139, 1650 (1965).
[6]P. Noelle, W. Pfeil, and D. Schwela, Nucl. Phys. B26, 461 (1971).
[7]F. A. Berends and D. L. Weaver, Nucl. Phys. B30, 575 (1971).
[8]K. M. Watson, Phys. Rev. 95, 228 (1954).
[9]F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. B4, 54 (1967).
[10]F. A. Berends and A. Donnachie, Phys. Letters 30B, 555 (1969).
[11]H. G. Hilpert *et al.*, Nucl. Phys. B8, 535 (1968). We use here a revised set of these data; cf. H. Butenschön, DESY Report No. R1-70, 1970 (unpublished).
[12]E. Lodi-Rizzini *et al.*, Lett. Nuovo Cimento 3, 697 (1970).
[13]P. E. Scheffler and P. L. Walden, Phys. Rev. Letters 24, 952 (1970).
[14]The π^+ and π^0 total photoproduction cross sections from Ref. 7 are within errors in agreement with the total photon-proton cross sections for hadron production, as recently measured by T. A. Armstrong *et al.*, Phys. Letters 34B, 535 (1971).
[15]T. Fujii *et al.*, Phys. Rev. Letters 26, 1672 (1971).
[16]Compilation by J. J. Beale, S. D. Ecklund, and R. L. Walker, Caltech Report No. CTSL-42; No. CALT-68-108 (unpublished).
[17]D. Schwela, Nucl. Phys. B26, 525 (1971), also points out inconsistencies between π^-/π^+ ratio data and the original data of Ref. 11.
[18]B. M. K. Nefkens (private communication). This asymmetry is defined as the ratio of the difference and the sum of polarized-target differential cross sections, measured for polarization along $\vec{q}\times\vec{k}$ and $\vec{k}\times\vec{q}$, where $\vec{q}$ and $\vec{k}$ are the three-momenta of pion and photon, respectively. A direct measurement of the recoil-proton polarization has been carried out at higher energy by J. R. Kenemuth and P. C. Stein, Phys. Rev. 129, 2259 (1963).
[19]P. Noelle and W. Pfeil, Nucl. Phys. B31, 1 (1971).
[20]B. Gittelman and W. Schmidt, Phys. Rev. 175, 1998 (1968).
[21]J. Favier *et al.*, Phys. Letters 31B, 609 (1970). We are grateful to Dr. C. Joseph for communicating the latest results of this experiment to us.
[22]N. Christ and T. D. Lee, Phys. Rev. 148, 1520 (1966).
[23]If one introduces the T violation in M_{1+}^0, the smallness of $|M_{1+}^0|$ allows for a large T-violating phase. The latter can shift the position of the bump or dip in Δ coming from the rapidly varying term in Eq. (2).
[24]A. I. Sanda and G. Shaw, Phys. Rev. Letters 26, 1057 (1971).
[25]This holds also for the backward region. Qualitatively this can already be seen from the T-invariant theoretical predictions in Figs. 2 and 3.
[26]Y. Hemmi *et al.*, Phys. Letters 32B, 137 (1970).

PHYSICAL REVIEW D VOLUME 4, NUMBER 7 1 OCTOBER 1971

Hadron Momentum Distribution in Deeply Inelastic *ep* Collisions*

T. T. Chou
Department of Physics, University of Denver, Denver, Colorado 80210

and

Chen Ning Yang
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 6 July 1971)

Extending previous speculations originating from the idea that a hadron is a spatially extended object with many internal degrees of freedom, we argue that in *ep* collisions in the deeply inelastic region, the proton partially fragments and partially pulverizes. The usefulness of studying experimentally the single-hadron spectrum for fixed and large q^2 and ν is discussed.

I. INTRODUCTION

This paper describes some speculations concerning the momentum distribution of the hadrons in the reaction

$$e+p\to e+\text{hadrons}. \qquad (1)$$

We concentrate especially on the distribution in the Bjorken limit,[1,2] $X=q^2/2M\nu$ fixed, $q^2\to\infty$. These speculations are made as extensions of the idea that a hadron is a spatially extended object with many internal degrees of freedom. In this sense, these speculations represent a continuation of previously discussed ideas about elastic scattering,[3] about processes[4] $ab\to cd$, and about the hypothesis of limiting fragmentation.[5]

In Sec. II, we shall describe the details of the speculation, leaving the arguments in support of such speculations for Sec. III. The rest of the paper consists of additional remarks and possible experimental tests.

