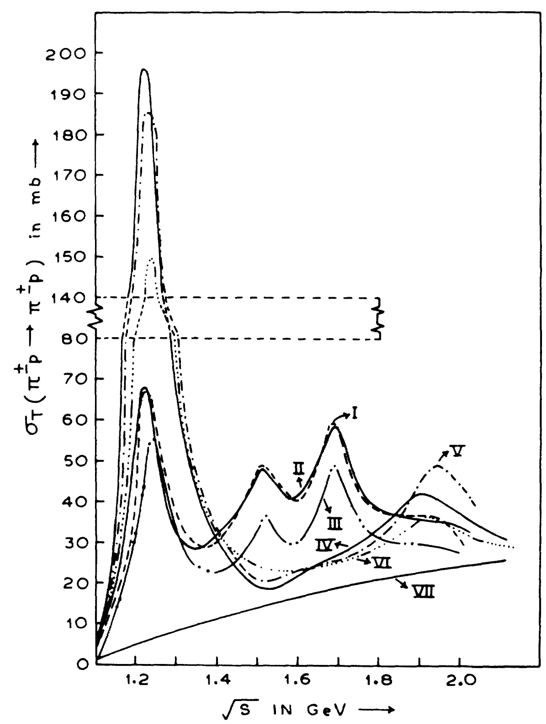

FIG. 2. Total cross section (σ_T) in mb for the processes $\pi^\pm p\to\pi^\pm p$ against $\sqrt{s}$ in GeV. Curves II, I, and III represent the experimental and the theoretical curves with the form factors $f_{LJ}^{(\pm)}(\mathrm{III})$ and $f_L^{(\pm)}(\mathrm{II})$, respectively, for the process $\pi^- p\to\pi^- p$. For the process $\pi^+ p\to\pi^+ p$, curve IV is experimental; curves V and VI correspond to the theoretical curves with the form factors $f_{LJ}^{(\pm)}(\mathrm{III})$ and $f_L^{(\pm)}(\mathrm{II})$, respectively. Curve VII stands for the common background.

assuming the form factors $f_{LJ}^{(\pm)}$(III) and $f_L^{(\pm)}$(II), are represented by the curves I and III, respectively, for the process $\pi^- p \to \pi^- p$ and by the curves V and VI, respectively, for the process $\pi^+ p \to \pi^+ p$. The smooth curve VII represents the common background. We see (curves I and V) that with the form factor $f_{LJ}^{(\pm)}$(III) the theoretical curves for both the processes are in good accord with the experimental curves, when the common background is incorporated. It is interesting to note that the same amount of background (curve VII) is required for both the processes. Similar is the case when the form factor $f_L^{(\pm)}$(II) is considered. Here again we have added the same background contribution as in the previous case. The results are depicted by the curves III and VI for the processes $\pi^- p \to \pi^- p$ and $\pi^+ p \to \pi^+ p$, respectively.

The systematic deviation that we are expecting is the common background which has to be added to the theoretical values in order to bring it closer to the experimental numbers. The fact that the same amount of background matches both the curves $\sigma_T(\pi^\pm p \to \pi^\pm p)$ simultaneously is very encouraging. At this point, if we assume that the s-channel contributions are duly generated by the set of resonance states considered, which is more or less true when the form factor $f_{LJ}^{(\pm)}$(III) is operative, we can identify the background as arising due to the following reasons:

(i) Pomeranchukon exchange in the t channel is allowed by the quantum numbers of both the processes.

(ii) There is a partial simulation of the ρ-Regge-pole effects due to the limited number of s-channel resonance states.

We conjecture further that a significant part of the ρ-Regge-pole effects are already generated by the s-channel resonances (see Sec. III E for further discussion on this point).

It is believed that the Pomeranchukon trajectory manifests itself in the low-energy region as a smoothly varying background.[14,26] From this point of view the fact that we have to assume the same amount of background for both the processes $\pi^\pm p \to \pi^\pm p$ is merely a statement of the principle $\sigma_T(\pi^- p) - \sigma_T(\pi^+ p) \sim 0$ at high energy.

