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Contribution to Kaon Decays of Direct Isospin-Violating Nonelectromagnetic Interactions*

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We discuss the effect of an isospin-violating nonelectromagnetic term U_3 on $K\to 2\pi$ and $K\to 3\pi$ decays. With the coefficient of the term U_3 compatible with electromagnetic mass differences, the contribution is too small to explain the $(K^+\to\pi^+\pi^0)/(K_1^0\to 2\pi)$ decay ratio and the deviations from the $|\Delta I|=\frac{1}{2}$-rule prediction of slope parameters in $K\to 3\pi$ decays.

I. INTRODUCTION

The explanation of the validity of the $|\Delta I|=\frac{1}{2}$ rule in nonleptonic weak interactions is an old problem in particle physics; in particular, taking the underlying Hamiltonian to be of the current $\times$ current form, we would expect to have $|\Delta I|=\frac{1}{2}$ and $|\Delta I|=\frac{3}{2}$ terms if the current is of the conventional Cabibbo form. Experimentally, the only clear violations of the $|\Delta I|=\frac{1}{2}$ rule appear in K decays,[1] and are small in magnitude, so the question arises of whether the effective weak Hamiltonian need have any $|\Delta I|>\frac{1}{2}$ components at all. In particular one may ask: Can the electromagnetic interactions, coupled with a $|\Delta I|=\frac{1}{2}$ weak interaction, induce an effective $|\Delta I|>\frac{1}{2}$ transition sufficiently large in magnitude to explain the observed deviation in K decays?

The idea is by no means new; the chief problem lies in the fact that the ratio of the decay matrix elements for $K\to\pi^+\pi^0$ and $K_1^0\to 2\pi$ is of the order of $\frac{1}{20}$, rather than the naive estimate of $\alpha/\pi\sim\frac{1}{400}$ one would make on the basis of one virtual photon loop being emitted and reabsorbed in K^+ decay. One explanation of this phenomenon is of course the addition to the Hamiltonian of a small admixture of $|\Delta I|=\frac{3}{2}$.[1] Another, due to Cabibbo,[2] is that perhaps $K_1^0\to\pi\pi$ is actually also suppressed by being forbidden in the limit of SU(3) symmetry, so that the ratio of K^+ to K_1^0 decay is larger than would be expected *a priori*. Several authors have studied this ratio, introducing electromagnetic corrections only in the form of keeping the π^+ and π^0 masses different. Sakurai,[3] using a model which satisfies the current-algebra constraints first obtained by Hara and Nambu,[4] obtains for the ratio of the amplitudes, when all particles are on the mass shell,

$$\frac{A(K^+\to\pi^+\pi^0)}{A(K_1^0\to\pi^+\pi^-)}=\frac{1}{2}\frac{M_{\pi^+}{}^2-M_{\pi^0}{}^2}{M_K{}^2-M_\pi{}^2}\,, \tag{1}$$

which is far too small. Note that the amplitude for $K_1^0\to 2\pi$ does in fact vanish in the SU(3) limit, but the K-π mass difference is so large as to make the suppression factor negligible. Clavelli,[5] Schechter,[6] and Okubo *et al.*[7] have introduced models which make the ratio in (1) larger by a factor of $M_K{}^2/M_\pi{}^2$ and thus bring it into agreement with the experiment. Their arguments rest on particular forms of the coupling of the weak spurion[5,6] or on a set of dispersion-relation assumptions[7]. However, as has been pointed out by Feynman,[8] although $\Delta M_\pi{}^2$ is usually attributed to electrodynamics, these deviations do not take electrodynamics explicitly into account. In Sec. II we will discuss the contribution of an isospin-violating nonelectromagnetic term in the Hamiltonian to the K^+ decay.

