vector form factor $f_+(s)$ and $\lambda_+$ is determined to be close to the $K^*$-dominance result. Our calculation is performed with mesons on the mass shell so that meson dominance should be good. The Callan-Treiman expressions for the sum and difference of the form factors follow in our approach at the proper off-shell points. Although we have used PCAC in the form of Eq. (15) for the pion, a similar form for the kaon has not been adopted in anticipation of correction to the model of Gell-Mann, Oakes, and Renner. The commutation relation (14) is the assumption and is true for extended models including the addition of the (1, 8) and (8, 1) representations to the symmetry-breaking Hamiltonian. In comparison with other current-algebra calculations, we agree with their results if the last term is neglected. This last term, however, is expected to be significant if $b = c = \epsilon_8/\epsilon_0$. By choosing $b$, we can achieve agreement with experiment.

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# Predictions on $\pi^- + p \to \rho^0 + n$ and Some Related Processes Using Vector-Meson Dominance*†

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In the usual application of vector-meson dominance to analyze the process $\pi^- + p \to \rho^0 + n$, only the amplitudes for transversely polarized $\rho$ are related to single-pion photoproduction. In this paper, it is shown that both the longitudinal and the transverse amplitudes for the process $\pi^- + p \to \rho^0 + n$ can be obtained from single-pion photoproduction amplitudes by assuming that the off-shell Ball amplitudes not only satisfy the constraints imposed by current conservation, but also are smooth in the vector-meson mass. The smoothness assumption is discussed in particle-exchange models in detail. We also extend our predictions to somewhat larger $|t|$ than in our previous work, and comparison is made with recent 15-GeV SLAC data on $\pi^- + p \to \rho^0 + n$. We also apply the same assumptions to analyze some related processes such as the electroproduction of a charged pion.

## I. INTRODUCTION

Part of the objective of the vector-meson-dominance model (VMD)[1] is to relate processes involving $\rho$ mesons to processes involving isovector photons. The idea of VMD is most easily understood in a theory in which the $\rho$ meson and the isovector part of the photon are coupled to the same conserved source.[2] However, since the $\rho$ is massive while the photon is massless, in practical applications of VMD one must, at one stage or another, assume smoothness of amplitudes in the vector-meson mass. Since the photon has only transverse polarization, in the usual applications of VMD only the transversely polarized $\rho$ ($\lambda = \pm 1$) amplitudes in the reaction $\pi^- + p \to \rho^0 + n$ are related to the photoproduction amplitudes $\gamma_V + p \to \pi^+ + n$, where $\gamma_V$ stands for isovector photon.[3] In these relations, it is assumed that the transverse helicity amplitudes for the $\rho$ production process are smooth in the vector-meson mass, and are related to the pion photoproduction amplitudes by a constant numerical factor obtainable from other experiments, e.g., $e^+e^- \to \rho^0$. Also, for a spin-1 object, the concept of transverse polarization is not Lorentz-invariant, and a question arises as to which frame we ought to use in testing various VMD relations.[4] This problem, together with the early predictions on $\pi^- + p \to \rho^0 + n$ using VMD, is briefly reviewed in Sec. II.

In the early development of the vector-meson-dominance model, it was speculated that the electromagnetic form factor of the $\pi^+$ may satisfy an unsubtracted dispersion relation,[5] and the vector-meson dominance of the electromagnetic form factor immediately led to the idea of vector-meson universality and conserved currents in a strong-interaction theory.[6] So one way to formulate the theory of VMD is to select a set of invariant amplitudes which are assumed to satisfy dispersion relations written in the vector-meson mass. After all, there is no more reason to assume the helicity amplitudes to be smooth in the vector-meson mass than to assume the invariant amplitudes to be smooth in the vector-meson mass. One advantage in using invariant amplitudes instead of helicity amplitudes is that unlike helicity amplitudes, invariant amplitudes are Lorentz scalars, and so the procedure is manifestly Lorentz-covariant. Furthermore, if one assumes that the invariant amplitudes not only satisfy the constraints imposed by current conservation, but also are smooth in the vector-meson mass, one in general obtains more information than that obtained from applying the smoothness assumption to the helicity amplitudes. In the case of $\pi^- + p \to \rho^0 + n$, one finds that the longitudinally polarized $\rho$ amplitudes can be related to the transversely polarized $\rho$ amplitudes. All this is illustrated in Sec. III using spinless nucleons for pedagogical purposes.

There are various sets of invariant amplitudes one can construct for $\pi^- + p \to \rho^0 + n$. However, the choice of invariant amplitudes is restricted when one considers only amplitudes that satisfy the Mandelstam representation and are free of kinematic singularities. A set of amplitudes that satisfy these criteria was given by Ball.[7,8]

Recently, predictions on $\pi^- + p \to \rho^0 + n$ were made independently by Achasov and Shestakov,[9] and by Sakurai and the present author[10] by assuming that the off-shell Ball amplitudes not only satisfy the constraints imposed by current conservation, but also are smooth in the vector-meson mass. However, the approaches employed by the former authors[9] are quite different from ours.[10] The main predictions of the former authors come from combining the "smoothness" assumption with a pure Regge-pole model, whereas we took advantage of the finite-energy sum-rule evaluation of the photoproduction amplitudes[11-13] and the pseudomodel of Jackson and Quigg for pion photoproduction[13]; as a result, the two predictions are quite different from each other. Our approach is studied in detail in Sec. IV, with the value of $|t|$ extended from the 0.12 $(\mathrm{GeV}/c)^2$ of our previous work[10] to 0.25 $(\mathrm{GeV}/c)^2$, and comparison with the recent 15-GeV/$c$ $\pi^- + p \to \rho^0 + n$ experiment is also made. Our model has no adjustable parameter, yet the calculated cross sections and density-matrix elements are in reasonable agreement with experiment.

In Sec. V we discuss our assumption in particle-exchange models in detail. It is found that although the assumption is satisfied in the electric Born model (one-pion exchange plus nucleon pole terms with a $\gamma_\mu$-type $\rho\bar{N}N$ coupling), it is not always valid in particle-exchange models. On the other hand, pure particle-exchange models have difficulty in analyzing photoproduction data,[14] and so the fact that they do not support our assumption need not be used as a serious objection to our model. In any case, we feel that this is an interesting way of making new predictions on $\pi^- + p \to \rho^0 + n$, and the justification of the assumption may rest on the comparison with experiment.

In Sec. VI we use the same assumption to study some related processes. Specifically, we study the electroproduction of pions and the $K^- + p \to \omega + \Lambda$ process. In Sec. VII we summarize our results.

## II. OLD PREDICTIONS ON $\pi^- + p \to \rho^0 + n$ USING VMD

We review here some early predictions on $\pi^- + p \to \rho^0 + n$ using VMD. These have been discussed quite extensively in the literature.[15] So our presentation here will be brief.

The vector-meson-dominance model can be used to relate the reaction

$$\pi^- + p \to \rho^0 + n \tag{1}$$

to the reaction

$$\gamma_V + p \to \pi^+ + n, \qquad (2)$$

where $\gamma_V$ stands for the isovector photon.[16] For the convenience of later discussions, we first define some quantities:

$k = \rho$ momentum in (1), photon momentum in (2).

$q$ = pion momentum.

$p'$ = initial nucleon momentum in (1), final nucleon momentum in (2).

$p$ = final nucleon momentum in (1), initial nucleon momentum in (2).

$P = \frac{1}{2}(p+p')$.

$\epsilon$ = polarization vector of $\rho$ in (1), of photon in (2).

$\lambda$ = polarization state of $\rho$ in (1), of photon in (2).

$\lambda_N'$ = polarization state of initial nucleon in (1), of final nucleon in (2).

$\lambda_N$ = polarization state of final nucleon in (1), of initial nucleon in (2).

$m$, $\mu$, $m_\rho$ = masses of $N$, $\pi$, and $\rho$, respectively.

$\frac{d\sigma(\rho)}{dt}$ = differential cross section for (1).

$\frac{d\sigma(\gamma_V)}{dt}$ = differential cross section for (2).

$\frac{d\sigma^{(\perp)}(\gamma_V)}{dt}, \frac{d\sigma^{(\parallel)}(\gamma_V)}{dt}$ = differential cross section for (2) with the photon polarized perpendicular and parallel to the scattering plane, respectively.

$\rho_{\lambda\lambda'}$ = elements of the $\rho$-meson density matrix. (Throughout this paper, $\rho_{\lambda\lambda'}^{(H)}$ and $\rho_{\lambda\lambda'}^{(J)}$ stand for the density matrix in the helicity frame and the Gottfried-Jackson frame, respectively.)

In the usual application of vector-meson dominance, only the production amplitudes for transversely polarized $\rho$ ($\lambda = \pm 1$) in (1) are compared to reaction (2), viz.,[3,15]

$$\rho_{11}\frac{d\sigma(\rho)}{dt} = \left[\frac{f_\rho}{e}\right]^2 \frac{|\vec{k}_\gamma|^2}{|\vec{q}|^2} \frac{d\sigma(\gamma_V)}{dt}, \qquad (3)$$

$$\frac{d\sigma^{(\perp)}(\gamma_V)/dt - d\sigma^{(\parallel)}(\gamma_V)/dt}{d\sigma^{(\perp)}(\gamma_V)/dt + d\sigma^{(\parallel)}(\gamma_V)/dt} = \frac{\rho_{1-1}}{\rho_{11}}, \qquad (4)$$

$$(\rho_{11} + \rho_{1-1})\frac{d\sigma(\rho)}{dt} = \left[\frac{f_\rho}{e}\right]^2 \frac{|\vec{k}_\gamma|^2}{|\vec{q}|^2} \frac{d\sigma^{(\perp)}(\gamma_V)}{dt}. \qquad (5)$$

In these relations, it is assumed that aside from the coupling constant (normalized as in Ref. 6), the helicity amplitudes of the transversely polarized $\rho$ are independent of the vector-meson mass. Equation (3) is satisfied quite well by experiment when $\rho_{11}$ is measured in the helicity frame[3,15]; however, Eqs. (4) and (5) seem not to be supported by the 4-GeV/$c$ experiment,[17,15] although interpretation of the experimental data is not yet conclusive.[18,19] There was also the question as to which frame we ought to use in testing these VMD relations, since the concept of transverse polarization is not Lorentz-invariant for a massive spin-1 object.[4] It was pointed out that Eq. (4) is satisfied by experiment if $\rho_{\lambda\lambda'}$ is measured in the Donohue-Högaasen (DH) frame.[4] However, most theoretical arguments seem to favor the helicity frame as the one in which to apply VMD.[20,21] As will be shown later, our model will also lead to the helicity frame as the one to use. In any case, Eq. (5) is the same for the helicity frame and the DH frame, and it is still not supported by experiment. On the other hand, it was argued[18] that higher partial waves in pion-pion scattering may be needed to fit the angular distribution of pion pairs in $\pi^- + p \to \pi^+ + \pi^- + n$ in the $\rho$ region, and more careful analysis of the data may help bring Eqs. (4) and (5) into better agreement with experiment.[22,23]

Equations (3), (4), and (5) involve only the transversely polarized $\rho$. However, if one writes down the invariant amplitudes for processes (1) and (2) with arbitrary vector-meson mass, and VMD is formulated by assuming these amplitudes to be smooth in the vector-meson mass,[24] more relations can be obtained. One finds, in fact, that the longitudinally polarized $\rho$ amplitudes can be expressed in terms of the transversely polarized $\rho$ amplitudes.[10] The basic idea is illustrated in Sec. III using scalar pion and nucleons.

