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CP Nonconservation and Inequalities Between $\mu^+\mu^-$ and 2γ Decay Rates of K_S^0 and K_L^0 †

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Assuming that the absorptive part of $K_L^0 \to 2\mu$ is due only to the on-mass-shell 2γ intermediate states, several inequalities between 2μ and 2γ decay states of K_S^0 and K_L^0 are derived without the assumption of CP conservation. These inequalities, together with the present experimental upper bound on $K_L^0 \to 2\mu$, imply that the branching ratio for $K_S^0 \to 2\mu$ should be greater than 5×10^{-7}.

I. INTRODUCTION

It has been pointed out by several authors[1,2] that if the absorptive part of $K_L^0 \to \mu^+\mu^-$ is assumed to be due only to the on-mass-shell 2γ intermediate state and if, in addition, CP conservation holds, then the usual quantum electrodynamics leads to the inequality

$$\frac{\text{rate}(K_L^0 \to 2\mu)}{\text{rate}(K_L^0 \to 2\gamma)} > \lambda^2, \tag{1}$$

where[1,3]

$$\lambda^2 = \frac{1}{2}\frac{\alpha^2}{v_\mu}\left(\frac{m_\mu}{m_K}\right)^2\left(\ln\frac{1+v_\mu}{1-v_\mu}\right)^2 \cong 1.2\times10^{-5} \tag{2}$$

and

$$v_\mu \cong 0.9 \tag{3}$$

is the velocity of $\mu^\pm$ in the rest system of the kaon. The present experimental upper bound on the branching ratio[4] is

$$\frac{\text{rate}(K_L^0 \to \mu^+\mu^-)}{\text{rate}(K_L^0 \to \text{all})} < 1.8\times10^{-9}, \tag{4}$$

while, according to (1), the theoretical lower bound for the same branching ratio should be $\cong 6 \times 10^{-9}$.

At first sight, this discrepancy may not seem to be too disturbing since in $K_L^0 \to \mu^+\mu^-$, besides 2γ there are also $2\pi\gamma$, 3π, and other on-mass-shell intermediate states, and furthermore, CP conservation is known to be violated. However, difficulty does arise on a dynamical level. At present, attempts to include $2\pi\gamma$, 3π, and other intermediate states in the absorptive part lead only to a small correction to the above theoretical lower bound,[5] and it seems quite difficult to explain the large difference between the theoretical and experimental bounds on $K_L^0 \to \mu^+\mu^-$ by using any simple theoretical model.

The purpose of this note is to examine the alternative possibility, i.e., the effect of CP nonconservation. As we shall see, there are definite tests which can be used to trace whether the present discrepancy is due to CP nonconservation or due to other reasons. In order to separate out the implications of different theoretical hypotheses, we shall assume, throughout our subsequent discussions, (i) that the absorptive part of the $K_L^0 \to 2\gamma$ amplitude is zero, (ii) that the absorptive part of the $K_L^0 \to \mu^+\mu^-$ amplitude is due only to the on-mass-shell 2γ intermediate state, and (iii) that both CPT invariance and quantum electrodynamics are valid, but CP conservation is not. As we shall see, under these assumptions, the lower bound given above by (1) no longer holds, and it is replaced by

$$[\text{rate}(K_S^0 \to \mu^+\mu^-)]^{1/2} \geq (\text{Re}\epsilon)^{-1}\{\lambda v_\mu[\text{rate}(K_L^0 \to 2\gamma)]^{1/2} - [\text{rate}(K_L^0 \to \mu^+\mu^-)]^{1/2}\} \tag{5}$$

and

$$[\text{rate}(K_S^0 \to \mu^+\mu^-)]^{1/2} \leq (\text{Re}\epsilon)^{-1}\{\lambda[\text{rate}(K_L^0 \to 2\gamma)]^{1/2} + [\text{rate}(K_L^0 \to \mu^+\mu^-)]^{1/2}\}, \tag{6}$$

where[6]

$$\text{Re}\epsilon \equiv \tfrac{1}{2}\langle K_L^0|K_S^0\rangle \cong 1.4\times10^{-3},$$

and λ and v_μ are given by Eqs. (2) and (3), respectively. For clarity, the derivation of the above formulas will be given in Sec. II.

We observe that by using (4) and the experimental value[7]

$$\frac{\text{rate}(K_L^0 \to 2\gamma)}{\text{rate}(K_L^0 \to \text{all})} = (4.7 \pm 0.6)\times 10^{-4},$$

the branching ratio for $K_S^0 \to \mu^+\mu^-$ should lie within

$$1\times 10^{-5} \geq \frac{\text{rate}(K_S^0 \to \mu^+\mu^-)}{\text{rate}(K_S^0 \to \text{all})} \geq 5\times 10^{-7}, \qquad (7)$$

which is consistent with its present experimental upper bound[8] of 7.3×10^{-6}.