The empirical estimation of the smooth background has been made in relation to the best fit of the theoretical curve with form factor $f_{LJ}^{(\pm)}$(III), to the experimental curve for the process $\pi^- p \to \pi^- p$.[27] This background curve VII, when extrapolated beyond 2.0 GeV, would lie between the curves $\sigma_{\text{bg}}(\pi^- p)$ and $\sigma_{\text{bg}}(\pi^+ p)$, as estimated earlier.[28] Furthermore, this estimation of the background is in accord with the conjecture that it might be the Pomeranchukon effect with some contamination due to the unaccounted t-channel effects. This can be demonstrated by writing down the FESR for the background as

$$\frac{1}{N^{n+1}} \int_0^N \nu^n \operatorname{Im} A_{\text{bg}}(\nu) d\nu = \beta_P \frac{N^{\alpha_P}}{\alpha_P + 1},$$

where for $\operatorname{Im} A_{\text{bg}}$ we write $K\sigma_{\text{bg}}(\nu)/4\pi$ and calculate α_P. Here ν and K have their usual significance and σ_{bg} represents the background. In the calculation we have taken $N \sim 1.66$. The value of α_P comes out to be 1.22. It is very tempting to interpret this 22% discrepancy as due to partial simulation of t-channel effects.

The smoothness of the background curve together with the "very little discrepancy" at the different rapidly varying regions of the $\sigma_T(\pi^- p)$ curve suggest that the contributions from different states are in correct phase. The main contributing states are $P_{33}(1238)$ at the first peak, $D_{13}(1515)$ and $P_{11}(1470)$ at the second peak, and $F_{13}(1690)$ and $D_{13}(1675)$ and to some extent $D_{15}(1675)$ at the third peak. States like F_{35} and F_{37} do not show up in these total cross sections. The results with the form factor $f_L^{(\pm)}$(II) are similar except that the magnitudes are somewhat lowered. Explicit calculation showed that the contribution of the state $D_{13}(1675)$ is suppressed in this case. It is noted that except for the first peak, a significant part of the amplitude $A(\pi^- p \to \pi^- p)$ is transmitted via the $I = \frac{1}{2}$ channel.

In the $\pi^+ p \to \pi^+ p$ process also, the best fit is achieved with the form factor $f_{LJ}^{(\pm)}$(III) as seen from a comparison of curve V with curve VI where the form factor $f_L^{(\pm)}$(II) is operative. Although some deviations do occur near the two peaks, the over-all agreement is not discouraging. The discrepancy near the first peak is only ~5%. The mismatch at the second peak is presumably due to the neglect of neighboring resonances whose experimental status is still in doubt. Another feature of the experimental curve which is not reflected in the theoretical curves is the shoulder in the neighborhood of the energy 1.6 GeV. This is attributed to too small a contribution mainly from the state $S_{31}(1630)$, and to some extent from the state $D_{33}(1670)$, to the $\sigma_T(\pi^+ p \to \pi^+ p)$. But the over-all picture that emerges from this study of the two elastic πN scattering processes seems to suggest that our findings are in conformity with the validity of the "resonance-plus-background" models.[29,30]

B. Energy Dependence of Total Cross Section for the Process $\pi^- p \to \pi^0 n$

Unlike the earlier two cases of elastic scattering where Pomeranchukon contamination is unavoidable, in $\pi^- p$ charge-exchange scattering we have a relatively cleaner process. A good test of our model is therefore possible in this case. The set of experimental points against which we will compare our results are in the momentum range $0.67 \leq p_\pi^{\rm lab} \leq 1.715$.[31-33] The total cross section is obtained by straightforward integration of the differential cross section. The calculations are done under three different conditions characterized by the three different form factors that are available. Our results together with the experimental points are shown in Fig. 3. Prominent features such as the shoulder around $p_\pi^{\rm lab} \sim 0.75$ GeV/c and the dip at $p_\pi^{\rm lab} \sim 0.85$ GeV/c are faithfully reproduced at least qualitatively by all three form factors. The peak at 1.0 GeV/c is again reproduced qualitatively in all three cases, but is displaced towards the low-energy region presumably because of a quite strong contribution of the state $S_{31}(1630)$. Here again the performance of the form factor $f_{LJ}^{(\pm)}$(III) is the best, as the curve III shows a reasonable degree of agreement. The curves I and II are displaced higher and lower, respectively, as expected from the preceding case. (We will find the same trend in Sec. III C.)

