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Parton Model with Variable Intermediate-State Parton Mass

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The parton model is modified by allowing the parton mass to change after absorbing the spacelike photon. Each prescription for the intermediate-state parton mass leads to a model with a definite scaling variable and to a scale-invariance-breaking factor which describes the approach to scaling. The model with Bloom *et al.*'s variable ω' has the required linear zero in νW_2 as $Q^2 \rightarrow 0$ for fixed ω, and has a scale-invariance-breaking factor which agrees with experiment for $1 \leq \omega' \leq 12$. The prescription that the parton mass must vanish after absorbing the photon leads to the scaling variable $\hat{\omega}$, which also occurs in the light-cone-singularity dominance model. Models which allow agreement of the sum rule for the mean squared charge per parton with experiment are given, as are models which have diffractive behavior for Q^2 near zero.

I. INTRODUCTION

The parton model of Feynman[1] and of Bjorken and Paschos[2] has the virtue of providing a simple intuitive picture of deep-inelastic electron scattering in which scaling is valid, but the disadvantages of being *ad hoc*, as well as (1) leading to sum rules which disagree with experiment, (2) not giving the required kinematic zero at $Q^2 = 0$ in νW_2, (3) therefore being exactly scale-invariant with no statement about the approach to scale invariance, (4) not giving modified scaling variables[3] in terms of which scaling occurs at lower Q^2, and (5) requiring a rather large contribution from configurations with large numbers of partons. The *ad hoc* nature of the parton model raises the question: Is it worth while to try to remedy the defects which we have just listed? We believe that it is, provided that the remedy is simple. An improved version of the model will be useful phenomenologically to summarize the data, and the changes in the model can provide clues which may be helpful in other, more fundamental approaches to inelastic electron scattering. The parton model has been modified by increasing the minimum number of partons from three to four,[4] and by introducing transverse momenta.[5] We propose to change the model in a different way by altering the model for electron-parton scattering. We retain the assumption that the parton scatters like a point object; the introduction of parton structure functions would make the model too arbitrary. We alter the assumption that the parton mass before and after the collision is small (or does not significantly change). This alteration allows us to remedy the defects listed above in a simple way. The change is consistent with the physical picture behind the model. According to the model, the parton mass squared ranges from zero to the proton mass squared. Since no particles with such a continuous mass distribution are known, we view the mass squared which occurs in the model as an effective mass squared for excitations (partons) in the nucleon, rather than as the mass squared of a free, on-shell particle. With this view, we can allow the mass squared of the parton to change after absorbing the virtual photon, so that a parton with fraction x_i of the proton energy-momentum P (in the infinite-momentum frame) need not retain its mass squared $x_i^2 M^2$, but can have a mass squared

$$(x_i P + q)^2 = x_i^2 M^2 + 2x_i M\nu - Q^2,$$

where $M\nu = P \cdot q$, and $q^2 = -Q^2$ is the square of the momentum transfer carried by the photon, prescribed in some different way. Since the matrix element relevant to inelastic electron scattering has real intermediate states and is an integral over absorptive electron-parton scattering matrix elements, we prefer the parton intermediate states to have positive energy (i.e., to have positive or zero mass squared). However, in view of the fact that we consider partons to be excitations in the nucleon, we will also allow the parton mass squared to be negative in intermediate states.

The experimental plot[6] of lines of constant νW_2 ("νW_2 contours," for short) indicates three regimes in deep-inelastic electron scattering: (1) a regime for $1 \leqslant \omega \lesssim 5$ ($\omega = 2M\nu/Q^2$) in which the νW_2 contours intercept the $2M\nu$ axis at about $-M^2$ and the appropriate scaling variable is

$$\omega' = (2M\nu + M^2)/Q^2 \text{ or } \hat{\omega} = M[(\nu^2 + Q^2)^{1/2} + \nu]/Q^2,$$

(2) a plateau for $5 \lesssim \omega \lesssim 10$ in which νW_2 is a constant, and (3) a regime for $10 \lesssim \omega$ in which the νW_2 contours are almost parallel to the $2M\nu$ axis and intercept it, if at all, at large negative values of

the order of $-20M^2$. This last regime can be considered almost a diffractive regime, since there the νW_2 contours are almost lines of constant Q^2.

In Sec. II, we calculate the structure functions for scattering of electrons off point partons in lowest order allowing change of mass of the parton after absorbing the virtual photon, and recalculate the parton-model structure functions in this context. In Sec. III, we modify the parton model to generate Bloom *et al.*'s scaling variable[3] ω', and then consider the case in which the mass squared of the parton goes to zero after absorbing the virtual photon. We find a new scaling variable $\hat\omega$ which interpolates between the Bjorken-Paschos variable (for $\hat\omega=\omega\to\infty$) and Bloom *et al.*'s variable (for $\hat\omega=\omega'=1$). In both cases, the structure functions are no longer exactly scale-invariant and νW_2 has a zero at $Q^2=0$. Scaling holds about equally well in terms of $\hat\omega$ and ω'. The model using ω' gives a good description of the approach to scaling. The sum rules for the scaling-limit structure functions for these cases are the same as in the original model. We point out that the light-cone-singularity dominance model leads to the scaling variable $\hat\omega$. The models of Sec. III are relevant for the small-ω regime $1\le\omega\lesssim 5$. In Sec. IV, we consider other constraints on the mass squared of the partons, and find examples in which the sum rules can be altered to give better agreement with experiment, and contours of constant νW_2 can be chosen to agree with data in the large-ω regime $10\lesssim\omega$. We conclude, in Sec. V, with some comments.