It will be evident that our arguments are based on imprecise extrapolations of known physical concepts. This is contrary to a contemporary fashion in our field to pursue at great lengths extrapolations of a mathematical nature. Preference for the one approach or the other is, at our present stage of understanding of hadronic collisions, a matter of taste. While both types of extrapolations are basically no more than guesses, the fruitfulness of which will have to be decided by experimental facts, we believe that extrapolations of physical concepts have a greater chance of providing a useful orientation to approach the very complex topic of many-body final states.

II. LIMITING PARTIAL FRAGMENTATION AND PULVERIZATION

We shall assume that in hadron-hadron collisions at very high energies limiting fragmentation results.[6] We shall also assume that for γ-hadron collisions at very high energies there is similar limiting fragmentation. In other words, the photon would fragment very much like a hadron does. (In this case, of course, one cannot go into the rest system of the photon. The limiting fragmentation is best viewed then in the center-of-mass system[7,8] in terms of the variables x and $p_\perp$.)

We shall only discuss the one-photon-exchange part of reaction (1). In other words, we shall only be discussing the reaction

$$\gamma_v + p \to \text{hadrons}, \tag{2}$$

with

$$\text{energy of } \gamma_v = \nu\,,$$
$$\text{momentum of } \gamma_v = (\nu^2 + q^2)^{1/2}\,, \tag{3}$$
$$(\text{mass of } \gamma_v)^2 = -q^2\,,$$

where γ_v is the virtual photon.

For the case $q^2 = 0$, the virtual photon is on the mass shell and the hadron distribution in (2) should be that of a double limiting fragmentation as in γp collisions. It has been argued[9] that for

$$q^2/2\nu < (\text{radius})^{-1}\,,$$

i.e.,

$$X \equiv q^2/2M\nu < \tfrac{2}{7}\,, \tag{4}$$

process (2) exhibits double limiting fragmentation. We believe this view is correct, since the reasoning was geometrical and expresses the fact that when (4) is satisfied the virtual photon can maintain *coherence* in its traversal through the target proton. [Actually, of course, since γ_v is not on the mass shell, the double limiting fragmentation process has to be a little bit changed from that of a real γp collision. The necessary change will become clearer when we discuss the general case where (4) is not necessarily satisfied.]

For the general case where X is not necessarily small, we speculate that in the laboratory system for (2), "part" of the target proton fragments in the Bjorken limit (i.e., $\nu \to \infty$ at fixed X). The resulting limiting fragments from this proton part will give, in the laboratory system, a total of

$$\sum_{\text{frag}} (e - p_\parallel) = M(1 - X). \tag{5}$$

The rest of the proton bears the brunt of the impact of the virtual photon and "pulverizes" into n hadronic pieces, each carrying a large longitudinal momentum in the original direction of the virtual photon. The pulverization multiplicity n is of the order

$$n = O(\nu^{\alpha(X)})\,, \tag{6}$$

where $\alpha(0) = 0$. E.g., a possible model is

$$\alpha(X) = aX, \quad a < \tfrac{1}{2}, \quad \text{and} \quad n \cong A(\nu/M)^{aX}\,, \tag{7}$$

where a and A are numerical constants. The condition $a \not> \frac{1}{2}$ is necessary since the multiplicity is limited for given ν and q^2 [for which the invariant mass of all hadrons together is $(M^2 + 2M\nu - q^2)^{1/2}$].