The roles of the different states in giving the shape of the curve are as follows: (i) The states $D_{13}(1515)$ and $S_{11}(1525)$, especially the former, are responsible for the shoulder at $p_\pi^{\rm lab} \sim 0.7$ GeV/c. (ii) The dip at 0.9 GeV/c is presumably due to the destructive interference between the tails of states like $D_{13}(1515)$ and $S_{11}(1525)$ with the state $S_{31}(1630)$. (iii) The states that contributed to the peak at $p_\pi^{\rm lab} \sim 1.0$ GeV/c are $S_{31}(1630)$, $D_{13}(1675)$, and to some extent $D_{15}(1675)$ and $D_{33}(1670)$. It is interesting to note that states such as $F_{15}(1690)$, $S_{11}(1715)$, $F_{35}(1880)$, and $F_{37}(1940)$ do not show up prominently in the total CEX cross-section curve.

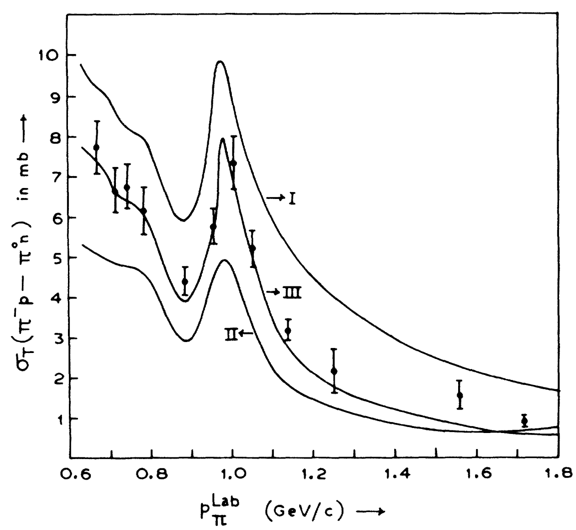

FIG. 3. Total cross section (σ_T) in mb for the $\pi^- p$ CEX process as a function of pion laboratory momentum in GeV/c. Theoretical curves I, II, and III are obtained with the form factors $f_L^{(\pm)}$(I), $f_L^{(\pm)}$(II), and $f_{LJ}^{(\pm)}$(III), respectively. Experimental points are taken from Refs. 31–33.

C. Energy Dependence of the Forward Charge-Exchange Differential Cross Section

Another quantity for which t-channel effects are expected to be considerable is the forward differential cross section. It therefore offers a good case where our model can be tested. We are considering $\pi^- p$ CEX processes for which a considerable number of experimental points over a wide range of energy are available.[34] A dispersion-theoretic[35] and a phase-shift analysis[36] are consistent with the experimental data. Figure 4 shows the extent of agreement of the experimental data points and the theoretical curves drawn assuming our s-channel-resonance model, under three different conditions explained below. The curve I is drawn with the form factor $f_L^{(\pm)}$(I) whereas curves II and III incorporate the form factors $f_L^{(\pm)}$(II) and $f_{LJ}^{(\pm)}$(III), respectively. The dashed branch of curve I corresponds to the situation where we do not take into account the Roper contribution. At this point we make an interesting observation that the peak around the momentum value $p_\pi^{\rm lab} \sim 0.7$ GeV/c, which is getting contributions from states such as $P_{11}(1470)$, $D_{13}(1515)$, and $S_{11}(1525)$, is due to an amplitude which seems to be predominantly imaginary. This is presumably because the contributing states are of N type and they have the mass spacing and widths which favor a constructive interference of the imaginary parts of the forward amplitude

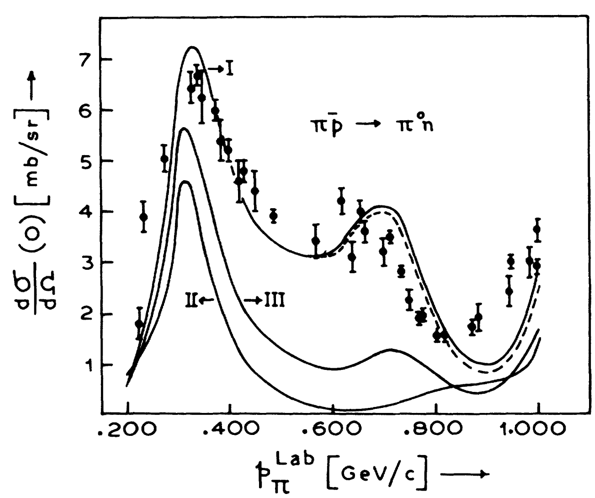

FIG. 4. Forward $\pi^- p$ CEX differential cross section in mb/sr as a function of pion laboratory momentum in GeV/c. Curves I, II, and III stand for the theoretical curves drawn with the form factors $f_L^{(\pm)}$(I), $f_L^{(\pm)}$(II), and $f_{LJ}^{(\pm)}$(III). Experimental points are taken from Ref. 34.