II. PARTON MODEL WITH VARIABLE PARTON MASS SQUARED

To compute the electron-point parton scattering structure functions, we consider the absorptive part of the forward Compton scattering amplitude in lowest order for the process

$$\gamma+\text{parton}\to\text{parton}'\to\gamma+\text{parton}$$

(see Fig. 1). In order to calculate the relevant matrix element,

$$\langle\text{parton}|j_\mu|\text{parton}'\rangle\langle\text{parton}'|j_\nu|\text{parton}\rangle,$$

we need a conserved current which produces transitions between partons of different masses. We expect the leading terms in the transition current to have the form $i\Phi^*\overleftrightarrow{\partial}^\mu\phi-2e\Phi^*A^\mu\phi+$ H.c. for spin-0 partons and $\bar\Psi\gamma^\mu\psi+$H.c. for spin-$\frac12$ partons, where ϕ (Φ) is a free spin-0 field for the parton (parton′) satisfying

$$(\Box+m^2)\phi=0,\quad(\Box+\mathfrak{M}^2)\Phi=0,\tag{1}$$

and ψ (Ψ) is a free spin-$\frac12$ field satisfying

$$(i\not\partial-m)\psi=0,\quad(i\not\partial-\mathfrak{M})\Psi=0.\tag{2}$$

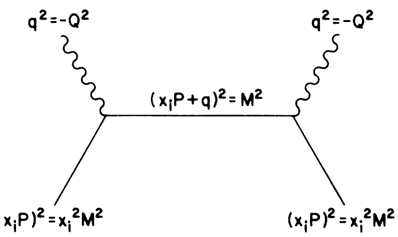

FIG. 1. Kinematics of electron-parton scattering.

For $\mathfrak{M}\ne m$, these leading terms are not conserved. The standard prescription for constructing the current from the free-field equations augmented by minimal electromagnetic coupling leads to

$$j^\mu(x)=i\Phi^*(x)\overleftrightarrow{\partial}^\mu\phi(x)-2e\Phi^*(x)A^\mu(x)\phi(x)-i(\mathfrak{M}^2-m^2)\partial_x^\mu\int\bar D(x-y)\Phi^*(y)\phi(y)d^4y+\text{H.c.}\tag{3}$$

for the spin-0 case, and

$$j^\mu(x)=\bar\Psi(x)\gamma^\mu\psi(x)-i(\mathfrak{M}-m)\partial_x^\mu\times\int\bar D(x-y)\bar\Psi(y)\psi(y)d^4y+\text{H.c.}\tag{4}$$

for the spin-$\frac12$ case, where $\Box\bar D(x)=\delta(x)$. These currents are exactly conserved, $\partial_\mu j^\mu=0$, but the terms proportional to $\mathfrak{M}-m$ violate spacelike commutativity, since local polynomials in ordinary free fields can have at most a finite order of differentiation, which is not the case for the terms with $\bar D$.[7] To discuss gauge invariance, consider the Lagrangian density

$$\mathfrak{L}=\mathfrak{L}_{0\ \text{matter}}+\mathfrak{L}_{0\ \text{EM}}+ej^\mu A_\mu+eJ^\mu A_\mu,\tag{5}$$

with $\mathfrak{L}_{0\ \text{matter}}$ the free Lagrangian for the fields ϕ and Φ for spin 0 or ψ and Ψ for spin $\frac12$, and J^μ equal to $i\phi^*\overleftrightarrow{\partial}^\mu\phi+i\Phi^*\overleftrightarrow{\partial}^\mu\Phi$ for spin 0 or $\bar\psi\gamma^\mu\psi+\bar\Psi\gamma^\mu\Psi$ for spin $\frac12$. Under combined gauge transformations of the first and second kind, $\mathfrak{L}(x)$ is not invariant, because of the nonlocal terms in j^μ; however, $\int\mathfrak{L}(x)d^4x$ is invariant, because j^μ is conserved, and thus the theory is gauge-invariant. The calculation of the contribution of the transition current to the structure functions in lowest order is straightforward. For spin-0 partons whose mass changes from M to $\mathfrak{M}$,

$$W_1(\nu,Q^2)=0,\tag{6a}$$

$$W_2(\nu,Q^2)=2M\,\delta(M^2+2M\nu-Q^2-\mathfrak{M}^2);\tag{6b}$$

for spin-$\frac12$ partons,

$$W_1(\nu,Q^2)=\nu\delta(M^2+2M\nu-Q^2-\mathfrak{M}^2),\tag{7a}$$

$$W_2(\nu,Q^2)=2M\,\delta(M^2+2M\nu-Q^2-\mathfrak{M}^2).\tag{7b}$$

The contribution to the nucleon structure functions from a parton of spin 0 or spin $\frac{1}{2}$ with charge Q_i and fractional linear momentum x_i before scattering is obtained from Eqs. (6) and (7) by multiplying by Q_i^2 and replacing M by $x_i M$. The nucleon structure function νW_2 is the same for spin-0 and spin-$\frac{1}{2}$ partons,

$$\nu W_2(\nu, Q^2) = \sum_N P(N) \left\langle \sum_i Q_i^2 \right\rangle_N \int_0^1 dx\, f_N(x) 2xM\nu\delta(x^2M^2 + 2xM\nu - Q^2 - \mathfrak{M}^2)\,; \tag{8}$$

