Discussion of Isotensor Terms and T Violation in Pion Photoproduction near the First Resonance*

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An analysis is made of the presently available experimental and theoretical information for pion photoproduction and π^- radiative capture near the first resonance with regard to the possibility of a violation of the $\Delta I \leq 1$ rule, and the possibility of a T-violating electromagnetic current. An isotensor term is only one of the several possible explanations of the recent π^- photoproduction data. The radiative-capture data near the first resonance support a substantial T violation in the magnetic dipole amplitude. However, it is not established that this T violation may be ascribed to a single isospin part of the amplitude. Some predictions for experimental tests are given which could clarify the situation.

I. INTRODUCTION

On the basis of the presently available π^- photoproduction data, interesting questions have recently been raised concerning the transformation properties of the electromagnetic current.

Comparing π^- and π^+ photoproduction data in the first resonance region, Sanda and Shaw[1,2] claimed evidence for an isotensor component causing a violation of the $\Delta I \leq 1$ rule. In a measurement of the inverse reaction (π^- radiative capture) Berardo *et al.*[3,4] found a striking disagreement with the direct reaction near the first resonance, which, taken at face value, would imply a violation of time-reversal invariance. This would be the first sizable effect of T violation in the hadron electromagnetic current, a possibility which was first suggested by Bernstein, Feinberg, and Lee[5] in order to explain CP violation in K^0 decays.

For π^+ and π^0 photoproduction from protons in the first resonance region, one has a fairly good knowledge both experimentally and theoretically, in particular through detailed multipole analyses. The purpose of this paper is to exploit this knowledge to predict what one may expect for π^- and π^0 photoproduction from neutrons and to compare the predictions with the most recent data. In particular, it is discussed how compelling the claims are for an isotensor term and T noninvariance. Section II deals with the isospin selection rule, and Sec. III with T noninvariance. Section IV summarizes the conclusions.

II. THE $\Delta I \leq 1$ RULE

A basic assumption in the analysis of pion photoproduction and other hadronic electromagnetic processes is that the electromagnetic current has the same isospin properties as the charge operator, i.e., the sum of an isoscalar and an isovector. This gives the $\Delta I \leq 1$ rule for photoproduction processes.

When one relaxes this rule, the most general isospin structure of the amplitudes (e.g., each helicity amplitude or each multipole amplitude) for the four pion photoproduction processes is

$$\begin{aligned}
(\tfrac{1}{2})^{1/2}A(\gamma p\to \pi^+ n) &= A^0+\tfrac{1}{3}A^1-\tfrac{1}{3}A^3-A^T \\
&= {}_pA^{1/2}-\tfrac{1}{3}\,{}_pA^{3/2},\\
A(\gamma p\to \pi^0 p) &= A^0+\tfrac{1}{3}A^1+\tfrac{2}{3}A^3+2A^T={}_pA^{1/2}+\tfrac{2}{3}\,{}_pA^{3/2},\\
(\tfrac{1}{2})^{1/2}A(\gamma n\to \pi^- p) &= A^0-\tfrac{1}{3}A^1+\tfrac{1}{3}A^3-A^T \\
&= {}_nA^{1/2}+\tfrac{1}{3}\,{}_nA^{3/2},\\
A(\gamma n\to \pi^0 n) &= -A^0+\tfrac{1}{3}A^1+\tfrac{2}{3}A^3-2A^T \\
&= -{}_nA^{1/2}+\tfrac{2}{3}\,{}_nA^{3/2},
\end{aligned} \tag{1}$$

where A^0 and A^1 are, respectively, the isoscalar and isovector amplitudes leading to the $I=\frac{1}{2}$ final state, and A^3 and A^T are, respectively, the isovector and isotensor amplitudes for the $I=\frac{3}{2}$ final state. As one can see, the $A^{1/2}$ amplitudes for proton and neutron targets differ. If the isotensor amplitude A^T is *not* zero, then the $A^{3/2}$ amplitudes differ as well.

In principle, one needs detailed experiments on all four reactions to test the $\Delta I \leq 1$ rule. At present, in the first resonance region, data are available for only three reactions, $\gamma p\to \pi^+ n$, $\gamma p \to \pi^0 p$, and $\gamma n \to \pi^- p$. Hence, any claim for a violation of the $\Delta I \leq 1$ rule must use, in addition, some specific knowledge of the reaction mechanism. The assumption underlying the claim of Sanda and

Shaw[1,2] is that the P_{33} resonance (on a slowly varying background) dominates pion photoproduction at low energies. They then suggested that the total-cross-section difference

$$\Delta = (k/q)[\sigma_T(\gamma n \to \pi^- p) - \sigma_T(\gamma p \to \pi^+ n)]$$

would be slowly varying with energy in the absence of an isotensor contribution because the major effect of the P_{33} resonance, $|M_{1+}^3|^2$, would cancel out. Since the data gave an indication that Δ was a rapidly varying function of energy near the first resonance, Sanda and Shaw concluded that this was striking evidence for the presence of an isotensor term. They also verified this evidence against differential cross-section measurements.

The most recent analyses of pion photoproduction from protons will now be used to reconsider the π^- total and differential cross-section data.

