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Aspects of the Scalar-Dominance Hypothesis*

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The scalar-dominance hypothesis, incorporating $\epsilon(700)$-$\epsilon'(1060)$ mixing, is applied to a large class of elastic, single-particle matrix elements of the energy-momentum trace operator.

I. INTRODUCTION

Among the diverse applications of scaling to particle physics is the concept of a scale-invariant world. This symmetry limit corresponds to the vanishing of all matrix elements of the energy-momentum trace operator $\theta \equiv \theta_\mu{}^\mu$. A recent study[1] of renormalizable field theories has displayed the existence of a scale current D^μ defined in terms of the symmetric energy-momentum tensor $\theta^{\mu\nu}$,

$$D^\mu(x) = x_\nu \theta^{\mu\nu}(x), \qquad (1)$$

with divergence

$$\partial_\mu D^\mu(x) = \theta(x) \qquad (2)$$

and charge

$$D(x^0) = \int x_\nu \theta^{0\nu}(x) d^3x. \qquad (3)$$

Thus, scale invariance can be described in terms of the vanishing of a current divergence. The idea of associating a symmetry with θ is clearly of interest. Taken together with studies of chiral invariance, it constitutes an important probe of the hadronic energy density.

Gell-Mann has conjectured[2] that the $\theta=0$ limit is correlated with the existence of a massless scalar particle, the ϵ meson. This is in analogy with the proposed Goldstone relization of SU(2) $\times$SU(2), where the pion mass vanishes, or of SU(3)$\times$SU(3), where the entire pseudoscalar octet has zero mass. The analogy suggests use of ϵ pole dominance of θ in the same way that the pion pole is used to dominate the axial-vector current divergence, i.e., the partial conservation of axial-vector current (PCAC). In practice, we generalize this to ϵ-ϵ' dominance, because these states are mixed. Even if the analogy between scale and chiral symmetry is not valid, ϵ-ϵ' pole dominance of θ might still be a useful approximation. In this case, pole dominance would be a consequence of a dynamical suppression of the cut contributions. Success of the ρ- and A_1-dominance hypotheses presumably depends on this feature.

To clarify and emphasize the relation between ϵ dominance and the limit of scale invariance, we examine now a simple model. The following calculation also serves to introduce certain parameters which appear throughout the paper. Consider the matrix element, taken between single nucleon states,[3] of $\theta^{\mu\nu}$,

$$\left(\frac{E_pE_q}{M^2}\right)^{1/2}\langle q,\lambda|\theta^{\mu\nu}(0)|p,\lambda'\rangle=\bar{u}(q,\lambda)\left(G_1(t)\frac{P^\mu\gamma^\nu+P^\nu\gamma^\mu}{2}+G_2(t)\frac{P^\mu P^\nu}{M}+G_3(t)\frac{k^\mu k^\nu-k^2g^{\mu\nu}}{M}\right)u(p,\lambda'),\tag{4a}$$

and of θ,

$$\left(\frac{E_pE_q}{M^2}\right)^{1/2}\langle q,\lambda|\theta(0)|p,\lambda'\rangle=F(t)\bar{u}(q,\lambda)u(p,\lambda'),\tag{4b}$$

where $k=p-q$, $t=k^2$, $P=\frac{1}{2}(p+q)$, M is the nucleon mass, $G_1(0)=1$, $G_2(0)=0$, and $F(0)=M$. In order to obtain explicit models for the form factors in Eq. (4a), we use pole contributions of the ϵ, ϵ', and f mesons, and for generality, a subtraction constant in each form factor. The coupling constants appearing in the pole contributions are defined from

$$(2\omega_k)^{1/2}\langle 0|\theta^{\mu\nu}(0)|\epsilon(k)\rangle=-\tfrac{1}{3}F_\epsilon(k^\mu k^\nu-g^{\mu\nu}k^2),\tag{5a}$$

$$(2\omega_k)^{1/2}\langle 0|\theta^{\mu\nu}(0)|\epsilon'(k)\rangle=-\tfrac{1}{3}F_{\epsilon'}(k^\mu k^\nu-g^{\mu\nu}k^2),\tag{5b}$$

$$(2\omega_k)^{1/2}\langle 0|\theta^{\mu\nu}(0)|f(k,\lambda)\rangle=f_Th^{\mu\nu}(k,\lambda),\tag{6}$$

where the spin-two polarization vector obeys $h^{\mu\nu}(k,\lambda)=h^{\nu\mu}(k,\lambda)$, $k_\mu h^{\mu\nu}(k,\lambda)=0$, $h_\mu{}^\mu(k,\lambda)=0$, and finally

$$\mathcal{L}=g_{\epsilon NN}\bar{N}N\epsilon+g_{\epsilon'NN}\bar{N}N\epsilon'+i\frac{g_1}{2M}f_{\mu\nu}\left(\bar{N}\frac{\gamma^\nu\overleftrightarrow{\partial}^\mu}{2}N+\bar{N}\frac{\gamma^\mu\overleftrightarrow{\partial}^\nu}{2}N\right)-\frac{g_2f_{\mu\nu}}{M^2}\bar{N}\frac{\overleftrightarrow{\partial}^\mu\overleftrightarrow{\partial}^\nu}{4}N.\tag{7}$$

We then find the following expression for the form factors:

$$G_i(t)=\frac{1}{M}\frac{f_Tg_i}{m_f^2-t}+G_i,\quad i=1,2$$

$$G_3(t)=\frac{M}{3}\left[-\frac{F_\epsilon g_{\epsilon NN}}{m_\epsilon^2-t}-\frac{F_{\epsilon'}g_{\epsilon'NN}}{m_{\epsilon'}^2-t}+\frac{1}{m_f^2}\frac{f_Tg_1}{m_f^2-t}+\left(\frac{1}{m_f^2}-\frac{1}{4M^2}\right)\frac{f_Tg_2}{m_f^2-t}\right]+G_3,\tag{8a}$$

where G_1, G_2, and G_3 are subtraction constants, and m_ϵ, $m_{\epsilon'}$, and m_f denote the masses of the ϵ, ϵ', and f mesons, respectively. Using the $t=0$ constraints for G_1 and G_2, and requiring that $F(t)$ be bounded by a constant for $t\to\infty$, we find from Eqs. (4a) – (8a)

$$F(t)=\frac{g_{\epsilon NN}F_\epsilon t}{m_\epsilon^2-t}+\frac{g_{\epsilon'NN}F_{\epsilon'}t}{m_{\epsilon'}^2-t}+M.\tag{8b}$$