W_1 vanishes for spin-0 partons, and for spin-$\frac{1}{2}$ partons,

$$W_1(\nu, Q^2) = \sum_N P(N) \left\langle \sum_i Q_i^2 \right\rangle_N \int_0^1 dx\, f_N(x)\nu\delta(x^2M^2 + 2xM\nu - Q^2 - \mathfrak{M}^2)\,, \tag{9}$$

where $P(N)$ is the probability that there are N partons in the nucleon $\sum_N P(N) = 1$, $f_N(x_1, \ldots, x_N)$ is the probability of finding partons with fractional linear momenta $x_1, \ldots, x_N$ in the N-parton configuration,

$$\int_0^1 dx_1 \cdots dx_N \delta\left(\sum_i^N x_i - 1\right) f_N(x_1, \ldots, x_N) = 1\,,$$

and

$$f_N(x) = \int_0^1 dx_2, \ldots, dx_N \delta\left(x + \sum_2^N x_i - 1\right) f_N(x, x_2, \ldots, x_N)\,,$$

with

$$\int_0^1 f_N(x) dx = 1\,.$$

Note that we require that only the transition current contribute to the structure functions, and we impose *ad hoc* the change in mass of the partons, without giving a dynamical basis for it.

The usual sum rule for the mean squared charge per parton,[2]

$$\int_0^1 F_2(x) dx = \sum_N P(N) \frac{\langle \sum_i Q_i^2 \rangle_N}{N}\,, \tag{10}$$

where

$$F_2(x) = \lim_{Q^2 \to \infty; x \text{ fixed}} \nu W_2(\nu, Q^2), \quad x = Q^2/2M\nu = 1/\omega$$

follows from the symmetry and normalization of $f_N(x_1, \ldots, x_N)$. For a different scaling variable this sum rule can be modified.

In Secs. III and IV, we will consider various prescriptions for $\mathfrak{M}^2$.

III. MODELS WITH SPACELIKE AND LIGHTLIKE INTERMEDIATE - STATE PARTONS

Bloom *et al.*[3] have pointed out that scaling occurs at lower Q^2 using a scaling variable ω',

$$\omega' = \frac{2M\nu + M^2}{Q^2} = 1 + \frac{s}{Q^2} = \omega + \frac{M^2}{Q^2}\,, \quad \omega = \frac{2M\nu}{Q^2}$$

than it does using the Bjorken-Paschos variable ω. If we replace the usual mass constraint by the constraint

$$x'^2M^2 + 2x'M\nu - Q^2 = \lambda x'^2 M^2\,,$$

and determine λ by the condition that it generate the scaling variable $\omega' = 1/x'$, we find $\lambda = -s/Q^2 < 0$. Thus the scaling variable of Bloom *et al.* requires spacelike intermediate-state partons. Nonetheless, the model with the scaling variable ω' has some interesting properties. The structure function νW_2 is

$$\nu W_2(\nu, Q^2) = \sum_N P(N) \left\langle \sum_i Q_i^2 \right\rangle_N \left(\frac{Q^2\omega' - M^2}{Q^2\omega' + M^2}\right) x' f_N(x')\,, \tag{11}$$

with $x' = 1/\omega' = Q^2/(2M\nu + M^2)$. The scale-invariance-breaking factor

$$B = \frac{Q^2\omega' - M^2}{Q^2\omega' + M^2} = \frac{\nu}{\nu + M}$$

in (11) approaches unity for large Q^2 and fixed ω'. In Fig. 2 we compare νW_2 and $B^{-1}\nu W_2$ vs Q^2 for six ranges of ω' using SLAC data.[8,3] We find that $B^{-1}\nu W_2$ is less Q^2-dependent in each range, and almost Q^2-independent for $\omega' \leq 12$. Thus B gives the observed scale-invariance breaking for $\omega' \leq 12$. Note that

$$\nu W_2(\nu, Q^2) \sim \sum_N P(N) \left\langle \sum_i Q_i^2 \right\rangle_N \frac{\omega Q^4}{2M^4} f_N\left(\frac{Q^2}{M^2}\right),$$

$$Q^2 \to 0, \quad \omega \text{ fixed}; \tag{12}$$

so in this model, $\nu W_2(\nu, Q^2)$ has the required linear zero as $Q^2 \to 0$ for fixed ω, but not in general. For $Q^2 \to \infty$ at fixed x, the usual parton-model result for νW^2 is recovered.