A. The Multipoles for Pion Photoproduction from Protons

Recently, energy-independent multipole analyses for pion photoproduction from protons have been carried out by Noelle, Pfeil, and Schwela[6] and the present authors.[7] The latter analysis used essentially all the existing data from 165-MeV to 450-MeV photon lab energy, using the real parts of some of the multipole amplitudes as parameters and fixing the phases via the Watson theorem[8] (which assumes T invariance) to be the same as the πN phase shifts. In the fit the unvaried multipole amplitudes for $l \leq 3$ were the dispersion-theoretic predictions of Berends, Donnachie, and Weaver[9] and the higher partial waves were taken in the Born approximation. Comparing the results of Refs. 6 and 7 with dispersion-theoretic predictions gives the following estimates of deviations from predictions possible at some energies (for the s and p waves):

${}_pE_{0+}^{1/2}$ deviates 20%, ${}_pE_{0+}^{3/2}$ deviates 50%,

${}_pE_{1+}^{1/2}$ deviates 15%,

${}_pE_{1+}^{3/2}$ deviates above resonance,

${}_pM_{1+}^{1/2}$ deviates 20%,

${}_pM_{1+}^{3/2}$ deviates up to 15% below resonance,

${}_pM_{1-}^{1/2}$ deviates 100%, ${}_pM_{1-}^{3/2}$ deviates 50%.

To assess the seriousness of any discrepancies in the π^- photoproduction data, one can now use the proton-target data analysis to fix the $I=\frac{3}{2}$ multipole amplitudes (and errors), and to estimate the errors on the theoretical predictions[9] for the $I=\frac{1}{2}$ multipole amplitudes, and then to evaluate the cross sections for photoproduction from neutron targets. In this way one uses as much experimental knowledge as possible, and one does not rely completely on dispersion-theoretic predictions as in Refs. 1 and 2. Errors of 10% were assigned to the A^0 and A^1 isospin parts of the s- and p-wave multipole amplitudes, and the errors found from the fits were assigned to the $I=\frac{3}{2}$ parts of the s and p waves. In addition, the multipole amplitude M_{1-}^0 and M_{1-}^1 of the P_{11} state were taken at the Born approximation with 100% errors. This was done because our knowledge of these multipoles[10] indicates that the Ref. 9 predictions are too large. One can now calculate with these multipoles total and differential cross sections with probable errors.

B. Total Cross Sections

The experimental results of the ABBHHM (Aachen-Berlin-Bonn-Hamburg-Heidelberg-München) bubble-chamber collaboration, as reanalyzed by Butenschön,[11] are compared to the theoretical total cross sections in Table I. (The Frascati data[12] are not included in the comparison as they are still preliminary.) The theoretical values are 8% to 34% higher (21% on the average). This discrepancy may be due to experiment or theory. In this energy range one has no other type of total cross-section measurements. At higher energy, however, Scheffler and Walden[13] performed angular distribution measurements in a counter experiment, from which they computed total cross sections by Moravcsik fits. Since the bubble-chamber experiment also gives cross sections in the 660–1250-MeV range, one can there compare the two experiments. The counter experiment gives, on the whole, larger cross sections (on the average 21% higher).

TABLE I. The experimental and theoretical total cross sections for the reaction $\gamma n \to \pi^- p$. The data are from Ref. 11. The theory uses the multipoles of Refs. 7 and 9 as explained in Sec. II.

Photon lab energy (MeV)	σ (experiment) (μb)	σ (theory) (μb)
210	$143\ ^{+23.1}_{-19.5}$	179 ± 18
230	$146.9^{+20.3}_{-15.8}$	199 ± 20
250	$192.5^{+25.9}_{-19.8}$	234 ± 24
270	$229.5^{+29.3}_{-21.5}$	267 ± 27
290	$233.1^{+29.2}_{-21.1}$	292 ± 30
310	$230.5^{+29.0}_{-21.0}$	295 ± 30
330	$216.5^{+26.5}_{-18.7}$	280 ± 29
350	$203.9^{+27.9}_{-21.6}$	222 ± 23
370	$159.4^{+21.7}_{-16.8}$	195 ± 20
390	$134.9^{+19.4}_{-15.5}$	146 ± 15

An over-all normalization error of a fixed percentage on the $\gamma n \to \pi^- p$ total cross sections could obscure the test of Sanda and Shaw, since it would simulate a dip in Δ. On the theoretical side, it should also be kept in mind that besides the difference in smoothly varying backgrounds, Δ also contains the difference in magnetic dipole terms.

$$|_n M_{1+}|^2 - |_p M_{1+}|^2 = \tfrac{8}{3}|M_{1+}^1||M_{1+}^0| + \tfrac{8}{3}|M_{1+}^3||M_{1+}^0|\cos(\delta_{33}-\delta_{13}). \quad (2)$$

The term proportional to $|M_{1+}^3|$ has a rapid energy variation and causes the bump near 290 MeV in the dispersion-theory prediction of Δ given by Sanda and Shaw. Since $|M_{1+}^0|$ is small in this energy region (~6×10^{-4} from Ref. 9), inclusion of corrections from higher $I=\frac{1}{2}$ resonances with isoscalar components (e.g., F_{15}) could change the theoretical prediction for Δ considerably (e.g., making it dip), without affecting the good agreement now obtained with the proton-target experiments. However, the position of the bump or dip due to the second term in Eq. (2) cannot be changed appreciably and one should, therefore, know the energy to about 5 MeV.