## III. SIMPLE MODEL

Consider reaction (1) with scalar pion and nucleons. We have three independent momenta, and the amplitude for the process can be written as

$$M_\lambda = \epsilon_\mu^{(\lambda)}(c_1 p^\mu + c_2 q^\mu + c_3 k^\mu). \qquad (6)$$

The $c_i$'s are the kinematic-singularity-free[25] invariant amplitudes for this process, and are in general functions of $s$, $t$, and the square of the vector-meson mass, $k^2$. The $s$-channel helicity amplitudes can now be expressed in terms of the $c_i$'s. At high $s$ and small $\theta$ ($\theta$ = the angle between $\vec{p}$ and $\vec{p}'$ in the $s$-channel center-of-mass system) such that the approximation

$$t = -\tfrac{1}{4}s\sin^2\theta \qquad (7)$$

is valid, we have

$$M_1 = (-\tfrac{1}{2}t)^{1/2}c_2, \qquad (8a)$$

$$M_0 = c_1 s/2m_\rho - c_2(t - \mu^2 + k^2)/2m_\rho. \qquad (8b)$$

Now the current-conservation relation reads

$$\tfrac{1}{2}(s - m^2 - k^2)c_1 - \tfrac{1}{2}(t - \mu^2 - k^2)c_2 + k^2 c_3 = 0 \qquad (9)$$

($k^2 = m_\rho{}^2$ for a $\rho$ on the mass shell). Using Eq. (9), Eq. (8b) can be written as

$$M_0 = -(c_2 + c_3) m_\rho . \qquad (10)$$

Note that $M_0 \to 0$ as $m_\rho \to 0$, as it must.

If we assume that the transverse helicity amplitude is smooth in the vector-meson mass, no relation is obtained between the transverse amplitude $M_1$ and the longitudinal amplitude $M_0$. However, if one considers the invariant amplitudes $c_i$ to be smooth in the vector-meson mass $k^2$, Eq. (9) actually gives two independent equations,

$$c_1(s - m^2) - c_2(t - \mu^2) = 0, \qquad (11a)$$

$$-\tfrac{1}{2}c_1 + \tfrac{1}{2}c_2 + c_3 = 0. \qquad (11b)$$

Equation (11a) is the usual gauge-invariance equation at $k^2 = 0$. This means that among the three amplitudes, only one is independent. We can therefore express the longitudinal amplitude $M_0$ in terms of the transverse amplitude $M_1$ as follows:

$$M_0 = -(-2t)^{-1/2} m_\rho M_1 . \qquad (12)$$

At first glance, $M_0$ seems to diverge at $t = 0$. But $M_1$ actually goes like $(-t)^{1/2}$, as can be seen from Eq. (8a), keeping in mind that the $c_i$'s are free of kinematic singularities.[25] Note also that by assuming only $c_2$ to be smooth in the vector-meson mass, one is immediately led to the conclusion that the VMD relations (3) and (4) should be written in the helicity frame, and all the experimental successes and failures of Eqs. (3), (4), and (5) discussed in Sec. II remain in our model. We have discussed the idealized world of spinless nucleons in this section. In the next section we shall discuss the realistic spin-$\frac{1}{2}$ nucleon case, and make a comparison of our model with experiment.

## IV. NEW PREDICTIONS ON $\pi^- + p \to \rho^0 + n$ USING VMD

For realistic (spin-$\frac{1}{2}$) nucleons, the amplitudes for the process $\pi^- + p \to \rho^0 + n$ can be written in terms of eight Ball amplitudes[7] defined below:

$$\begin{aligned} M_{\lambda_N';\lambda\lambda_N} = \bar{u}(p', \lambda_N')\gamma_5[ & B_1(\gamma\cdot\epsilon)(\gamma\cdot k) + 2B_2(\epsilon\cdot P) \\ & + 2B_3(\epsilon\cdot q) + 2B_4(\epsilon\cdot k) - B_5(\gamma\cdot\epsilon) \\ & + B_6(\epsilon\cdot P)(\gamma\cdot k) + B_7(\epsilon\cdot k)(\gamma\cdot k) \\ & + B_8(\epsilon\cdot q)(\gamma\cdot k)]u(p, \lambda_N). \end{aligned} \qquad (13)$$

Instead of the one current-conservation relation given in Eq. (9) for the scalar-nucleon case, we now have two. We have

$$\begin{aligned} & k^2 B_1 + sB_2 - (t - k^2 - \mu^2)B_3 + 2k^2 B_4 = 0, \\ & -2B_5 + sB_6 + 2k^2 B_7 - (t - k^2 - \mu^2)B_8 = 0. \end{aligned} \qquad (14)$$

The exact formulas relating $M_{\lambda_N';\lambda\lambda_N}$ to the $B_i$'s are quite complicated. However, these formulas can be simplified if one studies the $s$ dependence of the $B_i$'s at high $s$. The $s$ dependence of the $B_i$'s can be obtained by assuming the $t$-channel helicity amplitudes to behave like $s^\alpha$.[21] Here $\alpha$ may have nothing to do with the Regge trajectory, and in the pseudomodel of Jackson and Quigg,[13] $\alpha = 0$. Expressing the $B_i$'s in terms of the $t$-channel amplitudes, Le Bellac and Plaut[21] obtained the following $s$ dependence of the $B_i$'s:

$$B_1, B_2, B_6, B_8 \sim s^{\alpha-1}, \quad B_3 \sim s^\alpha, \qquad (15a)$$

and writing

$$B_5 = B_5^{(+)} + B_5^{(-)}, \qquad (15b)$$

where $B_5^{(+)}$ ($B_5^{(-)}$) corresponds to the exchange of systems with natural (unnatural) parity in the $t$ channel, they found

$$B_5^{(+)} \sim s^\alpha, \quad B_5^{(-)} \sim s^{\alpha-1}, \qquad (15c)$$

and

$$2B_5^{(+)} - sB_6 \sim s^{\alpha-2}, \quad 2B_5 - sB_6 \sim s^{\alpha-1}. \qquad (15d)$$

Then, using Eqs. (7), (14), and (15), and neglecting terms of order $1/s$, we obtain the following expressions for the helicity amplitudes[21]:

$$\begin{aligned} M_1 &= M_{+;0+} = (m_\rho s/2m)(B_7 + B_8), \\ M_2 &= M_{-;0+} = -[m_\rho(-t)^{1/2}/m](B_3 + B_4), \\ M_3^{(+)} &= M_{+;1+} + M_{+;-1+} = [(-\tfrac{1}{2}t)^{1/2}/m]B_5, \\ M_3^{(-)} &= M_{+;1+} - M_{+;-1+} = [(-\tfrac{1}{2}t)^{1/2}/m]sB_8, \\ M_4^{(+)} &= M_{-;1+} + M_{-;-1+} = -(2^{-1/2}/m)(-sB_1 + 2mB_5), \\ M_4^{(-)} &= M_{-;1+} - M_{-;-1+} \\ &= -(2^{-1/2}/m)(-sB_1 - 2tB_3 + 2mB_5). \end{aligned} \qquad (16)$$

Equation (7) is always valid for the range of $s$ and $t$ values considered in this paper. However, in general, the domain in which VMD is valid may be larger than that in which Eq. (7) is valid. For the sake of completeness, we include the more exact formulas for the $M_i$'s in terms of the $B_i$'s in Appendix A. Also, it can be easily seen from Eq. (16) that by assuming $B_1$, $B_3$, $B_5$, and $B_8$ to be independent of $k^2$ at fixed $s$ and $t$, one obtains the "old" VMD relations (3), (4), and (5), as already shown in Ref. 21.

Assuming all eight Ball amplitudes to be independent of $k^2$, the two equations in (14) are actually four independent equations[26]:

$$\begin{aligned} & sB_2 - (t - \mu^2)B_3 = 0, \\ & -2B_5 + sB_6 - (t - \mu^2)B_8 = 0, \\ & B_1 + B_3 + 2B_4 = 0, \\ & 2B_7 + B_8 = 0. \end{aligned} \qquad (17)$$

So, among the eight amplitudes only four are independent. We can therefore express the longitudinal amplitudes $M_1$ and $M_2$ in terms of the transverse amplitudes $M_3^{(\pm)}$ and $M_4^{(\pm)}$ as follows:

$$M_1 = [m_\rho/2(-2t)^{1/2}]M_3^{(-)}\,,$$
$$M_2 = -[m_\rho/2(-2t)^{1/2}](M_4^{(+)} - M_4^{(-)}). \tag{18}$$

"Complete" experiments on charged-pion photoproduction have not yet been performed. However, there are numerous phenomenological fits to the photoproduction of pion data.[11-13] We choose the "pseudomodel" of Jackson and Quigg[13] since their analysis was based mainly on finite-energy-sum-rule (FESR) calculations, and is therefore not very model-dependent.[27] It is convenient to express the $M_i$'s in terms of the $\phi_i$ they introduce, since the $\phi_i$'s can be related directly to the low-energy photoproduction data using FESR. In the pseudomodel of Jackson and Quigg, the $\phi_i$'s are related to the usual $t$-channel parity-conserving helicity amplitudes[28] $F_i^{(-)}$ by

$$\phi_i(\bar\nu, t) = \tfrac{1}{2}\mu\bar\nu\,\mathrm{Re}F_i^{(-)}(\bar\nu, t). \tag{19}$$

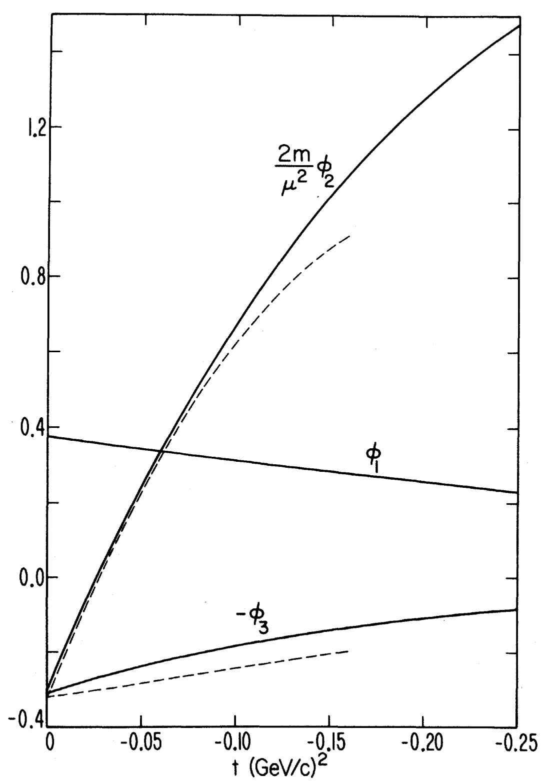


FIG. 1. Finite-energy-sum-rule amplitudes $\phi_i$. The solid curves for $\phi_2$ and $\phi_3$ are from Di Vecchia *et al.* (Ref. 12), evaluated at $\nu = 11.2$ GeV/$c$, and are used in this paper. The dashed curves for $\phi_2$ and $\phi_3$ are from Jackson and Quigg (Ref. 13). $\phi_1$ is from Quigg (private communication).