The closest we can come with positive-energy intermediate-state partons to the variable ω' is to choose $\mathfrak{M}^2 = 0$, i.e.,

$$\hat{x}^2 M^2 + 2\hat{x} M\nu - Q^2 = 0\,.$$

This choice leads to $\hat{x} = [(\nu^2 + Q^2)^{1/2} - \nu]/M$, and the scaling variable[9]

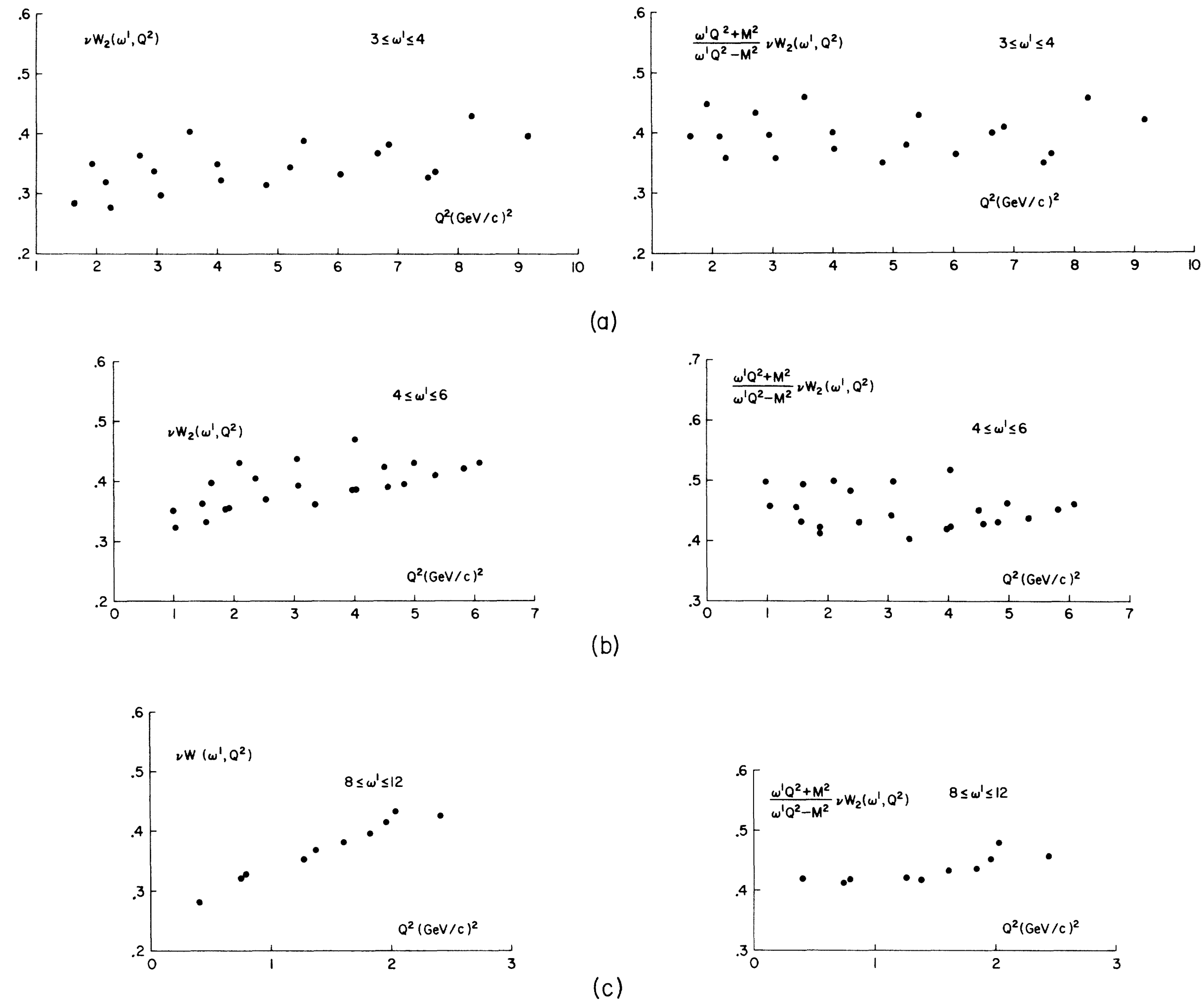

Fig. 2. *(Continued on following page.)*

$$\hat{\omega} = \frac{1}{\hat{x}} = \frac{M}{Q^2}[(\nu^2 + Q^2)^{1/2} + \nu]. \tag{13}$$

Lines of constant $1 \leq \hat{\omega} \leq \infty$ are straight lines in the $2M\nu$-Q^2 plane which interpolate between lines of constant ω' and constant ω as $\hat{\omega}$ varies from one to infinity (see Fig. 3). In this model,

$$\nu W_2(\nu, Q^2) = \sum_N P(N) \left\langle \sum_i Q_i{}^2 \right\rangle_N \frac{1}{\hat{\omega}} f_N\left(\frac{1}{\hat{\omega}}\right) \frac{\nu\hat{\omega}}{M+\nu\hat{\omega}} \tag{14}$$

$$\sim \sum_N P(N) \left\langle \sum_i Q_i{}^2 \right\rangle_N f_N\left(\frac{Q}{M}\right) \frac{\omega Q^2}{2M^2}, \quad Q^2 \to 0 \text{ at fixed } \omega \tag{15}$$

$$\to \sum P(N) \left\langle \sum_i Q_i{}^2 \right\rangle_N \frac{1}{\omega} f_N\left(\frac{1}{\omega}\right), \quad Q^2 \to \infty \text{ at fixed } \omega. \tag{16}$$

Thus, νW_2 is not exactly scale-invariant in this model and, assuming

$$\lim_{\omega\to\infty} \lim_{Q^2\to\infty} \nu W_2 > 0$$

has a $(Q^2)^{1/2}$ zero as $Q^2 \to 0$ for fixed ω, but becomes scale-invariant in the $Q^2 \to \infty$ limit. The approach to scaling in terms of the variable $\hat{\omega}$ is from below; however, quantitatively, the nonscaling factor is effectively equal to unity in the deep-inelastic region. Plots of νW_2 vs $\hat{\omega}$, ω', and ω for all of the deep-inelastic SLAC data show that scaling is comparably good in $\hat{\omega}$ and ω', and better in either $\hat{\omega}$ or ω' than in ω.