In Fig. 1 is plotted the experimentally determined Δ, using the latest determination of the π^+ total cross sections,[7,14] and the theoretical predictions for Δ and the background for which the dispersion theory of Ref. 9 has been used. It is seen that the dip is a 1-standard-deviation effect based on two points. If one fits the differential cross-section data of Butenschön together with the backward measurements of Fujii *et al.*[15] (see below), one can, in fact, obtain different π^- total cross sections and eliminate the dip.

So, from the total cross-section measurements, one sees that there is a clear discrepancy between dispersion theory and experiment. The conclusion that the discrepancy is due to an isotensor term is based on two experimental points.

From a discussion of the differential cross sections, we shall see that alternative solutions are possible.

C. Differential Cross Sections

One may compare the data of Ref. 11 with some scattered points from π^-/π^+ ratio experiments.[16] Using the π^+ data fits from Ref. 7, one can evaluate the π^- points. In general, they are again higher than the ABBHHM data.[17]

Examples of the neutron-target photoproduction predictions are shown in Figs. 2-7. It is seen that the theoretically expected differential cross sections are somewhat higher than the deuterium data, but considering the errors the discrepancy is not serious. The asymmetry data point is in good agreement (Fig. 5). The recoil polarization in π^-

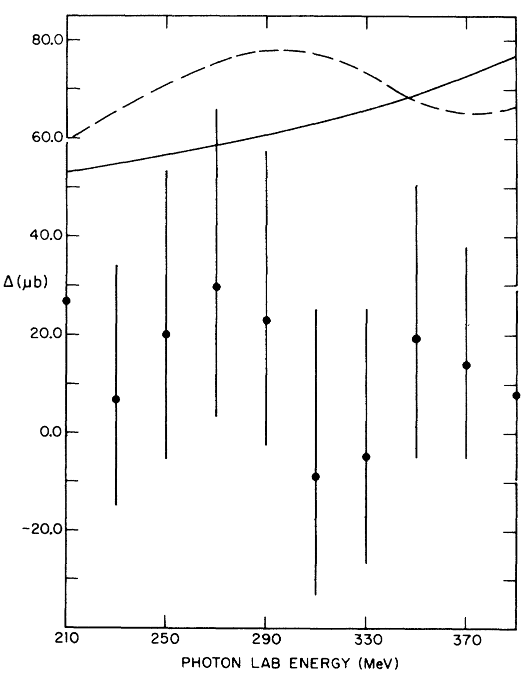

FIG. 1. The difference function Δ. Circles are the experimental points obtained from Refs. 7 and 11. The dashed curve is the dispersion-theoretic prediction (Ref. 7) for Δ, whereas the solid line gives Δ without the rapidly varying term in Eq. (2).

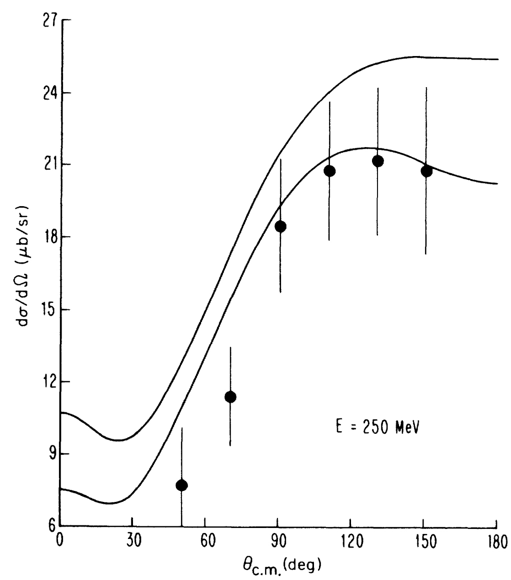

FIG. 2. Prediction for π^- differential cross section at photon lab energy of 250 MeV, using a dispersion-theoretic $I=\frac{1}{2}$ part and a phenomenological $I=\frac{3}{2}$ part. The two solid lines are the predictions with the multipole errors of Sec. II A.

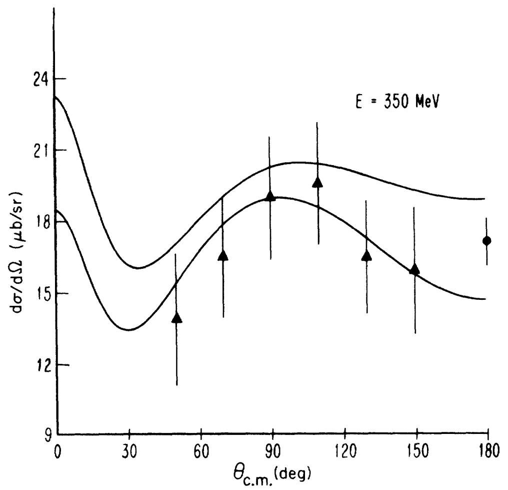

FIG. 3. Prediction for π^- differential cross section at photon lab energy of 350 MeV, using a dispersion-theoretic $I=\frac{1}{2}$ part and a phenomenological $I=\frac{3}{2}$ part. The two solid lines are the predictions with the multipole errors of Sec. II A.