The presence of an isospin-violating nonelectromagnetic term in the Hamiltonian has been speculated on by various authors, in order to eliminate certain divergences in higher-order weak interactions,[9] or as an effect arising from a Cabibbo rotation of an SU(2)$\otimes$SU(2)-symmetric strong-interaction Hamiltonian.[10] In all these cases the isospin-violating term is assumed or turns out to be the $\Delta I=1$, $I_3=0$, even-parity member of the $(3,\bar{3})\oplus(\bar{3},3)$ representation of SU(3)$\otimes$SU(3).[11] Following Oakes, we write the Hamiltonian density to be of the form

$$H=H_0-U_0+\sqrt{2}\,(1-\tfrac{3}{2}\sin^2\theta)\,U_8+\tfrac{1}{2}\sqrt{6}\,\sin^2\theta\,U_3, \tag{2}$$

where θ is the Cabibbo angle[12] and U_0, U_3, U_8 belong to the $(3,\bar{3})\oplus(\bar{3},3)$ representation. The contribution of such a term U_3 in the Hamiltonian to $\eta\to 3\pi$ decay and to $|\Delta I|=1$ electromagnetic mass differences has been evaluated by various authors.[13,14]

We will show that a U_3 term compatible with electromagnetic mass differences appears to be too small to explain the $(K^+\to\pi^+\pi^0)/(K_1^0\to 2\pi)$ decay ratio. Our arguments are model-dependent and hence not conclusive, but the discrepancy is very appreciable.

Recently there has seemed to be some evidence for the violation of the $|\Delta I|=\frac{1}{2}$ rule in $K\to 3\pi$ decays, which manifests itself in the deviations from the $|\Delta I|=\frac{1}{2}$ predictions of the ratio of slope parameters in $K^+\to 2\pi^0\pi^+$ and $K_2^0\to\pi^+\pi^-\pi^0$ to $K^+\to\pi^+\pi^+\pi^-$ decays. In Sec. III we consider the violation of the $|\Delta I|=\frac{1}{2}$ rule in a model for $K\to 3\pi$ decays and find once again that the proposed U_3 term's contribution is far too small to account for the deviations from the $|\Delta I|=\frac{1}{2}$ rule.

As we emphasize in the conclusion, the evidence appears to indicate an effective hadronic weak-interaction Hamiltonian containing an admixture of $|\Delta I|=\frac{1}{2}$ and $|\Delta I|>\frac{1}{2}$.

II. TWO-PION-DECAY MODES OF KAONS

Here, we would like to explore the effect of a possible virtual π^0-η transition on the $K^+\to\pi^+\pi^0$ decay, which, if dominant, would lead to an estimated amplitude

$$A(K^+\to\pi^+\pi^0)\sim A(K^+\to\pi^+\eta)\,\frac{\Delta_{\pi^0\eta}}{M_{\pi^0}{}^2-M_\eta{}^2}\,, \tag{3}$$

where $\Delta_{\pi^0\eta}$ is the π^0-η transition mass. If we simply use U-spin[15] arguments to evaluate $\Delta_{\pi^0\eta}$, we obtain

$$\Delta_{\pi^0\eta}=\tfrac{1}{3}[(M_{K^0}{}^2-M_{K^+}{}^2)+(M_{\pi^+}{}^2-M_{\pi^0}{}^2)]\,, \tag{4}$$

and hence $\Delta_{\pi^0\eta}/(M_{\pi^0}{}^2-M_\eta{}^2)\sim\frac{1}{90}$, which would fit experiment if the amplitude for $K^+\to\pi^+\eta$, which does not vanish in the limit of SU(3) symmetry, were approximately a factor of 4 larger than that for $K_1^0\to 2\pi$.

Riazuddin and Fayyazuddin[16] also considered this model and computed the decay rate in reasonable agreement with experiment. However, apart from questionable SU(3) assumptions arguing that since $K^+\to\pi^+\eta$ and $K_1^0\to 2\pi$ are both allowed by a pure $\Delta I=\frac{1}{2}$ rule, but since $\pi^0\eta$ is an $I=1$ state while the 2π mode is an $I=0$ state,

$$A(K^+\to\pi^+\eta)\simeq\sqrt{3}\,A(K_1^0\to 2\pi),$$

Riazuddin and Fayyazuddin used a value of $\Delta_{\pi^0\eta}$ considerably larger than (4), as obtained from a model for electromagnetic violation of charge independence of nuclear forces. We believe that their estimate is untenable.

