Inelastic-Compton-Scattering Inequalities in the Quark-Parton Model*

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On the assumption that partons are quarks, we derive several constraint relations (in the form of inequalities) for the structure functions of inelastic Compton scattering. Bounds on the structure functions of the inelastic Compton scattering with respect to those of inelastic electron-proton scattering are also discussed. All these inequalities must be true for any quark-parton model and, thus, furnish us with some means of testing the hypothesis of individual quark-parton identification. These are also very useful for obtaining information about the parton distribution function $P(N)$ and about the minimum number of partons N_0 in a nucleon. The constraint relations that we have derived here can be made much stronger if one uses a detailed quark-parton model. Assuming $\langle 1/N\rangle = 0.1$, which is consistent with the presently available data on both inelastic electron and inelastic neutrino scattering, we make a quantitative analysis of the structure functions.

I. INTRODUCTION

The parton idea[1] of hadrons has been found to produce various useful results in high-energy physics. In fact, it is emerging as a very powerful concept in inelastic scattering. The scale-invariant nature of inelastic e-p scattering has already been explained successfully by the parton model.[2,3] However, the nature of the partons is not known at all, and various models have been suggested. In the original discussion by Feynman,[1] the partons are assumed to be some sort of fundamental fields in the infinite-momentum frame. Bjorken and Paschos[2] then made a model where partons are thought of as quarks. This is a very interesting conjecture. If the partons are indeed found to be quark fields, it will unify our understanding of physics to a great extent.

Recently, several inequalities have been derived by one of the authors[4] to test the hypothesis of individual quark-parton association. This was done for the cases of inelastic electron-proton and electron-neutron scattering. It was found that the quark-parton identity puts severe constraints on the formalism of the parton model. A similar approach has been taken by Gourdin[5] for studying inelastic neutrino scattering.

The present paper is intended for extending similar work to other inelastic processes. Bjorken and Paschos[2] have argued that inelastic Compton scattering can produce a model-independent check on the fractional charge of the partons. We shall elaborate here on this topic and shall derive several inequalities for inelastic Compton scattering that must be satisfied if the partons are quarks. We shall also compare inelastic Compton and inelastic electron scattering from the proton and the neutron. All these processes are related in the parton model and, hopefully, from their study a complete picture will emerge.

Further, if any of our results is found to be contradicted by the experiments which will be performed in the near future, then the individual quark-parton association picture has to be abandoned; or, perhaps, a different and deeper understanding of the parton interactions will be needed.

II. PARTON-MODEL FORMULAS

In the parton picture the inelastic Compton scattering (see Fig. 1) is visualized as a sum of incoherent photon-parton scatterings. For a large electron-proton center-of-mass energy and a large momentum transfer squared of the photon, the lifetime of the intermediate states between the absorption and emission of the photon becomes much less than the lifetime of the virtual parton state in the proton. Consequently, the process can be treated as incoherent scattering. Also, in that case the processes where the photon is emitted from a different parton state can be neglected. In this picture of the Compton scattering a strict field-theoretic description may not be possible since we do not know how to omit the diagrams with delayed emitted photons.[6] However, the parton picture remains valid in its own idealization.

The differential cross section for the inelastic Compton scattering can then be written[2] as

$$\frac{d^2\sigma}{d\Omega dk'} = \sigma_R \frac{\nu}{kk'}\sum_N P(N)xf_N(x)\left\langle \sum_1^N Q_i{}^4\right\rangle, \qquad (2.1)$$

where k and k' are the incoming and outgoing energies of the photon in the laboratory frame,

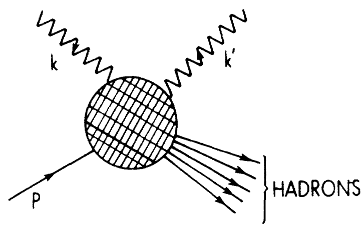

FIG. 1. Kinematics of inelastic Compton scattering.

$$\nu=(k-k')P/M, \quad x=\frac{Q^2}{2M\nu}=\frac{1}{\omega},$$

and

$$\sigma_R=\frac{\alpha^2}{4k'^2\sin^4(\frac{1}{2}\theta)}\left(1+R\frac{\nu^2}{2kk'}\right). \tag{2.2}$$