The validity of the above inequality (7) can be determined by an order-of-magnitude improvement of the present experimental limit on the branching ratio of $K_S^0 \to \mu^+\mu^-$. A violation of (7) can be used as a conclusive proof that the discrepancy between (1) and (4) cannot be attributed to CP nonconservation; other reasons must be sought to explain this difference. On the other hand, the validation of (7) implies that $[\text{rate}(K_S^0 \to \mu^+\mu^-)]^{1/2}$ is larger than $[\text{rate}(K_L^0 \to \mu^+\mu^-)]^{1/2}$ by, at least, a factor $\sim O(10^3)$; this strongly suggests that these decay rates do depend sensitively on the small CP-violating parameter Reϵ, and therefore CP nonconservation does play an important role in both $\mu^+\mu^-$ and 2γ decays of K_S^0 and K_L^0. Of course, even if this does turn our to be correct, one must still seek the detailed CP-violating mechanism that does produce such a large difference between the 2μ decay rates of K_S^0 and K_L^0.

II. INEQUALITIES

Let us define four complex numbers $a_\pm$ and $b_\pm$, related to the four decay amplitudes of K^0 and $\overline{K}^0$ into $\mu^+\mu^-$ in $CP=+1$ and -1 states by

$$\text{amp}(K^0 \to \mu^+\mu^-, CP=\pm1) \equiv b_\pm + i a_\pm \qquad (8)$$

and

$$\text{amp}(\overline{K}^0 \to \mu^+\mu^-, CP=\pm1) \equiv \pm(b_\pm^* + i a_\pm^*). \qquad (9)$$

[The $\pm$ sign in (9) is introduced so that if CP were conserved and if we had defined $|\overline{K}^0\rangle = CP|K^0\rangle$, then $a_\pm$ and $b_\pm$ would all be real.] Throughout this section, we shall include in all amplitudes the appropriate phase-space factors, so that the squares of their magnitudes are simply the transition rates. It is convenient to choose the phases of the initial and final states in these decays such that

$$CPT|K^0\rangle = |\overline{K}^0\rangle \qquad (10)$$

and for *free* $\mu^+\mu^-$ states

$$CPT|\mu^+\mu^-, CP=\pm1\rangle = \pm|\mu^+\mu^-, CP=\pm1\rangle; \qquad (11)$$

CPT invariance, then, implies that in (8) and (9) $a_\pm$, $a_\pm^*$ are the absorptive parts of the decay amplitudes and $b_\pm$, $b_\pm^*$ the corresponding nonabsorptive parts.

The decay amplitudes

$$L_\pm \equiv \text{amp}(K_L^0 \to \mu^+\mu^-, CP=\pm1)$$

and

$$S_\pm \equiv \text{amp}(K_S^0 \to \mu^+\mu^-, CP=\pm1)$$

can now be readily expressed in terms of the constants $a_\pm$, $b_\pm$ and their complex conjugates through the usual relations

$$|K_S^0\rangle = [2(1+|\epsilon|^2)]^{-1/2}[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\overline{K}^0\rangle]$$

and

$$|K_L^0\rangle = [2(1+|\epsilon|^2)]^{-1/2}[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\overline{K}^0\rangle].$$

Let $(L_\pm)_{\text{abs}}$ denote the absorptive part of $L_\pm$, i.e., the $a_\pm$-dependent part of $L_\pm$. One has then

$$(L_+)_{\text{abs}} = \sqrt{2}(-\text{Im}a_+ + i\epsilon\,\text{Re}a_+) \qquad (12)$$

and

$$(L_-)_{\text{abs}} = \sqrt{2}(i\,\text{Re}a_- - \epsilon\,\text{Im}a_-), \qquad (13)$$

in which, for clarity of presentation, we have *neglected* all terms that are proportional to ϵ^2. (The same approximation applies to all subsequent discussions as well.) By using the explicit expressions for $L_\pm$ and $S_\pm$, one can readily verify that these absorptive parts also satisfy

$$(L_+)_{\text{abs}} - \text{Re}L_+ = -\text{Re}(\epsilon S_+) + i\epsilon\,\text{Im}S_+ \qquad (14)$$

and

$$(L_-)_{\text{abs}} - i\,\text{Im}L_- = -i\,\text{Im}(\epsilon S_-) + \epsilon\,\text{Re}S_-. \qquad (15)$$

It is convenient to regard the right-hand sides of (14) and (15) as the two components of a 2-dimensional complex vector, whose magnitude is given by

$$[|-\text{Re}(\epsilon S_+) + i\epsilon\,\text{Im}S_+|^2 + |-i\,\text{Im}(\epsilon S_-) + \epsilon\,\text{Re}S_-|^2]^{1/2} = (\text{Re}\epsilon)[\text{rate}(K_S^0 \to 2\mu)]^{1/2}. \qquad (16)$$