Straightforward use of U spin in (4), however, may not be justified. Bell and Sutherland[17] have proposed an expansion of the π^0-η transition element which takes into account the PCAC (partially conserved axial-vector current) restrictions placed on the matrix elements. Their analysis, in this case, where the pion is the external particle, would lead to a value of $\Delta_{\pi^0\eta}$ smaller by a factor of $M_\pi{}^2/M_\eta{}^2$ than what we have given in (4). This is clearly incompatible with experiment. Of course the treatment of Ref. 17 is not itself unambiguous, but it does cast a serious doubt on the estimate of K^+ decay made above.

Now we would like to consider the decay as above, but allowing the π^0-η transition to be induced by the term U_3 in the Hamiltonian density (2). With such a term there is no need for the momentum-dependent expansion of Ref. 17, and the matrix element of U_3 between π^0 and η can be immediately evaluated from the results of Ref. 11, i.e.,

$$\langle\pi^0|\,U_3|\eta\rangle=-\frac{\beta(0)}{\sqrt{3}}=-\frac{\Delta M^2}{C\sqrt{3}}, \tag{5}$$

where

$$C=-\sqrt{2}\quad\text{and}\quad \Delta M^2=-0.24\ \text{GeV}^2;$$

one finds

$$\frac{\Delta_{\pi^0\eta}}{M_{\pi^0}{}^2-M_\eta{}^2}=\frac{\sqrt{6}}{2}\sin^2\theta\,\frac{\langle\pi^0|\,U_3|\eta\rangle}{M_\eta{}^2-M_{\pi^0}{}^2}\sim\frac{1}{40} \tag{6}$$

for $\sin\theta=0.24$. Therefore one only needs a slight enhancement of $K^+\to\pi^+\eta$ over $K_1^0\to 2\pi$ to obtain agreement with experiment.

We have tried evaluating the ratio of the amplitudes using Sakurai's simple pole model.[13] This means evaluating the diagrams in Fig. 1, where the weak-interaction spurion transforms like the sixth component of an octet and leads to a vector-meson–pseudoscalar-meson transition.

Calculating the values of the diagrams in Fig. 1 using Eq. (6) and the ones for $K_1^0\to 2\pi$ as given in Ref. 3, one finds for the ratio of amplitudes

$$\frac{A(K^+\to\pi^+\pi^0)}{A(K_1^0\to\pi^+\pi^-)}=\frac{1}{\sqrt{3}}\left(\frac{\sqrt{6}}{2}\sin^2\theta\right)\frac{\langle\pi^0|U_3|\eta\rangle}{M_\eta{}^2-M_\pi{}^2}\sim\frac{1}{65}, \tag{7}$$

i.e., $K^+\to\pi^+\eta$ is suppressed rather than enhanced with regard to $K_1^0\to 2\pi$ in this model, despite the SU(3) considerations.

III. THREE-PION-DECAY MODES OF KAONS

A significant violation of the $|\Delta I|=\frac{1}{2}$ rule has been detected in $K\to 3\pi$ decays. The ratio of the slope parameters as defined by Aubert[1] for $K^+\to\pi^+\pi^0\pi^0$ and $K^+\to\pi^+\pi^+\pi^-$ is found to be

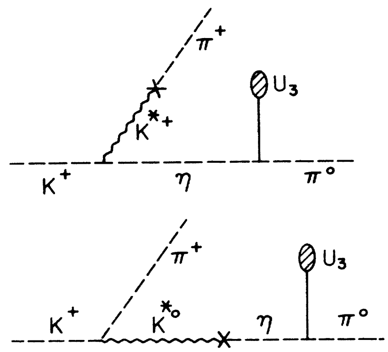

FIG. 1. Pole diagrams for $K^+\to\pi^+\pi^0$ decay.

-2.6 ± 0.12 and that for $K_2\to\pi^+\pi^-\pi^0$ to $K^+\to\pi^+\pi^+\pi^-$ is found[18] to be -3.0 ± 0.20, as compared with the $|\Delta I|=\frac{1}{2}$-rule prediction of -2.