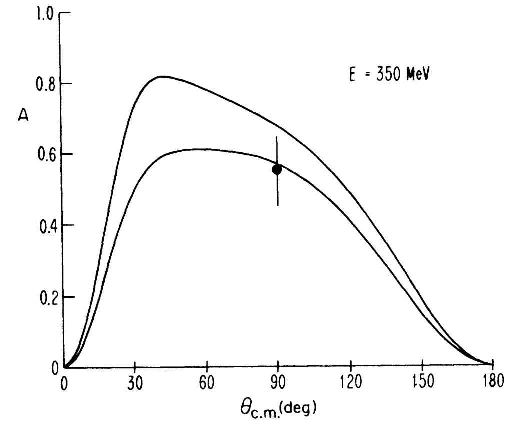

FIG. 5. Prediction for the asymmetry ratio A for polarized photons at photon lab energy of 350 MeV, using the same model as in Fig. 2.

photoproduction could also be measured as the polarized-target asymmetry in π^- radiative capture at one of the meson factories.[18]

If one wants to put the experimental and theoretical π^- photoproduction differential cross sections in closer agreement, one has three options:

(1) Renormalize the data upwards.

(2) Reduce the theoretical background terms, mainly ${}_nE_{0+}^{1/2}$ and ${}_nM_{1-}^{1/2}$.

(3) Decrease ${}_nM_{1+}^{3/2}$, the dominant resonant amplitude, by introducing an isotensor term.

The data would require about a 20% upward renormalization. For the second possibility, fits to the data of Refs. 11 and 15 were performed with ${}_nE_{0+}^{1/2}$ and ${}_nM_{1-}^{1/2}$ as variables. (The total cross sections obtained from this fit remove the dip in Δ.) The deviations between the theoretical and fitted values are shown in Table II. The deviations in ${}_nM_{1-}^{1/2}$ are not to be taken seriously since this multipole amplitude is hard to compute theoretically. On the other hand, the decrease in ${}_nE_{0+}^{1/2}$ would be more surprising since the theoretical values are known to give the correct threshold limit. Such deviation would indicate a steeper energy variation for the s wave in the threshold region than is customarily supposed.[19] The third

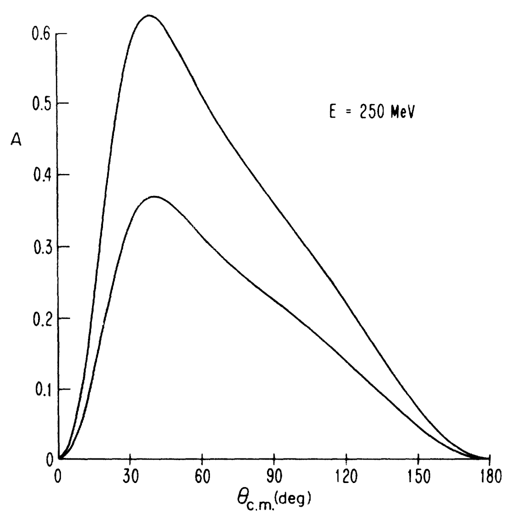

FIG. 4. Prediction for the asymmetry ratio A for polarized photons at photon lab energy of 250 MeV, using the same model as in Fig. 2.

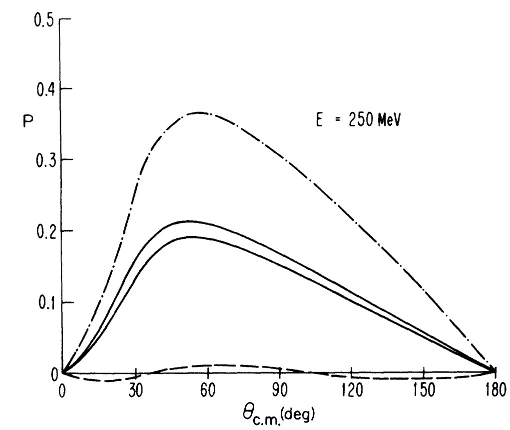

FIG. 6. Prediction for the recoil-proton polarization P at photon lab energy of 250 MeV, using the same model as in Fig. 2. Including a T-violating phase ($\tan^{-1}x=-20°$, same $|M_{1+}^{3/2}|$), one obtains the dashed curve for the recoil-proton polarization in $\gamma n\to\pi^- p$ and the dashed-dotted curve for the polarized-target asymmetry in $\pi^- p\to\gamma n$.

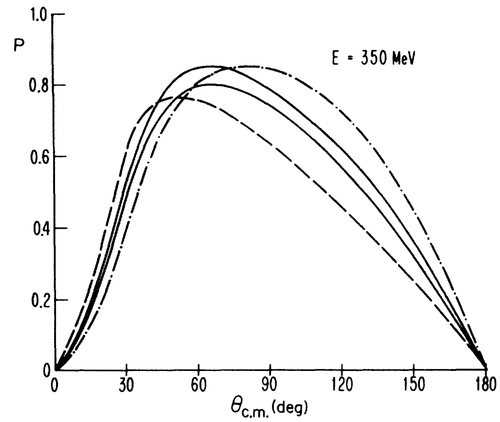

FIG. 7. Prediction for the recoil-proton polarization P at photon lab energy of 350 MeV, using the same model as in Fig. 2. Including a T-violating phase ($\tan^{-1}x = -20°$, same $|M_{1+}^{3/2}|$), one obtains the dashed curve for the recoil-proton polarization in $\gamma n \to \pi^- p$ and the dashed-dotted curve for the polarized-target asymmetry in $\pi^- p \to \gamma n$.

possibility gives decreases of 20% in $|_n M_{1+}^{3/2}|$, as suggested in Ref. 1.