while for the real part there is a large-scale cancellation. This. together with the fact that with the form factor $f_L^{(\pm)}(\mathrm{I})$ we get enhanced magnitudes of $\mathrm{Im}A$ $(t=0)$ in this momentum range [see Sec. III E], enables us to state that the good fit of curve I is accidental, in the sense that the deficiency in $\mathrm{Re}A(t=0)$ is compensated for by the enhanced magnitudes of $\mathrm{Im}A$ $(t=0)$. For the other two form factors, $f_L^{(\pm)}(\mathrm{II})$ and $f_{LJ}^{(\pm)}(\mathrm{III})$, the value of $\mathrm{Im}A$ $(t=0)$ is reasonable, and leads to a drop in the magnitudes of the values of $d\sigma(0)/d\Omega$. It seems therefore that the simulation of $\mathrm{Re}A$ by our limited number of direct-channel resonances is rather low at least in the forward direction for the π^-p CEX process. The partial simulation of the forward CEX amplitude is also reflected in similar studies of processes such as $\pi^-p \to \rho^0 n$ and $\gamma p \to \pi^+ n$.

The least agreement of the curve II is as usual mainly due to the general feature of the form factor $f_L^{(\pm)}(\mathrm{II})$ which gives lower magnitudes of the numbers. Explicit calculation shows that in all three cases the dominant contributors to the forward differential cross section are the states such as $P_{33}(1238)$, $D_{13}(1515)$, $F_{15}(1690)$, and to some extent the Roper contribution. In this case also the fit with the form factor $f_{LJ}^{(\pm)}(\mathrm{III})$ is better, although it falls short of requirements presumably to some extent because the limited number of direct-channel resonances that are being considered fail to simulate fully the t-channel effects even in this low-energy region.

D. π^-p Charge-Exchange Polarization

Although π^-p charge-exchange scattering data in the forward and near-forward directions in the intermediate and high-energy region are well accounted for by the single ρ-Regge-pole exchange on the t channel, they fail to explain the observed[37] nonzero polarization of the recoil neutron. A number of attempts[38-41] have been made to account for this nonzero polarization with varying degrees of success. Here we want to see how far the predictions of our model tally with the experimental data. We have calculated the polarization of the recoil neutron as a function of t in the region -0.2 $(\mathrm{GeV}/c)^2 \leq t \leq 0$ at two different momenta, namely, at 5.9 GeV/c and 11.2 GeV/c, assuming all three form factors separately. Figure 5 shows our result. The agreement with the form factor $f_{LJ}^{(\pm)}(\mathrm{III})$ in this instance also is the best compared to the other two form factors. The polarization is not sensitive to the variation of energy; in fact, there is a slight decrease of polarization with increase of energy in all the cases. This again is in conformity with the experimental evidence. We therefore conclude that there is an over-all agreement between the prediction of our model and the experimental data at least with the form factor $f_{LJ}^{(\pm)}(\mathrm{III})$.

E. Energy Dependence of the Dimensionless Function $\mathrm{Im}Kf^{(-)}$

It has been pointed out that the interference model where one essentially adds the Regge contributions to the resonance contributions is compatible with the experimental data only when there is either a large cancellation between nearby resonances or resonance contributions are small compared to Regge contributions.[12,20] Otherwise there is a possibility of double counting which is also indicated by FESR in the sense that the leading Regge contribution is the local average in energy of the full amplitude.[42-45] An elegant way of displaying this is through the dimensionless quantity $\mathrm{Im}Kf^{(-)}$.