Similarly, one may regard the two terms on the left-hand sides of (14) and (15) as the appropriate components of two other complex vectors: The magnitudes of these two complex vectors are given, respectively, by

$$L_{\text{abs}} \equiv [|(L_+)_{\text{abs}}|^2 + |(L_-)_{\text{abs}}|^2]^{1/2} \qquad (17)$$

and

$$[(\text{Re}L_+)^2 + (\text{Im}L_-)^2]^{1/2}. \qquad (18)$$

Since (18) is less than or equal to

$$[\text{rate}(K_L^0 \to 2\mu)]^{1/2},$$

the usual triangular inequality, implied by Eqs.

(14) and (15), leads to

$$[\text{rate}(K_L^0 \to 2\mu)]^{1/2} \geq |L_{\text{abs}} - (\text{Re}\epsilon)[\text{rate}(K_S^0 \to 2\mu)]^{1/2}|. \quad (19)$$

In an identical way, through the interchange of K_L^0 and K_S^0, one can establish

$$[\text{rate}(K_S^0 \to 2\mu)]^{1/2} \geq |S_{\text{abs}} - (\text{Re}\epsilon)[\text{rate}(K_L^0 \to 2\mu)]^{1/2}|, \quad (20)$$

where, in a manner similar to (17), S_{abs} denotes the total magnitude of the absorptive amplitudes in $K_S^0 \to 2\mu$ decay.

From the assumptions (i) and (ii), stated in Sec. I, it follows that the absorptive parts $(L_+)_{\text{abs}}$ and $(L_-)_{\text{abs}}$ of the $K_L^0 \to 2\mu$ amplitudes are proportional to the appropriate 2γ decay amplitudes of K_L^0:

$$\text{amp}(K_L^0 \to 2\gamma, CP=+1) = (\lambda v_\mu)^{-1}(L_+)_{\text{abs}} \quad (21)$$

and

$$\text{amp}(K_L^0 \to 2\gamma, CP=-1) = \lambda^{-1}(L_-)_{\text{abs}}, \quad (22)$$

where the factors λv_μ and λ denote, respectively, the $2\gamma \to \mu^+\mu^-$ amplitudes in $CP=+1$ and -1 channels; to the lowest order in α, they are given by (2) and (3). The quantity L_{abs}, related to the absorptive amplitudes in $K_L^0 \to 2\mu$ decay by (17), therefore satisfies the inequality

$$\lambda v_\mu \leq [\text{rate}(K_L^0 \to 2\gamma)]^{-1/2} L_{\text{abs}} \leq \lambda. \quad (23)$$

From (19) and (23), the inequalities (5) and (6) follow readily. (If Reϵ were zero, then (19) and (23) would lead to

$$[\text{rate}(K_L^0 \to 2\mu)]^{1/2} \geq \lambda v_\mu [\text{rate}(K_L^0 \to 2\gamma)]^{1/2},$$

which would reduce to (1) if, in addition, CP conservation were valid.)

The inequalities (4) and (5) imply that

$$[\text{rate}(K_L^0 \to 2\mu)]^{1/2} < (\text{Re}\epsilon)[\text{rate}(K_S^0 \to 2\mu)]^{1/2}.$$

Thus, upon neglecting terms $\sim O(\epsilon^2 |S_\pm|)$, the inequality (20) becomes simply

$$[\text{rate}(K_S^0 \to 2\mu)]^{1/2} \geq S_{\text{abs}}. \quad (24)$$

Hitherto, our assumptions (i) and (ii) have concerned only K_L^0 decay amplitudes. Because of the dominant $K_S^0 \to 2\pi$ decay mode, the extension of similar assumptions to K_S^0 decays is expected to be a worse approximation. For completeness, we note that if, in addition to (i) and (iii) stated in Sec. I, one makes the assumption that (iv) the $K_S^0 \to 2\gamma$ decay amplitude has also a zero absorptive part, then it is possible to derive two more inequalities:

$$[\text{rate}(K_S^0 \to 2\gamma)]^{1/2} \leq (\text{Re}\epsilon)^{-1}[\text{rate}(K_L^0 \to 2\gamma)]^{1/2} \quad (25)$$

and

$$[\text{rate}(K_S^0 \to 2\gamma)]^{1/2} \geq (\text{Re}\epsilon)[\text{rate}(K_L^0 \to 2\gamma)]^{1/2}. \quad (26)$$

Furthermore, if one makes the additional assumption that (v) the absorptive part of the $K_S^0 \to 2\mu$ amplitude is due only to the on-mass-shell 2γ intermediate state, then from (24), one obtains,[9] independently of CP conservation,

$$[\text{rate}(K_S^0 \to 2\mu)]^{1/2} \geq \lambda v_\mu [\text{rate}(K_S^0 \to 2\gamma)]^{1/2}. \quad (27)$$

We emphasize that these additional inequalities (25)–(27) are based on assumptions which are less certain than those required for (5) and (6).