The sum rules for νW_2 in the scaling limit are the same for both models in this section as in the usual case.

The assumption that the leading operator-product

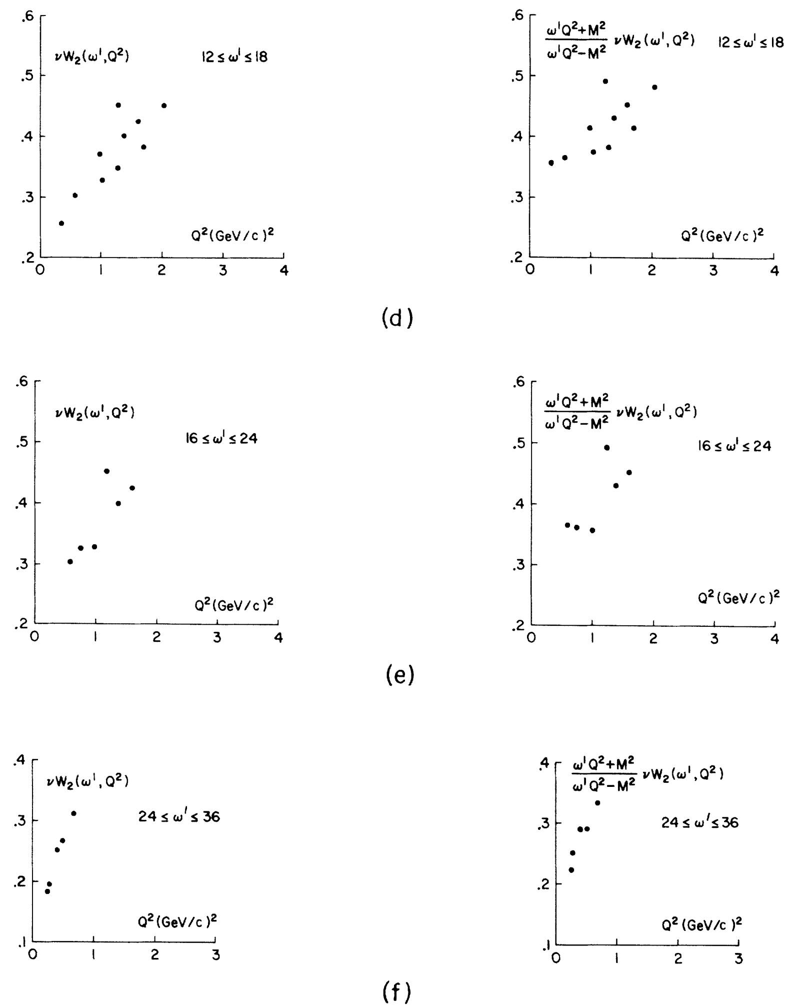

FIG. 2. Approach to scaling: Q^2 dependence of νW_2 (left) and B^{-1} νW_2 (right) for (a) $3 \le \omega' \le 4$, (b) $4 \le \omega' \le 6$, (c) $8 \le \omega' \le 12$, (d) $12 \le \omega' \le 18$, (e) $16 \le \omega' \le 24$, and (f) $24 \le \omega' \le 36$. SLAC data from Ref. 8 and the first article of Ref. 3 were used. $B = (Q^2\omega' - M^2)/(Q^2\omega' + M^2)$ from the model using the scaling variable of Bloom *et al.* Note the suppressed zeros for both axes of these graphs.

light-cone singularity dominates deep-inelastic electron scattering leads to scaling.[10] We point out here that the proper scaling variable which follows from light-cone dominance is $\hat{\omega}$ rather than ω. We show this for a simple example using the light-cone-dominance assumptions, without giving the specific details relevant to the structure functions W_1 and νW_2, since these details are given in the references cited. Consider an amplitude

$$\begin{aligned} M(p\cdot q, Q^2 = -q^2) &= \int d^4x\, e^{iq\cdot x} F(x^2, p\cdot x), \\ F(x^2, p\cdot x) &= \langle p | T(j(x)j(0)) | p\rangle, \end{aligned} \tag{17}$$

with $p^2 = M^2$. Working in the nucleon rest frame, $p = (M, \vec{0})$, introducing[11]

$$F(x^2, p\cdot x) = (x^2)^{-r}\hat{F}(x^2, p\cdot x),$$

and using $(\nu^2 + Q^2)^{1/2} = \nu + M/\hat{\omega}$,

$$\begin{aligned} M(p\cdot q, Q^2) = \frac{\pi}{iM(\nu^2+Q^2)^{1/2}} \int d\alpha\, d\beta\, \frac{\hat{F}(\alpha,\beta)}{\alpha^r} \Bigg(&\exp i\left\{\nu\left[\frac{\beta}{M} + \left(\frac{\beta^2}{M^2} - \alpha\right)^{1/2}\right] + \frac{M}{\hat{\omega}}\left(\frac{\beta^2}{M^2} - \alpha\right)^{1/2}\right\} \\ &- \exp i\left\{\nu\left[\frac{\beta}{M} - \left(\frac{\beta^2}{M^2} - \alpha\right)^{1/2}\right] - \frac{M}{\hat{\omega}}\left(\frac{\beta^2}{M^2} - \alpha\right)^{1/2}\right\}\Bigg), \end{aligned} \tag{18}$$