We now study the energy variation of the function $\mathrm{Im}Kf^{(-)}$ defined by Eq. (26) (Sec. II). The motivation is twofold. Firstly, we want to see the extent to which the s-channel resonances do (or do not) reproduce the t-channel effects in our model. Secondly, an examination of this much-studied function in terms of s-channel resonances should help to see in what way and to what extent the resonance states contribute to it, individually and collectively. Calculation of $\mathrm{Im}Kf^{(-)}$ is done under three different sets of form factors. The result is diagrammatically represented in Fig. 6. For the related purpose of a duality test we have calculated the same function taking into account only the ρ Regge pole [Eq. (27), Sec. II]. These are compared with the experimental curve,[20] which was drawn from the $\pi^\pm p \to \pi^\pm p$ total cross-section data, using the formula $\mathrm{Im}Kf^{(-)} = (4\pi)^{-1}K^2(\sigma_{(-)} - \sigma_{(+)})$, where $\sigma_{(-)}$ and $\sigma_{(+)}$ denote the total cross sections for the processes $\pi^-p \to \pi^-p$ and $\pi^+p \to \pi^+p$, respectively. As we have taken a limited number of particles in the representative set of s-channel resonances, we do not expect to reproduce the oscillations beyond ~2.0 GeV, and so the duality effects are not particularly marked in our theoretical curves.

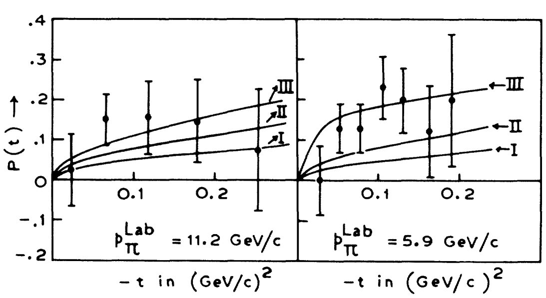

FIG. 5. Recoil-neutron polarization for the process π^-p CEX. Experimental points are taken from Ref. 37. The theoretical curves I, II, and III are drawn assuming the form factors $f_L^{(\pm)}(\mathrm{I})$, $f_L^{(\pm)}(\mathrm{II})$, and $f_{LJ}^{(\pm)}(\mathrm{III})$.

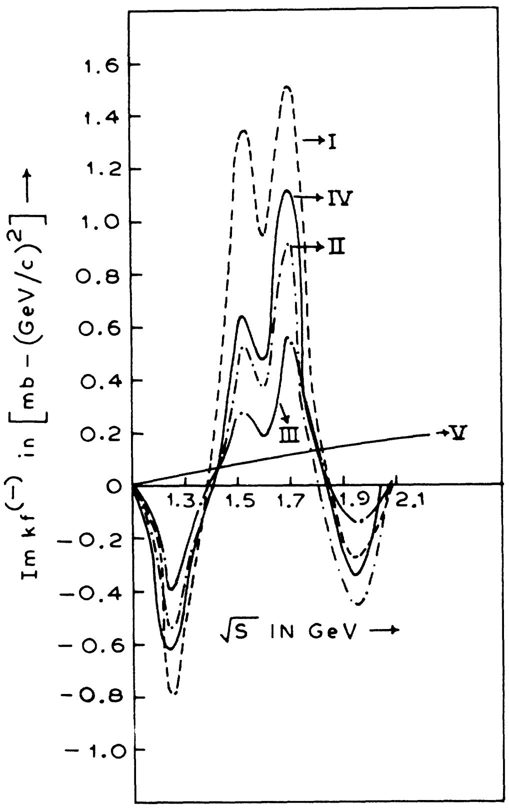

FIG. 6. Energy dependence of the dimensionless function $\mathrm{Im}Kf^{(-)}$. Curve IV is experimental (Ref. 20); curves I, II, and III stand for the theoretical curves drawn assuming the form factors $f_L^{(\pm)}(\mathrm{I})$, $f_L^{(\pm)}(\mathrm{II})$, and $f_{LJ}^{(\pm)}(\mathrm{III})$, respectively. The Regge extrapolated curve is represented by curve V.

