Unitary Symmetry and Duality in the Scattering of Hadrons

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A previously suggested method for suppression of exotic resonances in meson-meson scattering is extended to the case of the meson-baryon interaction. The diagonalizing matrix S is identified as one of the crossing matrices itself; this is also seen to be essential for the reproduction of the correct signature of the t-channel trajectories.

In a recent paper[1] we have suggested an explicit method for suppressing exotic resonances in meson-meson scattering, which avoids the use of group theory or a tensorial decomposition. Some indications were given also regarding the applicability of the method to the case of meson-baryon (*MB*) scattering (the case of interaction of Sakata-model triplets with mesons was considered in the paper referred to above). In this note we have considered an extension to the case of the *MB* interaction, prototypes of which are $\underline{8}+\underline{10}\to\underline{8}+\underline{10}$, $\underline{8}+\underline{8}\to\underline{8}+\underline{10}$, etc. In paper I we have observed that the crucial step in our process was the determination of the diagonalizing matrix S. Here we also proceed with the same motivation for the process $\underline{8}+\underline{10}\to\underline{8}+\underline{10}$. The (s,u) and (s,t) crossing matrices for this case are given by[2]

$$X_{us}=\begin{pmatrix} \frac{1}{5} & -\frac{1}{2} & -\frac{9}{20} & \frac{7}{4} \\ -\frac{2}{5} & \frac{3}{4} & -\frac{9}{40} & \frac{7}{8} \\ -\frac{2}{15} & -\frac{1}{12} & \frac{37}{40} & \frac{7}{24} \\ \frac{2}{5} & \frac{1}{4} & \frac{9}{40} & \frac{1}{8} \end{pmatrix}, \tag{1a}$$

$$X_{ts}=\begin{pmatrix} \frac{2}{5}\sqrt{5} & \frac{1}{2}\sqrt{5} & \frac{27}{20}\sqrt{5} & \frac{7}{4}\sqrt{5} \\ \frac{1}{5}\sqrt{2} & \frac{3}{8}\sqrt{2} & -\frac{81}{80}\sqrt{2} & \frac{7}{16}\sqrt{2} \\ \frac{1}{5}\sqrt{10} & \frac{1}{8}\sqrt{10} & \frac{9}{80}\sqrt{10} & -\frac{7}{16}\sqrt{10} \\ \frac{2}{15}\sqrt{7} & -\frac{1}{8}\sqrt{7} & -\frac{1}{20}\sqrt{7} & \frac{1}{12}\sqrt{7} \end{pmatrix}. \tag{1b}$$

The eigenvalues of X_{us} are seen to be (+1, +1, +1, −1). Then, as has been observed before, the determination of S reduces to the solution of the eigenvalue equation

$$X_{us}^T\xi_i=\pm\xi_i\,, \tag{2}$$

where ξ_i are the columns of the matrix S^T. In the present case we have obtained the following set of eigenvectors with the symmetry property indicated:

$$\left.\begin{matrix} -\frac{1}{2}A_8+A_{10} \\ -\frac{1}{6}A_8+A_{27} \\ +\frac{1}{2}A_8+A_{35} \end{matrix}\right\}\ \text{symmetric}$$
$$-\tfrac{16}{35}A_8-\tfrac{2}{7}A_{10}-\tfrac{9}{35}A_{27}+A_{35}\quad\text{antisymmetric}. \tag{3}$$

We can now write the most general functional form for these eigenvectors according to the prescription laid down in paper I:

$$\begin{aligned} -\tfrac{1}{2}A_8+A_{10}&=f'(s,t)+f'(u,t)+g(s,u)=X\quad(\text{say}),\\ -\tfrac{1}{6}A_8+A_{27}&=g'(s,t)+g'(u,t)+h(s,u)=Y,\\ +\tfrac{1}{2}A_8+A_{35}&=h'(s,t)+h'(u,t)+j(s,u)=Z,\\ -\tfrac{16}{35}A_8-\tfrac{2}{7}A_{10}-\tfrac{9}{35}A_{27}+A_{35}&=p(s,t)-p(u,t)=T. \end{aligned} \tag{4}$$