Viewed in the center-of-mass system of (2), the final hadrons consist of

(a) *limiting fragments* satisfying[7,8]

$$\sum x = 1 - X\,; \tag{8}$$

(b) *pulverization products* which in the coordinate x would appear to occupy the position $x = 0$, giving rise to a $\delta(x)dx$ distribution. According to our terminology,[8] the pulverization products exhibit the phenomena of pionization. (We introduce the word pulverization rather than use the word pionization because of the possible connotation that pionization is a radiative process. The pulverization products are incoherent products resulting from a violent momentum transfer.) The average value of x for each pulverization pion is perhaps approximately

$$(1 - X)/n \approx (1 - X)A^{-1}(M/\nu)^{aX}\,. \tag{9}$$

The same selection rule for quantum numbers applies to the fragments in (5) or (8) as that which applies in the usual fragmentation process. In other words, the fragmentation pieces of the resulting hadrons in reaction (2) carry a total $N = 1$, charge $= 1$, and $|I| = \frac{1}{2}$. The total charge of the pulverization product is thus zero, and they are mostly pions.

III. ARGUMENTS IN SUPPORT OF THE ABOVE SPECULATIONS

(1) In a limiting fragmentation process, *the fragmentation of a fast projectile is a gentle process requiring only an infinitesimal energy transfer, an infinitesimal longitudinal momentum trans-*

fer, and only a finite transverse momentum transfer. (This is easily checked, for example, in the process of an incoming π fragmenting into three outgoing π's.) Notice that these four-momentum transfers are also those required for an elastic scattering. Thus, the limiting fragmentation process is a generalization of elastic scattering, as emphasized in Sec. 8.7 of Ref. 5.

(2) The virtual photon in process (2) cannot fragment by the gentle process described above because it has an energy deficiency, compared with its momentum, given by

$$(\nu^2+q^2)^{1/2}-\nu\cong q^2/2\nu\ . \qquad (10)$$

(In other words, its four-momentum is spacelike.)

It is probably best to discuss the absorption of γ_v by the proton in the "brick-wall system" in which γ_v has no energy, as illustrated in Fig. 1(a).

In the Bjorken limit we are considering, the proton is fast in the brick-wall system, and it has a *tendency to fragment* in even a gentle collision. We speculate that upon the absorption of γ_v, which is a violent process, a part of it would still fragment in the usual way. In other words, the four-momentum of the proton in Figs. 1(a) and 1(b) divides into two parts[9a] (A) and (B) in the proportion of $1-y$ and y. Part (A) fragments in the usual limiting way, resulting in fragments that satisfy (5) and (8) with X replaced by y. [To show this, let $p_\parallel^b$ be the brick-wall-system longitudinal momentum of a fragment. Define $x^b=p_\parallel^b(m\nu/q)^{-1}$. Then $\sum x^b=1-y$. Transformation into the c.m. system of (2) yields

$$\sum x=1-y\ . \qquad (8')$$

That this is equivalent to the formula in the laboratory system,

$$\sum_{\text{frag}}(e-p_\parallel)=M(1-y), \qquad (5')$$

follows reasoning already discussed[8] in the literature. Notice that (8′) *and* (5′) *can be read as saying explicitly that a fraction* $1-y$ *of the proton fragments.*]

One may ask why does one envisage the division of the proton four-momentum into parts (A) and (B) in Fig. 1(b) where each part is, to leading order, a null four-vector? The reasons are provided by the discussion above in Sec. III, paragraph 1. Notice that in the limiting fragmentation of a projectile, *any* subgroup of the fragments would have a total four-momentum which is, to the leading order, a null four-vector.

Part (B) absorbs the four-momentum of γ_v, with a possible additional finite transverse three-momentum transfer from part (A) [cf. Fig. 1(b)].

For part (B) after the absorption of γ_v to be capable of materializing physically into hadrons, the final four-momentum must be timelike, i.e. [cf. Fig. 1(b)],

$$q-M\nu q^{-1}y\leqslant M\nu q^{-1}y\ .$$

Thus

$$1-X\geqslant 1-y\ .$$

(3) What value does $1-y$ take in the Bjorken limit? (Or should it have a probability distribution?) We speculate that there is the rule that $1-y$ takes its maximum value, i.e., $y=X$. In other words, as much of the hadron would fragment as is kinematically possible [cf. Fig. 1(c)].