It is clear that if a term U_3 is present in the Hamiltonian density, it will contribute to $K\to 3\pi$ decays through an η–π^0 transition. If, as before, the $|\Delta I|=\frac{1}{2}$ rule is supposed to be valid for nonleptonic weak interactions, the U_3 term will contribute a $|\Delta I|=\frac{3}{2}$ piece. In order to estimate the contribution of the U_3 term to the slope parameter in $K\to 3\pi$ decays, one needs to work in the framework of some sort of a model for $K\to 3\pi$ decays. Lovelace[19] considered the π-π scattering amplitude in the Veneziano[20] model with current-algebra constraints and applied this analysis to $K\to 3\pi$ and $\eta\to 3\pi$ decays. Sutherland[21] indicated some ambiguities in Lovelace's treatment and rectified a few. We will follow Sutherland's treatment. The relevant diagrams for $K_2^0\to\pi^+\pi^-\pi^0$ decay are shown in Fig. 2, where Fig. 2(a) involves $\pi\pi$ scattering at the strong vertex and Figs. 2(b) and 2(c) involve π-K scattering.

The matrix elements of the diagrams shown in Figs. 2(a), 2(b), and 2(c) in the above model are written as follows:

$$(\mathrm{a})=\frac{\langle\pi^0|H_W|K_2^0\rangle}{M_K^2-M_\pi^2}f^2\left[F(\alpha(t),\alpha(u))-F(\alpha(s),\alpha(t))-F(\alpha(s),\alpha(u))\right],\tag{8}$$

$$(\mathrm{b})=\frac{\langle\pi^0|H_W|K_2^0\rangle}{M_K^2-M_\pi^2}\frac{f^2}{2}\left[F(\alpha_{K^*}(u),\alpha(s))+F(\alpha_{K^*}(t),\alpha(s))\right],\tag{9}$$

$$(\mathrm{c})=\frac{\langle\pi^0|H_W|K_2^0\rangle}{M_K^2-M_\pi^2}\frac{f^2}{2}\left[F(\alpha_{K^*}(u),\alpha(t))-F(\alpha_{K^*}(s),\alpha(t))+F(\alpha_{K^*}(t),\alpha(u))-F(\alpha_{K^*}(s),\alpha(u))\right],\tag{10}$$

where

$$s=(p-p_-)^2,\quad t=(p-p_+)^2,\quad u=(p-p_0)^2,\tag{11}$$

and

$$f^2=26\quad\text{and}\quad F(x,y)=\frac{\Gamma(1-x)\Gamma(1-y)}{\Gamma(1-x-y)}.$$

Adding (a), (b), and (c) we get Sutherland's result

$$A(s,t,u)=\frac{\langle\pi^0|H_W|K_2^0\rangle}{M_K^2-M_\pi^2}f^2\Big[F(\alpha(t),\alpha(u))-F(\alpha(s),\alpha(t))-F(\alpha(s),\alpha(u))+\tfrac{1}{2}F(\alpha_{K^*}(t),\alpha(s))+\tfrac{1}{2}F(\alpha_{K^*}(u),\alpha(s))$$
$$+\tfrac{1}{2}F(\alpha_{K^*}(t),\alpha(u))-\tfrac{1}{2}F(\alpha_{K^*}(s),\alpha(u))+\tfrac{1}{2}F(\alpha_{K^*}(u),\alpha(t))-\tfrac{1}{2}F(\alpha_{K^*}(s),\alpha(t))\Big].\tag{12}$$

In the SU(3) limit $\alpha_{K^*}=\alpha_\rho$ we obtain zero at the center of the Dalitz plot. In the broken-symmetry situation we use the trajectories given by Lovelace for $\pi\pi$ and Kawarabayashi[22] for πK, i.e.,

$$\alpha(t)=0.48+0.89t,$$
$$\alpha_{K^*}(s)=0.28+0.89s.\tag{13}$$

Expanding $A(s,t,u)$ about Adler's zero, $\alpha(M_\pi^2)=\frac{1}{2}$, and using Aubert's definition

$$|A|^2=1+2a\frac{M_K}{M_\pi^2}(2T_0-T_{0\,\max})$$

for the slope parameter, we get $a=-0.24$.