The other factors have their usual meaning as given in Refs. 2 or 3. We only state that, by definition,

$$\sum_N P(N)=1 \tag{2.3}$$

and

$$\int_0^1 f_N(x)dx=1. \tag{2.4}$$

For convenience and in analogy to the inelastic e-p scattering, let us define

$$\nu W_2^{\gamma p}=\sum_N P(N)xf_N(x)\left\langle\sum_1^N Q_i^{\,4}\right\rangle_p. \tag{2.5}$$

For the inelastic e-p case, we have seen before that

$$\nu W_2^{ep}=\sum_N P(N)xf_N(x)\left\langle\sum_1^N Q_i^{\,2}\right\rangle_p. \tag{2.6}$$

Note that both $\nu W_2^{\gamma p}$ and νW_2^{ep} must be positive for the whole range of x.

Finally, our definition leads to

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\gamma p}=\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{ep}\frac{\nu^2}{kk'}\,\frac{W_2^{\gamma p}(\nu,Q^2)}{W_2^{ep}(\nu,Q^2)}. \tag{2.7}$$

For a symmetric distribution of the momenta among the partons, it can be shown[2] that

$$\int_0^1 xf_N(x)dx=\frac{1}{N}. \tag{2.8}$$

Following Ref. 2, two sum rules for the inelastic Compton scattering can be derived from the above equations:

$$\int_0^1 \nu W_2^{\gamma p}\,dx=\sum_N P(N)\left\langle\sum_1^N Q_i^{\,4}\right\rangle\Big/N$$

$$=\text{Average of the fourth power of charges of the partons} \tag{2.9}$$

and

$$\int_0^1 \nu W_2^{\gamma p}\frac{dx}{x}=\sum_N P(N)\left\langle\sum_1^N Q_i^{\,4}\right\rangle. \tag{2.10}$$

III. INEQUALITIES FOR INELASTIC γ-p AND γ-n SCATTERING

If the partons are identified with quarks, the nucleon can be thought of as being made up of three quarks ($\mathcal{P}\mathcal{P}\mathcal{N}$ quarks for proton and $\mathcal{P}\mathcal{N}\mathcal{N}$ quarks for neutron) surrounded by a cloud of quarks and antiquarks. Let N_1, N_2, and N_3 denote the number of $\mathcal{P}$-, $\mathcal{N}$-, and λ-type quarks, respectively, and $\overline{N}_1$, $\overline{N}_2$, and $\overline{N}_3$ the corresponding number of antiquarks in the proton cloud. Then

$$N_1+\overline{N}_1+N_2+\overline{N}_2+N_3+\overline{N}_3=N-3. \tag{3.1}$$

By charge symmetry

$$N_1^p=N_2^n,\quad N_2^p=N_1^n,\quad N_3^p=N_3^n$$

and

$$\overline{N}_1^p=\overline{N}_2^n,\quad \overline{N}_2^p=\overline{N}_1^n,\quad \overline{N}_3^p=\overline{N}_3^n, \tag{3.2}$$

where the superscripts p and n refer to proton and neutron, respectively. The fourth power of the $\mathcal{P}$ or $\overline{\mathcal{P}}$ quark charge is $\frac{16}{81}$, while for the others it is $\frac{1}{81}$, so that

$$\left\langle\sum_1^N Q_i^{\,4}\right\rangle_{\text{proton}}=\tfrac{10}{27}+\tfrac{5}{27}(N_1^p+\overline{N}_1^p)+\tfrac{1}{81}N$$

and

$$\left\langle\sum_1^N Q_i^{\,4}\right\rangle_{\text{neutron}}=\tfrac{5}{27}+\tfrac{5}{27}(N_2^p+\overline{N}_2^p)+\tfrac{1}{81}N. \tag{3.3}$$

A. Inequalities for $\nu W_2^{\gamma N}$

Since the number of quarks and antiquarks, $(N_1^p+\overline{N}_1^p)$ or $(N_2^p+\overline{N}_2^p)$, in the "quark cloud" of a nucleon can be anything between 0 and $N-3$, using (3.3) in (2.9), we obtain the following inequalities:

$$\frac{1}{81}+\frac{10}{27}\left\langle\frac{1}{N}\right\rangle\le\int_0^1 \nu W_2^{\gamma p}dx\le\frac{16}{81}-\frac{5}{27}\left\langle\frac{1}{N}\right\rangle \tag{3.4}$$

for the proton, and

$$\frac{1}{81}+\frac{5}{27}\left\langle\frac{1}{N}\right\rangle\le\int_0^1 \nu W_2^{\gamma n}dx\le\frac{16}{81}-\frac{10}{27}\left\langle\frac{1}{N}\right\rangle \tag{3.5}$$

for the neutron.