This rule is in the same spirit as the suggestion[5,8] that in hadron-hadron collisions, there is 100% fragmentation of each hadron, so that instead of (5) and (8) one has $\sum(e-p_\parallel)=M$ and $\sum x=1$. [If it were not for this rule, in a hadron-hadron collision both hadrons could, kinematically, fragment

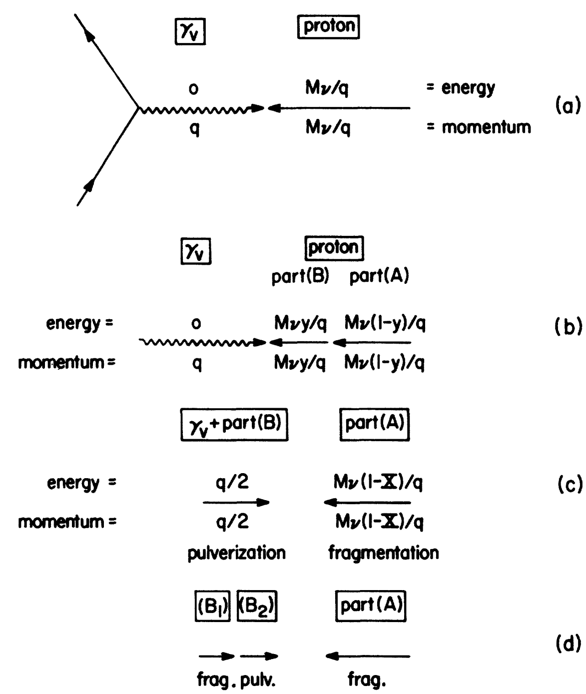

FIG. 1. The brick-wall system of ep collisions. [The proton energy is $M(1+\nu^2/q^2)^{1/2}\approx M\nu/q$ in the Bjorken limit.] Notice that in this frame of reference the virtual photon delivers a pure three-momentum impact without any energy transfer. All quantities are accurate only to $O(q)$. (a) Four-momenta of virtual photon and proton before collision. (b) Divison of proton four-momentum into parts (A) and (B). (c) Four-momenta after the absorption of virtual photon by proton leading to pulverization and partial fragmentation of the proton. (d) Division of part (B) after the absorption of γ_v into parts (B_1) and (B_2). This division is unlikely. See text.

only partially (i.e., $\sum x < 1$), leaving the rest of their energies for pionization.]

(4) What happens to part (B) of the proton after it absorbs γ_v? It has reversed the direction of its three-momentum [Fig. 1(c)], without having absorbed any energy [to the order $O(q)$]. We thus speculate that it is *pulverized by the shock of this violent absorption of* γ_v. In contrast to fragmentation, which is coherent, pulverization is an incoherent process.

It seems that the multiplicity of the pulverized pieces, n, should be an increasing function of ν and of X. Proposals (6) and (7) join on smoothly to the region $X < \frac{2}{7}$ where the multiplicity n is more like the multiplicity of the fragmentation of a real photon which is $\sim O(\ln\nu) \sim O(\nu^{\alpha})$, where $\alpha = 0+$. The increase of the multiplicity in the deeply inelastic region is in agreement with the view that a system with many degrees of freedom breaks up into more pieces for such collisions, as emphasized in Ref. 5.

(5) Why is it not the case that after the absorption of γ_v part (B) of the proton breaks off into a part (B_1) that fragments, leaving only the rest, part (B_2), to pulverize [cf. Fig. 1(d)]? It seems to us that that is highly unlikely. The fragments from (B_1) would have to move in the direction of γ_v in the brick-wall system, as if they had originated from a real photon. There simply is not the coherence in γ_v to manage such a concerted effort. [γ_v is very much different from a stable hadronic system such as a proton or pion. These latter systems in their rest frames of reference can be excited into *collective modes* of excitation, resulting in fragmentation (which is coherent). γ_v does not even possess such a rest frame of reference.]

(6) The fragmentation-pulverization of the proton by γ_v can be viewed in the proton rest system, as described in Sec. II and illustrated in Fig. 2. Notice that in the Bjorken limit the pulverization part is greatly accelerated by the absorption of γ_v. The rule proposed above in Sec. III, paragraph 3 is thus equivalent to the rule that the accelerated "stuff" should be minimized. This last rule seems to be in agreement with the general facts of high-energy phenomena.