Here

$$\nu = (s - m^2)/2m$$

and $\bar\nu$ is the upper limit of the FESR integral. Equation (19) is the essence of the pseudomodel of Jackson and Quigg. It relates the real part of the high-energy photoproduction amplitudes $F_i^{(-)}$ directly to the FESR $\phi_i$. The $s$-channel helicity amplitudes can now be expressed in terms of the $\phi_i$'s.[29] We have

$$M_3^{(+)} = (f_\rho/e)[2(-t)^{1/2}\phi_1 + (-t)^{1/2}\phi_3/m]/m\,\mu,$$
$$M_3^{(-)} = (f_\rho/e)[-4m(-t)^{1/2}\phi_4]/m\,\mu,$$
$$M_4^{(+)} = -(f_\rho/e)[(t/m)\phi_1 + 2\phi_3]/m\,\mu,$$
$$M_4^{(-)} = (f_\rho/e)[4\phi_2/\mu(\mu^2 - t)]. \tag{20}$$

The $\phi_i$'s used by Jackson and Quigg in the pseudomodel are taken from the FESR calculation by Fox,[30] which in turn is based on Walker's fit to the low-energy photoproduction data.[31] However, possibly because of the bias in Walker's fit towards $\pi^+$ data, the $\phi_i$'s used by Jackson and Quigg agreed well with the $\gamma + p \to \pi^+ + n$ data instead of with the $\gamma_V + p \to \pi^+ + n$ data,[16] although their $\phi_i$'s are supposed to be related to the isovector amplitudes. This means that the $\phi_i$'s they used do not fit the

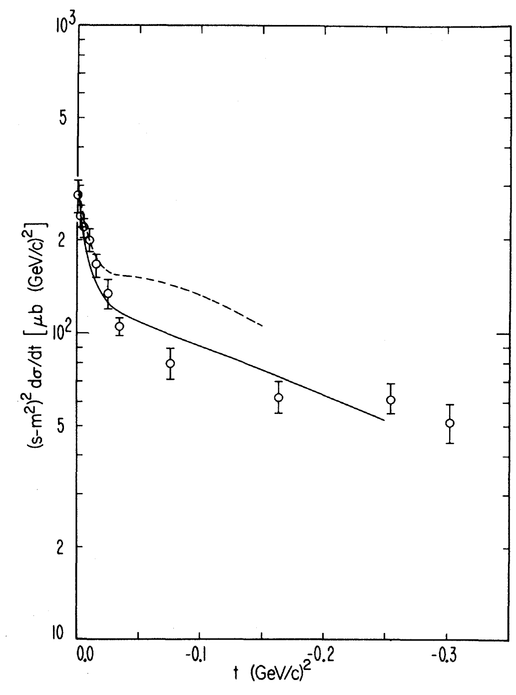


FIG. 2. Comparison of our calculation (solid curve), using the $\phi_2$ and $\phi_3$ of Di Vecchia *et al.* (Ref. 12) and $\phi_1$ of Quigg (solid curves of Fig. 1), and the calculation (dashed curve) of Jackson and Quigg (Ref. 13), to the $\gamma_V + p \to \pi^+ + n$ data (Ref. 32).

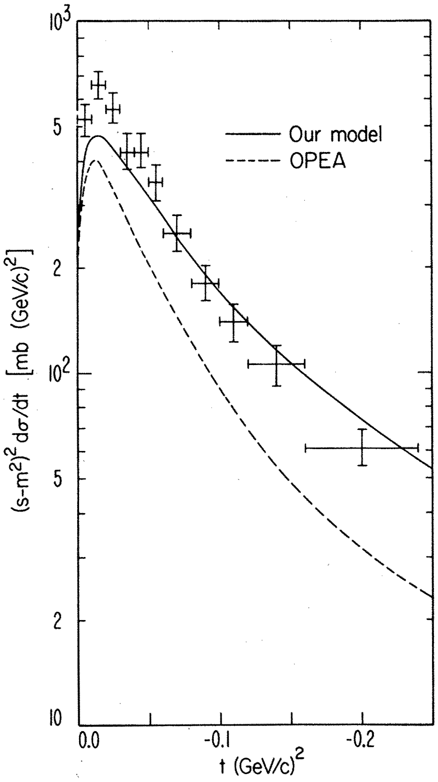


FIG. 3. Comparison of our prediction (solid curve) on the cross section for $\pi^- + p \to \rho^0 + n$ with the 11.2-GeV/$c$ experiment of Hyams *et al.* (Ref. 35). The experimental values are the cross sections for the reaction $\pi^- + p \to \pi^- + \pi^+ + n$ for the $(\pi^-\pi^+)$ mass range 600–960 MeV (see text). The OPEA prediction (Ref. 37) is also shown (dashed curve).

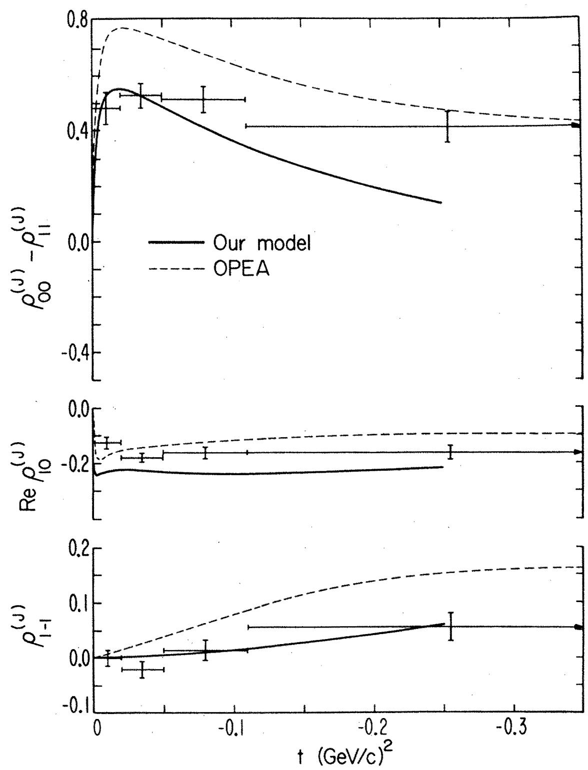


FIG. 4. Comparison of our predictions (solid curves) on the density-matrix elements (in the Jackson frame) for $\pi^- + p \to \rho^0 + n$ with the 11.2-GeV/$c$ experiment of Hyams *et al.* (Ref. 35). The experimental density matrix is normalized to $\rho_{00}^{(s\ \mathrm{wave})} + 2\rho_{11} + \rho_{00} = 1$ (see text). The OPEA predictions (Ref. 37) are also shown (dashed curves).

$\gamma_V + p \to \pi^+ + n$ data very well, especially for larger $|t|$. This can be seen from the $\pi^-/\pi^+$ ratio from deuterium.[32] On the other hand, Di Vecchia *et al.*[12] have also calculated $\phi_2$ and $\phi_3$ using FESR and Walker's fit, and although they parametrize their results in terms of Regge poles, the values of their $\phi_2$ and $\phi_3$ are consistent within errors with those of Fox.[33] Using the $\phi_2$ and $\phi_3$ of Di Vecchia *et al.*, and the $\phi_1$ evaluated by Fox, we find that we can obtain a better fit to the $\gamma_V + p \to \pi^+ + n$ data. We therefore use this set of $\phi_1$, $\phi_2$, and $\phi_3$ to predict the $\pi^- + p \to \rho^0 + n$ cross section. $\phi_4$, which goes like an $A_1$ exchange, is found to be small.[30] In Fig. 1 we plot the $\phi_1$, $\phi_2$, and $\phi_3$ used in our work together with those of Jackson and Quigg. The resulting photoproduction fits are shown in Fig. 2.

Using Eqs. (18) and (20), we can now write the longitudinal amplitudes as follows:

$$M_1 = 0,$$

$$M_2 = (f_\rho/e)[m_\rho(-2t)^{-1/2}/2m\mu] \times\{[4m/(\mu^2-t)]\phi_2 + 2\phi_3 + (t/m)\phi_1\}. \quad (21)$$

The conspiracy condition

$$\phi_2 + (\mu^2/2m)\phi_3 = O(t) \quad (22)$$

at $|t| \simeq 0$ eliminates the $(-t)^{-1/2}$ singularity in $M_2$. Using Eqs. (16) and (21), we calculate the cross section[34] and density matrix.

In Figs. 3 and 4, we compare our predictions with the experimental data from the CERN-Munich collaboration at $P_{\mathrm{lab}}^{(\pi)} = 11.2$ GeV/$c$.[35] We have considered only the $p$ wave when comparing the experimental results of Ref. 35 with our predictions. The authors of Ref. 35 normalize their density matrix using $\rho_{00}^{(s\ \mathrm{wave})} + 2\rho_{11} + \rho_{00} = 1$, which is to be contrasted with our normalization $2\rho_{11} + \rho_{00} = 1$. Absorptive-one-pion-exchange (OPEA) calculations suggest that $\rho_{00}^{(s\ \mathrm{wave})}/\rho_{11}$ may be as large as 10% in the Jackson frame.[36] In calculating the absolute magnitude of the cross sections, we have used $f_\rho^2/4\pi = 2$, in agreement with the colliding-beam results. We have also extended our predictions from $|t| < 0.12$ (GeV/$c$)$^2$ of our previous work[10] to $|t| < 0.25$ (GeV/$c$)$^2$. Also shown in Figs. 3 and 4 are the usual OPEA predictions for this process.[37]

The OPEA model usually has two adjustable parameters due to the lack of knowledge of the elastic $\rho$-$p$ scattering. Our model has no adjustable parameters, yet our prediction is at least as good as the OPEA predictions.