Here the upper bounds correspond to assuming all the partons beyond the first three to be $\mathcal{P}$ or $\overline{\mathcal{P}}$ quarks, while the lower bounds correspond to having no $\mathcal{P}$ or $\overline{\mathcal{P}}$ quarks in the cloud. To improve on these inequalities, we need more detailed knowledge about the parton configurations. However, similar inequalities[4] for the inelastic e-p scattering data tell us that $\langle 1/N\rangle<0.1$, while the CERN neutrino data give[5] $\langle 1/N\rangle>0.09$. So, to obtain some estimates for the inelastic Compton scattering, we choose $\langle 1/N \rangle = 0.1$. This gives us the following bounds for the mean fourth power of the parton charges [see Eq. (2.9)] in the nucleon:

$$0.05 \leq \int_0^1 \nu W_2^{\gamma p}\, dx \leq 0.18 \tag{3.6}$$

and

$$0.03 \leq \int_0^1 \nu W_2^{\gamma n}\, dx \leq 0.16\,. \tag{3.7}$$

Similar considerations for the inelastic e-p scattering produce the following bounds for the mean-square charge of the partons:

$$0.18 \leq \int_0^1 \nu W_2^{ep}\, dx \leq 0.41 \tag{3.8}$$

and

$$0.14 \leq \int_0^1 \nu W_2^{en}\, dx \leq 0.38\,. \tag{3.9}$$

One can also derive similar inequalities starting with Eq. (2.10). Since for a quark-parton model the minimum number of quarks in a nucleon must be three, these inequalities yield the following relations:

$$\int_0^1 \nu W_2^{\gamma N} \frac{dx}{x} \geq \frac{11}{27} \text{ for proton,} \qquad \geq \frac{2}{9} \text{ for neutron.} \tag{3.10}$$

B. Inequalities for $\nu W_2^{\gamma n} + \nu W_2^{\gamma p}$

For the sum of $\nu W_2^{\gamma p}$ and $\nu W_2^{\gamma n}$, somewhat stronger bounds than those implied by the previous section can be derived. This is because certain terms of Q^4 for proton and neutron cancel out due to charge symmetry.[7] We note that

$$\nu W_2^{\gamma p} + \nu W_2^{\gamma n} = 2\sum_N P(N) \left\langle \sum_1^N [(I_i^3)^4 + \tfrac{3}{2}(I_i^3)^2 Y_i^2 + \tfrac{1}{16} Y_i^4] \right\rangle x f_N(x)\,. \tag{3.11}$$

Utilizing the values of I^3 and Y for the quarks, we obtain from above

$$\frac{2}{81} + \frac{5}{9}\left\langle \frac{1}{N} \right\rangle \leq \int_0^1 (\nu W_2^{\gamma p} + \nu W_2^{\gamma n}) dx \leq \frac{17}{81}\,. \tag{3.12}$$

The upper bound should be true for any quark-parton model. This result indicates that perhaps the actual upper bounds in Eqs. (3.6) and (3.7) should be quite low. To obtain an estimate for the lower bound, we can approximate $\langle 1/N \rangle$ by 0.1. This gives a lower bound of 0.08.

Integrating the structure functions with dx/x, we obtain several similar inequalities. From these we can obtain an inequality for the minimum number of partons N_0 in a nucleon, namely,

$$3 \leq N_0 \leq \frac{81}{2}\left[\int_0^1 (\nu W_2^{\gamma p} + \nu W_2^{\gamma n}) \frac{dx}{x} - \frac{5}{9} \right]. \tag{3.13}$$

It was stressed before that the minimum number of partons plays an important role in parton models. In earlier work[3] N_0 was fixed to be four by looking at the data near $\omega \sim 1$. The above inequality and a similar inequality[4] derived for inelastic e-p scattering will now give us a better idea in determining N_0 for any quark-parton model. Since deuteron experiments may yield data directly for the sum of the neutron and proton structure functions, this inequality may furnish an important check on N_0.