An excellent test of the isotensor hypothesis is to measure the 90° excitation curve for $\gamma n \to \pi^0 n$, because options 1 and 2 predict much higher cross sections than option 3, the isotensor hypothesis. The expected excitation curves are shown in Fig. 8.

Total cross-section measurements of $\gamma n \to \pi^0 n$ would also be extremely helpful. If one forms the difference function

$$\Delta_T = \sigma(\gamma n \to \pi^- p) + \sigma(\gamma n \to \pi^0 n) - \sigma(\gamma p \to \pi^+ n) - \sigma(\gamma p \to \pi^0 p),$$

the term proportional to $|M_{1+}^3|$ vanishes and, in the absence of an isotensor term, only a smoothly varying background remains.[20]

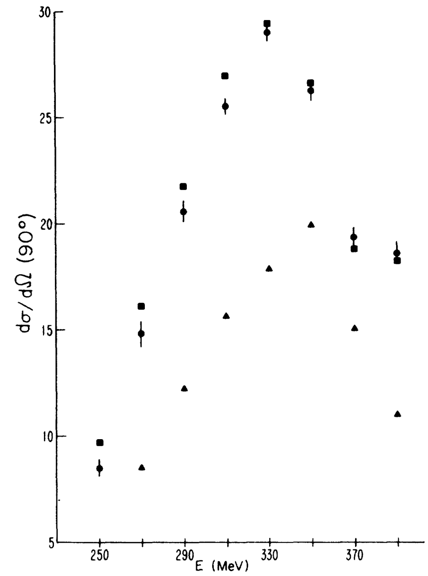

FIG. 8. The 90° excitation curve for π^0 photoproduction off neutrons. The circles are theoretical predictions based on dispersion theory for the $I=\frac{1}{2}$ part and phenomenological fits for the $I=\frac{3}{2}$ part. The squares are predictions from the $_nE_{0+}^{1/2}, {}_nM_{1-}^{1/2}$ fit to the data of Ref. 11. The triangles are predictions from the isotensor fit to the data of Ref. 11.

III. T INVARIANCE

Measurements of the inverse reaction $\pi^- p \to \gamma n$ have recently been performed by Berardo *et al.*[3] and Favier *et al.*[21] The former experiments measured angular distributions at two energies and the latter an excitation curve for center-of-mass angles near 30°. The angular distribution at 354-MeV photon lab energy shows a discrepancy with respect to the measurements[11,15] of the direct re-

TABLE II. A comparison of some two-parameter ($_nE_{0+}^{1/2}, {}_nM_{1-}^{1/2}$) fits to the differential cross sections of Refs. 11 and 15 with dispersion-theoretic predictions from Ref. 9. The Born approximation for $_nM_{1-}^{1/2}$ is also shown. The units are $\hbar = \mu = c = 1$.

Photon lab energy (MeV)	$10^3 \times {}_nE_{0+}^{1/2}$		$10^3 \times {}_nM_{1-}^{1/2}$		
	Theory	Fit	Theory	Born	Fit
210	−12.6	−5.71 ± 1.16	−2.21	−1.21	−7.98 ± 1.21
230	−11.9	−4.45 ± 3.18	−2.47	−1.21	−7.16 ± 3.57
250	−11.3	−5.80 ± 2.64	−2.69	−1.17	−5.98 ± 3.47
270	−10.7	−5.77 ± 1.93	−2.90	−1.10	−6.10 ± 2.79
290	−10.2	−3.65 ± 1.99	−3.11	−1.01	−4.10 ± 2.96
310	−9.74	−4.14 ± 1.94	−3.34	−0.90	−6.20 ± 2.12
330	−9.30	−4.65 ± 1.68	−3.61	−0.79	−4.60 ± 1.74
350	−8.88	−6.02 ± 2.24	−3.91	−0.67	−3.60 ± 2.15
370	−8.40	−2.48 ± 2.44	−4.26	−0.54	−5.08 ± 2.59
390	−8.11	−5.22 ± 3.16	−4.67	−0.42	−4.46 ± 3.32

action (see Fig. 9); whereas at 480 MeV the direct and inverse reactions are in reasonable agreement.

It is not possible, at present, to make an unambiguous interpretation of the discrepancy at 354 MeV. First, as seen in Sec. II C, the interpretation of the direct reaction has several options. Second, it is crucial to make a comparison of the two reactions at the same energy. The energy determinations of the inverse reaction and direct reaction have uncertainties of 6 MeV and at least 10 MeV, respectively. In order to demonstrate the importance of a comparison at the same energy, the experimental asymmetry ratio (as obtained in Ref. 4), i.e., the difference between the differential cross sections for the direct and inverse reactions divided by their sum,[22] is shown in Fig. 10 along with a fake ratio made by taking dispersion-theoretic predictions[7,9] at 340 MeV and 360 MeV as the direct and inverse reactions, respectively. In the remainder of this section the energy determinations are assumed to be correct.