In Figs. 5, 6, and 7 we compare our predictions with the SLAC data at $P_{\text{lab}}^{(\pi)} = 15\,\text{GeV}/c$.[23] Here the $s$-wave contribution in the $\rho$ region is obtained from the fit of Sonderegger and Bonamy[38] to the data of $\pi^- + p \to \pi^0 + \pi^0 + n$ and extrapolated to 15 GeV/$c$. Our prediction is compared with the data after the $s$-wave contribution obtained in this way is subtracted. The agreement between theory and experiment is not bad, considering the possible bias in the $s$-wave subtraction.[39] Several features are worth pointing out here. First, it was speculated by Avni and Harari[40] that unlike the photoproduction cross section, the $\pi^- + p \to \rho^0 + n$ cross section with transversely polarized $\rho$ does not show a peak even in the helicity frame, while the usual application of the VMD model (including ours) requires such a peak.[41] In Fig. 5 we see that the experimental value $2\rho_{11}^{(H)}(s-m^2)^2 d\sigma(\rho)/dt$ indicates a peak in the forward direction, supporting the usual prediction of VMD.[42] Secondly, in Fig. 7 we find that the 15-GeV/$c$ experiment satisfies the VMD relation (4) better[43] than the 4-GeV/$c$ experiment does.[17] Actually this is not so surprising, since an explicit calculation using the electric Born model[44] revealed that the asymmetry relation (4) can be violated by as much as 40% at 4 GeV/$c$, due to the effect of the finite $\rho$ mass, while at 15 GeV/$c$ relation (4) should be good to a fraction of one percent. It was also pointed out that the difficulty in the VMD relation (4) at 4 GeV/$c$ may be due to neglecting $d$-wave effects in the analysis of the $\pi^- + p \to \pi^- + \pi^+ + n$ data,[18,22,45] which may cause $\rho_{1-1}^{(H)}$ to become negative. A higher-statistics experiment with wider angular acceptance should help to clarify the $d$-wave effect.[46] Also note that the interesting feature in pion production with a polarized photon beam is that the asymmetry $[d\sigma^{(\perp)}(\gamma_V) - d\sigma^{(\parallel)}(\gamma_V)]/[d\sigma^{(\perp)}(\gamma_V) + d\sigma^{(\parallel)}(\gamma_V)]$ is positive in the range of $t$ values measured, indicating the dominance of natural-parity over unnatural-parity exchanges, and it attains the maximum value of unity at $t \simeq -\mu^2$. It is comforting to find in Fig. 7 that the same feature is present in the $\rho$-production asymmetry $\rho_{1-1}^{(H)}/\rho_{11}^{(H)}$.

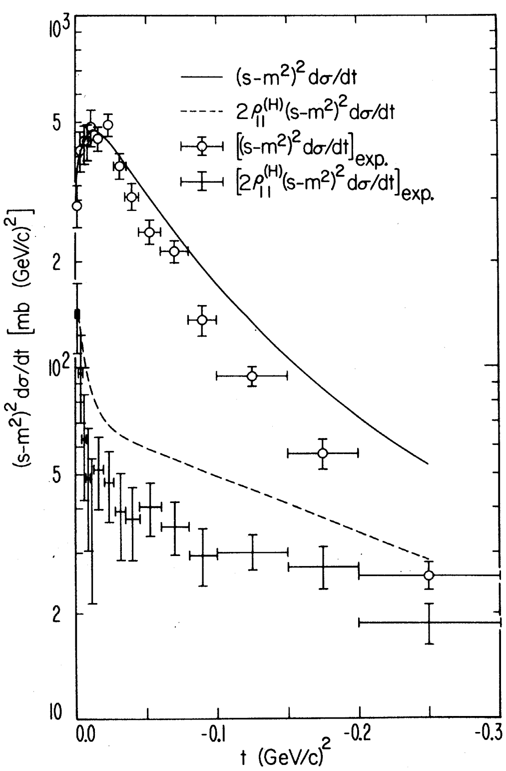


FIG. 5. Comparison of our predictions on the cross sections for $\pi^- + p \to \rho^0 + n$ with the 15-GeV/$c$ experiment of Bulos *et al.* (Ref. 23). The $s$-wave contribution is subtracted according to the fit of Sonderegger and Bonamy (see text).

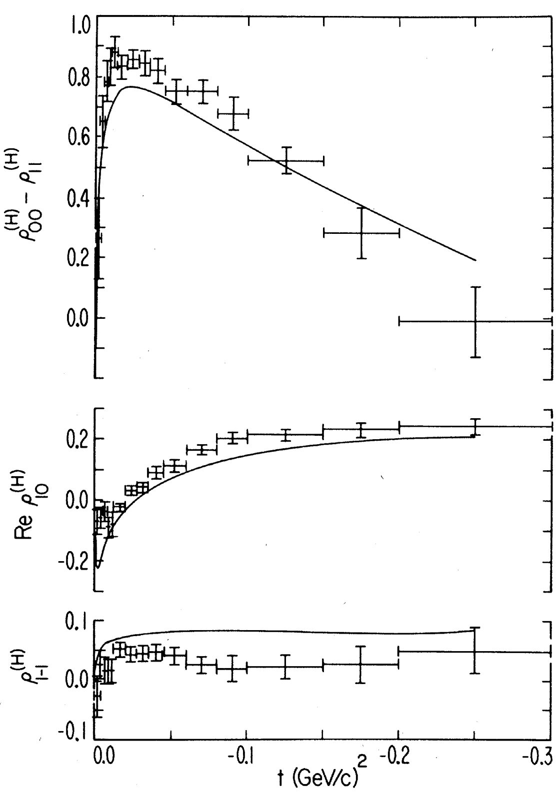


FIG. 6. Comparison of our predictions on the density-matrix elements (in the helicity frame) for $\pi^- + p \to \rho^0 + n$ with the 15-GeV/$c$ experiment of Bulos *et al.* (Ref. 23). The $s$-wave contribution is subtracted according to the fit of Sonderegger and Bonamy (see text).

## V. THE SMOOTHNESS ASSUMPTION

Having stated our results, we would like to discuss here the smoothness assumption we made in the Ball amplitudes. In the electric Born model, we readily verify that all the Ball amplitudes are independent of $k^2$ at high $s$. In this model, we have the matrix element

$$M\begin{Bmatrix}\gamma_V\\ \rho\end{Bmatrix} = \sqrt{2}\begin{Bmatrix}e\\ f_\rho\end{Bmatrix} g\bar{u}(p')\gamma_5\left[\frac{2\epsilon\cdot q-\epsilon\cdot k}{t-\mu^2}+\frac{\epsilon\cdot p+\frac{1}{2}(\gamma\cdot k)(\gamma\cdot\epsilon)}{s-m^2}-\frac{\epsilon\cdot p'-\frac{1}{2}(\gamma\cdot\epsilon)(\gamma\cdot k)}{u-m^2}\right]u(p). \quad (23)$$

At high $s$, and small $t$, the Ball amplitudes are, aside from the coupling constants,

$$B_1=-1/s,\quad B_2=1/s,\quad B_3=1/(t-\mu^2),\quad B_4=-1/[2(t-\mu^2)],\quad B_5=B_6=B_7=B_8=0. \quad (24)$$

Actually, the electric Born model is not used here simply as an illustration. It has been known for some time that the electric Born model gives a fairly good approximation to both the shape and the magnitude of the observed $\gamma+p\to\pi^++n$ cross section[47] for $|t|\lesssim 2\mu^2$ at high $s$. The relevance of the electric Born model to the high-energy pion photoproduction was also discussed in terms of dispersion relations and finite-energy sum rules.[39,48] In view of these studies, one is tempted to take the electric Born model more seriously, at least in the low-$|t|$ region, and the fact that the Ball amplitudes expressed in the electric Born model are smooth in $k^2$ lends some support to our assumption. Note, however, that our successful prediction for $d\sigma(\rho)/dt$ extends up to $|t|\simeq 0.25$ (GeV/$c$)$^2\simeq 12\ \mu^2$ where the electric Born model is no longer applicable.

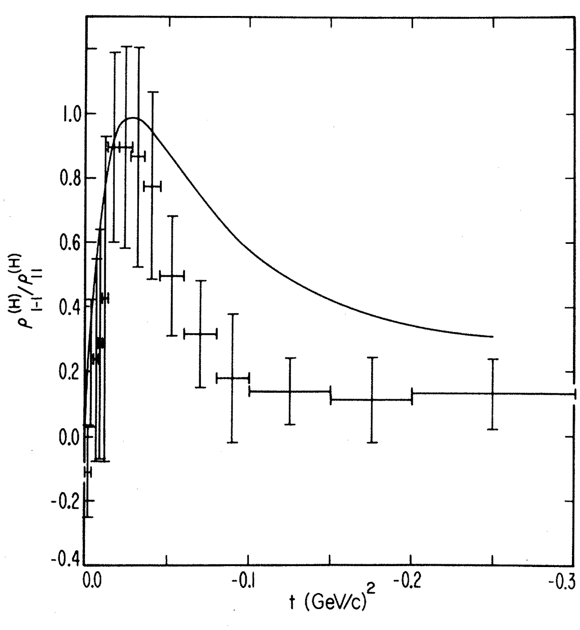


FIG. 7. Comparison of our prediction on $\rho^{(H)}_{1-1}/\rho^{(H)}_{11}$ with the 15-GeV/$c$ experiment of Bulos *et al.* (Ref. 23). The $s$-wave contribution is subtracted according to the fit of Sonderegger and Bonamy (see text).

We now turn to models based on higher-spin-particle exchanges. The contribution to the $B_i$'s from higher-spin exchanges can be readily obtained using Scadron's covariant-propagator method.[49] Covariant Reggeization of the amplitudes can be obtained by performing the Sommerfeld-Watson transformation on these amplitudes.[50] The expressions for the Ball amplitudes for higher-spin exchanges are given in Appendix B [especially Eqs. (B6), (B14), and (B16)]. In general, we see that for natural-parity exchange ($\pi_c$ and $A_2$ exchange), $B_2$, $B_7$, and $B_8$ depend strongly on $k^2$, while the other $B$'s do not depend on $k^2$ at high $s$.[51,52] For unnatural-parity exchange with $G(-1)^{I+J}=+1$ ($\pi$ exchange), all $B$'s are independent of $k^2$ at high $s$, and for unnatural-parity exchanges with $G(-1)^{I+J}=-1$ ($A_1$ exchange), $B_2$, $B_3$, and $B_4$ depend on $k^2$ while the other $B$'s do not. Using the formulas in Appendix B, we can easily show that the density-matrix elements for transversely polarized $\rho$'s are independent of $k^2$ at high $s$.[53] One may therefore be tempted to think that relations involving transversely polarized $\rho$'s may be more likely to be valid. However, it should be noted that our main difficulty is with relations (4) and (5) which involve only transverse amplitudes.

We see that, in general, particle-exchange models do not support the assumption that all Ball amplitudes are independent of $k^2$, especially when the exchanged particles are considered separately. On the other hand, it was argued that in order to understand photoproduction of charged pions in the pure Regge-pole model, a pion conspirator had to be invented to fit the data; however, even this conspiracy solution now has difficulty in reconciling with the idea of factorization.[14] In any case, one may not be forced to abandon the smoothness assumption in the Ball amplitudes, although particle-exchange models in general do not seem to support this assumption. The qualitative success of of our approach may simply reflect the fact that the amplitudes for (1) and (2) resemble the electric Born amplitudes. Alternatively, we may entertain the optimistic possibility that the smoothness assumption for the $B$'s is, in some sense, more basic than inferences drawn from various model calculations.