C. Inequalities for the Ratio of $\nu W_2^{\gamma n}/\nu W_2^{\gamma p}$

The ratio of the structure functions for inelastic Compton scattering can be written as

$$\frac{\nu W_2^{\gamma n}}{\nu W_2^{\gamma p}} = \frac{\sum_N P(N) x f_N(x) [\frac{5}{27}(1 + N_2 + \bar{N}_2) + \frac{1}{81} N]^{\text{neutron}}}{\sum_N P(N) x f_N(x) [\frac{10}{27} + \frac{5}{27}(N_1 + \bar{N}_1) + \frac{1}{81} N]^{\text{proton}}}\,. \tag{3.14}$$

From this we can derive that

$$\frac{1}{16} + \frac{255}{256} \frac{\sum_N P(N) x f_N(x)}{\sum_N P(N) x f_N(x)(N - \frac{15}{16})} \leq \frac{\nu W_2^{\gamma n}}{\nu W_2^{\gamma p}} \leq 16 - 17 \frac{\sum P(N) x f_N(x)}{\sum P(N) x f_N(x)(1 + \frac{1}{30} N)}\,. \tag{3.15}$$

This crudely shows that $\nu W_2^{\gamma n}/\nu W_2^{\gamma p}$ must be greater than $\frac{1}{16}$ and must be less than 16, for all ω. To obtain a better estimate of the bounds, we shall require detailed information about $P(N)$ and $f_N(x)$. However, several integrated inequalities can be derived without any additional assumptions. This can be achieved by noticing that

$$\frac{\nu W_2^{\gamma p}}{\sum_N P(N) x f_N(x)(N + 30)} \geq \frac{1}{81}, \tag{3.16}$$

so that

$$\tfrac{1}{16} \nu W_2^{\gamma p} + \tfrac{255}{256} \times \tfrac{1}{81} \sum_N P(N) x f_N(x) \leq \nu W_2^{\gamma n} \leq 16 \nu W_2^{\gamma p} - \tfrac{510}{81} \sum_N P(N) x f_N(x)\,. \tag{3.17}$$

Integrating over x, we obtain two inequalities,

$$\int_0^1 (\nu W_2^{\gamma n} - \tfrac{1}{16} \nu W_2^{\gamma p}) \frac{dx}{x} \geq 0.01 \tag{3.18}$$

and

$$\int_0^1 (\nu W_2^{\gamma p} - \tfrac{1}{16} \nu W_2^{\gamma n}) \frac{dx}{x} \geq 0.39\,. \tag{3.19}$$

One should note that in deriving these inequalities we have not used Eq. (2.8). We further observe that these inequalities can also be derived from the relations

$$\tfrac{10}{27} + \tfrac{1}{81}\langle N\rangle \leqslant \int_0^1 \nu W_2^{\gamma p}\frac{dx}{x} \leqslant \tfrac{16}{81}\langle N\rangle - \tfrac{5}{27} \tag{3.20}$$

and

$$\tfrac{5}{27} + \tfrac{1}{81}\langle N\rangle \leqslant \int_0^1 \nu W_2^{\gamma n}\frac{dx}{x} \leqslant \tfrac{16}{81}\langle N\rangle - \tfrac{10}{27}, \tag{3.21}$$

which follow from (2.5).

IV. INEQUALITIES FOR INELASTIC γ-p AND e-p SCATTERING

In the previous section we have considered bounds on the γ-p and γ-n inelastic Compton scattering. Let us now see what we can learn from a comparison between γ-p and e-p inelastic scattering. The ratio of the structure functions for γ-p and e-p inelastic scattering can be written as

$$\frac{\nu W_2^{\gamma p}}{\nu W_2^{ep}} = \frac{\sum_N P(N)xf_N(x)[\frac{10}{27}+\frac{5}{27}(N_1+\overline{N}_1)+\frac{1}{81}N]}{\sum_N P(N)xf_N(x)[\frac{2}{3}+\frac{1}{3}(N_1+\overline{N}_1)+\frac{1}{9}N]}. \tag{4.1}$$