A detailed discussion of the introduction of T noninvariance into photopion production via the electromagnetic current has been given by Christ and Lee,[22] who derived results for the general many-channel case and then specialized to photoproduction near the first resonance. By applying the familiar derivation of the Watson theorem,[8] separately to the T-invariant and T-noninvariant parts of the transition matrix, one easily rederives their results as shown below. Elastic unitarity, using states with definite angular momentum, parity, and isospin, gives for photopion production the relation

$$\langle \pi N_2|T|\gamma N_1\rangle - \langle \gamma N_1|T|\pi N_2\rangle^* = i\langle \pi N|T|\pi N_2\rangle^* \langle \pi N|T|\gamma N_1\rangle, \tag{3}$$

where * indicates complex conjugate. The transition operator can be decomposed into

$$T = T_1 + T_2, \tag{4}$$

where T_1 has the usual T-conserving behavior and T_2 the opposite behavior. T_1 derives from the hadronic electromagnetic current J_μ, and T_2 from K_μ, where under charge conjugation[5]

$$CJ_\mu C^{-1} = -J_\mu, \quad CK_\mu C^{-1} = K_\mu. \tag{5}$$

If Eq. (3) is applied to T_1 and its time-reversal behavior is used, one finds

$$2i\,\mathrm{Im}\langle \pi N_2|T_1|\gamma N_1\rangle = i\langle \pi N|T|\pi N_2\rangle^* \langle \pi N_1|T_1|\gamma N_1\rangle, \tag{6}$$

so for a photoproduction multipole amplitude $M(1)$ resulting from the matrix element of T_1, one obtains

$$M(1) = \pm|M(1)|e^{i\delta}, \tag{7}$$

where δ is the πN scattering phase shift for the same final state. For T_2 the left-hand side of Eq. (6) becomes twice the real part of $\langle \pi N_2|T_2|\gamma N_1\rangle$, and so for the same multipole amplitude as in Eq. (7), one obtains for the T-violating part $M(2)$ the relation

$$M(2) = \pm i|M(2)|e^{i\delta}. \tag{8}$$

One has for every multipole $M = M(1) + M(2)$. For example, the $M_{1+}^{3/2}$ multipole [assuming a plus sign in Eq. (7) and a minus sign in Eq. (8)] for the direct

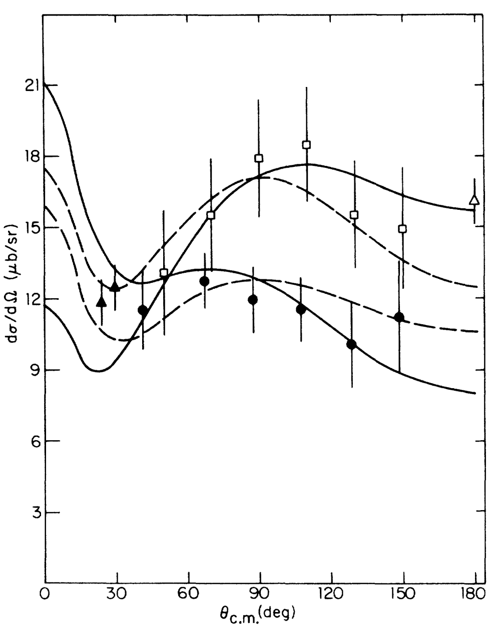

FIG. 9. The direct and inverse reactions at $E = 354$ MeV (□ Ref. 11, △ Ref. 15, ● Ref. 3, and ▲ Ref. 21). The curves give the predictions as discussed in the text; solid curves for $\tan^{-1}x = -20°$, dashed curves for $\tan^{-1}y = 20°$.

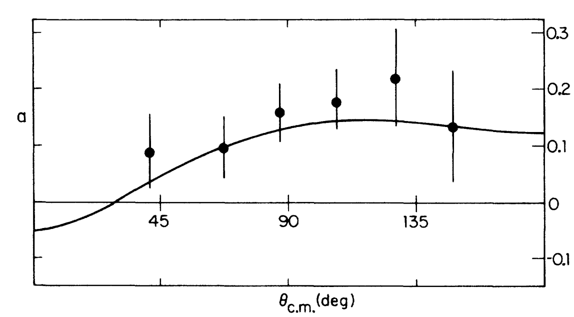

FIG. 10. The experimental T-violation asymmetry function (from Ref. 4). The curve is a faked asymmetry function caused by an energy difference, using the same multipoles as in Fig. 2.

and inverse reactions is given by

$$\gamma N_1 \to \pi N_2: \quad M_{1+}^{3/2} = [\,|M_{1+}^{3/2}(1)| - i\,|M_{1+}^{3/2}(2)|\,]e^{i\delta_{33}},$$
$$\pi N_2 \to \gamma N_1: \quad M_{1+}^{3/2} = [\,|M_{1+}^{3/2}(1)| + i\,|M_{1+}^{3/2}(2)|\,]e^{i\delta_{33}}. \tag{9}$$

Letting $x = -|M_{1+}^{3/2}(2)|/|M_{1+}^{3/2}(1)|$, one can rewrite Eq. (9) as [x is defined to absorb the ± sign between $M(2)$ and $M(1)$ in Eqs. (7) and (8)]

$$\gamma N_1 \to \pi N_2: \quad M_{1+}^{3/2} = |M_{1+}^{3/2}(1)|(1+x^2)^{1/2}\exp[i(\delta_{33}+\tan^{-1}x)],$$
$$\pi N_2 \to \gamma N_1: \quad M_{1+}^{3/2} = |M_{1+}^{3/2}(1)|(1+x^2)^{1/2}\exp[i(\delta_{33}-\tan^{-1}x)], \tag{10}$$

and similarly for any other multipole amplitude. From Eq. (10) one notices that the absolute value of a particular isospin (0), (1), (3), or (T) multipole is the same for the direct and inverse reactions, even including T-violating effects in the above way. Of course, T violation in the above way makes the magnitudes of the isospin combination for the direct and inverse reactions, in general, not equal for a particular multipole. The total cross section for the direct and inverse reactions can, therefore, be different, and the present data indicate this to be the case at 354 MeV, but not at 480 MeV.