Let us pass to the scale-invariant limit. Setting $F(t)=0$ for all t, we obtain the relations

$$g_{\epsilon NN}F_\epsilon+g_{\epsilon'NN}F_{\epsilon'}-M=0,$$

$$m_{\epsilon'}^2(g_{\epsilon NN}F_\epsilon-M)+m_\epsilon^2(g_{\epsilon'NN}F_{\epsilon'}-M)=0,\tag{9}$$

$$Mm_\epsilon^2m_{\epsilon'}^2=0.$$

If we demand $m_\epsilon=0$, $m_{\epsilon'}\neq0$, and use Eq. (5b), we find as the unique solution to Eqs. (9)

$$M=g_{\epsilon NN}F_\epsilon,\tag{10a}$$

$$F_{\epsilon'}=0.\tag{10b}$$

We interpret Eqs. (10a) and (10b) as implying ϵ pole dominance in the scale-invariant limit provided ϵ is the only particle whose mass vanishes.[4] Suppression of cut contributions is indicated by $F_{\epsilon'}=0$. If we were to enlarge our calculation by including several scalar poles in order to better approximate the cut contribution, we would find a solution with the same qualitative properties: The single scalar meson which becomes massless in the scale-invariant limit must have couplings as in Eq. (10a), and all the other scalar mesons decouple [as in Eq. (10b)].

In this paper, we present a detailed study of the scalar-meson pole-dominance hypothesis applied to single-particle, elastic matrix elements of θ.[5] The real world is not scale-invariant, so corrections to Eqs. (10a) and (10b) are necessary. For reasons of simplicity, we employ as corrections only the use of physical masses, and the phenomenon of ϵ-ϵ' mixing. This approach is also motivated by our feeling that relations like Eq. (10a) constitute a significant method for determining parameters like F_ϵ and $F_{\epsilon'}$, and thus deserve a comprehensive study which employs consistent approximations. It is worth remembering that similar relations do not occur for the axial-vector currents and the associated limit of chiral symmetry.

II. DERIVATIONS

We plan to apply the scalar-dominance hypothesis to a number of meson and baryon states. Included among these are high-spin particles. Therefore, we must define the coupling constants which are to be used in the arbitrary-spin problem, and also derive the appropriate scalar-dominance formulas.

A. Mesons

Consider the matrix elements of θ taken between single-particle states of a spin-s meson. The spin-s polarization vector is denoted by $\phi_{\mu_1\cdots\mu_s}(q,\lambda)$, where q is the momentum, and λ is the helicity. It obeys the following constraints[6]:

$$(q^2-m^2)\phi_{\mu_1\cdots\mu_s}(q,\lambda)=0, \tag{11a}$$

$$\phi_{\cdots\mu_i\cdots\mu_j}(q,\lambda)=\phi_{\cdots\mu_j\cdots\mu_i}(q,\lambda), \tag{11b}$$

$$g^{\mu_i\mu_j}\phi_{\cdots\mu_i\cdots\mu_j}(q,\lambda)=0, \tag{11c}$$

$$q^{\mu_i}\phi_{\cdots\mu_i\cdots}(q,\lambda)=0. \tag{11d}$$

In Eq. (11a), m is the mass of the meson. The general form of the matrix element of θ, taken between the spin-s meson states, is

$$(4\omega_p\omega_q)^{1/2}\langle q,\lambda|\theta(0)|p,\lambda'\rangle = \phi^{\mu_1\cdots\mu_s\dagger}(q,\lambda)\Gamma_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_s}\phi^{\nu_1\cdots\nu_s}(p,\lambda'), \tag{12}$$

where $\Gamma_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_s}$ contains $(s+1)$ terms, each a form factor multiplied by a kinematic coefficient. The only quantities available for constructing the kinematic coefficients in $\Gamma_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_s}$ are q^μ, p^μ, and $g^{\mu\nu}$. The constraints (11b) – (11d) imply that, at zero momentum transfer $[t=(p-q)^2=0]$, $\Gamma_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_s}$ consists of just one nonvanishing term,

$$\Gamma_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_s}(p=q)=g_{\mu_1\nu_1}g_{\mu_2\nu_2}\cdots g_{\mu_s\nu_s}E(t)|_{t=0}\,, \tag{13}$$

where $E(t)$ is a Lorentz-invariant form factor. Next, we determine the pole contribution to $E(t)$ of a scalar particle σ (e.g., $\sigma=\epsilon,\epsilon'$), which we treat in the narrow-resonance approximation. In general, the scalar meson σ couples to a pair of spin-s mesons in more than one way. The only such coupling relevant to our calculation is defined by

$$\mathcal{L}(x)=g_{\sigma ss}\,T^{\mu_1\cdots\mu_s}(x)T_{\mu_1\cdots\mu_s}(x)\sigma(x)\,, \tag{14}$$

where $g_{\sigma ss}$ has the dimension of energy. The quantity $T_{\mu_1\cdots\mu_s}(x)$ displays the configuration-space dependence of the spin-s meson, and is related to $\phi_{\mu_1\cdots\mu_s}(q,\lambda)$ by

$$T_{\mu_1\cdots\mu_s}(x)=\sum_{k,\lambda}(2\omega_k)^{-1/2}\left[\phi_{\mu_1\cdots\mu_s}(k,\lambda)a_{k,\lambda}e^{-ik\cdot x}+\text{H.c.}\right]. \tag{15}$$

The starting point for the calculation of the σ-pole contribution to $E(t)$ is the reduction formula

$$(4\omega_p\omega_q)^{1/2}\langle 0|\theta(0)|p\lambda';\bar{q}\lambda\rangle = -i\int d^4x\, e^{-i\bar{q}\cdot x}\phi^{\mu_1\cdots\mu_s}(\bar{q},\lambda)(2\omega_p)^{1/2}(\vec{\Box}_x+m^2)\langle 0|[\theta(0),T_{\mu_1\cdots\mu_s}(x)]\theta(-x_0)|p\lambda'\rangle. \tag{16}$$

Upon inserting the σ intermediate states, and using Eqs. (13), (14), and a relation analogous to Eqs. (5a) and (5b), we obtain

$$\text{Im}E(t)=m_\sigma^2F_\sigma g_{\sigma ss}\pi\,\delta(m_\sigma^2-t)\,. \tag{17}$$

Thus, in a model with ϵ-ϵ' dominance and no subtractions, we find $E(0)=F_\epsilon g_{\epsilon ss}+F_{\epsilon'}g_{\epsilon' ss}$. Alternatively, from the vanishing of the self-stress, we have

$$2\omega_q\langle 0,\lambda|\theta(0)|0,\lambda'\rangle=2m^2\phi^{\mu_1\cdots\mu_s\dagger}(0,\lambda)\,\phi_{\mu_1\cdots\mu_s}(0,\lambda')\,. \tag{18}$$