In this expression $(N_1+\overline{N}_1)$ can vary from zero to $(N-3)$. We, therefore, obtain by rearrangement

$$\frac{1}{9}+\frac{8}{3}\frac{\sum_N P(N)xf_N(x)}{\sum_N P(N)xf_N(x)(4N-3)} \leqslant \frac{\nu W_2^{\gamma p}}{\nu W_2^{ep}} \leqslant \frac{4}{9} - \frac{1}{3}\frac{\sum_N P(N)xf_N(x)}{\sum_N P(N)xf_N(x)(4N-3)}, \tag{4.2}$$

which implies that the ratio of the structure functions must be between $\frac{1}{9}$ and $\frac{4}{9}$ for all values of ω. Bjorken and Paschos (BP) considered a model where all types of quarks are present with equal probability in the cloud. They obtained lower and upper bounds for the ratio $\nu W_2^{\gamma p}/\nu W_2^{ep}$ to be $\frac{1}{3}$ and $\frac{5}{9}$, respectively. Since they used a detailed quark-parton model, we would expect them to obtain stronger bounds than those given by (4.2), which have been derived on very general grounds. However, their upper bound seems to be weaker than that given by (4.2). To understand this, we show that in the BP model the upper bound for the ratio $\nu W_2^{\gamma p}/\nu W_2^{ep}$ is, in fact, much smaller than $\frac{5}{9}$. This is seen as follows. In the BP model

$$\left\langle \sum_1^N Q_i^2 \right\rangle_p = \tfrac{2}{9}N + \tfrac{1}{3} \text{ and } \left\langle \sum_1^N Q_i^4 \right\rangle_p = \tfrac{2}{27}N + \tfrac{5}{27},$$

so that one can write

$$\left\langle \sum_1^N Q_i^4 \right\rangle_p = \beta \left\langle \sum_1^N Q_i^2 \right\rangle_p + \tfrac{2}{27}(1-3\beta)(N-3) + (\tfrac{11}{27}-\beta), \tag{4.3}$$

where β is an arbitrary constant. For $\beta \geqslant \frac{11}{27}$, the sum of the last two terms on the right-hand side of (4.3) is negative, while for $\beta \leqslant \frac{1}{3}$ it is positive. Hence,

$$\tfrac{1}{3}\left\langle \sum_1^N Q_i^2 \right\rangle_p < \left\langle \sum_1^N Q_i^4 \right\rangle_p < \tfrac{11}{27}\left\langle \sum_1^N Q_i^2 \right\rangle_p. \tag{4.4}$$

Thus in the BP model the ratio $\nu W_2^{\gamma p}/\nu W_2^{ep}$ lies between $\frac{1}{3}$ and $\frac{11}{27}$, which is consistent with our bounds.

Following Sec. III C we can now derive two integrated inequalities, which does not require any knowledge of $P(N)$ or $f_N(x)$. We observe that

$$\frac{\nu W_2^{ep}}{\sum_N P(N)xf_N(x)(N+6)} \geqslant \frac{1}{9}. \tag{4.5}$$

Then, Eq. (4.2) produces the following inequality:

$$\tfrac{1}{9}\nu W_2^{ep} + \tfrac{2}{27}\sum_N P(N)xf_N(x) \leqslant \nu W_2^{\gamma p} \leqslant \tfrac{4}{9}\nu W_2^{ep} - \tfrac{1}{108}\sum_N P(N)xf_N(x). \tag{4.6}$$

Integrating over x, we obtain our final results,

$$\int_0^1 (\nu W_2^{\gamma p} - \tfrac{1}{9}\nu W_2^{ep})\frac{dx}{x} \geqslant \frac{2}{27} \tag{4.7}$$

and

$$\int_0^1 (\nu W_2^{ep} - \tfrac{9}{4}\nu W_2^{\gamma p})\frac{dx}{x} \geqslant \frac{1}{48}. \tag{4.8}$$

Similar results can be derived for the neutron structure functions as well.