## VI. SOME RELATED PROCESSES

### A. Electroproduction of Pions

Our model can be readily applied to study the electroproduction of pions. The cross section for pion electroproduction (one-photon-exchange approximation) can be written as follows[54]:

$$\frac{d\sigma}{dE'd\Omega_e dt d\phi}=\Gamma_t\left[\frac{d\sigma_{11}}{dt}-\epsilon\cos2\phi\frac{d\sigma_{1-1}}{dt}+\epsilon\frac{d\sigma_{00}}{dt}+2\epsilon^{1/2}(1+\epsilon)^{1/2}\cos\phi\frac{d\sigma_{10}}{dt}\right]. \quad (25)$$

Here[54-56]

$$\Gamma_t=\frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{s-m^2}{2m}\frac{1}{(-k^2)}\frac{1}{1-\epsilon}, \quad (26)$$

$$\epsilon=\frac{(-k^2/|\vec{k}_L|^2)\cot^2(\frac{1}{2}\theta)}{2+(-k^2/|\vec{k}_L|^2)\cot^2(\frac{1}{2}\theta)}, \quad (27)$$

and

$$s^2\frac{d\sigma_{\lambda\lambda'}}{dt}=\frac{m^2}{16\pi}\sum_{\lambda_N',\lambda_N}(g^*_{\lambda_N';\lambda\lambda_N}g_{\lambda_N';\lambda'\lambda_N}+g^*_{\lambda_N';\lambda'\lambda_N}g_{\lambda_N';\lambda\lambda_N}). \quad (28)$$

$E$ and $E'$ are the energies of the initial and final electrons in the lab system, $k^2$ is the square of the mass of the virtual photon, $|\vec{k}_L|$ is the magnitude of the virtual-photon momentum in the lab system, and $\theta$ is the lab angle between the initial and final electron momentum. $\Omega_e$ is the laboratory solid angle of the final electron, $t$ is the invariant momentum transfer to the proton, and $\phi$ the angle seen in the laboratory between the electron momentum and the momentum of the pion-nucleon system; $g_{\lambda_N';\lambda\lambda_N}$ are the helicity amplitudes for pion production by virtual photons in the $s$-channel pion-nucleon center-of-mass system, and $s$ is the square of the pion-nucleon center-of-mass energy.

In our model, $d\sigma_{\lambda\lambda'}/dt$ can be expressed in terms of the $\pi^-+p\to\rho^0+n$ helicity amplitudes $M_{\lambda_N';\lambda\lambda_N}$ as follows[56]:

$$\begin{aligned}
s^2\frac{d\sigma_{11}}{dt}&=\frac{m^2}{8\pi}\left[\frac{e}{f_\rho}\right]^2\left[\frac{m_\rho^2}{m_\rho^2-k^2}\right]^2\sum_{\lambda_N',\lambda_N}M^*_{\lambda_N';1\lambda_N}M_{\lambda_N';1\lambda_N},\\
s^2\frac{d\sigma_{1-1}}{dt}&=\frac{m^2}{8\pi}\left[\frac{e}{f_\rho}\right]^2\left[\frac{m_\rho^2}{m_\rho^2-k^2}\right]^2\sum_{\lambda_N',\lambda_N}M^*_{\lambda_N';1\lambda_N}M_{\lambda_N';-1\lambda_N},\\
s^2\frac{d\sigma_{10}}{dt}&=\frac{m^2}{8\pi}\left[\frac{e}{f_\rho}\right]^2\left[\frac{m_\rho^2}{m_\rho^2-k^2}\right]^2\frac{(-k^2)^{1/2}}{m_\rho}\,\tfrac{1}{2}\sum_{\lambda_N',\lambda_N}(M^*_{\lambda_N';1\lambda_N}M_{\lambda_N';0\lambda_N}+M^*_{\lambda_N';0\lambda_N}M_{\lambda_N';1\lambda_N}),\\
s^2\frac{d\sigma_{00}}{dt}&=\frac{m^2}{8\pi}\left[\frac{e}{f_\rho}\right]^2\left[\frac{m_\rho^2}{m_\rho^2-k^2}\right]^2\frac{(-k^2)}{m_\rho^2}\sum_{\lambda_N',\lambda_N}M^*_{\lambda_N';0\lambda_N}M_{\lambda_N';0\lambda_N}.
\end{aligned} \quad (29)$$

For $e+p\to e+n+\pi^+$, we should include the $\omega$ contribution. However, if we restrict ourselves to small $|t|$, this is not a bad approximation, as can be seen from the $\pi^-/\pi^+$ ratio. For larger $|t|$, we can form suitable averages of the $e+p\to e+n+\pi^+$ and $e+n\to e+p+\pi^-$ cross sections to obtain the isovector cross section.[57] So, again using Eqs. (20) and (21), we can calculate the differential cross section for pion electroproduction from the photoproduction amplitudes. In Figs. 8 and 9, we plotted the quantities $s^2 d\sigma_{00}/dt$ and $s^2 d\sigma_{10}/dt$ as a function of $t$ and $k^2$.[58] One notices the rise of the cross section with $|k^2|$ as $|k^2|$ increases from zero, and gradually the $k^2$ dependence in the $\rho$ propagator takes over and the cross section decreases. This feature is also true in the VMD prediction of $\sigma_s$ in inelastic $e$-$p$ scattering.[59] $s^2 d\sigma_{11}/dt$ and $s^2 d\sigma_{1-1}/dt$ are not plotted here. Their $k^2$ dependence is only in the $\rho$ propagator and so they decrease monotonically with increasing $|k^2|$.

In the usual application of VMD to electroproduction of pions, it is assumed that the space part of the electromagnetic matrix element $\langle\pi^+n|j_\mu|p\rangle$ is related to the hadronic matrix element $\langle\pi^+n|J_\mu^{(\rho)}|p\rangle$ via[60-62]

$$\langle\pi^+n|j_i|p\rangle=\frac{m_\rho^2/f_\rho}{m_\rho^2-k^2}\langle\pi^+n|J_i^{(\rho)}|p\rangle,\quad i=1,2,3 \quad (30)$$

and the time-component matrix element is obtained using the current-conservation equation

$$|\vec{k}|\langle\pi^+n|J_3^{(\rho)}|p\rangle=k_0\langle\pi^+n|J_0^{(\rho)}|p\rangle. \quad (31)$$

Our model, of course, agrees with this assumption.

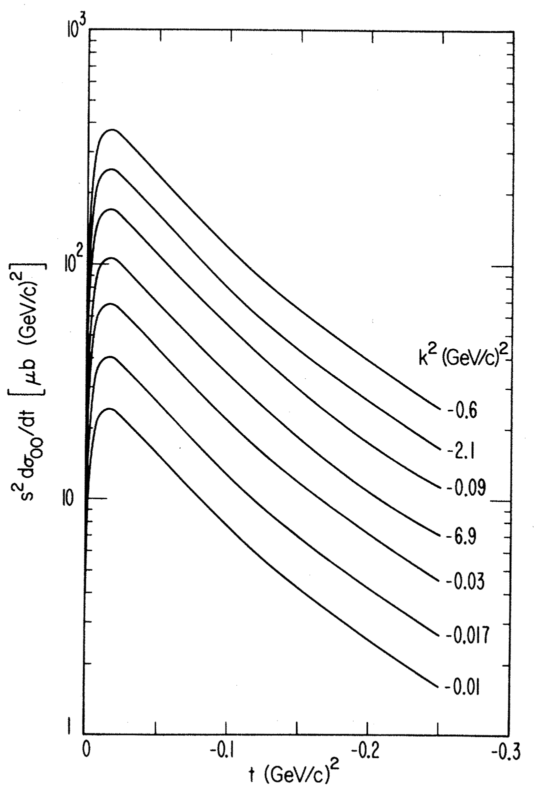


FIG. 8. Our prediction on $s^2\, d\sigma_{00}/dt$ for electroproduction of pions [see Eq. (25)].

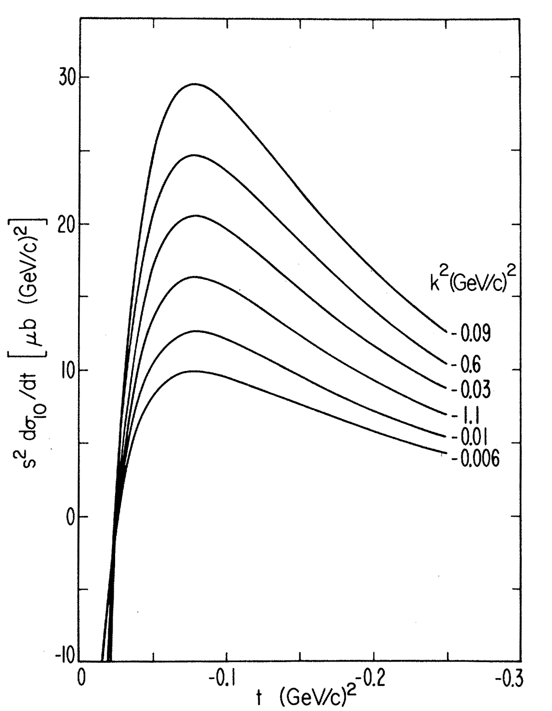


FIG. 9. Our prediction on $s^2\, d\sigma_{10}/dt$ for electroproduction of pions [see Eq. (25)].

Electroproduction of pions also offers additional tests to the choice of frame in the vector-meson-dominance model.[61] At present, there are not enough data for high-$s$ pion electroproduction, and so comparison with our predictions was not made.