Our result, for the case of an arbitrary-spin meson, is the scalar-dominance formula

$$2m^2=F_\epsilon g_{\epsilon ss}+F_{\epsilon'}g_{\epsilon' ss}\,. \tag{19}$$

B. Baryons

The derivation for the case of an arbitrary-spin baryon is similar to that just given for an arbitrary-spin meson. Hence, we shall be brief. The spin-s baryon is described in momentum space by a quantity, $u_{\mu_1\cdots\mu_n}(q,\lambda)$, where $s=n+\frac{1}{2}$. The constraints on $u_{\mu_1\cdots\mu_n}(q,\lambda)$ are those of Eqs. (11a) –

– (11d) except for the replacement of (11c) and (11d) by[6]

$$(q\cdot\gamma - M)u_{\mu_1\cdots\mu_n}(q,\lambda)=0, \tag{11c$'$}$$

$$\gamma^{\mu_i}u_{\cdots\mu_i\cdots}(q,\lambda)=0, \tag{11d$'$}$$

where M is the baryon mass. The general form of the trace matrix element taken between the spin-s baryon states is

$$\left(\frac{E_pE_q}{M^2}\right)^{1/2}\langle q,\lambda|\theta(0)|p,\lambda'\rangle = \bar{u}^{\mu_1\cdots\mu_n}(q,\lambda)\Gamma'_{\mu_1\cdots\mu_n,\nu_1\cdots\nu_n}u^{\nu_1\cdots\nu_n}(p,\lambda'). \tag{20}$$

At $t=0$, it follows from Eqs. (11b), (11c$'$), and (11d$'$) that $\Gamma'_{\mu_1\cdots\mu_n,\nu_1\cdots\nu_n}$ has the form given in Eq. (13) for the meson vertex function $\Gamma_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_s}$. We define the σ-baryon-baryon coupling relevant to our calculation by

$$\mathcal{L}(x)=f_{\sigma BB}\bar{\psi}^{\mu_1\cdots\mu_n}(x)\psi_{\mu_1\cdots\mu_n}(x)\sigma(x)\ , \tag{21}$$

where $\psi_{\mu_1\cdots\mu_s}(x)$ describes the spin-s baryon in configuration space. Aside from the condition

$$\langle 0,\lambda|\theta(0)|0,\lambda'\rangle = M\bar{u}^{\mu_1\cdots\mu_n}(0,\lambda)u_{\mu_1\cdots\mu_n}(0,\lambda'),$$

which follows from the vanishing of the self-stress, the rest of the derivation parallels the meson case. We find that ϵ-ϵ' dominance takes the form

$$M=f_{\epsilon BB}F_\epsilon+f_{\epsilon' BB}F_{\epsilon'}\ . \tag{22}$$

III. APPLICATIONS

This section consists of applications of Eqs. (19) and (22) to all cases for which sufficient data are available. The main restriction in the use of these equations involves the coupling constants $g_{\epsilon ss}$, $g_{\epsilon' ss}$, $f_{\epsilon BB}$, and $f_{\epsilon' BB}$. Except for the $\frac{1}{2}^+$ baryon and 0^- meson systems, we have no knowledge of any of these quantities. One way to partially overcome this difficulty is to apply Eqs. (19) and (22) to particles lying within given SU(3) multiplets. Upon using SU(3) symmetry to approximate the coupling constants, we reduce the number of unknowns. We defer discussion of the assumption that the SU(3) couplings provide reasonable estimates until Sec. IV.

With the introduction of SU(3) into our calculations, we consider the ϵ-ϵ' mixing in more detail. We define an ϵ-ϵ' mixing angle ϕ from[5]

$$\begin{aligned}\epsilon&=\sigma_0\cos\phi-\sigma_8\sin\phi,\\ \epsilon'&=\sigma_0\sin\phi+\sigma_8\cos\phi\ ,\end{aligned} \tag{23}$$

where σ_0 transforms as a unitary singlet, and σ_8 transforms as the eighth component of an octet. Also, it is convenient to define coupling strengths F_0, F_8,

$$\begin{aligned}F_\epsilon&=F_0\cos\phi-F_8\sin\phi,\\ F_{\epsilon'}&=F_0\sin\phi+F_8\cos\phi.\end{aligned} \tag{24}$$

Notice that F_0 and F_8 have the dimension of energy. From Eqs. (23) and (24), we see that if $\phi\to 0$ in the limit of SU(3) invariance, the quantities ϵ, ϵ', F_ϵ, $F_{\epsilon'}$, are to be identified, respectively, with the quantities σ_0, σ_8, F_0, F_8. The main purpose of the calculations to follow is to obtain estimates of the basic constants, F_0 and F_8. We shall consider first the baryons, then the mesons.

A. Baryons

(a) *Octets.* We use the obvious notation, N, Λ, Σ, Ξ, for the members of an arbitrary baryon octet. The coupling of σ_0, σ_8 to a baryon octet is given in terms of dimensionless coupling constants f_0, f_8, and an F/D parameter α,

$$\mathcal{L}=f_0\bar{B}B\sigma_0+2f_8\bar{B}[\alpha\underline{D}+(1-\alpha)\underline{F}]B\cdot\underline{\sigma}, \tag{25}$$

where $\underline{\sigma}$ is the scalar-meson octet to which σ_8 is assigned.[7] Equations (22), (24), and (25) imply

$$\alpha=\frac{3(M_\Sigma-M_\Lambda)}{3(M_\Sigma-M_\Lambda)-2(M_\Xi-M_N)}\ . \tag{26}$$

Notice that in this model, the value of α depends only upon the baryon masses. Another interesting quantity is the ratio

$$\frac{F_8}{F_0}=\frac{\sqrt{3}\,(M_\Sigma-M_\Lambda)}{2\alpha(M_\Sigma+M_\Lambda)}\frac{f_0}{f_8}\ . \tag{27}$$

In most cases, we have no knowledge of the coupling constants f_0 and f_8. However, because it is only the ratio of f_0 and f_8 which appears in Eq. (27), we can proceed in our analysis by making one further assumption – that the coupling of the ϵ' to nonstrange baryons vanishes.[5] This assumption is motivated by the small value of the $\epsilon'\to\pi\pi$ decay width, which in terms of the quark model suggests that the ϵ' consists solely of strange quarks. Other isoscalar states believed to have this property are the ϕ and f' mesons. Thus, we use the canonical value of the ϵ-ϵ' mixing angle, $\phi_c=\cot^{-1}\sqrt{2}$. With this assumption, we can deduce that

$$\frac{f_0}{f_8}=-(\tfrac{2}{3})^{1/2}(3-4\alpha)\ , \tag{28}$$

and finally, that[8]

$$\frac{F_8}{F_0}=\frac{M_\Sigma-M_\Lambda+2(M_\Xi-M_N)}{\sqrt{2}\,(M_\Sigma+M_\Lambda)}\ . \tag{29}$$

Values of F_8/F_0 for the collection of observed baryon octets are presented in Table I(a).