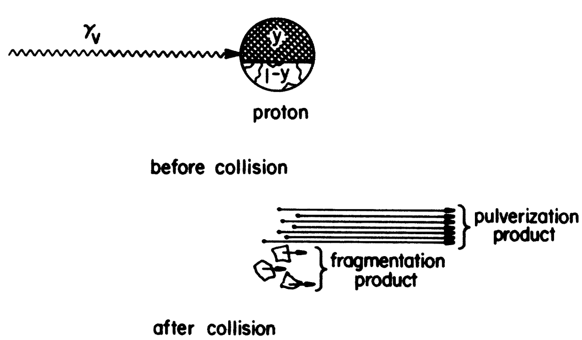

FIG. 2. Fragmentation and pulverization of the proton by a virtual photon as viewed in the lab system. This drawing is schematic. See Sec. VI.

Consistent with the selection rule that inhibits the acceleration of quantum numbers N and $\vec{I}$, one arrives at the selection rule that the pulverization products are mostly pions with a total charge of zero.

IV. THE DISTRIBUTION FUNCTIONS $\underline{\rho}_n$

In this section we consider, for *fixed* q^2 and *fixed* ν, the case of infinite incoming electron energy. As discussed in Ref. 5, the hadronic matter in the laboratory system approaches a limiting distribution. Consider now the exclusive part of this distribution. It is a restricted distribution and we shall denote it by $(\sigma_n | q^2, \nu)$ to explicitly emphasize its dependence on q^2 and ν. The arguments of $(\sigma_n | q^2, \nu)$ are the three-momenta $\vec{p}_1\vec{p}_2 \cdots \vec{p}_n$. For $(\sigma_n | q^2, \nu)$ not to vanish, these arguments must satisfy the conditions

$$\begin{aligned} &\sum e = M + \nu , \\ &\sum p_\parallel = \nu , \qquad (11) \\ &(\sum p_\perp)^2 = q^2 . \end{aligned}$$

Thus any $(\sigma_n | q^2, \nu)$ or $(\rho_n | q^2, \nu)$ is restricted to a finite region of momentum space $\vec{p}_1 \cdots \vec{p}_n$. [The integral of $(\sigma_n | q^2, \nu)$ over all q^2 and ν gives σ_n, the unrestricted limiting exclusive distribution.]

We now normalize and define r_n by

$$r_n(q^2, \nu; \vec{p}_1 \cdots \vec{p}_n) = \left(\frac{d\sigma}{dq^2 d\nu}\right)^{-1} \frac{d\sigma}{dq^2 d\nu d\vec{p}_1 \cdots d\vec{p}_n} , \qquad (12)$$

where r_n is the normalized n-particle distribution (exclusive and inclusive) when the proton absorbs a γ_v of energy ν, longitudinal momentum ν, and transverse momentum $\sqrt{q^2}$ (longitudinal is here defined relative to the incoming electron). r_n is normalized so that

$$\int r_1 d^3p = \text{average multiplicity of the particle specified in } r_1 \text{ for the collision described above,} \qquad (13)$$

etc.

Like $(\sigma_n | q^2, \nu)$ and $(\rho_n | q^2, \nu)$ above, r_n is nonzero only in a finite region of momentum space $\vec{p}_1 \cdots \vec{p}_n$.

The postulate of limiting partial fragmentation of Sec. II states that

$$\underline{\rho}_n(p_1 \cdots p_n) = \lim r_n \qquad (14)$$

exists in Bjorken limit. This limit represents the n-particle distribution in the *limiting partial fragmentation* of the proton. The pulverizing part has infinite momentum in the laboratory system. Thus it plays no part in $\underline{\rho}_n$.

The region where $\underline{\rho}_n$ is nonvanishing consists of the region

$$\underline{\mathrm{R}}_n \quad \text{where } \sum (e - p_{\parallel}) < M(1 - X) , \tag{15}$$

and its boundary

$$\underline{\mathrm{S}}_n \quad \text{where } \sum (e - p_{\parallel}) = M(1 - X) . \tag{16}$$

The function $\underline{\rho}_n$ has two parts,

$$\underline{\rho}_n = \underline{\sigma}_n + \underline{\tau}_n , \tag{17}$$

where

$\underline{\tau}_n$ = function in $\underline{\mathrm{R}}_n$,

and

$\underline{\sigma}_n$ = δ function on $\underline{\mathrm{S}}_n$.