### B. $K^- + p \to \omega + \Lambda$

Since both $\omega$ and $\Lambda$ decay in this reaction, the assumption of smoothness in the Ball amplitudes actually leads to relations among the joint-decay density-matrix elements.[63] We define the joint-decay density matrix as follows:

$$\rho_{\Lambda\Lambda'}^{\omega\omega'} = \frac{\sum_p T(K^- p \to \omega\Lambda)\, T^*(K^- p \to \omega'\Lambda')}{\sum_{p,\omega,\Lambda} |T(K^- p \to \omega\Lambda)|^2}. \qquad (32)$$

Here $p$, $\omega$, and $\Lambda$ represent the $s$-channel helicity states of $p$, $\omega$, and $\Lambda$, respectively. $T(K^- p \to \omega\Lambda)$ is the amplitude for the reaction $K^- + p \to \omega + \Lambda$. Again using the relations (18) and using the high-energy approximation (7), one obtains the following relations for the joint-decay distribution:

$$-4\,\mathrm{Im}\,\rho_{\frac{1}{2}-\frac{1}{2}}^{1-1} + (8t/m_\omega^2)\,\mathrm{Im}(\rho_{\frac{1}{2}-\frac{1}{2}} + \bar{\rho}_{\frac{1}{2}-\frac{1}{2}})$$
$$= -(8/m_\omega)(-2t)^{1/2}\,\mathrm{Im}\,\rho_{\frac{1}{2}-\frac{1}{2}}^{10},$$
$$-(2/m_\omega)(-2t)^{1/2}\,\mathrm{Im}\,\bar{\rho}_{10} + \mathrm{Im}\,\bar{\rho}_{1-1} = 0, \qquad (33)$$
$$-(4/m_\omega)(-2t)^{1/2}\,\mathrm{Im}\,\rho_{-\frac{1}{2}\frac{1}{2}}^{10}$$
$$= \mathrm{Im}(4\rho_{\frac{1}{2}-\frac{1}{2}} + 3\bar{\rho}_{\frac{1}{2}-\frac{1}{2}} - \rho_{\frac{1}{2}-\frac{1}{2}}^{1-1} - \rho_{-\frac{1}{2}\frac{1}{2}}^{1-1}),$$

where

$$\bar{\rho}_{\Lambda\Lambda'} = \rho_{\Lambda\Lambda'}^{11} + \rho_{\Lambda\Lambda'}^{-1-1} - 2\rho_{\Lambda\Lambda'}^{00},$$
$$\bar{\rho}_{\omega\omega'} = \rho_{\frac{1}{2}\frac{1}{2}}^{\omega\omega'} - \rho_{-\frac{1}{2}-\frac{1}{2}}^{\omega\omega'},$$
$$\rho_{\Lambda\Lambda'} = \sum_\omega \rho_{\Lambda\Lambda'}^{\omega\omega}.$$

Here the $\rho_{\Lambda\Lambda'}^{\omega\omega'}$ are considered in the $\omega$ helicity frame; i.e., the $z$ axis is taken to be in the direction of the $\omega$ momentum in the $s$-channel center-of-mass system, and the $y$ axis is perpendicular to the scattering plane.

The joint-decay density-matrix elements for $K^- + p \to \omega + \Lambda$ were measured by Flatté[64] and by Schreiner.[65] Unfortunately, the imaginary parts of the joint-decay density-matrix elements in general are small, and with the statistics of the existing data, they are compatible with zero within experimental errors. So, with the present experimental data on $K^- + p \to \omega + \Lambda$, Eqs. (33) are trivially satisfied.

## VII. DISCUSSION

In the usual application of VMD to the $\pi^- + p \to \rho^0 + n$ process, only the transversely polarized $\rho$ amplitudes are related to the photoproduction amplitudes. It is shown that assuming that the Ball amplitudes not only satisfy the constraints imposed by current conservation, but also are smooth in the vector-meson mass, we can express the longitudinally polarized $\rho$ amplitudes in terms of the transversely polarized $\rho$ amplitudes, which, in turn, can be related to the photoproduction amplitudes. More generally, assuming a certain set of invariant amplitudes for processes involving vector mesons to be smooth in the vector-meson mass, one can usually obtain relationships between the transverse and longitudinal amplitudes.[66] In the case of $\pi^- + p \to \rho^0 + n$, the assumption that the Ball amplitudes are smooth in the vector-meson mass is studied in particle-exchange models in detail. It is shown that this assumption is satisfied in the electric Born model, although in particle-exchange models this assumption is not always valid. On the practical side, we have shown that the basic smoothness assumption for the Ball amplitudes leads to completely nontrivial relations among the various helicity amplitudes. With the exception of $\rho_{1-1}^{(H)}$, our predictions, which have no adjustable parameters, are in satisfactory agreement with the high-statistics data of Refs. 23 and 35. We have also used these assumptions to study the electroproduction of pions and the reaction $K^- + p \to \omega + \Lambda$. These experiments will provide additional tests of our assumptions.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor J. J. Sakurai for suggesting this problem to me, for continued and patient guidance, and for many stimulating discussions. I am grateful to C. Quigg, G. Fox, and H. Lynch for helpful communications. I would also like to take this opportunity to thank the members of the UCLA Physics Department for their hospitality during my stay there.

## APPENDIX A

In deriving Eqs. (16) we have used the large-$s$, small-$|t|$ approximation, Eq. (7). However, the domain in which VMD is valid may be larger than that in which Eq. (7) is valid. There have been extensive investigations on the domain of validity of VMD in inelastic $e$-$p$ scattering[67]; they all give more or less the same result. From a simple kinematical consideration, one finds that in order for $s$, $t$, and $u$ to be approximately the same for processes (1) and (2), we must have

$$s + t \gg 2m^2 + m_\rho^2 + \mu^2. \tag{A1}$$

But the domain in which Eq. (A1) is satisfied is generally larger than that in which Eq. (7) is valid. So, for the sake of completeness, we write down the exact expressions for the $M_i$'s in terms of the $B_i$'s:

$$\begin{aligned}
M_1 &= \{-B_2\eta_-[k(p_0+p_0') + k_0(p+p'\cos\theta)] - 2B_3\eta_-(kq_0 - k_0 q\cos\theta) + B_5(\xi_+k_0 + \eta_+k) \\
&\quad - \tfrac{1}{2}B_6(\xi_+k + \eta_+k_0)[k(p_0+p_0') + k_0(p+p'\cos\theta)] - B_8(\xi_+k+\eta_+k_0)(kq_0 - k_0q\cos\theta)\}(1/m_\rho)\cos\tfrac{1}{2}\theta, \\
M_2 &= \{B_2\eta_+[k(p_0+p_0') + k_0(p+p'\cos\theta)] + 2B_3\eta_+(kq_0 - k_0q\cos\theta) + B_5(-\xi_-k_0 + \eta_-k) \\
&\quad - \tfrac{1}{2}B_6(-\xi_-k + \eta_-k_0)[k(p_0+p_0') + k_0(p+p'\cos\theta)] - B_8(-\xi_-k + \eta_-k_0)(kq_0 - k_0q\cos\theta)\}(1/m_\rho)\sin\tfrac{1}{2}\theta, \\
M_3^{(+)} &= -[-B_1(\eta_-k - \xi_-k_0) - B_5\xi_+]\sqrt{2}\sin\tfrac{1}{2}\theta, \\
M_4^{(+)} &= -[-B_1(\eta_+k + \xi_+k_0) + B_5\xi_-]\sqrt{2}\cos\tfrac{1}{2}\theta, \\
M_3^{(-)} &= -[-B_1(\eta_-k - \xi_-k_0) + 2(B_2p' - 2B_3q)\eta_-\cos^2(\tfrac{1}{2}\theta) - B_5\xi_+ + 2(\tfrac{1}{2}B_6p' - B_8q)(\xi_+k + \eta_+k_0)\cos^2(\tfrac{1}{2}\theta)]\sqrt{2}\sin\tfrac{1}{2}\theta, \\
M_4^{(-)} &= -[-B_1(\eta_+k + \xi_+k_0) + 2(-B_2p' + 2B_3q)\eta_+\sin^2(\tfrac{1}{2}\theta) + B_5\xi_- + 2(\tfrac{1}{2}B_6p' - B_8q)(\xi_-k - \eta_-k_0)\sin^2(\tfrac{1}{2}\theta)]\sqrt{2}\cos\tfrac{1}{2}\theta.
\end{aligned} \tag{A2}$$

Here all quantities are expressed in the $s$-channel center-of-mass system.

$k$ = magnitude of the space momentum of the vector meson;

$k_0$ = energy of the vector meson;

$(p, p')$ = magnitude of the space momentum of the (final, initial) nucleon in reaction (1), of the (initial, final) nucleon in reaction (2);

$(p_0, p_0')$ = energy of the (final, initial) nucleon in (1), of the (initial, final) nucleon in (2);

$\theta$ = the angle between $\vec{p}$ and $\vec{p}\,'$ in the $s$-channel center-of-mass system.

$$\begin{aligned}
\eta_\pm &= \frac{(p_0+m)^{1/2}(p_0'+m)^{1/2}}{2m}\left[\frac{p'}{p_0'+m} \pm \frac{p}{p_0+m}\right], \\
\xi_\pm &= \frac{(p_0+m)^{1/2}(p_0'+m)^{1/2}}{2m}\left[1 \pm \frac{pp'}{(p_0+m)(p_0'+m)}\right].
\end{aligned} \tag{A3}$$

In the calculations in this paper, with $p_{\text{lab}}^{(\pi)} = 11.2$ and 15 GeV/$c$, and with $|t|$ going up to 0.25 (GeV/$c$)$^2$, Eq. (A2) and Eq. (7) are essentially indistinguishable. Finally, we write down the formula relating $\rho_{\lambda\lambda'}^{(H)}$ and $d\sigma/dt$ to the $M_i$'s defined in Eqs. (16):

$$\begin{aligned}
&\rho_{11}^{(H)} = (|M_3^{(+)}|^2 + |M_4^{(+)}|^2 + |M_3^{(-)}|^2 + |M_4^{(-)}|^2)/X ,\\
&\rho_{1-1}^{(H)} = (|M_3^{(+)}|^2 + |M_4^{(+)}|^2 - |M_3^{(-)}|^2 - |M_4^{(-)}|^2)/X,\\
&\rho_{00}^{(H)} = 4(|M_1|^2 + |M_2|^2)/X,\\
&\mathrm{Re}\rho_{10}^{(H)} = 2\mathrm{Re}(M_3^{-*}M_1 + M_4^{-*}M_2)/X,\\
&\frac{d\sigma}{dt} = \frac{m^2}{16\pi s^2}X.
\end{aligned} \tag{A4}$$

Here,

$$X = 4(|M_1|^2 + |M_2|^2) + 2(|M_3^{(+)}|^2 + |M_4^{(+)}|^2 + |M_3^{(-)}|^2 + |M_4^{(-)}|^2). \tag{A5}$$