That the total cross section is affected only near the resonance and not elsewhere may be understood from Eq. (10). Assuming the usual isospin properties, the full M_{1+} multipole for the direct and inverse reactions may be written

$$\gamma n \to \pi^- p: \quad M_{1+} = \sqrt{2}\,|{}_nM_{1+}^{1/2}|e^{i\phi}\left[1 + \frac{1}{3}\frac{|{}_nM_{1+}^{3/2}|}{|{}_nM_{1+}^{1/2}|}e^{i(\psi-\phi)}\right],$$
$$\pi^- p \to \gamma n: \quad M_{1+} = \sqrt{2}\,|{}_nM_{1+}^{1/2}|e^{i\phi'}\left[1 + \frac{1}{3}\frac{|{}_nM_{1+}^{3/2}|}{|{}_nM_{1+}^{1/2}|}e^{i(\psi'-\phi')}\right], \tag{11}$$

where from Eqs. (10) $\psi = \delta_{33} + \tan^{-1}x$, $\psi' = \delta_{33} - \tan^{-1}x$, and the phases ϕ and ϕ' are similarly defined with the δ_{13} phase shift, say $\delta_{13} \pm \tan^{-1}y$. Using dispersion theory as a rough guide, the magnitudes of the M_{1+} multipoles in Eqs. (11) are approximately proportional to $[1+\cos(\psi-\phi)]^{1/2}$ and $[1+\cos(\psi'-\phi')]^{1/2}$, so the T-violating phases $\tan^{-1}x$ and $\tan^{-1}y$ affect $|M_{1+}|$ most strongly when $\delta_{33} - \delta_{13} = 90°$, which occurs near the resonance because $\delta_{13} \cong -2°$. Thus, near the resonance $|M_{1+}|$ in the direct and inverse reactions may be quite different. The same argument does not hold for E_{1+} since $|E_{1+}^{3/2}|$ is small near the resonance. Away from the resonance the difference in $|M_{1+}|$ between direct and inverse reactions will decrease. Whether this difference becomes negligible depends on the sizes of $\tan^{-1}x$ and $\tan^{-1}y$.

The reasonable agreement between the direct and inverse reactions at 480 MeV indicates that no T-violating phase affects the cross section, and that T violation in partial waves other than M_{1+} need not be discussed at present.

A discussion will now be made of the possible values of $\tan^{-1}x$ and $\tan^{-1}y$ allowed by the present data. This discussion will still be qualitatively correct if the data are renormalized (one of the options proposed in Sec. II C). In that case there will be a discrepancy at 480 MeV, leading one to consider T-violating effects in other partial waves at that energy and near the resonance (accounting for part of the now larger discrepancy). Therefore, the effect required from $\tan^{-1}x$ and $\tan^{-1}y$ need not be increased.

The magnitudes of M_{1+} in direct and inverse reactions depends on $\tan^{-1}x - \tan^{-1}y$, whereas the phases are functions of both T-violating angles separately. Changing the phases affects interference terms in the cross sections which show up mainly near the forward and backward directions.

To determine $\tan^{-1}x$ and $\tan^{-1}y$, ${}_nM_{1-}^{1/2}$ and ${}_nE_{0+}^{1/2}$ were determined from fits to the data for the direct and inverse reactions for a large range of fixed $|{}_nM_{1+}^{3/2}|$ and x and y values. The most satisfactory result was obtained for $|{}_nM_{1+}^{3/2}| = 0.9\,|{}_pM_{1+}^{3/2}|$, $\tan^{-1}x = -14°$, $\tan^{-1}y = 6°$, ${}_nE_{0+}^{1/2} = -6.00\times10^{-3}$, and ${}_nM_{1-}^{1/2} = -1.00\times10^{-3}$; the remaining multipoles being the combination of dispersion-theoretic and phenomenological multipoles used in Sec. II.

It was found to be quite satisfactory to blame all T violation on the $I=\frac{3}{2}$ part of M_{1+}, but fits are less convincing with only a T-violating $I=\frac{1}{2}$ part (see Fig. 9). The above results show that there is considerable freedom in distributing the T violating phases over the ${}_nM_{1+}^{1/2}$ and ${}_nM_{1+}^{3/2}$ multipoles.

Constraints on this freedom are imposed by the proton data. Variation of the phases without destroying the possibility of good fits to the proton data is allowed, e.g., the P_{33} phase in the analyses of Refs. 6 and 7 differ sometimes by 7°. On the whole it is easier to move the phase of ${}_pM_{1+}^{3/2}$ upward than downward with respect to the P_{33} phase of Ref. 7.