In Ref. 5, an estimate of the magnitudes of F_0 and F_8 was obtained by using a value of $g_{\epsilon NN}$ determined from a model of low-energy πN and NN scattering. From the value $g_{\epsilon NN}{}^2/4\pi=10$, we find $F_0=128$ MeV and $F_8=33$ MeV. The nucleon "mass radius" is given in the ϵ-ϵ' pole model by[9]

$$\langle r^2\rangle_N=\frac{6F_\epsilon g_{\epsilon NN}}{M_N m_\epsilon{}^2}+\frac{6F_{\epsilon'}g_{\epsilon' NN}}{M_N m_{\epsilon'}{}^2}\ . \qquad (30)$$

The results of the above analysis give $\langle r_N\rangle\cong 0.7\times 10^{-13}$ cm.

A higher symmetry whose content overlaps some of the results found in this section is SU(6). Let us assign the $\frac{1}{2}^+$, $\frac{3}{2}^+$ baryons to $\underline{56}$, and the unmixed scalar-meson states σ_0, σ_8 to $\underline{1}$, $\underline{35}$, respectively. Recall how well the SU(6) F/D parameter for the coupling of the pseudoscalar-meson octet to the $\frac{1}{2}^+$ baryon octet agrees with both experimental estimates and bootstrap calculations. The scalar-meson case is much less impressive – the SU(6) prediction of $\alpha=0.0$ compares poorly with the value $\alpha=-0.44$ exhibited in Table I(a).

(b) *Decuplets*. Denoting the decuplet baryons by Δ, Y, Ξ^*, Ω, we write the SU(3) couplings of σ_0 and σ_8 to a baryon decuplet as

$$\mathcal{L}(x)=f_0'\sigma_0(\overline{\Delta}\Delta+\overline{Y}Y+\overline{\Xi}{}^*\Xi^*+\overline{\Omega}\Omega)+f_8'\sigma_8(\overline{\Delta}\Delta-\overline{\Xi}{}^*\Xi^*-2\overline{\Omega}\Omega), \qquad (31)$$

where f_0', f_8' are dimensionless. Equations (22) and (31) imply

$$\frac{F_8}{F_0}=\frac{M_\Delta-M_Y}{M_Y}\frac{f_0'}{f_8'}\ . \qquad (32)$$

The decoupling of ϵ' from nonstrange particles implies

$$\frac{f_0'}{f_8'}=-\cot\phi_c=-\sqrt{2} \qquad (33)$$

from which

$$\frac{F_8}{F_0}=\frac{\sqrt{2}\,(M_Y-M_\Delta)}{M_Y}\ . \qquad (34)$$

Numerical values are given in Table I(b).

TABLE I. Baryon mass analysis. The mass values (in MeV) are taken from Ref. 15. For those cases in (a) for which the $S=-2$, $T=\frac{1}{2}$ baryon mass is not given by Ref. 15, we use the Gell-Mann–Okubo relation to calculate it. The spin parity (J^P) of each SU(3) multiplet is given in the first column. The symbol α refers to the F/D parameter defined in Eq. (26).

(a) Octets

J^P	N	Λ	Σ	Ξ	α	F_8/F_0
$\frac{1}{2}^+$	938.9	1115.6	1193.1	1318.0	−0.44	0.26
$\frac{3}{2}^-$	1520	1670	1670	1820	0.00	0.13
$\frac{1}{2}^-$	1550	1670	1750	1830	−0.75	0.13
$\frac{5}{2}^+$	1688	1820	1905	1994	−0.71	0.13
$\frac{5}{2}^-$	1680	1835	1765	1955	0.28	0.094

(b) Decuplets

J^P	Δ	Y	F_8/F_0
$\frac{3}{2}^+$	1236	1385	0.15
$\frac{7}{2}^+$	1930	2030	0.07

B. Mesons

Before examining individual cases, we define several tri-meson coupling constants. The SU(3)-invariant interaction of the scalar nonet (S), σ_0, $\underline{\sigma}$, with some *different* meson nonet (M), ϕ_0, $\underline{\phi}$, depends on four constants, g_0, g_1, g_2, and g_3,[5,10] defined by

$$\mathcal{L}_{MMS}=g_0\sigma_0\phi_0{}^2+g_1\sigma_0\underline{\phi}^2+\sqrt{3}\,g_2 d_{ijk}\sigma_i\phi_j\phi_k+g_3\phi_0\underline{\sigma}\cdot\underline{\phi}\ . \qquad (35)$$

For the self-interactions of the scalar nonet,[5] we need three constants, h_0, h_1, and h_2:

$$\mathcal{L}_{SSS}=h_0\sigma_0{}^3+h_1\sigma_0\underline{\sigma}^2+\sqrt{3}\,h_2 d_{ijk}\sigma_i\sigma_j\sigma_k\ . \qquad (36)$$

All seven of the coupling constants just defined have the dimension of energy. With these definitions, we proceed to analyze the meson spectrum.

(a) $J^P=0^-$. If we use Eq. (19) to obtain pion and kaon mass relations, and solve these for F_0 and F_8, we find

$$F_0=\frac{m_\pi{}^2+2m_K{}^2}{3g_1}\ ,\quad F_8=\frac{2(m_\pi{}^2-m_K{}^2)}{3g_2}\ . \qquad (37)$$

The pseudoscalar mesons are unique in being the only system for which couplings to scalar mesons can be determined directly from decay widths. In Ref. 5, a value $g_2=-0.97$ GeV is inferred from the decay width $\Gamma(\pi_N\to\pi\eta)=50$ MeV of the scalar meson $\pi_N(980)$. The negative sign of g_2 is consistent with taking F_0 and F_8 positive. An estimate for g_1 comes from the relation

$$g_{\epsilon\pi\pi}{}^2+g_{\epsilon'\pi\pi}{}^2=g_1{}^2+g_2{}^2. \qquad (38)$$

The coupling constant $g_{\epsilon'\pi\pi}$ is not known accurately. However, it is probably safe to deduce from the tentative data on decay widths that $|g_{\epsilon\pi\pi}|^2\gg|g_{\epsilon'\pi\pi}|^2$. Therefore, to a good approximation, $g_1\cong(g_{\epsilon\pi\pi}{}^2-g_2{}^2)^{1/2}$. We find $g_1=1.12$ GeV upon using the width $\Gamma(\epsilon\pi\pi)=300$ MeV as input. From Eq. (37), it follows that $F_0=152$ MeV and $F_8=156$ MeV. This result is in obvious disagreement with solutions derived from the baryon data.