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$$K_L^0 \to 3\pi \text{ (on-mass-shell)} \to \pi^0 \text{ (off-mass-shell)} \to 2\gamma.$$

This yields an absorptive amplitude whose magnitude is found to be only ~10^{-3} times the magnitude of the observed amplitude for $K_L^0 \rightarrow 2\gamma$; therefore, the resulting fractional change in the lower bound (1) is expected to be of a similarly small magnitude.

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Weak Radiative Decay of the Σ^+ and Λ Hyperons*

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Unitarity is found to give a reliable, model-independent lower bound on the branching ratio [rate$(\Lambda \rightarrow n\gamma)$/rate$(\Lambda \rightarrow \text{all})$] $> 8.5\times 10^{-4}$. This value is nearly as large as the experimentally determined branching ratio for the decay $\Sigma^+ \rightarrow p\gamma$. Dispersion-theoretic techniques supplemented with current algebra, PCAC, and pole models are then used to determine the real as well as imaginary parts of the amplitudes for the radiative decays of the Λ and Σ^+ hyperons. Input consists of the experimental nonleptonic decay amplitudes and pion-nucleon phase shifts. The theoretical predictions are very sensitive to the hyperon magnetic moments, and until these are better known, these results for the radiative decays can only be qualitatively compared with experiment.

I. INTRODUCTION

The radiative hyperon decays, $\Sigma^+ \rightarrow p\gamma$, $\Sigma^0 \rightarrow n\gamma$, $\Lambda \rightarrow n\gamma$, $\Xi^- \rightarrow \Sigma^-\gamma$, $\Xi^0 \rightarrow \Sigma^0\gamma$, and $\Xi^0 \rightarrow \Lambda\gamma$, are interesting because they can provide insight into the nature of the nonleptonic weak interactions. They are presumed to result from the combined effect of electromagnetic and weak interactions, so that working to lowest order in the electromagnetic interaction one has in them a probe of the nonleptonic weak interaction. Given the scarcity of experimentally accessible nonleptonic processes, each source of information on them is particularly precious. At present only $\Sigma^+ \rightarrow p\gamma$ has been seen, with a branching ratio[1]

$$(\Sigma^+ \rightarrow p\gamma)/(\Sigma^+ \rightarrow \text{all}) = (1.43 \pm 0.26)\times 10^{-3}\,.$$

The experiment of Gershwin *et al.*[1] to determine the asymmetry parameter of the decay $\Sigma^+ \rightarrow p\gamma$ has added impetus to theoretical efforts to account for these processes. The asymmetry parameter α is determined from the correlation between the final proton momentum and the polarization of the initial Σ^+. It is a measure of the relative magnitudes of the s- and p-wave amplitudes. Gershwin *et al.* found α to be $-1.03^{+0.52}_{-0.42}$. Since most theoretical models predict that α is approximately zero, this measurement is very challenging to theorists. For a summary of the predictions which various techniques have given when applied to the radiative hyperon decays, see the review article by Tanaka.[2]

The principal new result to be presented here is in fact virtually model-independent. Unitarity can be used particularly effectively because the only purely-hadronic intermediate state that is energetically accessible is the $N\pi$ state. Knowing experimentally the photoproduction and $\Lambda \rightarrow N\pi$ amplitudes enables us to give the unitarity lower limit[2a]:

$$\text{branching ratio } (\Lambda \rightarrow n\gamma)/(\Lambda \rightarrow \text{all}) \gtrsim 8.5\times 10^{-4}\,.$$

This is quite a stunning result, being only a factor of 2 smaller than the experimental branching ratio for $\Sigma^+ \rightarrow p\gamma$ given above. The corresponding unitarity lower limit for $\Sigma^+ \rightarrow p\gamma$ turns out to be "unnaturally" small, as we shall see, leading to

$$\text{branching ratio } (\Sigma^+ \rightarrow p\gamma)/(\Sigma^+ \rightarrow \text{all}) \gtrsim 6.9\times 10^{-6}\,.$$

With the incentive of a possibly large rate for $\Lambda \rightarrow n\gamma$, we proceed to make a model-dependent estimate of the real part of the amplitudes. For this we exploit our knowledge of the imaginary parts by assuming the amplitudes $\Lambda \rightarrow n\gamma$ and $\Sigma \rightarrow p\gamma$ obey unsubtracted dispersion relations in the mass squared of the initial particle. In the dispersion integral, however, we need the absorptive part of the amplitude as a function of the initial hyperon mass. Approximating the full absorptive part at all energies by the contribution of the nucleon-pion intermediate state alone, even at masses for which other hadronic intermediate states are ener-