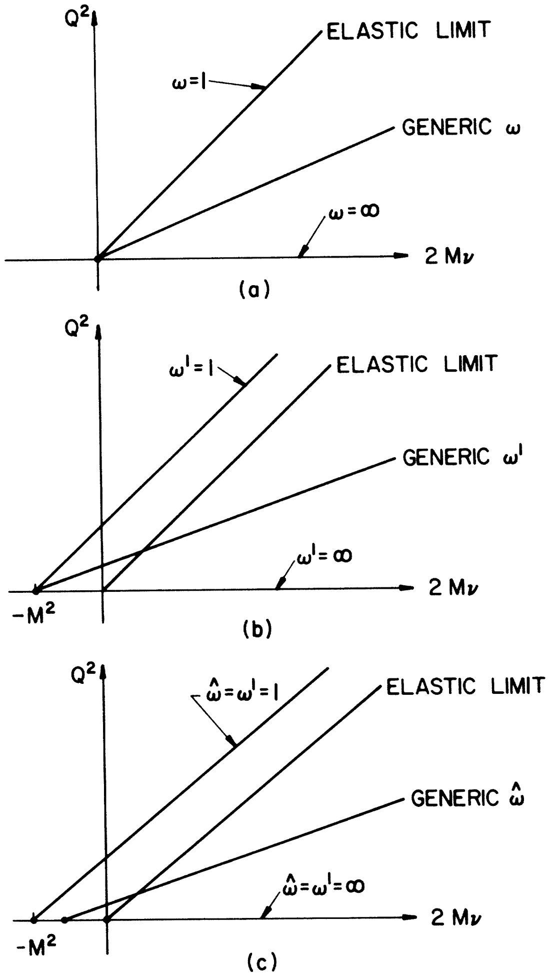

FIG. 3. Lines of constant (a) ω, (b) ω', and (c) $\hat{\omega}$ in the $2M\nu$-Q^2 plane.

which is still exact. The light-cone-dominance technique for the limit $\nu \to \infty$, $\hat{\omega}$ fixed, avoids the expansion of $(\nu^2+Q^2)^{1/2}$ that is necessary using fixed ω. For $\nu \to \infty$, $\hat{\omega}$ fixed, and assuming light-cone dominance, i.e., dominance of the α^{-r} singularity, we have the following replacements: $\hat{F}(\alpha, \beta) \to \hat{F}(0, \beta)$, the first exponential oscillates to zero, the second exponential approaches $-\exp[i(\nu\alpha m/2\beta - \beta/\hat{\omega})]$, and

$$M(p\cdot q, Q^2) \to \frac{\text{const}\,\nu^{r-1}}{\nu + M/\hat{\omega}} \int d\beta \frac{\hat{F}(0,\beta)}{\beta^r} e^{-i\beta/\hat{\omega}} + O(\nu^{r-3}). \tag{19}$$

Thus $\nu^{2-r}M$ is $\nu\hat{\omega}/(M+\nu\hat{\omega})$ times a function of $\hat{\omega}$ only plus terms of order ν^{-1}, and the scale-invariance-breaking factor which occurs here is similar to the factor [Eq. (14)] that occurs in the $\mathfrak{M}^2=0$ version of the parton model.

IV. OTHER MODELS

The variables ω' and $\hat{\omega}$ approach ω for $Q^2 \to \infty$ with ω fixed. The associated non-scale-invariant factors approach unity in the same limit, and the sum rules of the usual parton model are unmodified. This situation is general: Whenever a scaling variable $\tilde{\omega} \to \omega$, and the nonscaling factor approaches unity, in the usual scaling limit, the sum rules will remain the same. To change these two limiting behaviors requires a mass-squared constraint in which $\mathfrak{M}^2$ grows when $2M\nu$ or $Q^2 \to \infty$.

A. Model with Modified Sum Rules

Let $\mathfrak{M}^2 = \alpha Q^2 + \tilde{x}^2 M^2$. Then the new scaling variable is

$$\tilde{\omega} = \frac{1}{\tilde{x}} = \frac{1}{1+\alpha}\omega. \tag{20}$$

The sum rule (10) becomes

$$\int_0^1 F_2(\tilde{x})d\tilde{x} = (1+\alpha)\int_0^{(1+\alpha)^{-1}} F_2(x)dx = \sum_N P(N)\frac{\langle \sum_i Q_i^2\rangle N}{N}. \tag{21}$$