The failure of the mass relation, Eq. (19), to adequately describe the pseudoscalar octet has already been noted in several places.[5,11] However, we wish to direct attention to the quite reasonable value of $F_0=152$ MeV. This solution is consistent with the baryon analysis, especially in view of the substantial uncertainties in the data to which these calculations are subject. Unless the value of F_0 determined from the pseudoscalar analysis is a complete accident, it seems fair to conclude that the large ratio of F_8/F_0 found there arises from the abnormally large value of F_8. We shall return to this point in the Conclusion.

An estimate of the coupling constant g_0 follows from the mass relation[12]

$$m_{\eta'}{}^2=F_0 g_0 \ . \tag{39}$$

From Eq. (37), we find

$$g_0=\frac{3m_{\eta'}{}^2 g_1}{m_\pi{}^2+2m_K{}^2} \ . \tag{40}$$

This relation should give a reliable estimate of g_0/g_1 because it depends on the value of F_0 and not F_8. Numerically, Eq. (40) gives $g_0=6.34$ GeV. The ratio $g_0:g_1:g_2::6.34:1.12:-0.97$, obtained from the mass relations, differs appreciably with the corresponding SU(3)×SU(3)-symmetry prediction, $-2:1:-\sqrt{2}$.

(b) $J^P=1^-, 1^+, 2^+$. Our analysis of the 1^-, 1^+, 2^+ nonets resembles the treatment of the baryon multiplets given earlier. Denoting the members of an arbitrary meson octet as π, K, and η, we can write down, but cannot numerically solve, equations like (37) because we do not know g_1 and g_2 in general. However, decoupling the ϵ' from nonstrange particles and using canonical mixing, we can deduce that

$$\frac{F_8}{F_0}=\frac{2\sqrt{2}\,(m_K{}^2-m_\pi{}^2)}{2m_K{}^2+m_\pi{}^2} \ . \tag{41}$$

Numerical solutions of Eq. (41) are given in Table II. Applying Eq. (41) to the 0^- mesons, we find the anomalously large ratio $F_8/F_0=1.26$. This is in rough agreement with the values of F_0 and F_8 estimated in the preceding section from $0^+\to 0^-0^-$ decay data.

(c) $J^P=0^+$. Our knowledge of the scalar mesons is deficient relative to other meson systems of comparable mass. Existence of the $\epsilon(700)$, $\epsilon'(1060)$, and $\pi_N(980)$ mesons is generally accepted, although in each case there is need for further experimental work. The tendency of mesons to occur in nonets leads one to wonder about the existence of the κ meson. An accurate determination of the κ mass, m_κ, is important to our work because, given the other scalar masses as well, we can calculate the ϵ-ϵ' mixing angle, ϕ. We list in Table III this dependence of ϕ upon m_κ. Our assumption that the nonstrange members of the scalar nonet are indeed $\epsilon(700)$, $\epsilon'(1060)$, and $\pi_N(980)$ places fairly sharp restrictions on m_κ. Current data imply that the ϵ-ϵ' mixing angle lies in the interval $20°\lesssim\phi\lesssim 40°$. If so, the mass of the κ is restricted to $940\lesssim m_\kappa\lesssim 1010$ MeV.

The scalar-nonet mass relations have the important property of depending on just *three* coupling parameters, h_0, h_1, and h_2 [see Eq. (36)]. This happens because the mesons which dominate the trace, θ, belong to the very multiplet whose matrix elements are being considered. Upon writing the mass relations, Eq. (19), in terms of the SU(3) coupling constants, we find

$$\begin{aligned}
2m_\kappa{}^2&=2h_1F_0-3h_2F_8,\\
m_{\pi_N}{}^2&=h_1F_0+3h_2F_8,\\
m_\epsilon{}^2&=3h_0F_0\cos^2\phi+h_1F_0\sin^2\phi\\
&\qquad-2h_1F_8\sin\phi\cos\phi-3h_2F_8\sin^2\phi,\\
m_{\epsilon'}{}^2&=3h_0F_0\sin^2\phi+h_1F_0\cos^2\phi\\
&\qquad+2h_1F_8\sin\phi\cos\phi-3h_2F_8\cos^2\phi.
\end{aligned} \tag{42}$$

The equations in (42) express the full content of the scalar-meson mass relations. Because there are four mass relations in terms of just three trimeson coupling constants, we can solve for F_8/F_0 *without making any assumption as to the value of the mixing angle,* ϕ. We find

TABLE II. Meson mass analysis. The mass values (in MeV) are taken from Ref. 15.

J^P	π	K	F_8/F_0
1^-	765	892.1	0.27
1^+	1070	1247	0.27
2^+	1300	1413	0.15

TABLE III. Analysis of the scalar nonet. The notation used is: m_κ is the mass of the κ meson, ϕ is the ϵ-ϵ' mixing angle, and h_0, h_1, h_2 are coupling constants defined in Eq. (36). The mass values (in MeV) are taken from Ref. 15.

m_κ (MeV)	ϕ (deg)	F_8/F_0	h_0/h_1	h_0/h_2
790	79.3	0.16	0.50	0.77
820	68.3	0.28	0.45	1.5
850	60.6	0.34	0.40	2.1
880	53.7	0.36	0.36	2.6
910	47.1	0.36	0.32	3.4
940	40.4	0.34	0.28	5.1
960	35.6	0.32	0.25	8.7
980	30.5	0.29	0.23	∞
1000	24.7	0.24	0.20	−5.5
1020	17.4	0.18	0.18	−1.8
1040	2.9	0.03	0.16	−0.19

$$\frac{F_8}{F_0}=\frac{3m_{\epsilon'}{}^2-(4m_\kappa{}^2-m_{\pi_N}{}^2)+\sin^2\phi[2(4m_\kappa{}^2-m_{\pi_N}{}^2)-3(m_\epsilon{}^2+m_{\epsilon'}{}^2)]}{\sin2\phi(m_{\pi_N}{}^2+2m_\kappa{}^2)}\ . \tag{43}$$

It is also interesting to solve for the ratios

$$\frac{h_0}{h_1}=\frac{3(m_\epsilon{}^2+m_{\epsilon'}{}^2)+m_{\pi_N}{}^2-4m_\kappa{}^2}{3(m_{\pi_N}{}^2+2m_\kappa{}^2)}\ , \tag{44}$$

and

$$\frac{h_1}{h_2}=\frac{3}{2\sin2\phi}\ \frac{3m_{\epsilon'}{}^3-4m_\kappa{}^2+m_{\pi_N}{}^2+\sin^2\phi[2(4m_\kappa{}^2-m_{\pi_N}{}^2)-3(m_\epsilon{}^2+m_{\epsilon'}{}^2)]}{m_{\pi_N}{}^2-m_\kappa{}^2}\ . \tag{45}$$

The ratios (43)–(45) depend only on the masses of the nine scalar mesons. In view of our present uncertainty regarding the κ meson, we have listed in Table III the ratios F_8/F_0, h_0/h_1, and h_0/h_2 for a range of allowed values for m_κ.