All of these are straightforward generalizations of the corresponding ideas in Ref. 5. We shall come back to these ideas in Sec. VI.

In the same way that one derives the sum rule Eq. (2) of Ref. 8, we now have

$$\left[\int \underline{\rho}_1 d^3p (e - p_{\parallel})\right]_{\text{proton}} + [\text{same}]_{\pi} + [\text{same}]_{\Lambda} + \cdots = M(1 - X) . \tag{18}$$

V. REMARKS

(1) For

$$\nu + p \to \mu^- + \text{hadrons} \tag{19}$$

the speculations are identical to the above except for the following:

(a) The pulverization pions will have a total charge of +1.

(b) For the case $q^2 \cong 0$, there are two possible processes originating from Figs. 3(a) and 3(b).

Figure 3(a) leads to a double (limiting) fragmentation process entirely similar to the corresponding one in ep collisions. For this latter case it is convenient to describe the outgoing electron together with the fragments from the virtual photon as fragmentation products from the projectile electron. We write in the notation of Ref. 8

$$e \xrightarrow{p} e + \text{hadrons} . \tag{20}$$

Similarly for a νp collision at $q^2 \approx 0$ we expect the limiting fragmentation of the neutrino:

$$\nu \xrightarrow{p} \mu^- + (\text{hadrons})^+ , \tag{21}$$

with the total charge of the hadrons equal to +1. In the same process the target proton undergoes its own limiting fragmentation:

$$p \xrightarrow{\nu} (\text{hadrons})^+ . \tag{22}$$

No charge transfer takes place between the two fragmentation processes (21) and (22).

Figure 3(b), on the other hand, leads to a charge transfer, and results in, for the case $q^2 \approx 0$,

$$\nu \xrightarrow{p} \mu^- \quad (\text{no hadrons}) \tag{23}$$

and

$$p \xrightarrow{\nu} (\text{hadrons})^{++} . \tag{24}$$

The laboratory momentum of μ^- divided by the incoming neutrino momentum is approximately unity. Notice that (24) is kinematically the same as a limiting fragmentation process, but there is a net charge transfer. Neglecting electromagnetic interactions and taking the weak interactions only to the lowest order we speculate that the unusual fragmentation process (23), (24) has a finite cross section at infinite energy. The questions of the validity of these approximations at *very high* energies and whether experimentally (23) and (24) would have a finite cross section at such energies are deep and difficult and are beyond the scope of this paper.

(2) There have been many discussions[10,11] in the literature on topics in ep collisions related to our speculations. We shall not discuss these here except for the parton[11] model. The ideas that we have pursued (Ref. 3–5) and that we extended in the present paper follow the general concept that there

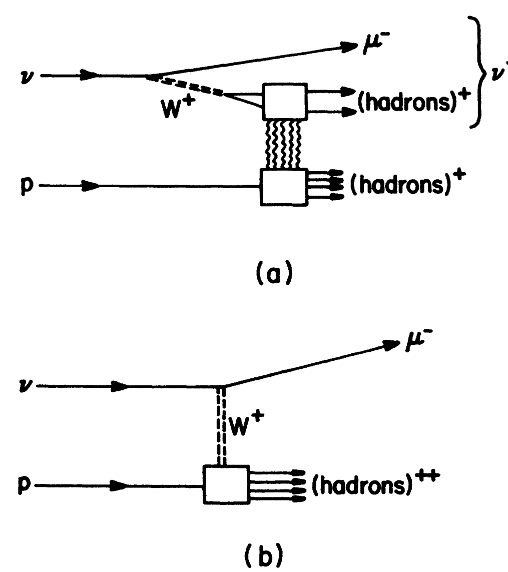

FIG. 3. (a) Double fragmentation process in νp collisions: $\nu \xrightarrow{p} \mu^- + (\text{hadrons})^+$ and $p \xrightarrow{\nu} (\text{hadrons})^+$. (b) νp collision at $q^2 \approx 0$ with charge transfer: $\nu \xrightarrow{p} \mu^-$ and $p \xrightarrow{\nu} (\text{hadrons})^{++}$.