## APPENDIX B

We derive here the expressions for the Ball amplitudes in the particle-exchange model, using the covariant-propagator approach of Scadron.[49] For natural-parity exchange, there are two couplings at the nucleon vertex. There are various ways of writing these couplings. However, since the nucleon vertex involves only the nucleon momentum in the $t$-channel center-of-mass system, this will not affect the $k^2$ dependence of the Ball amplitudes. We follow Scadron and choose the nucleon vertex coupling to be

$$g_1 P^\mu + g_2\gamma^\mu \tag{B1}$$

with

$$P = \tfrac{1}{2}(p' - p). \tag{B2}$$

The quantities $k$, $q$, $p$, $p'$, and $\epsilon$ are defined in the same way as in the text, except that here everything is considered in the $t$-channel center-of-mass system. The $g_i$'s are coupling constants. The boson vertex is

$$g_3\epsilon^{\nu\alpha\sigma\tau}Q_\sigma\Delta_\tau\epsilon_\nu , \tag{B3}$$

with

$$Q = \tfrac{1}{2}(k - q) \tag{B4}$$

and

$$\Delta = p' + p = k + q. \tag{B5}$$

According to Ref. 49, the contribution to the Ball amplitudes of a spin-$J$ isobar of mass $M$, in the physical region of the $t$ channel near $t = M^2$, can be written as follows:

$$\begin{aligned}
(t-M^2)B_1 &= g_1g_3\frac{t-4m^2}{2}\frac{c_J}{J}\mathcal{P}_J' - g_2g_3 2m\frac{c_J}{J}\mathcal{P}_J',\\
(t-M^2)B_2 &= -g_1g_3\frac{t+k^2-\mu^2}{4}\frac{c_J}{J}\mathcal{P}_J' - g_2g_3 m\frac{t+k^2-\mu^2}{4}\left[\frac{(t+k^2-\mu^2)^2}{4t} - k^2\right]\frac{c_J}{J^2}\mathcal{P}_{J-1}'',\\
(t-M^2)B_3 &= -g_1g_3\frac{s-u}{8}\frac{c_J}{J}\mathcal{P}_J' - g_2g_3\frac{c_J}{J^2}\frac{m}{2}\left\{\left[\frac{(t+k^2-\mu^2)^2}{4t} - k^2\right]\left[\frac{s-u}{4}\right]\mathcal{P}_{J-1}'' + k^2\mathcal{P}_J''\right\},\\
(t-M^2)B_4 &= g_1g_3\left[-\frac{t-4m^2}{4} + \frac{s-u}{8}\right]\frac{c_J}{J}\mathcal{P}_J' + g_2g_3\frac{c_J}{J^2}\frac{m}{2}\left\{2J\mathcal{P}_J' + \left[\frac{(t+k^2-\mu^2)^2}{4t} - k^2\right]\left[\frac{s-u}{4}\right]\mathcal{P}_{J-1}'' - \frac{t-k^2-\mu^2}{2}\mathcal{P}_J''\right\},\\
(t-M^2)B_5 &= -g_1g_3 m\left[\frac{s-u}{2}\right]\frac{c_J}{J}\mathcal{P}_J' - g_2g_3\frac{c_J}{J^2}\left\{J\left[\frac{s-u}{2}\right]\mathcal{P}_J' + \tfrac{1}{2}t\left[\frac{(t+k^2-\mu^2)^2}{4t} - k^2\right]\left[\frac{s-u}{4}\mathcal{P}_{J-1}'' - \mathcal{P}_J''\right]\right\},\\
(t-M^2)B_6 &= -g_1g_3 2m\frac{c_J}{J}\mathcal{P}_J' - g_2g_3\frac{c_J}{J^2}\left\{2J\mathcal{P}_J' + \tfrac{1}{2}t\left[\frac{(t+k^2-\mu^2)^2}{4t} - k^2\right]\mathcal{P}_{J-1}''\right\},\\
(t-M^2)B_7 &= -g_2g_3\frac{t+\mu^2-k^2}{4}\frac{c_J}{J^2}\mathcal{P}_J'',\\
(t-M^2)B_8 &= -g_2g_3\frac{t+k^2-\mu^2}{4}\frac{c_J}{J^2}\mathcal{P}_J''.
\end{aligned} \tag{B6}$$

Here

$$c_J = \frac{\pi^{1/2}}{2^J} \frac{\Gamma(J+1)}{\Gamma(J+\frac{1}{2})} . \tag{B7}$$

$\mathcal{P}_J$ is the solid Legendre polynomial,

$$\mathcal{P}_J \equiv (pk)^J P_J(-\cos\theta_t), \tag{B8}$$

$P_J$ is the Legendre polynomial, and $\theta_t$ is the angle between $k$ and $p$. The prime on $\mathcal{P}_J$ means differentiation with respect to the argument $(-\cos\theta_t)$. The Regge prescription is [68]

$$\frac{c_J \mathcal{P}_J}{M^2 - t} \to (pk\cos\theta)^\alpha \frac{1 \pm e^{-i\pi\alpha}}{2\sin\pi\alpha} \pi\alpha' . \tag{B9}$$

It should be noted that

$$pk\cos\theta = \tfrac{1}{4}(s-u). \tag{B10}$$

So, at high $s$, the solid Legendre polynomial is independent of $k^2$.

For unnatural-parity exchange with isospin $I$ and $G$-parity $G$, we have, for $G(-1)^{I+J} = +1$,

$$g_4 \gamma^5 P^\mu \tag{B11}$$

at the nucleon vertex. The boson vertex has, in general, two couplings. There are many ways of writing these two couplings. We choose not to use the form given by Ref. 49 because $\epsilon \cdot k = 0$ was used to obtain the form given in Ref. 49. We write the boson vertex as

$$\epsilon_\nu [g_5 g^{\nu\alpha} - g_6 (2q+k)^\nu Q^\alpha] . \tag{B12}$$

The part $2q+k$ (instead of $Q$ given in Ref. 49) conforms with the $\pi\pi\gamma$ vertex derived from the usual interaction Lagrangian. The boson vertex (B12) in general is not gauge-invariant. However, if one takes the point of view that in pure Regge-pole theory (i.e., no cuts), all Regge poles should separately satisfy gauge-invariance constraints, [69] the boson vertex couplings will be related by

$$g_5 = k\cdot(2q+k) g_6. \tag{B13}$$

For the Ball amplitudes, we have

$$B_1 = B_5 = B_6 = B_7 = B_8 = 0,$$

$$(t-M^2)B_2 = -g_4 g_5 \frac{1}{2}\frac{c_J}{J}\mathcal{P}_J',$$

$$(t-M^2)B_3 = -g_4 g_5 \frac{t+k^2-\mu^2}{4t}\frac{t-4m^2}{4}\frac{c_J}{J}\mathcal{P}_{J-1}' - g_4 g_6 c_J \mathcal{P}_J, \tag{B14}$$

$$(t-M^2)B_4 = -g_4 g_5 \frac{t+\mu^2-k^2}{4t}\frac{t-4m^2}{4}\frac{c_J}{J}\mathcal{P}_{J-1}' + g_4 g_6 \tfrac{1}{2} c_J \mathcal{P}_J.$$

For unnatural parity with $G(-1)^{I+J} = -1$, the nucleon vertex is given by

$$g_7 \gamma^5 \gamma^\mu . \tag{B15}$$

Again using Eq. (B12) for the boson vertex, we obtain the following Ball amplitudes:

$$B_1 = 0,$$

$$(t-M^2)B_2 = -g_7 g_5 \frac{m}{2}\frac{t+k^2-\mu^2}{t}\frac{c_J}{J^2}\mathcal{P}_J'',$$

$$(t-M^2)B_3 = -\frac{m}{t}\left[-g_7 g_5 \frac{c_J}{J^2}\mathcal{P}_J' + g_7 g_5 \frac{(t+k^2-\mu^2)^2}{4t}\frac{t-4m^2}{4}\frac{c_J}{J^2}\mathcal{P}_{J-1}'' + g_7 g_6 (t+k^2-\mu^2)\frac{c_J}{J}\mathcal{P}_J'\right],$$

$$(t-M^2)B_4 = \frac{m}{t}\left[-g_7 g_5 \frac{c_J}{J^2}\mathcal{P}_J{}' - g_7 g_5\left(\frac{(t+k^2-\mu^2)^2}{4t} - k^2\right)\frac{t-4m^2}{4}\frac{c_J}{J^2}\mathcal{P}_{J-1}{}'' + g_7 g_6 \frac{t+k^2-\mu^2}{2}\frac{c_J}{J}\mathcal{P}_J{}'\right],$$

$$(t-M^2)B_5 = g_7 g_5 \frac{c_J}{J^2}\mathcal{P}_J{}', \tag{B16}$$

$$(t-M^2)B_6 = -g_7 g_5 \frac{c_J}{J^2}\mathcal{P}_J{}'',$$

$$(t-M^2)B_7 = -g_7 g_5 \frac{t+\mu^2-k^2}{2t}\frac{t-4m^2}{4}\frac{c_J}{J^2}\mathcal{P}_{J-1}{}'' + g_7 g_6 \frac{c_J}{J}\mathcal{P}_J{}',$$

$$(t-M^2)B_8 = -g_7 g_5 \frac{t+k^2-\mu^2}{2t}\frac{t-4m^2}{4}\frac{c_J}{J^2}\mathcal{P}_{J-1}{}'' - 2g_7 g_6 \frac{c_J}{J}\mathcal{P}_J{}'.$$

For $\pi^- + p \to \rho^0 + n$, the relevant natural-parity trajectory is $\pi_c$ (if it exists) and $A_2$. The unnatural-parity trajectory with $G(-1)^{I+J} = +1$ is $\pi$, and with $G(-1)^{I+J} = -1$ is $A_1$.

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[6]See, for example, J. J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969).

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[13]J. D. Jackson and C. Quigg, Phys. Letters 29B, 236 (1969); also Nucl. Phys. B22, 301 (1970).

[14]M. Le Bellac, Phys. Letters 25B, 524 (1967); M. Aderholz *et al.*, *ibid.* 27B, 174 (1968).

[15]See, for example, the review article by D. Schildknecht, Z. Physik 229, 278 (1969); F. Gilman, Stanford Linear Accelerator Center Report No. SLAC-PUB-589, 1969 (unpublished); and R. Diebold, in *High Energy Physics,* edited by K. T. Mahanthappa, W. D. Walker, and W. E. Brittin (Colorado Associated University Press, Boulder, 1970).

[16]In practice, the $\gamma_V$ amplitudes can be obtained by analyzing the following reactions:

$$\gamma + p \to \pi^+ + n, \quad \gamma + p \to \pi^0 + p, \quad \text{and} \quad \gamma + n \to \pi^- + p.$$

In the case of cross sections, one can usually approximate $d\sigma(\gamma_V + p \to \pi^+ + n)$ by

$$\tfrac{1}{2}[d\sigma(\gamma + p \to \pi^+ + n) + d\sigma(\gamma + n \to \pi^- + p)].$$

[17]R. Diebold and J. A. Poirier, Phys. Rev. Letters 22, 255 (1969); 22, 906 (1969); L. J. Gutay *et al.*, *ibid.* 22, 424 (1969).

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[20]H. Fraas and D. Schildknecht, Nucl. Phys. B6, 395 (1968); C. F. Cho and J. J. Sakurai, Phys. Letters 30B, 119 (1969); W. Potter and J. Sullivan, Nuovo Cimento 68A, 623 (1970); S. G. Brown, Phys. Rev. D 1, 207 (1970); J. T. Donohue, *ibid.* 1, 1972 (1970).

[21]M. Le Bellac and G. Plaut, Nuovo Cimento 64A, 95 (1968).

[22]However, in a recent 15-GeV/$c$ $\pi^- + p \to \rho^0 + n$ experiment at SLAC (Ref. 23), preliminary results show that there is no compelling reason to believe that the angular distribution may not be described adequately by an $s$ and a $p$ wave.

[23]F. Bulos *et al.*, Phys. Rev. Letters 26, 1453 (1971); 26, 1457 (1971).

[24]The more precise meaning of "smoothness" is the following: We assume the invariant amplitudes $B_i$ to satisfy unsubtracted dispersion relations in the vector-meson mass $k^2$, and let the dispersion integral be saturated by vector mesons; hence

$$B_i^{(\gamma_V)}(k^2) = [m_\rho^2/(m_\rho^2 - k^2)](e/f_\rho)B_i^{(\rho)}.$$

[25]A. C. Hearn, Nuovo Cimento 21, 333 (1961).