As the dynamical origin of the resonances is the cross-channel forces (as ensured by unitarity), it is expected that the form factor with which crossing symmetry is preserved should reproduce the experimental curve better – i.e., in our case the curve $\mathrm{Im}Kf^{(-)}$ as a function of $\sqrt{s}$ in the resonance model should be better represented by the form factors $f_L^{(\pm)}(\mathrm{II})$ and $f_{LJ}^{(\pm)}(\mathrm{III})$ than by the form factor $f_L^{(\pm)}(\mathrm{I})$, since (as mentioned earlier) $f_L^{(\pm)}(\mathrm{I})$ has no crossing symmetry, while $f_L^{(\pm)}(\mathrm{II})$ and $f_{LJ}^{(\pm)}(\mathrm{III})$ have this property. This is clearly displayed in Fig. 6. Comparison with the experimental curve shows us that curve III tallies best, as we have achieved a reasonable compromise between qualitative and quantitative agreements, except for the dip at $\sqrt{s} \sim 1.9$ GeV, which is getting contributions mainly from the states $F_{37}(1940)$ and $F_{35}(1880)$ and is apparently off by some amount. The curve II drawn with the form factor $f_L^{(\pm)}(\mathrm{II})$ also has a satisfactory qualitative agreement. The discrepancy is mainly due to the particular form of parametrization of the form factor $f_L^{(\pm)}(\mathrm{II})$ which somewhat suppresses the $(L+1)$-wave interactions. Like the former case, here also there is a discrepancy around the energy region $\sqrt{s} \sim 1.9$ GeV where the states $F_{37}(1940)$ and $F_{35}(1880)$ contribute, as a result of our neglect of s-channel resonances beyond 1.94 GeV (e.g., G_{17}, etc). The graph I, on the other hand, where the form factor $f_L^{(\pm)}(\mathrm{I})$ is operative, has the least qualitative as well as quantitative agreement. Qualitative agreement has been destroyed because of the enhanced contributions of states such as $D_{13}(1515)$ and $S_{11}(1525)$, especially the former, relative to the other intermediate states. In general the numbers obtained with the form factor $f_L^{(\pm)}(\mathrm{I})$ are of somewhat higher magnitudes. This is why the very good agreement of the forward differential cross section, as a function of $\sqrt{s}$, with the experimental points has to be viewed suspiciously [as was mentioned in Sec. III C].

Explicit calculation showed that, in general, states such as $S_{11}(1525)$, $S_{31}(1630)$, $D_{33}(1630)$, $D_{15}(1675)$, and $P_{31}(1905)$ give very small contributions, while the dominant contributors are the states $P_{33}(1238)$, $F_{15}(1690)$, $D_{13}(1515)$, $F_{35}(1880)$, and $F_{37}(1940)$. The Roper contribution at its peak is about (10–12)% in the cases when $f_L^{(\pm)}(\mathrm{I})$ and $f_L^{(\pm)}(\mathrm{II})$ are operative, whereas with $f_{LJ}^{(\pm)}(\mathrm{III})$ it is ~20%. A second remark is that the $(L+1)$ wave is dominant compared to the $(L-1)$-wave interactions. [However, with the form factor $f_L^{(\pm)}(\mathrm{I})$ we have somewhat pronounced contributions from $(L-1)$-wave interactions as well.] Thus by studying the behavior of the function $\mathrm{Im}Kf^{(-)}$ against $\sqrt{s}$, we can infer the following:

(i) The t-channel effects are partially generated by the limited set of s-channel resonances, as the oscillations are more or less about the ρ-Regge extrapolated curve and not about the zero axis.

(ii) Out of the list of particles mentioned in Table I, states like $P_{33}(1238)$, $F_{15}(1690)$, $D_{13}(1515)$, $F_{35}(1880)$, and $F_{37}(1940)$ have appreciable imaginary parts of the amplitudes for the charge-exchange process $\pi^- p \to \pi^0 n$.

IV. SUMMARY AND CONCLUSION

We have attempted an examination of the relativistic hadron coupling scheme, which scored some success in analyzing the decay modes of different higher-lying baryonic states, for the off-mass-shell processes like $\pi N \to \pi' N'$. This parameter-free analysis is in effect a study of the relativistically invariant form factors. This is done in the direct-channel-resonance model, in the spirit of duality, whose validity at all angles seems less in doubt presently[46] than earlier.