The ratios h_0/h_1 and h_0/h_2 given in Table III may be compared with their SU(3)×SU(3)-symmetric counterparts, $-\frac{2}{3}$ and $\sqrt{2}$, respectively. There is no apparent agreement between the pole-model ratios and those predicted by chiral symmetry.

IV. SYMMETRY BREAKING

One can interpret the calculations presented in Sec. III in terms of two distinct kinds of symmetries, SU(3) and scale invariance. Within the context of the pole model, the assumption of SU(3) invariance enables us to relate various coupling constants. In addition, the very use of the scalar-dominance model may be associated with the existence of a scale-invariant limit. In nature both symmetries are broken. The implication of this is the subject of the present section.

First, we discuss the problem of estimating the effects of SU(3) breaking. One practical method for accurately determining coupling constants is to obtain precise values of decay widths. This motivates us to consider the $0^+\to0^-0^-$ decays. We define, in terms of the unmixed particles σ_0 and σ_8, the couplings

$$\mathcal{L}_0=g_{\sigma_0\eta\eta}\sigma_0\eta^2+g_{\sigma_0\pi\pi}\sigma_0\underline{\pi}^2+g_{\sigma_0KK}\sigma_0\overline{K}K\ , \tag{46a}$$

$$\begin{aligned}\mathcal{L}_8=&g_{\pi_N\pi\eta}\underline{\pi}_N\cdot\underline{\pi}\eta+g_{\pi_NKK}\underline{\pi}_N\cdot\overline{K}\underline{\tau}K+g_{\sigma_8\eta\eta}\sigma_8\eta^2\\&+g_{\sigma_8\pi\pi}\sigma_8\underline{\pi}^2+g_{\sigma_8KK}\sigma_8\overline{K}K+g_{\kappa K\pi}(\overline{\kappa}\underline{\tau}K\cdot\underline{\pi}+\text{H.c.})\\&+g_{\kappa K\eta}(\overline{\kappa}K\eta+\text{H.c}).\end{aligned} \tag{46b}$$

If the symmetry breaking of SU(3) transforms solely as the eighth component of an octet, we may express the ten coupling constants of Eqs. (46a) and (46b) in terms of six independent parameters.[13] For instance, if α,β,γ are members of octets as in Eq. (46b), we can write

$$g(\gamma\to\alpha+\beta)=g_8(\gamma\to\alpha+\beta)+\sum_N X_N\begin{pmatrix}8&8&N\\\gamma&0&\gamma\end{pmatrix}\begin{pmatrix}8&8&N\\\alpha&\beta&\gamma\end{pmatrix}, \tag{47}$$

where g_8 is the D-type SU(3)-invariant coupling, the quantities in parentheses are SU(3) isoscalar factors, and the index N goes over $\underline{1}$, $\underline{8}_{DD}$, and $\underline{27}$. A similar relation, in terms of two constants, $\overline{g}_0$ and Y_8, holds for the couplings of Eq. (46a). Thus, we can derive four sum rules:

$$\begin{aligned}&\frac{2}{\sqrt3}g_{\pi_NKK}+\frac{1}{\sqrt3}g_{\kappa K\pi}+g_{\kappa K\eta}-g_{\pi_N\pi\eta}=0,\\&\frac{4}{\sqrt3}g_{\kappa K\pi}+g_{\sigma_8KK}-2g_{\sigma_8\pi\pi}-\frac{1}{\sqrt3}g_{\pi_NKK}=0,\\&2g_{\kappa K\eta}+g_{\sigma_8KK}+\frac{1}{3\sqrt3}g_{\pi_NKK}+\frac{2}{3\sqrt3}g_{\kappa K\pi}-2g_{\sigma_8\eta\eta}=0,\\&3g_{\sigma_0\eta\eta}+g_{\sigma_0\pi\pi}-2g_{\sigma_0KK}=0.\end{aligned} \tag{48}$$

We can also solve for the six parameters. Those associated with SU(3) invariance are[14]

$$\begin{aligned}&\overline{g}_0=\tfrac{1}{3}(g_{\sigma_0KK}+g_{\sigma_0\pi\pi}),\\&\overline{g}_8=\frac{1}{3}\left(\frac{6}{\sqrt3}g_{\kappa K\pi}+g_{\sigma_8KK}-2g_{\sigma_8\pi\pi}\right),\end{aligned} \tag{49}$$

whereas those associated with symmetry breaking are[14]

$$\begin{aligned}&Y_8'=\tfrac{1}{3}(2g_{\sigma_0\pi\pi}-g_{\sigma_0KK}),\\&X_{27}'=\frac{1}{40}\left(g_{\pi_N\pi\eta}-\frac{2}{\sqrt3}g_{\pi_NKK}\right),\\&X_{DD}'=\frac{1}{10}\left(g_{\pi_N\pi\eta}+\frac{2}{\sqrt3}g_{\kappa K\pi}-\frac{8}{3}g_{\sigma_8\pi\pi}+\frac{4}{3}g_{\sigma_8KK}\right),\\&X_1'=\tfrac{1}{8}(3g_{\sigma_8\pi\pi}+2g_{\sigma_8KK}+g_{\sigma_8\eta\eta})\ .\end{aligned} \tag{50}$$

Unfortunately, our current empirical knowledge of scalar-meson couplings allows neither verification of the sum rules (48) nor evaluation of the parameters $\overline{g}_0,\ldots,X_{27}$. However, this situation is expected to improve. Accurate measurement of the widths $\Gamma(\epsilon'\pi\pi)$, $\Gamma(\epsilon'K\overline{K})$, $\Gamma(\kappa K\pi)$, and evaluation of g_{π_NKK} from data pertaining to $K\overline{K}$ states near threshold will provide sufficient information for

Eqs. (48)–(50) to be put to use.