The loss of the interval $(1+\alpha)^{-1} < x < 1$ does not decrease the sum rule significantly, since $F_2(x)$ is small for $x \sim 1$. Thus choice of $\alpha > 0$ allows the sum rule to be increased. Using data in the deep-inelastic region, we estimate that $\alpha = 1.7$ produces the quark-model value $\frac{1}{3}$ for the sum rule, and $\alpha = 0.33$ gives the infinite quark-antiquark sea value $\frac{2}{9}$. Since increasing α decreases the range of integration, the left-hand side of (21) cannot be made arbitrarily large. We find that the left-hand side of (21) attains a maximum value of 0.356 for $\alpha = 5.7$. Similar estimates using deep-inelastic neutron data lead to $\alpha = 0.82$ for the quark-model value $\frac{2}{9}$ for the sum rule. The maximum value is 0.32 for $\alpha \gtrsim 9$. The quark-model values for the sum rules for proton and neutron can be fit fairly well simultaneously with $\alpha = 1.26$ which gives 0.31 and 0.25 for the proton and neutron sum rules to be compared with the quark-model values $\frac{1}{3}$ and $\frac{2}{9}$, respectively.[12,13]

B. Models with Special Behavior near $Q^2 = 0$

Here we give models for the third regime mentioned in the Introduction: (1) a model in which the νW_2 contours are parallel to the $2M\nu$ axis; and (2) a model with νW_2 contours whose intercepts on the $2M\nu$ axis go to minus infinity for $Q^2 \to 0$. We will see that these two models allow nonzero νW_2 for large ω without contributions from N-parton distributions for large N.

1. Model with νW_2 Contours Dependent on Q^2 Only for Small Q^2

Let $\mathfrak{M}^2=(\alpha_1\tilde{x}+\alpha_2)Q^2+\tilde{x}2M\nu+(\tilde{x}^2+\gamma\tilde{x})M^2$. Then the new scaling variable is

$$\tilde{\omega}=\frac{1}{\tilde{x}}=-\frac{\alpha_1Q^2+\gamma M^2}{(1+\alpha_2)Q^2}\,. \tag{22}$$

Choose $\gamma<0$, $\alpha_1<0$. The structure function

$$\nu W_2=\sum_N P(N)\Big\langle\sum_i Q_i{}^2\Big\rangle_N \times\frac{2(1+\alpha_2)Q^2M\nu}{|\alpha_1Q^2+\gamma M^2|^2}f_N\left(\frac{(1+\alpha_2)Q^2}{|\alpha_1Q^2+\gamma M^2|}\right) \tag{23}$$

has the following limits:

$$F_2(x)=\sum_N P(N)\Big\langle\sum_i Q_i{}^2\Big\rangle_N f_N\left(\frac{1+\alpha_2}{|\alpha_1|}\right)\frac{1+\alpha_2}{|\alpha_1|^2}\frac{1}{x} \tag{24}$$

and

$$\lim_{x\to 0}F_2(x)\text{ diverges}\,. \tag{25}$$

Thus the zero in F_2 at $x=0$ has been removed and even converted to a divergence without requiring contributions from large N. Associated with this divergence is the divergence of

$$\lim_{\nu\to\infty;Q^2\text{ fixed}}\nu W_2\,. \tag{26}$$

The zero at $Q^2=0$ is present, since

$$\lim_{Q^2\to 0;x\text{ fixed}}\nu W_2(\nu,Q^2)=0\,. \tag{27}$$

2. Model with νW_2 Contours Parallel to the $2M\nu$ Axis in the Limit $Q^2\to 0$

Let $\mathfrak{M}^2=\alpha Q^2+(\beta\tilde{x}^2+\tilde{x})2M\nu+(\tilde{x}^2+\gamma\tilde{x})M^2$. Then

$$\tilde{x}=-\frac{\gamma}{2\beta}\frac{M^2}{2M\nu}-\frac{1}{2\beta}\frac{1}{2M\nu}[(\gamma M^2)^2-4\beta(1+\alpha)2M\nu Q^2]^{1/2}\,. \tag{28}$$

The intercept of the νW_2 contours on the $2M\nu$ axis is $(-\gamma M^2)/(\beta\tilde{x})\to-\infty$, $\tilde{x}\to 0$, if $\gamma/\beta>0$. The variable $\tilde{x}\geq 0$ if $\beta(1+\alpha)<0$. Thus, to satisfy these two conditions, and increase the sum rule (21), we require $\gamma<0$, $\beta<0$, and $\alpha>0$. The structure function is

$$\nu W_2(\nu,Q^2)=\sum_N P(N)\Big\langle\sum_i Q_i{}^2\Big\rangle_N 2\tilde{x}M\nu \times[(\gamma M^2)^2-4\beta(1+\alpha)2M\nu Q^2]^{-1/2}f_N(\tilde{x})\,, \tag{29}$$

with $\tilde{x}$ given by (28). The limits of interest are

$$F_2(x)=\sum_N P(N)\Big\langle\sum_i Q_i{}^2\Big\rangle_N\frac{1}{2|\beta|}f_N\left(\left(\frac{1+\alpha}{|\beta|}x\right)^{1/2}\right), \tag{30}$$

$$\lim_{x\to 0}F_2(x)=\sum_N P(N)\Big\langle\sum_i Q_i{}^2\Big\rangle_N\frac{1}{2|\beta|}f_N(0)\,, \tag{31}$$

which again can be nonvanishing, but in this case finite, without contributions from large N. The limit $\nu\to\infty$, Q^2 fixed of $\nu W_2(\nu,Q^2)$ is also finite and nonvanishing without contributions from large N. Again the zero at $Q^2=0$ is present.