It is found that the off-mass-shell extension of the form factors $f_L^{(\pm)}(\mathrm{II})$ and $f_{LJ}^{(\pm)}(\mathrm{III})$, especially the latter, is fairly successful. With the form factor

$f_L^{(\pm)}$(I) we get some of the qualitative features but it generally gives quite high numbers for the quantities calculated. This is in short the trend of our findings with the three form factors. The form factor $f_{LJ}^{(\pm)}$(III) seems to have the maximum coverage in terms of agreement with the experimental points. This is seen in Secs. III B and III D. In the case of the two elastic processes also (Sec. III A) the agreement is satisfactory although arbitrary to the extent of the arbitrariness of the smoothly varying background, whose estimation is not entirely *ad hoc*. The built-in damping mechanism is inadequate only in the case of the form factor $f_L^{(\pm)}$(II), at least in the energy range we are considering. This is indicated by the rise of the $\sigma_T(\pi^- p$ CEX) curve (curve II, Fig. 3) at an energy beyond the resonance region. This deficiency in the form factor $f_L^{(\pm)}$(II) can be traced back to the form of parametrization of the form factor.

An interesting observation is made in the study of the energy variation of the function $\mathrm{Im}Kf^{(-)}$. It seems to indicate that the form factors $f_L^{(\pm)}$(II) and $f_{LJ}^{(\pm)}$(III), in which crossing symmetry is preserved, have the property of tuning the contributions from different direct-channel resonances (at least the imaginary part of the amplitude) in a better way than the form factor $f_L^{(\pm)}$(I), which does not have this property.

On the basis of the performance of the form factor $f_{LJ}^{(\pm)}$(III) as mentioned above, we finally make the following comments bearing on the concept of duality which is intimately connected with our line of approach. Study of the sensitive function $\mathrm{Im}Kf^{(-)}$ reveals that the simulation of the t-channel effects by the direct-channel resonances is fairly satisfactory, at least as far as the imaginary part of the forward amplitude is concerned, as the resonance curve oscillates more or less about the Regge extrapolated curve and not about the zero line, which is evident from Fig. 6. But the forward differential cross section for the CEX process seems to indicate that the real part of the forward amplitude is not so adequately generated by the direct-channel resonances (Sec. III C). We hope that away from the forward direction the s-channel-resonance description of the scattering process would give better agreement. This view is supported partially by the agreement of our polarization predictions. The over-all investigation, which is essentially confined to the low-energy region where t-channel effects are small but not completely absent, seems to indicate that the agreement with experiment obtained in this direct-channel-resonance model could be enhanced by considering more resonances.

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[1]A. N. Mitra, Nuovo Cimento 61A, 344 (1969).
[2]A. N. Mitra, Nuovo Cimento 64A, 603 (1969).
[3]S. Das Gupta and B. K. Mohana, Nuovo Cimento 64A, 325 (1969).
[4]D. K. Choudhury and A. N. Mitra, Phys. Rev. D 1, 351 (1970).
[5]D. L. Katyal and A. N. Mitra, Phys. Rev. D 1, 338 (1970).
[6]R. Mehrotra and A. N. Mitra, Phys. Rev. D (to be published).
[7]K. Sen Gupta and V. K. Gupta, Phys. Rev. D (to be published).
[8]A. N. Mitra, in *Lectures in High-Energy Theoretical Physics*, edited by H. H. Aly (Gordon and Breach, London, 1970).
[9]C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958).
[10]A. N. Mitra, I. C. T. P. Report No. Ic/70/58 (unpublished).
[11]D. K. Choudhury, D. L. Katyal, and A. N. Mitra, Nuovo Cimento (to be published).
[12]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).
[13]G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).
[14]H. Harari, Phys. Rev. Letters 20, 1395 (1968).
[15]K. Igi and S. Matsuda, Phys. Rev. Letters 18, 625 (1967); A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters 24B, 181 (1967).
[16]L. Durand, Phys. Rev. 166, 1680 (1968).
[17]P. W. Coulter, E. S. Ma, and G. L. Shaw, Phys. Rev. Letters 23, 106 (1969).
[18]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).
[19]V. Barger and M. Olsson, Phys. Rev. 151, 1123 (1966).
[20]C. B. Chiu and A. Stirling, Phys. Letters 26B, 236 (1968).
[21]The masses M_J and the widths Γ_J of the intermediate resonance states are taken from Ref. 5.
[22]V. Singh, Phys. Rev. 129, 1889 (1963).
[23]R. Blankenbecler and R. L. Sugar, Phys. Rev. 168, 1597 (1968).
[24]The J index of $f_{LJ}^{(\pm)}$ (III) is always implied wherever omitted.
[25]F. Arbab and C. B. Chiu, Phys. Rev. 147, 1045 (1966).
[26]P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968).