These equations, when coupled with the constraint that A_{27} and A_{35} in the s channel and A_{27} in the t channel do not contain any poles, yield

$$\begin{aligned} A_{27}&=0,\\ A_{35}&=\tfrac{18}{5}g(u,t),\\ A_{10}&=\tfrac{26}{21}g(s,u)-\tfrac{24}{5}[g'(u,t)+g'(s,t)],\\ A_8&=\tfrac{10}{21}g(s,u)-6[g'(u,t)+g'(s,t)]. \end{aligned} \tag{5}$$

A similar method can be followed for $\underline{8}+\underline{8}\to\underline{8}+\underline{10}$. At this stage it is interesting to make a simple observation. We form the following linear combinations:

(a) $10X+27Y+35Z$,

(b) $35Z+30X+81Y$,

(c) $5Z-3Y-10X$,

which also possess the property of being eigenvectors with eigenvalue +1, i.e., symmetry under (s,u) interchange. Then it is easily seen that (a), (b), (c), and T of Eq. (4) are simply proportional to the rows of X_{ts}. That is, X_{ts} is one of the possible solutions of S. This phenomenon has been seen to occur also in the case of $\underline{3}+\underline{8}\to\underline{3}+\underline{8}$ and $\underline{8}+\underline{8}\to\underline{8}+\underline{10}$.

Under the above circumstances it seems plausible to formulate the following lemma:

Lemma. For any hadron-hadron scattering the crossing matrix connecting those two channels with identical isospin states is always diagonalizable by a second one connecting one of the above channels and the third one. That is, if X_{ts} and X_{us} stand for our usual crossing connections, then

$$X_{ts}X_{us}=\eta X_{ts}\,.$$

The proof of the above lemma is facilitated by following the observation made by Dyson[3] that the crossing matrices can be represented by $6j$ symbols or Racah coefficients, and by making use of an elegant property of these coefficients which was discovered by Rose and Yang.[4]

Let us consider the process $a+b \to c+d$ and the crossed one $a+d \to c+b$; then

$$X_{ts} = (-1)^{a+b}(2S+1)W(astd; bc)\,,$$

$$X_{us} = (-1)^{a-c}(2S+1)W(abdc; su)\,.$$

The following property of Racah coefficients,[4]

$$\sum_{l_3}(-1)^{j+l_3+j_3}(2l_3+1)\begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix}\begin{Bmatrix} j_1 & l_1 & j \\ j_2 & l_2 & l_3 \end{Bmatrix} = \begin{Bmatrix} j_1 & j_2 & j_3 \\ l_2 & l_1 & j \end{Bmatrix}\,,$$

yields

$$(X_{ts})_{ij}(X_{us})_{jk} = (-1)^{i}(X_{ts})_{ik} \tag{6}$$

if and only if $b = d$. It should be kept in mind that the above calculation is done in the domain of SU(2) symmetry and can be easily extended to the case of SU(3), as the Clebsch-Gordan coefficients of SU(3) have the same property[5] as those of SU(2). In fact, the steps remain the same but for the terminology. Equation (6) suggests that eigenvalues – or, in other words, the symmetry properties – of the rows of X_{ts} (i.e., the eigenvectors) are determined by their respective dimension in the irreducible representation of the SU(2) group. It has been observed that this rule plays the same role even in the case of SU(3). It further suggests that we need not be cognizant about the external states involved in the scattering. We have observed that states labeled by the irreducible representations $\underline{1}$, $\underline{8}_{ss}$, $\underline{27}$ occur always with eigenvalue +1, and those labeled by $\underline{10}$, $\underline{10}$, $\underline{8}_{aa}$ with eigenvalue −1, whatever be the scattering process under consideration.

In conclusion it can be remarked that the simple structure of S obtained above is intimately connected with the signature of the t-channel Regge trajectories, as the physical amplitudes of the t channel, being eigenvectors of X_{su}, are either symmetric or antisymmetric under (s, u) interchange. This implies that they are of the form $f(s,t) \pm f(u,t)$, where f is a dual function, and in the limit $s \to \infty$, t fixed yield the Regge behavior with proper signature.

A further remark regarding the application of the method of baryon-antibaryon scattering is worth mentioning. It has been shown[6] that there are inherent difficulties in the construction of the amplitude in the $B\overline{B}$ case, under the assumption of global duality and absence of exotic resonances. Our present analysis is also not immediately applicable to this case.

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