Since ${}_nM_{1+}^{1/2}$ consists of two parts, the T-violating phase may arise from either or both of the two terms. Depending on this the T-violating phases of ${}_pM_{1+}^{1/2}$ will be different, i.e., the phase may be moved upward or downward.[23] If there is no isotensor term, the phase of ${}_pM_{1+}^{3/2}$ is fixed once the phase of ${}_nM_{1+}^{3/2}$ is determined, so it has to move downward (maximally by 20°). If one ascribes T

violation exclusively to an isotensor part,[24] the phase moves upward (by less than 20°) and moreover $|_pM_{1+}^{3/2}|>|_nM_{1+}^{3/2}|$.

The last possibility is certainly the easiest to introduce. The most difficult option is to assume no isotensor term and to ascribe the full T-violating phase of $-20°$ exclusively to M_{1+}^3. Even in this case it is possible to fit the proton data at 354 MeV (as extrapolated from 350 MeV), when using the above-mentioned reduced value for $_pM_{1+}^{3/2}$. Of course, the values for the other multipoles are considerably different from the ones found in Ref. 7, as were the recoil-proton polarization predictions. The χ^2 is, however, better. So the reduced value of $_nM_{1+}^{3/2}$ does not necessarily mean an isotensor term in the T-violating case. When keeping the T-violating phase constant and using a reduced $_pM_{1+}^{3/2}$ also at neighboring energies, it does not seem possible to find fits statistically as acceptable as the ones of Ref. 7. Of course, a variation of the T-violating phase may occur, but with radiative-capture data at only one energy near the resonance this cannot be settled.

Extension of the data of Ref. 21 to other angles would alleviate this situation, but at the moment their data are just near 30°, which is lower than the data of Ref. 11. The sensitivity to interference effects is particularly large in the forward region[25] with some T-violating fits crossing over here and others not crossing over (e.g., see Fig. 9). Therefore, one also needs to know the direct reaction at these forward angles in order to make full use of the present data. The present π^- data leave too much freedom to pin down the isospin character of the T violation.

Other experimental quantities like the asymmetry ratio for linearly polarized photon cross sections and recoil-proton polarization provide additional useful constraints on the multipole magnitudes and phases. In particular, the latter quantity can be measured in the direct reaction and as the polarized-target asymmetry in the inverse reaction. If there is no T violation, the two measurements should be equal in size. In the presence of T violation, effects as indicated in Figs. 6 and 7 may arise. Unfortunately, rather precise measurements are necessary, particularly near the resonance.

In addition, the simplicity of having only one or two T-violating phases changes if the data for the direct reaction must be renormalized as discussed in Sec. II. It is, then, likely that the value of $|_nM_{1+}^{3/2}|$ required would increase, since it is in some sense an "average" between direct and inverse reactions, and the introduction of an isotensor term would be unnecessary.

IV. CONCLUSIONS

There are discrepancies between the present $\gamma n\to\pi^-p$ data and our estimates, which were obtained by using partly phenomenological information from the proton-target data and partly dispersion theory (and always assuming T invariance).

These discrepancies can be removed by (1) renormalized data or by (2) modifying the estimates either by changing some background multipoles, or by introducing an isotensor term. Precise π^-/π^+ experiments could provide information on the quality of the data; measurement of an excitation curve for $\gamma n\to\pi^0 n$ (e.g., also via ratios, cf. Ref. 26) can decide between the isotensor option and the other two.

Settlement of the above question is required for an interpretation of the apparent T violation seen by comparing π^- photoproduction and π^- radiative capture. The size of this discrepancy and its statistical relevance can be established only by having accurate data on both reactions at the *same* energy. For its interpretation, it is essential to have *full* angular distributions at several energies near the first resonance. In particular, forward and backward points give severe constraints.

Note added in proof. In Contribution No. 298 to the 1971 International Symposium on Electron and Photon Interactions at High Energies, the ABBHHM collaboration reported new results (using an improved method of extracting the neutron target data) based on three times the number of events of their previous analyses.[11] The new data are higher than the previous results and do not need an isotensor current. In order to settle the isotensor question beyond doubt, precise π^-/π^+ and π^0 ratio experiments should be performed.

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Hadron Momentum Distribution in Deeply Inelastic *ep* Collisions*

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Extending previous speculations originating from the idea that a hadron is a spatially extended object with many internal degrees of freedom, we argue that in *ep* collisions in the deeply inelastic region, the proton partially fragments and partially pulverizes. The usefulness of studying experimentally the single-hadron spectrum for fixed and large q^2 and ν is discussed.

I. INTRODUCTION

This paper describes some speculations concerning the momentum distribution of the hadrons in the reaction

$$e+p\to e+\text{hadrons}. \qquad (1)$$

We concentrate especially on the distribution in the Bjorken limit,[1,2] $X=q^2/2M\nu$ fixed, $q^2\to\infty$. These speculations are made as extensions of the idea that a hadron is a spatially extended object with many internal degrees of freedom. In this sense, these speculations represent a continuation of previously discussed ideas about elastic scattering,[3] about processes[4] $ab\to cd$, and about the hypothesis of limiting fragmentation.[5]

In Sec. II, we shall describe the details of the speculation, leaving the arguments in support of such speculations for Sec. III. The rest of the paper consists of additional remarks and possible experimental tests.

It will be evident that our arguments are based on imprecise extrapolations of known physical concepts. This is contrary to a contemporary