With the U_3 term, apart from the diagrams we have shown in Fig. 2, we get an additional contribution which violates the $|\Delta I|=\frac{1}{2}$ rule. This contribution comes from the following two diagrams (Fig. 3). Sutherland[21] has shown that the contribution to the slope parameter coming from the graph, Fig. 3(b), which involves η-π scattering at the strong vertex, is of the order of 10% of Fig. 3(a) at the center of the Dalitz plot. So, for a rough estimate we will include only the term (a), which can be written as

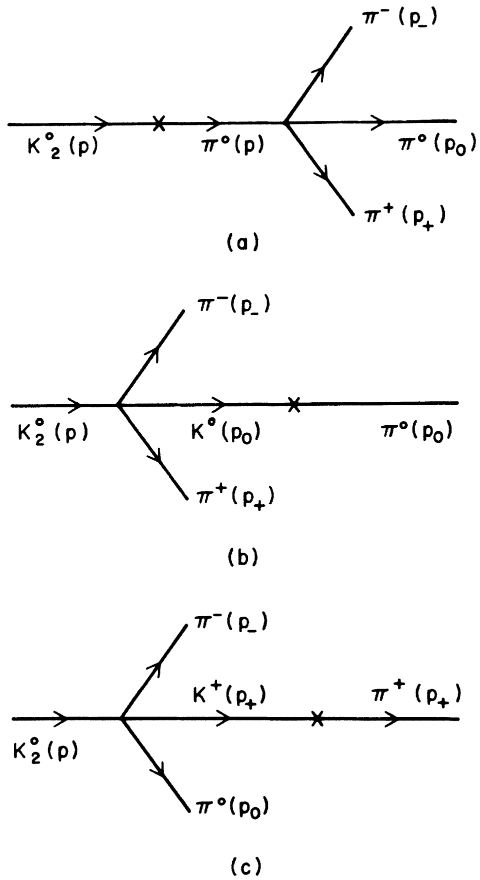

FIG. 2. Pole diagrams for $K_2^0 \to \pi^+\pi^-\pi^0$ decay.

$$(\mathrm{a}) = \frac{\langle \eta^0 | H_W | K_2^0 \rangle}{M_K^2 - M_\eta^2}\, \frac{\sqrt{6}}{2} \sin^2\theta \frac{\langle \eta^0 | U_3 | \pi^0 \rangle}{M_K^2 - M_\pi^2} f^2 \times [F(\alpha(t), \alpha(u)) - F(\alpha(s), \alpha(t)) - F(\alpha(s), \alpha(u))]. \qquad (14)$$

Now, $\langle \eta^0 | H_W | K_2^0 \rangle = (\frac{1}{3})^{1/2} \langle \pi^0 | H_W | K_2^0 \rangle$. Using Eq. (5), expanding around Adler's zero, and adding the diagrams shown in Fig. 2, we get for the slope parameter $a = -0.237$. Hence the modification due to the presence of the U_3 term is very small.

IV. CONCLUSIONS AND DISCUSSION

We saw in Sec. II that an η pole in the decay $K^+ \to \pi^+\pi^0$ may provide a possible explanation for the anomalously large ratio of $(K^+ \to \pi^+\pi^0)/(K_1^0 \to 2\pi)$, particularly if the η-π^0 transition is due to the term U_3. The result depends rather sensitively upon Oakes's estimate of the coefficient of the U_3 term. Osborn and Wallace[14] have computed this term from electromagnetic mass differences in the meson octet and baryon octet, and have found that Oakes's estimate is too large