The relations (48)–(50) are valid for the coupling of any even-parity meson system to 0^-0^- pairs, e.g., $2^+ \to 0^-0^-$. Upon using existing data for tensor-meson decays[15] (assuming the A_2 is an unsplit entity), we have found the symmetry-breaking parameters (50) to be no larger than 10% of the SU(3) couplings (49). The relation between SU(3) symmetry breaking in tensor-meson and scalar-meson decays is not known at this time. If it turns out that they are comparable, then the SU(3) assumption used in our analysis is a reasonable one.

Correction factors to a symmetry broken away from its Goldstone limit are expected to be qualitatively governed by the physical mass of the associated Goldstone particle.[16] We examine a "hard-ϵ" model here to bring out some points relevant to this comment. We define an off-shell three-point function,

$$\delta_{ab}\, T^{\mu\nu}(q,p) = \int dx\, dy\, e^{iq\cdot x - ip\cdot y} \langle 0 | T V_a^\mu(x)\theta(0)V_b^\nu(y)|0\rangle \,, \tag{51}$$

where V_a^μ is a vector current, and $a, b = 1, 2, 3$, and vacuum intermediate states are not included in (51). The only independent Ward identity involving $T^{\mu\nu}$ is

$$q_\mu T^{\mu\nu}(q,p) = 0. \tag{52}$$

A useful representation for $T^{\mu\nu}$ is given by

$$T^{\mu\nu}(q,p) = \Delta_V^{\mu\sigma}(q)\Delta_V^{\nu\rho}(p)\Delta_s(t)\Gamma_{\sigma\rho}(q,p) \,, \tag{53}$$

where $\Delta_V^{\mu\nu}$, Δ_s are vector-current and scalar propagators, respectively, $t=(p-q)^2$, and $\Gamma_{\sigma\rho}$ is a polynomial in p and q. Equations (52) and (53) imply

$$q^\sigma \Gamma_{\sigma\rho}(q,p) = 0. \tag{54}$$

The lowest-order polynomial expansion of $\Gamma_{\sigma\rho}(q,p)$ consistent with both the Ward identity (54) and crossing symmetry is

$$\Gamma_{\sigma\rho}(q,p) = (g_{\sigma\rho}\, q\cdot p - p_\sigma q_\rho)X, \tag{55}$$

where X is an arbitrary constant. This procedure constitutes the "smoothness" hypothesis.[17] Upon treating the ρ contribution to the vector-current propagator in the narrow-resonance approximation, and passing to the mass shell, $p^2, q^2 \to m_\rho^2$, we find

$$(4\omega_q\omega_p)^{1/2}\langle \rho_a(q,\lambda)|\theta(0)|\rho_b(p,\lambda')\rangle = -\delta_{ab} X g_\rho^2 \Delta_s(t)\epsilon_\sigma^\dagger(q,\lambda)\epsilon_\eta(p,\lambda')[g^{\sigma\eta}(m_\rho^2 - \tfrac{1}{2}t) - p^\sigma q^\eta] \tag{56}$$

for the matrix element of θ between ρ-meson states. The constant g_ρ is defined by

$$(2\omega_p)^{1/2}\langle 0|V_a^\mu(0)|\rho_b(p,\lambda)\rangle = g_\rho\,\delta_{ab}\,\epsilon^\mu(p,\lambda) \,. \tag{57}$$

Let us examine the effect of hard-ϵ corrections on the single-pole model. Using the known $t=0$ constraint, Eq. (18), we can write Eq. (56) as

$$(4\omega_p\omega_q)^{1/2}\langle \rho_a(q,\lambda)|\theta(0)|\rho_b(p,\lambda')\rangle = \delta_{ab}\frac{2m_\epsilon^2}{m_\epsilon^2 - t}\,\epsilon_\sigma^\dagger(q,\lambda)\epsilon_\eta(p,\lambda')[g^{\sigma\eta}(m_\rho^2 - \tfrac{1}{2}t) - p^\sigma q^\eta] \,. \tag{58}$$

In general, we may write

$$(4\omega_p\omega_q)^{1/2}\langle \rho_a(q,\lambda)|\theta(0)|\rho_b(p,\lambda')\rangle = \delta_{ab}\epsilon_\sigma^\dagger(q,\lambda)\epsilon_\eta(p,\lambda')[g^{\sigma\eta}F(t) + p^\sigma q^\eta G(t)] \,. \tag{59}$$

Computing the ϵ pole contribution to this matrix element for an $\epsilon\rho\rho$ coupling as defined in Eq. (14), we find

$$F(t) = \frac{2g_{\epsilon\rho\rho}m_\epsilon^2 F_\epsilon}{m_\epsilon^2 - t} \,. \tag{60}$$

When we let $t \to m_\epsilon^2$ and compare with the appropriate term in Eq. (58), we obtain $F_\epsilon g_{\epsilon\rho\rho} = 2m_\rho^2 - m_\epsilon^2$. Compare this with the uncorrected single-pole model equation, $F_\epsilon g_{\epsilon\rho\rho} = 2m_\rho^2$. Thus, at least in this model, corrections are appreciable because m_ϵ is not a small parameter. However, it is not clear that the correction we have just calculated is really accurate. The validity of assumptions such as "smoothness" have no *a priori* justification. The model-dependent nature of corrections makes it difficult to obtain accurate results even when a small parameter, like the pion mass, is present.[18]

V. CONCLUSION

This paper consists of a comprehensive study of the scalar-dominance hypothesis applied to elastic, single-particle matrix elements of θ. We fully realize that, in view of the large ϵ mass, the validity of scalar dominance is suspect. To our knowledge, however, no other approach yet exists which can uniformly be applied to as large a class of examples as we have considered here. By incorporating ϵ-ϵ' mixing into the calculation, and using physical values of quantities like mass throughout, we have, at least in part, taken symmetry-breaking effects into account. The question we must now answer is–what has been learned?