is "stuff" in the hadron.[12] If "partons" are "stuff" then the present paper is also in the spirit of the parton model. One would identify part (B) of the proton in Sec. III, paragraph 2 as the recoiling parton. It has a "mass" equal to MX. However, it has been repeatedly emphasized that the spirit of the parton model resides in the concept that partons are *pointlike unbreakable* particles.[11] If that is the essential spirit of the parton model, then the present speculation about *pulverization* seems to be contradictory to the spirit of the parton model. If one argues that the pointlike unbreakable feature of the parton is meant only in the sense of the impulse approximation, one is perhaps arguing for a description of hadronic matter complementary to our speculations with different emphasis for different physical questions. To sharpen up such complementary views it would then seem useful to us to raise in the context of the parton model the following questions: (i) Is there limiting partial fragmentation? (ii) Is there a "parton quark number" selection rule for parts (A) and (B)? (iii) What is the hadron multiplicity for parts (A) and (B)?

VI. EXPERIMENTAL TESTS

How does one test these speculations? To discuss this question let us first emphasize that Fig. 2 is only schematic. The separation of the pulverization and fragmentation products is clear as illustrated only in the limit of very large ν. [The momenta of fragments are of the order $O(1)$, and that of the pulverization products $O(\nu^{1-\alpha})$.]

Perhaps a more feasible test is to study the single-particle distribution which was designated $r_1(q^2, \nu; \vec{p})$ in Sec. IV above. One would be studying [13] the single-hadron inclusive distribution for fixed and large q^2 and ν. To be more specific, one measures experimentally the quantity

$$r_1(q^2, \nu; \vec{p}) = \left(\frac{d\sigma}{d\nu dq^2}\right)^{-1} \frac{d\sigma}{dq^2 d\nu d^3p} , \qquad (12')$$

where $\vec{p}$ is the laboratory three-momentum of a hadron fragment, say a pion or a proton. Notice that r_1 satisfies (13). In the following we shall concentrate on the case where $\vec{p}$ is the laboratory three-momentum of a proton. One asks whether r_1 approaches a limit $\underline{\rho}_1(\vec{p})$ in the Bjorken limit as stated in (14). This limit, if it exists, represents a limiting partial fragmentation of the proton. In particular, it has two parts,

$$\underline{\rho}_1 = \underline{\sigma}_1 + \underline{\tau}_1 , \qquad (17')$$

where $\underline{\sigma}_1$ is a δ function on the surface $\underline{S}_1$:

$$e - p_{\parallel} = M(1-X) \qquad (25)$$

and $\underline{\tau}_1$ is a function defined in the region

$$e - p_{\parallel} < M(1-X) . \qquad (26)$$

Physically, $\underline{\sigma}_1$ represents a process in which[14] (1) is described by

$$\gamma_v + p \to p + \text{pulverization products} , \qquad (27)$$

and $\underline{\tau}_1$ represents a process in which (1) becomes

$$\gamma_v + p \to p + \text{other fragments} + \text{pulverization products} .$$

Two remarks are perhaps appropriate here:

(a) In hadron-hadron collisions it is much easier to study whether the one-particle momentum distribution approaches a limit than to study whether pionization exists, or to study correlations between particles. Similarly, we believe it is easier for the present problem to study the existence of the limit $\underline{\rho}_1$ rather than to study the validity of Fig. 2.

(b) How much do such kinematic regions (25) and (26) depend on the concepts of partial fragmentation and pulverization? The answer is very much. Formula (25) of course explicitly states a *partial* limiting fragmentation. But it also depends on our speculations about pulverization. For example, if pulverization products have a multiplicity that is given by (7) but with $a = \frac{1}{2}$ [which is higher than given by (7)], then the sum of the laboratory-system contributions to $e - p_{\parallel}$ from the pulverization products would contribute finitely to and therefore would vitiate (25). The existence of the δ function $\underline{\sigma}_1$ on the paraboloid (25) is an especially sensitive test of the idea of partial fragmentation plus pulverization, since it is a nontrivial statement that the σ_n defined over (11) should yield, when $\nu \to \infty$ in the Bjorken limit, a limiting single-particle distribution $\underline{\rho}_1$ with a δ-function part $\underline{\sigma}_1$.