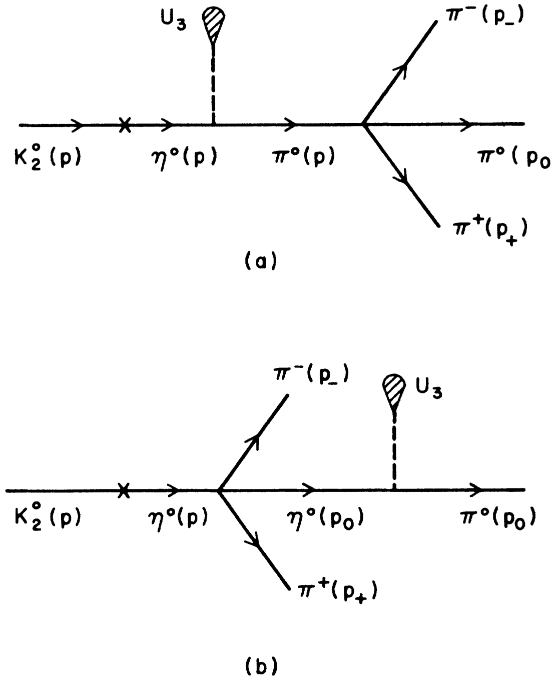

FIG. 3. U_3 contribution to $K_2^0 \to \pi^+\pi^-\pi^0$ decay.

by a factor of 3. This would lead to an overestimate of the decay rate by a factor of about 10. Their method of computing the coefficient of the U_3 term in this fashion, we believe, is questionable. What Osborn and Wallace have done is to assume that the tadpole contribution to $|\Delta I| = 1$ electromagnetic mass differences is solely given by the nonelectromagnetic U_3 term, while electromagnetism gives the usual nontadpole self-energy contribution to the mass differences. But in reality there is a tadpole electromagnetic term in $\Delta I = 1$ electromagnetic mass-difference relations coming from subtraction in $I = 1$ forward spin-nonflip Compton amplitude of virtual photons on hadrons. Several attempts have been made to compute the subtraction term using finite-energy sum rules (FESR), neglecting the presence of a fixed pole and saturating the sum rule only by keeping the low-lying states for baryons and pseudoscalar mesons.[23] The subtraction term also comes out to be very sensitive to the form of the electromagnetic form factors used. Thus, for the computation of the coefficient of the U_3 term in this manner, we need a better understanding of the subtraction constant than is heretofore available. However, it seems unlikely that the coefficient of the U_3 term would come out as large or larger than Oakes's value, in which case we would not have the U_3 term to account for the large $(K^+ \to \pi^+\pi^0)/(K_1^0 \to 2\pi)$ ratio.

Recently Wallace[24] computed the ratio of $K^+ \to \pi^+\pi^0$ to $K_1^0 \to 2\pi$ decay, with the U_3 term given in Ref. 14 (i.e., the one which is supposed to be consistent with electromagnetic mass differences), in a model in which $K^+ \to \pi^+\pi^0 K^0$ at the electromagnetic and U_3 vertex with K^0 subsequently decaying weakly into vacuum. Matrix elements of K^0 decaying to vacuum are then related to the amplitude of $K_1^0 \to 2\pi$ using soft-pion techniques. He finds for the ratio

$$A(K^+ \to \pi^+\pi^0)/A(K_1^0 \to \pi^+\pi^-) \sim \frac{1}{38}.$$

Apart from the ambiguity in applying the soft-pion technique, he has once again neglected the tadpole contribution coming from electromagnetism itself.

In Sec. III we found that the U_3 term seems to be incapable of giving a violation of the $|\Delta I| = \frac{1}{2}$ rule in $K \to 3\pi$ decays. Although our results are model-dependent, they do indicate that a U_3 term with a coefficient that would be consistent with electromagnetic mass differences gives deviations from the $|\Delta I| = \frac{1}{2}$ rule in $K \to 2\pi$ and $K \to 3\pi$ decays which are too small to explain the experimental data. The evidence thus appears to indicate an effective hadronic weak-interaction Hamiltonian containing an admixture of $|\Delta I| = \frac{1}{2}$ and $|\Delta I| > \frac{1}{2}$ pieces.

ACKNOWLEDGMENT

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