The basic parameters in the ϵ-ϵ' pole model are F_ϵ and $F_{\epsilon'}$, or equivalently, F_0, F_8, and the scalar mixing angle, ϕ. The role of F_ϵ in scale invariance is analogous to the role of F_π in SU(2)×SU(2) chiral invariance. The most secure of our results is that F_0 is substantially larger than F_8. We find that, aside from the 0^- meson octet, the set of known particle masses implies $0.10 \lesssim F_8/F_0 \lesssim 0.26$. Of the examples considered, three are of particular interest – the $\frac{1}{2}^+$ baryon and 0^- meson octets, and the 0^+ meson nonet. The $\frac{1}{2}^+$ baryon analysis[5] yields an estimate of magnitudes, $F_0 = 128$ MeV and $F_8 = 33$ MeV. This implies $F_\epsilon = 91$ MeV, if we use the canonical mixing angle $\phi_c = \cot^{-1}\sqrt{2}$. The similarity in magnitude of F_C and F_π has been noted in Ref. 5. Use of the canonical mixing angle ϕ_c in the baryon analysis is plausible, but nevertheless, very much an assumption. A significant way to relax this assumption comes from our study of the scalar-meson nonet. We find that if the κ mass is determined accurately, then the values of ϕ *and* F_8/F_0 follow immediately. For instance, we see in Table III that $m_\kappa = 960$ MeV gives $\phi = 35.6°$, $F_8/F_0 = 0.32$, whereas $m_\kappa = 1000$ MeV gives $\phi = 24.7°$, $F_8/F_0 = 0.24$. The agreement between the values of F_8/F_0 derived from the $\frac{1}{2}^+$ baryon octet and 0^+ meson nonet for reasonable values of m_κ is impressive, and lends credence to our results. Theorists have long been impatient to know what the mass of the κ is. We are too, but with something quite different in mind. There is little reason to expect that κ dominance of the strangeness-changing vector current will be a particularly useful concept. Rather, it is more likely that the κ will find its greatest use in enhancing our knowledge of the evasive scalar nonet. For this reason, we urge continued experimental effort on this problem.

Now we consider the 0^- octet. Our results here are somewhat puzzling. It is easy for us to reject the anomalously large value of F_8/F_0 found in the 0^- analysis since there are convincing arguments against applying scalar dominance (with no subtraction constant) to pseudoscalar matrix elements of θ.[5,11] Nevertheless, this analysis, based on known particle masses and decay widths, implies a value for F_0 which agrees very well with the value found in the baryon analysis. The possibility that this agreement is more than just a coincidence warrants further investigation, and could provide a clue as to the mechanism which appears to relate chiral symmetry with scale invariance.

The remaining parts of our calculation involve higher-mass multiplets whose analysis requires use of the canonical mixing angle, Tables I(a)–(b) and II reveal that F_8/F_0 exhibits a slow decrease as the average multiplet mass increases. This is caused by the tendency of SU(3) mass splittings to remain roughly constant. However, relations like Eq. (34), taken literally and applied to all conceivable masses, imply something quite different –that SU(3) mass splittings must increase in order to keep F_8/F_0 fixed. We interpret this as signaling the inadequacy of either the ϵ-ϵ' pole model, or the specific assumption of a canonical mixing angle, as applied to too large a range of masses. It is comforting to note that multiplets whose average mass $\overline{M}$ lies in the range $800 \lesssim \overline{M} \lesssim 1400$ MeV give a consistent picture and imply that $F_0/F_8 \cong 5$.

As a concluding remark, we wish to point out that the sum rules, Eq. (48), although a peripheral part of the scalar-dominance calculation, should not be overlooked. The significance of coupling-constant sum rules equals that of the Gell-Mann–Okubo mass formulas. To our knowledge, only two systems, the baryon decays $\frac{3}{2}^+, \frac{3}{2}^- \to 0^- \frac{1}{2}^+$, have been studied so far.[19] The former is in impressive agreement with experiment, whereas the latter still awaits more precise data. The sum rules (48) would constitute the first practical tests of octet dominance applied to meson decays.

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Sum Rule for the Σ Term from the Bjorken Limit and Mass Dispersion Relations*

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We present a sum rule for the Σ term derived from the assumptions of Bjorken limit, precocious asymptopia, and finite-mass dispersion relations. We use the commutation relations of the gluon model as input for a numerical evaluation.

I. INTRODUCTION

The successes of current algebra and partial conservation of axial-vector current (PCAC)[1] have led to the introduction of an underlying SU(3)×SU(3) symmetry[2] for the strong interactions. Mainly because the pion mass is so anomalously small, it has been postulated that the SU(2)×SU(2) subgroup is a much better symmetry than the subgroup SU(3). In view of certain difficulties encountered in this "strong PCAC" approach, Brandt and Preparata recently proposed a "weak PCAC" scheme[3] in which SU(3) is a good symmetry, while the pion-pole domination of matrix elements of the axial-vector current divergence (PCAC) is merely a dynamical accident.

Thus arises the question of the relative accuracy of SU(2) and SU(3). In a $(3,3^*)+(3^*,3)$ theory[4] of broken SU(3)×SU(3), such a question is decided by the value of a single parameter c, which ranges from $-\sqrt{2}$ for perfect SU(2)×SU(2) to 0 for perfect SU(3). In this framework, the so-called "Σ term" encountered in the Ward identity for pion-nucleon scattering is proportional to $\sqrt{2}+c$, and should be very small (of the order of 10 MeV) if strong PCAC is indeed correct. The actual value of the Σ term thus provides a sensitive test for distinguishing between the two possibilities.

Earlier determinations[5] of this quantity were inconclusive. Recently Cheng and Dashen[6] performed an accurate evaluation of the Σ term, and obtained the result $|\Sigma| = 110$ MeV. However, their derivation[7] makes use of an expansion in ϵ_2, the SU(2)×SU(2)-breaking parameter, and this approximation might not be good if ϵ_2 is large, as we now believe.[8]

It has been suggested by the SLAC electroproduction experiments[9] that asymptotic behavior sets in quite early (around 2.5 GeV^2) for large values of the photon mass and energy (the phenomenon of precocious asymptopia). This motivates the use of *finite* dispersion relations[10] in treating current correlation functions. Tests of Callan-Gross sum rules[11] in these experiments favor the gluon model of hadrons, and a formalism of Reggeized symmetry breaking[12] has been developed which extracts from the gluon model excellent numerical predictions of experimental results.

In this paper we use these techniques to derive and numerically evaluate a sum rule that relates the Σ term to the asymptotic behavior of the current correlation function, which is given by certain commutators via the Bjorken limit.[13] We abstract the commutators from the gluon model.

The sum rule is derived in Sec. II, numerically evaluated in Sec. III, and discussed in Sec. IV. The Appendix is devoted to some results from the gluon model.

II. DERIVATION OF THE SUM RULE

The so-called Σ term,

$$i\,\delta^4(x)\Sigma_{\alpha\beta}(x) \equiv \delta(x_0)\,[\partial_\mu A_\alpha^\mu(x), A_\beta^0(0)]\,, \qquad (1)$$

appears in the Ward identity