V. CONCLUSION

We have summarized here several inequalities for inelastic Compton scattering that must be satisfied if the partons are identified as quarks. The basic ingredients in deriving these inequalities have been the I^3 and Y properties of the quarks, and the fact that neutron and proton form an isospin doublet. We have then made a comparative study of the inequalities for inelastic Compton and inelastic e-p scattering. This analysis serves two purposes. First, it gives us some ideas about the numerical estimates of the structure functions for inelastic Compton scattering. Since the experimental data have not yet been obtained, we presently have no other way to estimate these values. Second, if the parton idea is correct, it must hold for all the inelastic processes and if partons are quarks, all these phenomena should produce a consistent quark-parton picture. Our analysis in this paper and that of Ref. 4 form a consistent set of constraint equations for the model which assumes partons as quarks. We should note that in deriving these relations we have not assumed any functional form for $P(N)$ or $f_N(x)$. We have also not assumed a special quark-parton model like the BP model. So all these constraint relations must be satisfied for any quark-parton model. The experiment of inelastic Compton scattering would thus be quite fruitful for understanding the structure of the hadrons.

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[7]Similarly, one finds that

$$\nu W_2^{\gamma p} - \nu W_2^{\gamma n} = 2\sum_N P(N)\left\langle \sum_1^N [2(I_i^3)^3 Y_i + \tfrac{1}{2} I_i^3 Y_i^3]\right\rangle x f_N(x),$$

which yields the inequality

$$\frac{20}{27}\left\langle\frac{1}{N}\right\rangle - \frac{5}{27} \le \int_0^1 (\nu W_2^{\gamma p} - \nu W_2^{\gamma n})dx$$
$$\le \frac{5}{27} - \frac{10}{27}\left\langle\frac{1}{N}\right\rangle.$$

This inequality can also be derived from Eqs. (3.4) and (3.5).

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s-Channel Analysis of πN Scattering in a Scheme of Higher Baryon Couplings

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A relativistic-hadron-coupling scheme recently proposed by Mitra and co-workers is applied to the process $\pi N \to \pi' N'$. This scheme, which is a relativistic extension of an $SU(6)\otimes O_3$ framework of $\bar{B}B_L P$ couplings, is characterized by the appearance of a relativistically invariant form factor, which is endowed with several desirable properties such as Regge universality, crossing symmetry, etc. So far three distinct choices, termed I, II, III, are available, which satisfy these criteria in varying degrees. The calculation of $\pi N \to \pi' N'$ scattering is done in a pure s-channel model where the various baryon resonances act as the propagators. The effect of other channels (t and/or u) is not considered in the calculation, in the spirit of duality. Specifically, the following processes are considered: (i) the two elastic $\pi^\pm p \to \pi^\pm p$ processes, which have a nonresonant background, and (ii) the $\pi^- p$ charge-exchange process, which does not have such a background. The quantities calculated are the following: $\sigma_T(\pi^\pm p \to \pi^\pm p)$, $\sigma_T(\pi^- p \to \pi^0 n)$, $d\sigma(0)/d\Omega(\pi^- p \to \pi^0 n)$, the polarization $P(t)$ for $\pi^- p$ charge exchange, and the sensitive dimensionless quantity $\mathrm{Im}Kf^{(-)}$. It is found that the above scheme gives a satisfactory account of the data, especially with form factor III, which fits the various details quite accurately. The role of duality in respect of the simulation of the t channel is discussed, especially in relation to the behavior of the quantities $d\sigma(0)/d\Omega$ and $\mathrm{Im}Kf^{(-)}$.

I. INTRODUCTION

In a few recent communications[1,2] it was shown how a relativistically invariant model of hadron couplings based on $SU(6)\otimes O_3$ could be constructed at the phenomenological level with a relativistically invariant form factor for each supermultiplet transition. The fair success of the model in the case of hadron decays[3-5] warrants a more stringent test in terms of the off-mass-shell extension of the form factor. For this purpose we have taken up the familiar $\pi N \to \pi' N'$ process to be studied in the light of the above model of hadron couplings. Similar studies for allied processes such as pion photoproduction[6] and vector-meson production[7] have been carried out recently with encouraging results. Apart from this, we are also interested to find out the extent of simulation of the t-channel effects by purely s-channel resonance contributions and so we are primarily concerned with the differ-