ACKNOWLEDGMENT

The paper was written when one of us (T. T. C.) was visiting the State University of New York at Stony Brook. He wishes to thank the members of the Institute for Theoretical Physics for the hospitality he enjoyed.

*Partially supported by the U. S. Atomic Energy Commission under Contract No. AT(30-1)3668B.

[1]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[2]We used the following notations: M = proton mass, $E - E' = \nu$ = laboratory energy loss of the electron, q^2 = (four-momentum transfer from electron)^2, $X = q^2/2M\nu$, e and $p_{\parallel}$ = laboratory energy and longitudinal momentum of a hadron, where the laboratory three-momentum of γ_ν is defined as longitudinal.

[3]T. T. Chou and Chen Ning Yang, Phys. Rev. 170, 1591 (1968); Phys. Rev. Letters 20, 1213 (1968).

[4]T. T. Chou and Chen Ning Yang, Phys. Rev. 175, 1832 (1968); B. Lee, Phys. Rev. D 1, 2361 (1970); S. J. Chang, *ibid.* 2, 2886 (1970).

[5]J. Benecke, T. T. Chou, Chen Ning Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

[6]D. B. Smith, R. J. Sprafka, and J. A. Anderson, Phys. Rev. Letters 23, 1064 (1969); R. W. Anthony, C. T. Coffin, E. S. Meanley, J. E. Rice, K. M. Terwilliger, and N. R. Stanton, *ibid.* 26, 38 (1971); L. G. Ratner, R. J. Ellis, G. Vannini, B. A. Babcock, A. D. Krisch, and J. B. Roberts, *ibid.* 27, 68 (1971). See also M. S. Chen, R. R. Kinsey, T. W. Morris, R. S. Panvini, L. L. Wang, T. F. Wong, S. L. Stone, T. Ferbel, P. Slattery, B. Werner, J. W. Elbert, and A. R. Erwin, *ibid.* 26, 1585 (1971).

[7]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).

[8]T. T. Chou and Chen Ning Yang, Phys. Rev. Letters 25, 1072 (1970).

[9]J. D. Bjorken, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, Stanford, California, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968); H. T. Nieh, Phys. Rev. D 1, 3161 (1970).

[9a]*Note added in proof.* After we submitted this paper for publication it was pointed out to us that already in S. Drell and T.-M. Yan, Phys. Rev. Letters 24, 855 (1970), there was a discussion of the separation of the target proton into two parts. The background for Drell and Yan's discussion is different from ours. So are some essential features of the conclusions. See Sec. V.

[10]H. Cheng and T. T. Wu, Phys. Rev. Letters 22, 1409 (1969); S. D. Drell, D. Levy, and T.-M. Yan, Phys. Rev. D 1, 2402 (1970) and earlier papers; H. T. Nieh and Jiunn-Ming Wang, Phys. Rev. Letters 26, 1139 (1971).

[11]R. P. Feynman (unpublished); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); S. Drell and T.-M. Yan, Phys. Rev. Letters 24, 855 (1970).

[12]T. T. Wu and Chen Ning Yang, Phys. Rev. 137, B708 (1965); N. Byers and Chen Ning Yang, *ibid.* 142, 976 (1966).

[13]The following paper already reported some preliminary experiments of this kind: F. W. Brasse, W. Fehrenbach, W. Flauger, K. H. Frank, J. Gayler, V. Korbel, J. May, P. D. Zimmermann, and E. Gaussauge, DESY Report No. 71/19 (unpublished).

[14]It may sound strange that in (27) the final proton is only a fraction $1-X$ of the original proton. What happens to its mass? Did it grow from $M(1-X)$ to M? Yes it did, and the change is made, in any coordinate system where the original proton is fast, by an *infinitesimal* transfer of energy and of longitudinal momentum. This same situation obtains in, e.g., $\pi p \to \pi p^*$ where the excitation of the proton requires an infinitesimal energy and an infinitesimal longitudinal momentum transfer in the c.m. system or in the projectile system. The same situation also obtains for every fragment in a usual limiting fragmentation process. (In other words, masses can be altered readily in high-energy collisions. See also Sec. III above.)