[27]The background is the amount which we have added to the theoretical curve to make it coincide with the experimental curve, disregarding a small discrepancy in the locations of the peaks and dips.
[28]F. N. Dikmen, Phys. Rev. Letters 18, 798 (1967).
[29]G. T. Hoff, Phys. Rev. Letters 18, 816 (1967).
[30]F. N. Dikmen, Phys. Rev. Letters 22, 622 (1969).
[31]F. Bulos et al., Phys. Rev. Letters 13, 558 (1964).
[32]A. S. Carroll et al., Proc. Roy. Soc. (London) A289, 513, 1966.
[33]V. V. Barmin et al., Zh. Eksperim. i Teor. Fiz. 46, 142 (1964) [Soviet Phys. JETP 19, 102 (1964)].
[34]The various references are listed in Ref. 35.
[35]G. Höhler et al., Phys. Letters 21, 223 (1966).
[36]P. Bareyre et al., Phys. Letters 18, 342 (1965).
[37]P. Bareyre et al., Phys. Letters 165, 1730 (1968); CEN-Saclay-Orsay-Pisa Collaboration, ibid. 23, 501 (1966).
[38]R. J. N. Phillips, Nuovo Cimento 45A, 245 (1966).
[39]R. K. Logan and L. Sertorio, Phys. Rev. Letters 17, 834 (1966).
[40]H. Hogadsen and W. Fischer, Phys. Letters 22, 516 (1966).
[41]G. Altarelli et al., Nuovo Cimento 48A, 245 (1966).
[42]R. Aviv and D. Horn, Phys. Rev. Letters 21, 704 (1968).
[43]V. Barger and L. Durand, Phys. Letters 26B, 588 (1968).
[44]C. B. Chiu, R. J. Eden, and C.-I Tan, Phys. Rev. 170, 490 (1968).
[45]H. Goldberg, Phys. Rev. 171, 1485 (1968).
[46]S. Y. Chu and A. W. Hendry, Phys. Rev. Letters 25, 313 (1970).

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How to See the Light Cone*

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New experiments which probe the behavior of products of electromagnetic currents at short and lightlike distances are investigated. They require the detection of a massive μ pair in electron-positron annihilation or in electroproduction. The former reaction is studied in detail and the (considerable) background is discussed. It is shown that, in suitable kinematic regions, one can approach both the Bjorken-Johnson-Low limit and the scaling limit, and put to severe test many recent ideas regarding products of currents. In the Bjorken-Johnson-Low limit one can test models of equal-time commutation relations and measure the spectral functions of an axial-vector current. In the scaling region one can measure the matrix elements of the bilocal operators appearing in the light-cone expansion of the product of electromagnetic currents, and test the tensorial structure of this expansion which emerges from quark models. Finally, in the inclusive reaction, where a massive μ pair is produced in addition to any number of hadrons, the large hadronic mass limit of the cross section is completely determined in terms of a commutator of bilocal operators.

I. INTRODUCTION

Recent experimental and theoretical advances have led to increased interest in the properties of products of local current operators separated by short or lightlike distances. Much of the interest arises from the remarkable regularities discovered in the SLAC-MIT[1] experiments on deep-inelastic electron-nucleon scattering. In the so-called scaling region one is probing the commutator of two electromagnetic currents in the vicinity of the light cone; the scaling behavior observed for the structure functions has important implications for the light-cone structure of the commutator. The structure is controlled by a collection of local operators which appear in the most singular terms on the light cone; experiments of the SLAC-MIT type provide information on these theoretically interesting objects. These theoretical issues have been discussed from a variety of points of view. In particular, it has seemed an attractive idea to investigate the light-cone structure in models, with a view to abstracting general features and then abandoning the specifics of the models. This approach has been emphasized by Gell-Mann,[2] who considered the situation for the free-quark model; it has been pursued also for quark models with strong interactions mediated by SU(3) singlet mesons.[3,4] Our discussion here will be based in part on the structure implied by the quark-vector-gluon model.[3]

If current commutators have any simple features, they are likely to show up in regions where the currents are separated by short or lightlike dis-