[26]Achasov and Shestakov (Ref. 9) also studied the consequences of Eqs. (17). However, their main predictions come from combining Eqs. (17) with relations derived from pure Regge-pole models and, as a result, their predictions are very different from ours.

[27]In his recent talk [see *Proceedings of the Philadelphia Conference on Meson Spectroscopy,* edited by C. Baltay and A. Rosenfeld (Columbia Univ. Press, New York, 1970)], G. L. Kane mentioned that at one stage in our prediction on $\pi^- + p \to \rho^0 + n$, we used amplitudes obtained from absorption-type arguments. However, we used the pseudomodel of Jackson and Quigg, and as these authors emphasized, the pseudomodel is derived mainly from finite-energy sum rules and is essentially model-independent.

[28]J. S. Ball, W. R. Frazer, and M. Jacob, Phys. Rev. Letters 20, 518 (1968).

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[30]G. C. Fox (unpublished).

[31]R. L. Walker, Phys. Rev. 182, 1729 (1969).

[32]A. M. Boyarski *et al.*, Phys. Rev. Letters 21, 1767 (1968).

[33]Actually the $\phi_2$ and $\phi_3$ of Di Vecchia *et al.* (Ref. 12) are weakly dependent on $s$ at finite $t$. We evaluated $\phi_2$ and $\phi_3$ with $\nu = 11.2$ GeV/$c$. The difference between the $\phi_2$ and $\phi_3$ evaluated at $\nu = 11.2$ GeV/$c$ and those evaluated at $\nu = 15$ GeV/$c$ is less than 5% for $\phi_2$, and less than 10% for $\phi_3$ for $|t| < 0.25$ (GeV/$c$)$^2$.

[34]Our normalization is such that when the initial nucleon spin is averaged,

$$s^2 \frac{d\sigma}{dt} = \frac{m^2}{8\pi} \sum_{\lambda_N, \lambda_N', \lambda_\rho} |M_{\lambda_N'; \lambda_\rho \lambda_N}|^2 .$$

[35]B. D. Hyams *et al.*, Nucl. Phys. B7, 1 (1968).

[36]L. J. Gutay *et al.*, Ref. 18.

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[38]P. Sonderegger and P. Bonamy, in *Proceedings of the Lund International Conference on Elementary particles,* edited by G. von Dardel (Berlingska, Lund, Sweden, 1970).

[39]The $s$-wave density-matrix element $\rho_{00}^{(s\ \text{wave})}$ obtained from the fit of Sonderegger *et al.*, and extrapolated to 15 GeV/$c$, is found to approach the maximum values allowed by the inequality

$$(\text{Re}\rho_{00}^{s-p\ \text{interference}})^2 < \rho_{00} \rho_{00}^{(s\ \text{wave})}$$

(Ref. 23). A smaller $\rho^{(s\ \text{wave})}$ will raise the experimental values for $2\rho_{11}^{(H)} (s - m^2)^2\, d\sigma(\rho)/dt$, improving the agreement between our prediction for this transverse cross section and experiment.

[40]Y. Avni and H. Harari, Phys. Rev. Letters 23, 262 (1969).

[41]See Eq. (3). Also see C. F. Cho and J. J. Sakurai (Ref. 20) and A. Dar, Nucl. Phys. B19, 259 (1970).

[42]J. H. Scharenguivel *et al.* [Phys. Rev. Letters 24, 332 (1970)] also studied the 2.7-, 4.1-, 8.0-, and 11.2-GeV/$c$ data, and found that there are indications of peaking in the low-$|t|$ region in the cross section for transversely polarized (in the helicity frame) $\rho$. However, they obtained their result through extrapolation from higher $|t|$ values.

[43]Remember that our curve in Fig. 7 is a good representation of the photoproduction asymmetry $[d\sigma^{(\perp)}(\gamma_V) - d\sigma^{(\parallel)}(\gamma_V)]/[d\sigma^{(\perp)}(\gamma_V) + d\sigma^{(\parallel)}(\gamma_V)]$.

[44]C. F. Cho and J. J. Sakurai, Ref. 20.

[45]It was also pointed out by Schmidt [Phys. Rev. 188, 2458 (1969)] and by Manweiller and Schmidt [Phys. Letters 33B, 366 (1970)] that the reason why the usual application of VMD failed to predict $\rho_{1-1}^{(H)}$ and the polarization cross section for $\pi^- + p \to \rho^0 + n$ correctly might be due to the sensitivity of $\rho_{1-1}^{(H)}$ to the ratio $F_1(k^2)/F_\pi(k^2)$, where $F_1$ and $F_\pi$ are the nucleon and pion form factors, respectively.

[46]A 17.2-GeV/$c$ experiment with high statistical precision has just been completed at CERN and the analysis of the data is under way (Peter Schlein, private communication).

[47]A. M. Boyarski *et al.*, Phys. Rev. Letters 20, 300 (1968); 21, 1767 (1968).

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[50]H. F. Jones and M. D. Scadron, Nucl. Phys. B4, 267 (1968).

[51]We consider only the case when all the couplings $g_i$ are finite and independent of $s$.

[52]The $s$ dependence of the solid Legendre polynomials defined by Eq. (B8) can be obtained by considering their power of $\cos\theta$ [cf. Eq. (B10)]. So $\mathcal{P}_J \propto s^J$, $\mathcal{P}_{J-1} \propto s^{J-1}$, $\mathcal{P}_{J-1}' \propto s^{J-2}$, etc.

[53]See Ref. 21; see also G. Eilam, G. Berlad, and A. Dar, Nucl. Phys. B27, 415 (1971).

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[56]We always limit ourselves to the kinematic region in which $s + t \gg 2m^2 + k^2 + \mu^2$ holds.

[57]One may also use the $\phi_i$'s used by Jackson and Quigg (Ref. 13) to evaluate the cross section for $e + p \to e + n + \pi^+$, since they seem to represent the $\gamma + p \to \pi^+ + n$ data quite well.

[58]These quantities are not very sensitive to $s$, although we have evaluated them at $s \simeq 20$ (GeV/$c$)$^2$.

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[62]$j_\mu$ stands for the electromagnetic current density and $J_\mu^{(\rho)}$ stands for the source density of the neutral $\rho$-meson field.

[63]For a discussion on joint-decay density-matrix elements, see H. Pilkuhn, *The Interaction of Hadrons* (Wiley, New York, 1967), p. 236.

[64]S. M. Flatté, Phys. Rev. 155, 1517 (1967).

[65]P. Schreiner, private communication.

[66]Recently, P. D. Mannheim and S. Nussinov [Nuovo Cimento A1, 619 (1971)] used this assumption to study the decay of a boson into a vector meson and a pion, to

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[68]H. F. Jones and M. D. Scadron, Phys. Rev. 171, 1809 (1968).

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# *CP* Nonconservation and Inequalities Between $\mu^+\mu^-$ and $2\gamma$ Decay Rates of $K_S^0$ and $K_L^0$ †

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Assuming that the absorptive part of $K_L^0 \to 2\mu$ is due only to the on-mass-shell $2\gamma$ intermediate states, several inequalities between $2\mu$ and $2\gamma$ decay states of $K_S^0$ and $K_L^0$ are derived without the assumption of *CP* conservation. These inequalities, together with the present experimental upper bound on $K_L^0 \to 2\mu$, imply that the branching ratio for $K_S^0 \to 2\mu$ should be greater than $5\times10^{-7}$.

## I. INTRODUCTION

It has been pointed out by several authors[1,2] that if the absorptive part of $K_L^0 \to \mu^+\mu^-$ is assumed to be due only to the on-mass-shell $2\gamma$ intermediate state and if, in addition, *CP* conservation holds, then the usual quantum electrodynamics leads to the inequality

$$\frac{\text{rate}(K_L^0 \to 2\mu)}{\text{rate}(K_L^0 \to 2\gamma)} > \lambda^2, \tag{1}$$

where[1,3]

$$\lambda^2 = \tfrac{1}{2}\frac{\alpha^2}{v_\mu}\left(\frac{m_\mu}{m_K}\right)^2\left(\ln\frac{1+v_\mu}{1-v_\mu}\right)^2 \cong 1.2\times10^{-5} \tag{2}$$

and

$$v_\mu \cong 0.9 \tag{3}$$

is the velocity of $\mu^\pm$ in the rest system of the kaon. The present experimental upper bound on the branching ratio[4] is

$$\frac{\text{rate}(K_L^0 \to \mu^+\mu^-)}{\text{rate}(K_L^0 \to \text{all})} < 1.8\times10^{-9}, \tag{4}$$

while, according to (1), the theoretical lower bound for the same branching ratio should be $\cong 6\times10^{-9}$.

At first sight, this discrepancy may not seem to be too disturbing since in $K_L^0 \to \mu^+\mu^-$, besides $2\gamma$ there are also $2\pi\gamma$, $3\pi$, and other on-mass-shell intermediate states, and furthermore, *CP* conservation is known to be violated. However, difficulty does arise on a dynamical level. At present, attempts to include $2\pi\gamma$, $3\pi$, and other intermediate states in the absorptive part lead only to a small correction to the above theoretical lower bound,[5] and it seems quite difficult to explain the large difference between the theoretical and experimental bounds on $K_L^0 \to \mu^+\mu^-$ by using any simple theoretical model.

The purpose of this note is to examine the alternative possibility, i.e., the effect of *CP* nonconservation. As we shall see, there are definite tests which can be used to trace whether the present discrepancy is due to *CP* nonconservation or due to other reasons. In order to separate out the implications of different theoretical hypotheses, we shall assume, throughout our subsequent discussions, (i) that the absorptive part of the $K_L^0 \to 2\gamma$ amplitude is zero, (ii) that the absorptive part of the $K_L^0 \to \mu^+\mu^-$ amplitude is due only to the on-mass-shell $2\gamma$ intermediate state, and (iii) that both *CPT* invariance and quantum electrodynamics are valid, but *CP* conservation is not. As we shall see, under these assumptions, the lower bound given above by (1) no longer holds, and it is replaced by

$$[\text{rate}(K_S^0 \to \mu^+\mu^-)]^{1/2} \geq (\text{Re}\,\epsilon)^{-1}\{\lambda v_\mu[\text{rate}(K_L^0 \to 2\gamma)]^{1/2} - [\text{rate}(K_L^0 \to \mu^+\mu^-)]^{1/2}\} \tag{5}$$

and

$$[\text{rate}(K_S^0 \to \mu^+\mu^-)]^{1/2} \leq (\text{Re}\,\epsilon)^{-1}\{\lambda[\text{rate}(K_L^0 \to 2\gamma)]^{1/2} + [\text{rate}(K_L^0 \to \mu^+\mu^-)]^{1/2}\}, \tag{6}$$

where[6]

$$\text{Re}\,\epsilon \equiv \tfrac{1}{2}\langle K_L^0|K_S^0\rangle \cong 1.4\times10^{-3},$$

and $\lambda$ and $v_\mu$ are given by Eqs. (2) and (3), respec-