V. COMMENTS

The models using Bloom *et al.*'s scaling variable ω' and the $\mathfrak{M}^2=0$ scaling variable $\hat{\omega}$ (Sec. III) improve the usual parton model in the region $1\leq\omega\lesssim 5$ giving a zero in νW_2 at $Q^2=0$ and νW_2 contours in agreement with experiment. The model with $\hat{\omega}$ has some theoretical justification from the light-cone-singularity analysis; the model with ω' leads to a good description of the approach to scaling. The model (which we can call asymptotically diffractive) with contours asymptotically parallel to the $2M\nu$ axis for $Q^2\to 0$ (Sec. IV B 2) allows improvement of the sum rule for the mean squared parton charge, gives the kinematic zero in νW_2 at $Q^2=0$, and avoids the necessity of contributions from N-parton configurations for large N in order to produce a nonvanishing small-x limit of $F_2(x)$.

These explorations with the parton model suggest, in general, that the precise choice of scaling variable can have a far-reaching effect on the predictions of the model, affecting the five properties we listed in the Introduction, and, in particular, that the light-cone-singularity analysis is most relevant in the domain $1\lesssim\hat{\omega}<5$ where the variable $\hat{\omega}$ gives νW_2 contours which agree with experiment, and an asymptotically diffractive model is more relevant for the large-ω domain. It remains to explore these suggestions in the context of more basic approaches to inelastic electron scattering; in particular, to justify the empirically verified scale-invariance breaking associated with the variable ω'.

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[1]R. P. Feynman, in *High Energy Collisions,* Third International Conference held at State University of New York, Stony Brook, 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

[2]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[3]E. D. Bloom *et al.*, SLAC Report No. SLAC-PUB-796, 1970 (unpublished). We used $R=\sigma_L/\sigma_T=0.18$. See also E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

[4]C. W. Gardiner and D. P. Majumdar, Phys. Rev. D 2, 151 (1970).

[5]C. W. Gardiner and D. P. Majumdar, Phys. Rev. D 2, 2040 (1970).

[6]O. Nachtmann, CERN Report No. CERN-TH-1221, 1970 (unpublished), and Nucl. Phys. B28, 283 (1971).

[7]The Bjorken-Paschos parton model gives local structure functions for any distributions of partons f_N. This locality follows from the fact that the structure functions for the scattering of an electron off a parton of mass xM are local, and the nucleon structure functions are an integral with weight $f_N(x)$ of the parton structure functions. [If a set $\{\phi(\alpha_i,p,q)\}$ are local in the variable x conjugate to q for all α_i, then any integral $\int\rho(\alpha_i)\phi(\alpha_i,p,q)\pi d\alpha_i$ is also local in x.] Locality continues to hold when transverse momenta are introduced as in Ref. 5. For our case, $\Box j^\mu(x)$ is local, with j^μ given by (3) for spin 0 and (4) for spin $\frac{1}{2}$, but these currents do not go smoothly to the usual equal-mass currents for $\mathfrak{M}\to m$, and give experimentally unacceptable extra Q^4 factors for the structure functions. We prefer to tolerate the nonlocal terms proportional to $\mathfrak{M}-m$. We use the results found here as an ansatz for cases in which the partons become spacelike in the intermediate state.

[8]E. D. Bloom *et al.*, Phys. Rev. Letters 23, 930 (1969).

[9]Bloom and Gilman (Ref. 3, footnote 6) refer to work of Feynman using this scaling variable. Y. Frishman (private communication) has informed us that Horn has also considered this scaling variable. The requirement $\mathfrak{M}^2=a^2$ leads to the variable $\tilde{\omega}=M[(\nu^2+Q^2+a^2)^{1/2}+\nu]/Q^2$ which was discussed by H. Fritzsch, in Proceedings of the Coral Gables Conference, 1971 (unpublished).

[10]R. A. Brandt, Phys. Rev. Letters 23, 1260 (1969); Phys. Rev. D 1, 2808 (1970); Phys. Letters 33B, 312 (1970); R. A. Brandt and G. Preparata, Phys. Rev. Letters 25, 1530 (1970); CERN Report No. CERN-TH-1208, 1970 (unpublished); R. A. Brandt and C. A. Orzalesi, N.Y.U. report, 1970 (unpublished); L. S. Brown, in *Boulder Lectures in Theoretical Physics*, edited by K. T. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, to be published), Vol. XII; Y. Frishman, Phys. Rev. Letters 25, 966 (1970); B. L. Ioffe, Phys. Letters 30B, 123 (1969); R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970); H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970); C. A. Orzalesi and P. Raskin, N.Y.U. report, 1971 (unpublished); and J. Sucher and C.-H. Woo, Phys. Rev. Letters 27, 696 (1971).

[11]The details of the regularization necessary to define integrals containing $(x^2)^{-r}$ are not relevant to the issue that $\hat{\omega}$ is the proper scaling variable in the light-cone-dominance approximation.

[12]M. Gell-Mann [in Proceedings of the Coral Gables Conference, 1971 (unpublished)] has emphasized that the sum rules of the parton model in the naive form we consider here do not follow from a less naive approach based on light-cone commutators. This latter approach allows a different way to modify the sum rules.

[13]We thank S. Meshkov for suggesting a simultaneous fit of the proton and neutron sum rules.